A generalization of quantum mass function: Quaternion mass function and the distance of it

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***Abstract*—To handle uncertainties and process complex information from different sources, quantum mass function, an efficient method has been proposed to address this issues. On the basis of the quantum mass function, many methods has been designed to indicate the differences among quantum evidences. Nevertheless, they are developed by quantum evidence theory to process traditional basic probability assignments (QBPAs) and not applicable in measuring quaternion BPAs (QTBPAs). Therefore, in this paper, a specific customized method is proposed for the generalized form of quantum mass function, namely quaternion mass function, to accurately demonstrate the distances among disparate evidences given as QTBPAs (QED). Moreover, it is a pioneer to investigate the differences between pieces of evidences in the plane space of quaternion which is reliable and strictly satisfies the axioms of distance. Besides, if QTBPAs degenerate into QBPAs, QED also degenerate into quantum evidential evidence, which indicates the consistency in this new standard of measuring distances. Consequently, QED is derived from the quantum evidential distance and possesses an extensive capability to indicate dissimilarities among QTBPAs. Several numerical examples are offered to check the validity and practical availability of QED.**
Keywords: quaternion evidence theory, quaternion basic probability assignment, quaternion evidential distance, Decision making, pattern recognition

I. INTRODUCTION

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II. PRELIMINARIES

Definition 1. *(Frame of discernment)*

Let Ω be a non-empty set whose elements are mutually exclusive, and then it is called the frame of discernment which is defined as:

$$\Omega = \{x_1, x_2, x_3, ..., x_i, ..., x_n\} \tag{1}$$

The power set of the frame of discernment is denoted by 2^Ω, which is defined as:

$$2^\Omega = \{\emptyset, x_1, x_2, ..., x_n, ..., \{x_1, x_2\}, ..., \Omega\} \tag{2}$$

Definition 2. *(Mass function)*

A basic probability function is called a mass function which is defined as:

$$m : 2^\Omega \to [0, 1] \tag{3}$$

And the properties it satisfies are defined as:

$$m(\emptyset) = 0 \tag{4}$$

$$\sum_{A \subset 2^\Omega} m(A) = 1 \tag{5}$$

Definition 3. *(Quantum mass function)*

With respect to set Ω, a mapping of every element in the set is defined as:

$$\mathbb{Q} : 2^\Omega \to \mathbb{C} \tag{6}$$

And the properties it satisfies are defined as:

$$\mathbb{Q}(\emptyset) = 0 \tag{7}$$

$$\sum_{A \subseteq 2^\Omega} |\mathbb{Q}(A)|^2 = 1 \tag{8}$$

$$\mathbb{Q}(A) = m(A)e^{i\theta} \quad A \in 2^\Omega \tag{9}$$

And the quantum mass function can be expressed into the another form which is defined as:

$$\mathbb{Q}(A) = x + yi \quad A \in 2^\Omega \tag{10}$$

III. NEW DISTANCE BETWEEN QUATERNION BASIC PROBABILITY ASSIGNMENTS

In this section, a new measurement of distance which is a generalization of the complex evidential distance in complex evidence theory is proposed to give an accurate indicator of the differences among different evidences in the form of QTBPA. The parts of real number and imaginary number are both taken into consideration which guarantees a high accuracy in distinguishing similarities of QTBPAs.

A. Quaternion mass function

In the frame of discernment Ω, a quaternion mass function is established in the level of four dimensions, a mapping of every element in the set is defined as:

$$\mathbb{N} : 2^\Omega \to \mathbb{QT} \tag{11}$$

And the properties it satisfies are defined as:

$$\mathbb{N}(\emptyset) = 0 \tag{12}$$

$$\sum_{B\subseteq 2^{\Omega}} |\mathbb{N}(B)|^2 = 1 \tag{13}$$

$$\mathbb{N}(B) = m(B)e^{\mu\theta} \quad B \in 2^{\Omega} \tag{14}$$

Where μ represents imaginary part $\mu_x i + \mu_y j + \mu_z k$ having $i = j = k = \sqrt{-1}$ and $m(B)$ is a measurement of the $\mathbb{N}(B)$. Besides, θ lies in the range of $[-\pi, \pi]$. And the quaternion mass function can be expressed into the another form which is defined as:

$$\mathbb{N}(B) = a + bi + cj + dk \tag{15}$$

And it satisfies the property which is defined as

$$\sqrt{a^2 + b^2 + c^2 + d^2} \in [0, 1] \tag{16}$$

The relationships between the two form of the quaternion are given as:

$$m(B) = \sqrt{a^2 + b^2 + c^2 + d^2} = ||q|| \tag{17}$$

$$\theta = \arctan(\frac{\mathbf{v}}{a}) \tag{18}$$

in which $a = m(B)\cos\theta$ and $\sqrt{b^2 + c^2 + d^2} = m(B)\sin\theta = \mathbf{v}$. Moreover, the square of the length of quaternion mass function $\mathbb{N}(B)$ is defined as:

$$|\mathbb{N}(B)|^2 = \mathbb{N}(B)\mathbb{N}(B)^* = a^2 + b^2 + c^2 + d^2 \tag{19}$$

$$\mathbb{N}(B)^* = a - bi - cj - dk \tag{20}$$

where $\mathbb{N}(B)^*$ is called a conjugate quaternion of $\mathbb{N}(B)$. According to the definitions given above, some conclusions can be deduced as:

$$m(B) = |\mathbb{N}(B)| \tag{21}$$

$$\theta = \angle\mathbb{N}(B) \tag{22}$$

if the imaginary part of the quaternion is not offered, then the quaternion mass function degenerates into the form mass function and $m(B) = |a|$.
For any $B \in 2^{\Omega}$, because the quaternion mass function is a generalized form of mass function, if $|\mathbb{N}(B)|$ has a value which is bigger than zero, then it is defined as a focal element of the quaternion mass function. And on account of that QTBPA is a generalization of BPA, the bigger of the $|\mathbb{N}(B)|$ is, the higher level of belief the proposition B possesses. Vice versa.

B. Differences among QTBPAs

Example 1: Three QTBPAs are given under the frame of discernment $\Omega = \{A, B, C, D, E, F, G\}$

$$\mathbb{N}_1 : \mathbb{N}_1(A, B, C) = 0.6614e^{\mu\arctan(0.8660)}$$
$$\mathbb{N}_1(\Omega) = 0.8080e^{\mu\arctan(1.866)}$$
$$\mathbb{N}_1 : \mathbb{N}_1(D, E, F) = 0.6614e^{\mu\arctan(0.8660)}$$
$$\mathbb{N}_1(\Omega) = 0.8080e^{\mu\arctan(1.866)}$$
$$\mathbb{N}_1 : \mathbb{N}_1(A, B, C, D) = 0.6614e^{\mu\arctan(0.8660)}$$
$$\mathbb{N}_1(\Omega) = 0.8080e^{\mu\arctan(1.866)}$$

It can be easily concluded that the evidence $\mathbb{N}_1$ possesses a much higher level of similarity to evidence $\mathbb{N}_3$ than evidence $\mathbb{N}_2$ does.

Therefore, the proposed distance of QBPAs is supposed to satisfy the relationship which is defined as:

$$d(\mathbb{N}_1, \mathbb{N}_3) < d(\mathbb{N}_2, \mathbb{N}_3)$$

However, how to properly indicate resemblances of QTBPAs to deduce the distance among evidences is still a problem to be solved. As a result, the method to measure similarities in traditional evidence theory of Jousselme $et\ al.$ is utilized to provide theoretical support in measuring the distance of QTBPAs.

Definition 4. *(Matrix of similarities of QTBPAs)*

Let A and B be two QTBPAs in the frame of discernment Ω, the matrix of similarities of two QTBPAs is defined as:

$$\widetilde{\mathbb{D}}(A, B) = \frac{|A \cap B|}{|A \cup B|} \tag{23}$$

where $A \cap B$ represents the intersection of two QTBPAs in their constitution of propositions.

C. the method of measuring distances between QTBPAs

Definition 5.

$$d_{QTBPA}(\mathbb{N}_1, \mathbb{N}_2) = \sqrt{\frac{|(\overrightarrow{\mathbb{N}}_1 - \overrightarrow{\mathbb{N}}_2)^T \widetilde{\mathbb{D}} (\overrightarrow{\mathbb{N}}_1 - \overrightarrow{\mathbb{N}}_2)|}{\sum_{A\subset 2^{\Omega}} |\mathbb{N}_1(A)|^2 + \sum_{B\subset 2^{\Omega}} |\mathbb{N}_2(B)|^2}} \tag{24}$$

IV. Conclusion

The conclusion goes here.

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References

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