A novel conflict management considering the optimal discounting weights using the BWM method in Dempster-Shafer evidence theory

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Abstract

Dempster-Shafer evidence theory (DST) is an effective tool for data fusion. In this theory, how to handle conflicts between evidences is still a significant and open issue. In this paper, the best-worst method (BWM) is extended to conflict management in DST. Firstly, a way to determine the best and worst basic probability assignment (BPA) is proposed. Secondly, a novel strategy for determining the optimal weights of BPA using the BWM method is developed. Compared to traditional measure-based conflict management methods, the proposed method has three better performances: (1) A consistency ratio is considered for BPA to check the reliability of the comparisons, producing more reliable results. (2) The final fusion result has less uncertainty, which is more conducive to improve the performance of decision making. (3) The

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number of BPA comparisons performed during operation (in conflict management) is reduced (especially matrix-based). A practical application in motor rotor fault diagnosis is used to illustrate the effectiveness and practicability of the proposed methodology.
Keywords: Dempster-Shafer theory, The BWM method, Conflict management, Basic belief assignments, Deng relative entropy, Fault diagnosis

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1. Introduction

How to measure and deal with the uncertainty in the uncertain environment to sustain effective decision-making in different fields has attracted considerable attention. One of the most important theoretical tools is the Dempster-Shafer theory [1, 2] (DST), which provides a feasible and effective framework to express and process uncertain information. DST has been widely used depending on its applicability and flexibility, such as evidential reasoning [3], classification [4], decision making [5, 6, 7], target recognition [8, 9], fault diagnosis [10], and so on [11, 12]. However, when there is a high degree of conflict between evidence, the traditional Dempster's combination rule (DCR) will obtain counter-intuition conclusions [13, 14]. Therefore, how to model and further process uncertain and inaccurate information with conflict evidences in DST is still an open key issue to support decision making.

According to previous work on DST, the traditional Dempster's conflict coefficient k [1] ignores the global consistency between the evidence and has certain limitations. Scholars have done a lot of research to overcome this problem, and frequently concentrate on two main strategies for the conflict management [15]. One is to improve DCR and reallocate conflicts. They believe that the main reason for the counter-intuitive conclusion is the normalization step in the DCR [16], such as Dubois and Prade [17] proposed the disjunctive and non-normalization combination method, which can solve the problem of high conflicts between evidence to a certain extent. However, the disadvantage is its lack of the normalization advantage of DCR. Therefore, the combination rule [18] that combines the two methods is usually constructed as disjunction rules and a weighted sum of conjunction rules is developed. Lefèvre [19, 20] developed a general framework to unify a variety of traditional combination rules. Yager [21] removed the normalization factor and proposed a fusion method based on non-standardized combination rules. Besides, discussion and allocation of conflicting concepts to manage conflict, such as Smets [22] assigned all conflicts to the empty set to avoid conflicts, Daniel [23] considering the potential conflicts and so on [24]. Furthermore, some novel strategies have also been developed, such as Deng consider the combination of biological and evolutionary evidence and proposed evolutionary rules [25], and so on [26, 27].

However, the evidence obtained by some of the above methods may be very different from the initial evidence [18], which can be regarded simply as false. The well-known disadvantage of this strategy is that, on the one hand, it may lose some good properties of traditional DCR itself, such as relevance, commutativity and almost not idempotent [28]. On the other hand, it may take more time to complete the adjustment process, which produces inaccuracies in the quality allocation [29]. Another idea is to modify the contradictory evidence before fusion (such as conflict management [30, 31, 32, 33] and improve the correlation coefficient of the belief function [34]). Therefore, retaining the traditional DCR method, the data is preprocessed before fusing the evidence, which is the preference in this paper. Jousselme et al. [35] considered the measurement conflict from the nonintersecting part of the evidence. Murphy [30] proposed a weighted average method to deal with the evidence of conflict. Although this method can achieve better evidence focusing effect, it lacks the specificity of evidence to a certain extent. To overcome this disadvantage, Deng [36] developed a weighted average to calculate the similarity between the evidence and Liu designed a two-dimensional conflict model combining Dempster's conflict coefficient k and distance to quantify the conflict, including from the perspective of the correlation coefficient between evidences [37]. However, as a two-dimensional measurement method, it has some disadvantages that the calculation is complicated. Then, several researchers further extended this method. For instance, Jiang [32] and Xiao [38, 39] introduced a new correlation coefficient, taking into account the difference and non-interaction of focal elements. Besides, there are some novel strategies for measuring the consistency of evidence [40, 41]. For instance, Deng [42] uses vector notation to represent BPA based on information quality and source credibility function to obtain the best quality subset, Tsallis entropy [43], divergence measures [44, 45], and so on [46, 47].

According to the above analysis, most existing methods of modifying the evidence model to manage conflicts always require multiple comparative measurements of evidence correlation between each pair of BPAs, such as conflict coefficient needs to construct the correlation matrix and then determine the weights of BPA. However, although the existing conflict measurement methods have made breakthroughs based on view as the consistency between the evidence, they have not considered the consistency of evidence comparison in conflict management. Meanwhile, it always advantages and disadvantages and has remained room for improvement. From this perspective, it is meaningful to reduce the number of comparisons between evidences in conflict management, which reduces the inaccuracy of the results to a certain extent.

To fill the above-mentioned, a method to determine the weights of BPA by using the best-worst method (BWM), to manage the conflict between evidence is proposed. Meanwhile, during to sequence analysis of BPAs based on evidence distance, a concept of the best and the worst BPA is developed. Based on the asymmetry of belief relative entropy, Deng relative entropy [48] is using to measure the relative conflict value between BPAs. On the one hand, the number of measurements between evidence will reduce by using the BWM method [49]. There is unnecessary to calculate all conflict matrices, which reduces the amount of calculation in conflict management to obtain more reasonable results. On the other hand, the results of each pair of BPA may incompletely consistent. Therefore, the consistency of pairwise comparison is developed based on the BWM method to obtain more reliable results. Numerical cases can prove the correlation between analysis and application of BPA and apply the proposed method to decision-making in data science applications. Besides, we compared the BPA weights and fusion results obtained by the proposed method with the well-known classic conflict management methods, and a motor rotor fault diagnosis is demonstrated based on the proposed method.

The main contributions of this work are included as follows:

(i) The proposed method extended the BWM method to evidence conflict management, which reduces the uncertainty of the weight determination between evidence by decreasing the number of measurements between evidence to obtain more reasonable fusion results.

(ii) The proposed method provides a way to determine the best and worst BPA based on the distance of evidence.

(iii) This model considers the consistency ratio of the reference comparison to obtaining the optimal discounting weights.

The remainder of this article is organized as follows. Section 2 briefly introduces the basic knowledge of DST, some existing measures to manage conflicts between evidence, Deng relative entropy, and the BWM method. A new BPA weight determination method and its properties are proposed and analyzed in Section 3. A numerical example illustrates the effectiveness of the proposed method in Section 4. In Section 5, a fault diagnosis algorithm is designed based on the proposed method and applied to solve a motor rotor fault diagnosis. Finally, Section 6 concludes this work.

2. Preliminaries

How to manage and measure uncertainty information plays a significant role in decision-making problems [50, 51, 52]. As one of the effective theories of data fusion in uncertain environments, DST is widely used in many fields [53, 54]. In this section, some basic concepts of DST, including the evidence measurement methods used, Deng relative entropy and BWM methods will be briefly introduced.

2.1. Dempster-Shafer evidence Theory

The basic concepts and definitions of DST [1, 2] will be described as follows.

Definition 2.1 Let Ψ be a set of n mutually exclusive and exhaustive hypothesis defined by [1, 2]:

$$\Psi = \{L_1, L_2, ..., L_i, ..., L_n\} \tag{1}$$

where is called the frame of discernment (FOD), and the power set 2^Ψ is defined as:

$$2^\Psi = \{\emptyset, \{L_1\}, \{L_2\}, \ldots, \{L_N\}, \{L_1, L_2\}, ..., \{L_1, L_2, ..., L_i\}, \ldots, \Psi\} \tag{2}$$

where $\emptyset$ represents an empty set. If $A \in 2^\Psi$, A is called a hypothesis, and any proposition corresponds to a subset of Ψ, satisfying:

$$m(\emptyset) = 0, 0 \leq m(A) \leq \ 1, A \subseteq \Psi, \sum_{A \subseteq \Psi} m(A) = 1 \tag{3}$$

Then, $m : 2^\Psi \to [0, 1]$ is called the mass function, which is also known as the basic probability assignment (BPA). For an $A \in \Psi$, if $m(A) > 0$, then A is called a focal element.

Definition 2.2 The belief function of $A \in \Psi$ is defined as:

$$Bel : P(\Psi) \to [0, 1] \ and \ Bel(A) = \sum_{B \subseteq A} m(B) \tag{4}$$

and the plausibility function of $A \in \Psi$ is defined as:

$$Pl : P(\Psi) \to [0, 1] \ and \ Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \tag{5}$$

Bel(A) presents the sum of all subset probabilities and *Pl(A)* means the sum of the probabilities of assuming that the intersection is not empty. Therefore, *Bel(A)* and *Pl(A)* are the lower limit and the upper limit, respectively. Both imprecision and uncertainty can be represented by them, which is illustrated in Figure 1.

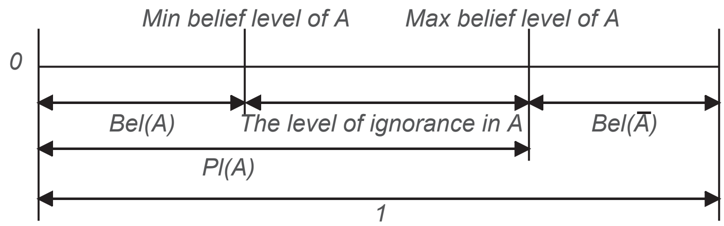

Figure 1: The relation between Bel and Pl.

Definition 2.3 There are two independent BPAs m_1, m_2 in Ψ, DCR is defined as [1, 2], represented by $m = m_1 \oplus m_2$, and it is calculated as follows.

$$m\left(A_i\right) = \frac{1}{1-K} \sum_{A_j \cap A_h = A_i} m_1\left(A_j\right) \cdot m_2\left(A_h\right) \tag{6}$$

with

$$K = \sum_{A_j \cap A_h = \emptyset} m_1\left(A_j\right) \cdot m_2\left(A_h\right) = 1 - \sum_{A_j \cap A_h \neq \emptyset} m_1\left(A_j\right) \cdot m_2\left(A_h\right) \tag{7}$$

where K reflects the degree of the conflict between m_1 and m_2.

Definition 2.4 Discounting of BPA.

A discounting coefficient $\alpha \in [0, 1]$ represents the weight (reliability) of the evidence, then the discounted evidence m^{α} can be defined as follows [2]:

$$m^{\alpha}\left(\Psi\right) = \alpha m\left(\Psi\right) + (1-\alpha) \tag{8}$$

$$m^{\alpha}\left(A\right) = \alpha m\left(A\right) \quad \forall A \subset \Psi \; and \; A \neq \Psi \tag{9}$$

2.2. Existing conflict management

When using DCR to fuse evidence, sometimes conflicts between BPAs will result in counter-intuitive conclusions [13, 27]. To overcome this problem, Murphy [30] proposed a method of evenly distributing the quality. However, the weight of each piece of evidence is often different in reality. Therefore, it is very necessary and important to analyze and determine the weight of each evidence before fusion. Suppose there are two BPAs m_1, m_2 in FOD $\Psi = \{L_1, L_2, ..., L_i, ..., L_n\}$, some existing conflict measures and conflict coefficients for belief functions are briefly introduced.

Definition 2.5 Jousselme et al.'s distance [35]

The distance between m_1 and m_2 is defined as:

$$d\left(m_1, m_2\right) = \sqrt{\frac{1}{2}\left(\vec{m}_1 - \vec{m}_2\right)^T \bar{D}\left(\vec{m}_1 - \vec{m}_2\right)} \tag{10}$$

where $\vec{m}_1$ and $\vec{m}_2$ represent the vector form of BPAs, $\bar{D}$ represents a $(2^n \times 2^n)$ matrix composed of

$$D\left(A_j, A_h\right) = \frac{|A_j \cap A_h|}{|A_j \cup A_h|} \tag{11}$$

where $A_j \in m_1, A_h \in m_2$, and belong to 2^Ψ. $d_{BPA} \in [0, 1]$. It is generally considered that the larger value, the greater conflict between the pieces of evidence.

Definition 2.6 Song et al.'s correlation coefficient [55]

$$cor(m_1, m_2) = \frac{< \tilde{m}_1, \tilde{m}_2 >}{||\tilde{m}_1|| \cdot ||\tilde{m}_2||} \tag{12}$$

where $\tilde{m}_1 = m_1 D$ and $\tilde{m}_2 = m_2 D$. D is defined in Eq.(11). Then, Song et al.'s conflict coefficient:

$$K_{cor}(m_1, m_2) = 1 - cor(m_1, m_2) \tag{13}$$

Definition 2.7 Jiang's correlation coefficient [32]

The correlation coefficient between two pieces of evidence m_1 and m_2 is defined as:

$$r_{BBA}(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1) \cdot c(m_2, m_2)}} \tag{14}$$

where

$$c(m_1, m_2) = \sum_{i=1}^{2^{|n|}} \sum_{j=1}^{2^{|n|}} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{15}$$

where A_i, A_j is the focal elements in the power concentration of the frame, $\frac{|A_i \cap A_j|}{|A_i \cup A_j|}$ is the modulus calculation, $i, j = 1, 2, ..., 2^n$.

Jiang proposed the correlation coefficient r, then the conflict coefficient between two pieces of evidence m_1 and m_2 is represented by k_r, which is defined as:

$$k_{r(m_1,m_2)} = 1 - c(m_1, m_2) = 1 - \sum_{i=1}^{2^{|n|}} \sum_{j=1}^{2^{|n|}} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{16}$$

Definition 2.8 Xiao's correlation coefficient [38]

Xiao [38] proposed a new measurement of the correlation coefficient Ecc between two pieces of evidence, which is defined as:

$$Ecc(m_1, m_2) = [cos\theta(\vec{m}_1, \vec{m}_2)]^2 = \left[\frac{(\vec{m}_1, \vec{m}_2)}{\vec{m}_1 \vec{m}_2}\right]^2 \tag{17}$$

where

$$(\vec{m}_1, \vec{m}_2) = \sum_{i=1}^{2^{|n|}} \sum_{j=1}^{2^{|n|}} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{18}$$

and

$$\vec{m}_1 = [(\vec{m}_1, \vec{m}_2)]^2 = \left[\sum_{i=1}^{2^{|n|}} \sum_{j=1}^{2^{|n|}} m_1 (A_i) \, m_2 (A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \right]^{\frac{1}{2}} \tag{19}$$

Therefore, Xiao's conflict coefficient between the two pieces of evidence of data sources m_1 and m_2 is defined as:

$$k_{ECC(m_1,m_2)} = 1 - Ecc\,(m_1, m_2) = 1 - \left[\frac{(\vec{m}_1, \vec{m}_2)}{\vec{m}_1, \vec{m}_2} \right]^2 \tag{20}$$

2.3. Deng relative entropy

In addition to the conflict measurement method between evidences introduced above, Deng [48] proposed a new relative entropy to measure the difference between BPA.

Definition 2.9 When BPA degenerates to probability, Deng relative entropy is equal to K-L divergence, which is based on the generalization of K-L divergence and is defined as [48]:

$$\tilde{D}_r = (m_1 || m_2) = \sum_i m_1(A_i) \log \frac{m_1(A_i)}{m_2(A_i)} \tag{21}$$

Although Deng relative entropy is similar in form to K-L divergence, it uses mass functions rather than probability distribution functions. Therefore, Deng relative entropy is the average of the logarithmic difference between m_1 and m_2, which should satisfy the properties as follows.

(1) Non-negative: $\tilde{D}_r = (m_1 || m_2) > 0$.

(2) Asymmetry: $\tilde{D}_r(m_1 || m_2) \neq \tilde{D}_r(m_2 || m_1)$.

According to the characteristics of asymmetry and the information divergence, it is contributed for us to measure the relative conflict value between

BPAs in this work. For instance, the conflict increment between m_1 and m_2 may different between m_2 and m_1.

2.4. The BWM method

Considering incomplete and uncertain information, the complexity of the decision-making environment is also increasing. Consequently, multi-criteria decision making (MCDM) methods have attracted extensive attention in various fields and Rezaei proposed BWM method recently [49]. Because of the advantages of requiring less pairwise comparisons and reducing the inconsistency of results, it has received a lot of attention and extended to uncertain environments [56, 57], such as Z number [58] and D number [59]. According to [49], the main steps of the BWM method as follows.

Step 1. Build a set of decision criteria.

In this step, decision-makers (DMs) determine a set of suggestions to decide with n decision criteria $\{c_1, c_2, ..., c_n\}$.

Step 2. Determine the best criterion and the worst criterion by DMs in this step, which is represented as c_B and c_W, respectively.

Step 3. Calculate the best-to-others vector.

It represents the preference of the best criterion over all the other criteria specified by numbers among 1-9, is defined as:

$$\hat{A}_B = (\hat{a}_{B1}, \hat{a}_{B2}, ..., \hat{a}_{Bn}) \tag{22}$$

where a_{Bj} represents the preference of best criterion to criterion j, $j = 1, 2, ..., n$. Meanwhile, $a_{BB} = 1$.

Step 4. Calculating the others-to-worst vector, which represents the preference of the other criterion over all the worst criteria specify by number among 1-9, is defined as:

$$\hat{A}_W = (\hat{a}_{1W}, \hat{a}_{2W}, ..., \hat{a}_{nW}) \tag{23}$$

where a_{jW} represents the preference of the criterion j to the worst criterion, $j = 1, 2, ..., n$. Meanwhile, $a_{WW} = 1$.

Step 5. Determine the optimal weights $(\hat{w}_1^*, \hat{w}_2^*, ..., \hat{w}_n^*)$.

For each pair of weight ratio, defined as $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$. The problem of determining the optimal weight of the criterion is also a constrained optimization problem, i.e., while all j is minimized and the maximum absolute difference $\left|\frac{w_B}{w_j} - a_{Bj}\right|$ and $\left|\frac{w_j}{w_W} - a_{jW}\right|$.

Therefore, the constrained optimization problem modeled for calculating and determining the optimal weight of each criterion can obtain as follows:

$$\begin{aligned} &\min \max_j \left\{\left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_W} - a_{jW}\right|\right\} \\ &s.t \begin{cases} \sum_{j=1}^{n} w_j = 1 \\ w_j \geq 0 \\ j = \{1, 2, ..., n\} \end{cases} \end{aligned} \tag{24}$$

Then, the above model also can be transformed into the problem as follows:

$$\begin{aligned} &\min \xi \\ &s.t \begin{cases} \left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi \\ \left|\frac{w_j}{w_W} - a_{jW}\right| \leq \xi \\ \sum_j w_j = 1 \\ w_j \geq 0 \end{cases} \end{aligned} \tag{25}$$

By solving the above model, the optimal weights $(\hat{w}_1^*, \hat{w}_2^*, ..., \hat{w}_n^*)$ and ξ^* can be obtained.

The key indicator to measure the degree of pairwise comparison is the consistency ratio (CR). However, the consistency ratio needs to be calculated if j cannot be consistent. According to [49], it is determined that a_{BW} belongs to the maximum value of different possible values. The consistency index is shown in Table 1. Furthermore, it is worth observing that the result matrix is reciprocal, which requires $a_{ij} = 1/a_{ji}$ and $a_{ii} = 1$.

Table 1: Consitency index table

a_{BW}	1	2	3	4	5	6	7	8	9
Consistency index (max ξ)	0	0.44	1	1.63	2.3	3	3.73	4.47	5.23

Therefore, according to Table 1, CR can be calculated as follows.

$$\gamma_{CR} = \frac{\xi^*}{\max\{\xi\}} \tag{26}$$

It is obvious that $\xi^* \in [0, 1]$, and the larger ξ^*, the lower the consistency.

However, according to previous studies, it is not difficult to find that most of the existing studies are based on the BWM method for weight analysis of decision criteria under imprecise information. Meanwhile, combining the BWM method and evidence theory can effectively deal with uncertain circumstances, which has great application potential and prospects in dealing with fuzzy and uncertain information.

3. Proposed method

To improve the space for conflict management between evidence, a new method to determine the weights of BPA is proposed. Here, we designed the best and worst BPA model and weight determination method. Specifically, most existing methods of modifying the evidence model to manage conflicts often require multiple measurements of evidence correlation. However, many existing measurement methods always have advantages and disadvantages and have room for improvement in accuracy. From this perspective, the proposed method reduces the number of measurement conflicts between evidence, which reduces the complexity of the entire algorithm. Furthermore, Deng relative entropy is used to measure the relative conflict value between BPAs and the consistency ratio are considered to obtain more reliable results.

3.1. Best and worst BPA models

According to the relevant literature, most of the existing BWM methods and their extensions are based on decision-making. DMs subjectively identified decision criteria, then determine the weights of the criteria to optimal decision-making. However, this subjective judgment has the disadvantage of ambiguity and inaccuracy. In recent years, scholars have successively extended BWM to evidence theory. Although Fei's method [60] takes the belief function as the best and worst criteria for decision-making, it can be used for more subjective suggestions by decision makers to a certain extent. However, the BPA value of each standard still needs to be given by the DMs to obtain the maximum plausibility value. This section provides a new way for calculating the best and worst BPA based on the distance between BPA.

The algorithm flowchart of the best and worst BPA method is shown in Figure 2, and a brief calculation process is introduced as follows.

Step 1. According to the distance between evidence, a distance matrix

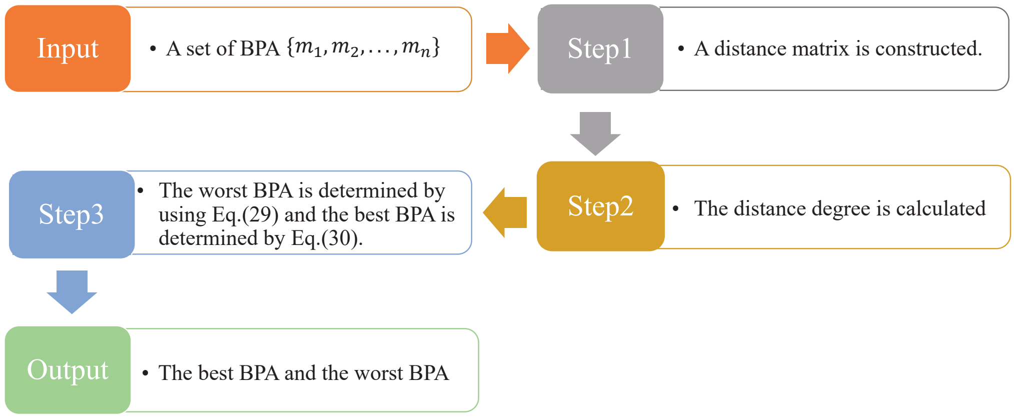

Figure 2: Flowchart of the best and worst BPA method

D is established, which is defined as follows:

$$M_D = \begin{bmatrix} d(m_1, m_1) & \ldots & d(m_1, m_n) \\ \ldots & \ldots & \ldots \\ d(m_n, m_1) & \ldots & d(m_n, m_n) \end{bmatrix} \tag{27}$$

Step 2. The distance degree of m_j is calculated as:

$$CD(m_j) = \sum_{i=1, j=1}^{n} d(m_i, m_j) \tag{28}$$

Step 3. In this work, according to measuring the conflict between evidence in evidence theory, it is more meaningful to determine the evidence with the largest conflict (the worst BPA). Therefore, the best BPA is determined based on the worst BPA. The definition as follows, respectively.

$$m_W = \max_j \frac{CD(m_j)}{\sum\limits_{j=1}^{n} CD(m_j)} \tag{29}$$

$$m_B = \max_i d(m_W, m_i) \tag{30}$$

where $i = \{1, 2, ..., n\}$.

3.2. The new method to determine the weight of BPA

The proposed method is to determine the optimal weights of BPA based on the BWM method, then manages the conflicts between evidence by discounting BPA in this section. Compared with conflict management methods by calculating the conflict correlation coefficient (especially matrix-based), on the one hand, it has the advantage of fewer calculations what means less complexity than other discounting methods when measuring conflict between evidence. On the other hand, it calculates the BPA consistency ratio from the perspective of measuring the consistency of the preference comparison to obtain more reliable results.

According to reference [49], the evidential comparison matrix by using Deng relative entropy can be obtained as follows:

$$ECM = \begin{pmatrix} \tilde{D}_r(m_1||m_1) & \tilde{D}_r(m_1||m_2) & ... & \tilde{D}_r(m_1||m_n) \\ \tilde{D}_r(m_2||m_1) & \tilde{D}_r(m_2||m_2) & ... & \tilde{D}_r(m_2||m_n) \\ ... & ... & ... & ... \\ \tilde{D}_r(m_n||m_1) & \tilde{D}_r(m_n||m_2) & ... & \tilde{D}_r(m_n||m_n) \end{pmatrix} \tag{31}$$

Meanwhile, Deng relative entropy reference comparisons is shown in Figure 3, where m_{ij} is euqal to $\tilde{D}_r(m_i||m_j)$, $i, j \in \{1, 2, ..., n\}$. There is unnecessary to calculate all conflict matrices, only need $2n - 3$ comparisons more less than other discounting method $n^2/2$ comparisons (such as Deng et al. [36], Jiang [32] and Xiao's methods [38]).

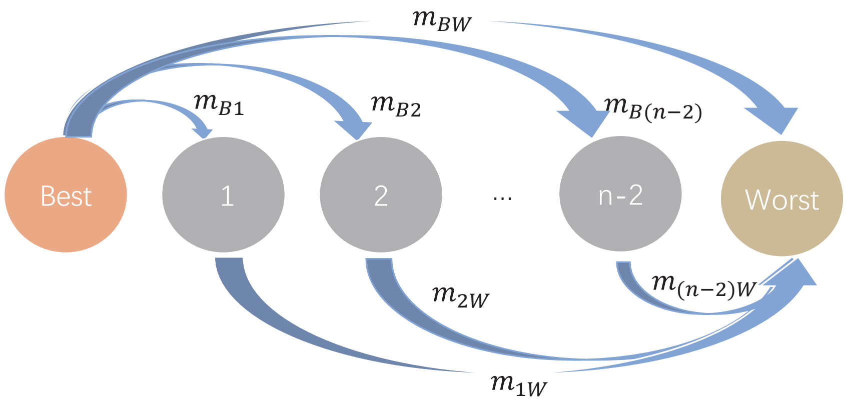

Figure 3: Reference comparisons of BPA

The proposed method must satisfy the properties as follows.

(1) The Deng relative entropy matrix of BPA should be given preference to be reciprocal. It is required to satisfy that $\tilde{D}_r(m_i||m_j) = 1/\tilde{D}_r(m_j||m_i)$ and $\tilde{D}_r(m_i||m_i) = 1$.

(2) If $FOD = \{m_1, m_2, ..., m_n\}$, and $m_i = m_j$, then $\tilde{D}_r = (m_1||m_2) = \tilde{D}_r(m_2||m_1) = 1$. Meanwhile, $\tilde{D}_r(m_B||m_B) = \tilde{D}_r(m_W||m_W) = 1$.

The overall structure of the proposed method is shown in Figure 4, and the main steps are as follows:

Step 1. Input a set of BPA $\{m_1, m_2, ..., m_n\}$

Step 2. According to the best and worst BPA models, using Eq.(29) to obtain the worst BPA m_W and using Eq.(30) to obtain the best BPA m_B.

Step 3. Execute the Deng relative entropy reference comparisons for the best BPA. One of the key steps of the proposed method is the reference comparison of BPA. According to Deng relative entropy, using Eq.(21) to determine the preference of the best BPA to other BPAs. The resulting BPA

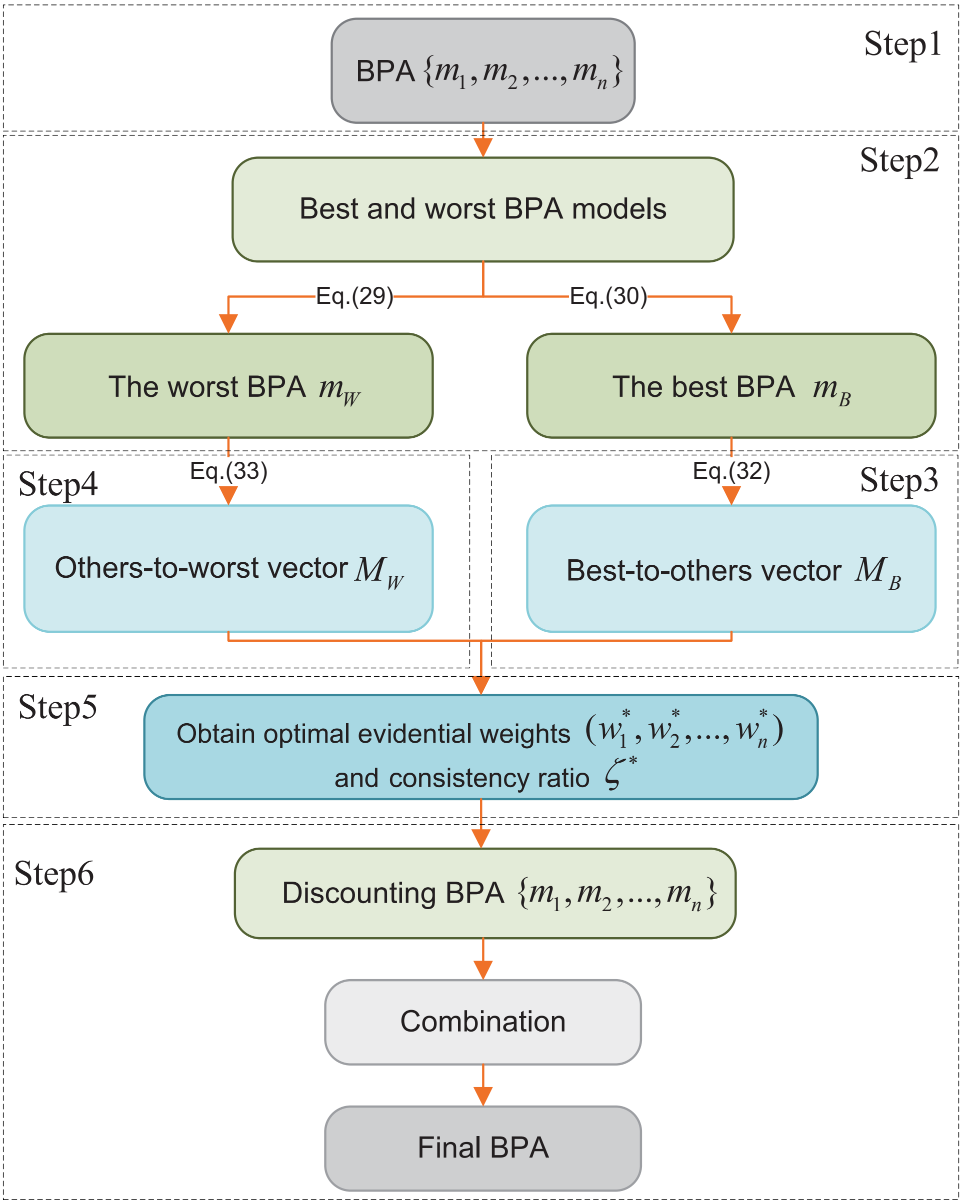

Figure 4: Overall structure of the proposed method

best-to-other vector is defined as:

$$M_B = (\tilde{D}_r(m_B||m_1), \tilde{D}_r(m_B||m_2), ..., \tilde{D}_r(m_B||m_n)) \tag{32}$$

where $\tilde{D}_r(m_B||m_j)$ represents the preference of the best BPA m_B over BPA j, $j = \{1, 2, ..., n\}$. It is can be known that $\tilde{D}_r(m_B||m_B) = 1$

Step 4. Execute the Deng relative entropy reference comparisons for the worst BPA.

$$M_W = (\tilde{D}_r(m_1||m_W), \tilde{D}_r(m_2||m_W), ..., \tilde{D}_r(m_n||m_W))^T \tag{33}$$

where $\tilde{D}_r(m_i||m_W)$ represents the preference of BPA m_i over the worst BPA m_W, $i = \{1, 2, ..., n\}$. It is can be known that $\tilde{D}_r(m_W||m_W) = 1$

Step 5. Find the optimal weights $(w_1^*, w_2^*, ..., w_n^*)$ and consistency ratio (ζ^*). The key indicator to measure the degree of pairwise comparison is the consistency ratio (CR). However, compare with other discounting methods of conflict management, if m_j cannot be consistent $w_B/w_j = \tilde{D}_r(m_B||m_j)$ and $w_j/w_W = \tilde{D}_r(m_j||m_W)$, the consistency ratio needs to be calculated to obtain more reliable weights.

According to previous steps, a solution for constrained optimization needs to determine the weights of BPA. Based on the best and worst BPA model and [49], the following model is used to determine the optimal BPA weights.

$$\min \xi$$
$$s.t \begin{cases} \left|\frac{w_B}{w_j} - \tilde{D}_r(m_B||m_j)\right| \leq \xi \\ \left|\frac{w_j}{w_W} - \tilde{D}_r(m_j||m_W)\right| \leq \xi \\ \sum\limits_j w_j = 1 \\ w_j \geq 0 \\ j = \{1, 2, ..., n\} \end{cases} \tag{34}$$

Therefore, the optimal weights $(w_1^*, w_2^*, ..., w_n^*)$ and CR(ζ^*) of BPA can be obtained. Meanwhile, according to [49], the consistency ratio for BPA BWM can be defined and obtained as follows.

$$\eta_{CR} = \frac{\zeta^*}{\max\{\zeta\}} \tag{35}$$

Step 6. Discounting of BPAs. Based on the optimal weight obtained by previous work, BPAs were discounting according to Eq.(8). Then, combine the pieces of evidence with the DCR and calculate the fusion results.

4. Numerical example

In this Section, a comparative analysis with other evidence combination methods of conflict management in a numerical example by [61], to illustrate the feasibility and effectiveness of the proposed method.

Example 4.1 Assume there are five BPAs in FOD $\Psi = \{A_1, A_2, A_3\}$, respectively:

$$m_1 : m_1(A_1) = 0.7, m_1(A_2) = 0.15, m_1(A_3) = 0.15$$
$$m_2 : m_2(A_1) = 0.5, m_2(A_2) = 0.5$$
$$m_3 : m_3(A_1) = 0.7, m_3(A_2) = 0.15, m_3(A_3) = 0.15$$
$$m_4 : m_4(A_1) = 0.7, m_4(A_2) = 0.15, m_4(A_3) = 0.15$$
$$m_5 : m_5(A_1) = 0.2, m_5(A_2) = 0.8$$

It is obvious that $m_1 = m_3 = m_4$, and has the value of 0.7, which is larger than A_2 and A_3, indicates A_1. Meanwhile, $m_2(A_1) = m_2(A_2)$, and m_5 has a value of 0.8, indicates A_2. Therefore, it is necessary to integrate conflicting evidences, and improve the accuracy of decision-making through evidence conflict management method.

Step 1 and **Step 2**: According to the best and worst BPA models, the worst and best BPA are obtained.

(1) The distance matrix D is constructed as:

$$M_D = \begin{bmatrix} 0 & 0.3041 & 0 & 0 & 0.5895 \\ 0.3041 & 0 & 0.3041 & 0.3041 & 0.3 \\ 0 & 0.3041 & 0 & 0 & 0.5895 \\ 0 & 0.3041 & 0 & 0 & 0.5895 \\ 0.5895 & 0.3 & 0.5895 & 0.5895 & 0 \end{bmatrix}$$

(2) The distance degree of m_j is calculated as:

$$CD(m_1) = 0.8936;\ CD(m_2) = 1.2124;\ CD(m_3) = 0.8936$$
$$CD(m_4) = 0.8936;\ CD(m_5) = 2.0685.$$

(3) Therefore, the worst and best BPA are m_5 and m_1, respectively.

Step 3: The Deng relative entropy reference comparisons for the best BPA m_1 is calculated as:

$$\begin{aligned} M_{m_1} &= (\tilde{D}_r(m_1||m_1), \tilde{D}_r(m_1||m_2), \tilde{D}_r(m_1||m_3), \tilde{D}_r(m_1||m_4), \tilde{D}_r(m_1||m_5)) \\ &= (1, 5.1769, 1, 1, 5.7478) \end{aligned}$$

Step 4: The Deng relative entropy reference comparisons for the worst BPA m_5 is calculated as:

$$\begin{aligned} M_{m_5} &= (\tilde{D}_r(m_1||m_5), \tilde{D}_r(m_2||m_5), \tilde{D}_r(m_3||m_5), \tilde{D}_r(m_4||m_5), \tilde{D}_r(m_5||m_5)) \\ &= (5.7478, 0.2231, 5.7478, 5.7478, 1) \end{aligned}$$

Step 5: Following our model (34), the optimal weights is calculated as:

$$w^*(m_1) = 0.2766;\ w^*(m_2) = 0.0467;\ w^*(m_3) = 0.3113;$$
$$w^*(m_4) = 0.3113;\ w^*(m_5) = 0.0541.$$

and consistency ratio is calculated as:

$$\eta_{CR} = 0.03.$$

Step 6: According to Eq.(8), discounting of BPAs is calculated as:

$$m(A_1) = 0.6636; \quad m(A_2) = 0.2015; \quad m(A_3) = 0.1349.$$

Then, the fusion resluts are obtained with the DCR:

$$m(A_1) = 0.9971; \quad m(A_2) = 0.0026; \quad m(A_3) = 0.0003.$$

The comparison between the proposed method and other management evidence conflict methods, the results are shown in Table 2. It is obvious that the traditional DCR [1] cannot handle the conflict and assigns more belief to A_3, obtained the wrong result. Yager [21] and Smets [62], which are improvement methods on the classic DCR, assign some of the beliefs to multi-element propositions and empty sets, respectively. Therefore, there is the disadvantage of the focusing effect. It may need a lot of evidence to obtain better results. Compared with other methods that need to construct a conflict matrix and modify the evidence model (Jiang [32]; Murphy [30]; [36]; Ma et al. [63]; Sun et al. [64]; Mi and Kang [61]) $n^2/2 = 25/2$ comparisons, the proposed method only requires $2n-3 = 7$ comparisons, which reduces the uncertainty of comparison to a certain extent. However, comparing with other conflict management methods (such as Chen [29]), although other methods seem to have the same purpose, the highest value $m(\{A_1\})$ is obtained by the proposed method, which illustrates effective conflict management. Furthermore, explore it from the perspective of consistency in conflict management, and consistency ratio $\eta_{CR} = 0.03$ is close to zero, which indicates a high consistency.

Table 2: Comparison results of the proposed method with some existing methods

Methods	$m(\{A_1\})$	$m(\{A_2\})$	$m(\{A_3\})$	$m(\{A_1, A_3\})$	Ψ	$\emptyset$
Dempster [1]	0	0.2000	0.8000	-	-	-
Yager [21]	0	0.0003	0.0014	-	0.9983	-
Smets et al.[62]	0	0.0003	0.0014	-	-	0.9983
Jiang [32]	0.9708	0.0028	0.0257	0.0007	-	-
Murphy [30]	0.9175	0.0090	0.0721	0.0015	-	-
Deng et al. [36]	0.6830	0.0293	0.2797	0.0081	-	-
Ma et al. [63]	0.6107	0.0917	0.3667	-	-	-
Sun et al. [64]	0.4036	0.1329	0.1369	-	0.3266	-
Mi and Kang [61]	0.4898	0.1020	0.4082	-	-	-
Chen [29]	0.9774	0.0224	0.0002	-	-	-
Proposed method	0.9971	0.0026	0.0003	-	-	-

5. Application in fault diagnosis

5.1. Problem statement

In this section, the practical application of motor rotor fault diagnosis [32] is illustrated the practicability and effectiveness of the proposed method. In this application, three sensors are located in different positions to obtain acceleration, velocity, and displacement of the motor rotor. Then, the four states of the motor rotor ("normal operation", "unbalance", "misalignment" and "pedestal looseness") can be obtained for decision making.

5.2. Implementation of proposed method

According to the data obtained in [32], and modeled as BPA: m_1,m_2 and m_3 in FOD $\Psi = \{A_1, A_2, A_3, A_4\}$, as shown below:

$$m_1 : m_1(A_1) = 0.06, m_1(A_2) = 0.68, m_1(A_3) = 0.02, m_1(A_4) = 0.04, m_1(\Psi) = 0.20$$
$$m_2 : m_2(A_1) = 0.02, m_2(A_2) = 0, m_2(A_3) = 0.79, m_2(A_4) = 0.05, m_2(\Psi) = 0.14$$
$$m_3 : m_3(A_1) = 0.02, m_3(A_2) = 0.58, m_3(A_3) = 0.16, m_3(A_4) = 0.04, m_3(\Psi) = 0.20$$

Where, m_1, m_2, and m_3 respectively represent the three evidences from the sensor, A_1="normal operation", A_2="unbalanced", A_3="misalignment" and A_4="pedestal looseness". The four states establish FOD = $\{A_1, A_2, A_3, A_4\}$. In this work, the threshold for deciding the four states is set to 0.7 based on [32]. When analyzing three pieces of evidence, each piece of evidence has a different direction although $m_2(A_2) > 0.7$. However, it is difficult to make a decision only on BPAs m_1, m_2 and m_3. Therefore, it is necessary to integrate the three-piece of evidence and a conflict management method to improve decisions.

Step1 and Step2: According to the best and worst BPA models, the worst and best BPA are obtained.

(1) The distance matrix D is constructed as:

$$M_D = \begin{bmatrix} 0 & 0.7276 & 0.1249 \\ 0.7276 & 0 & 0.6063 \\ 0.1249 & 0.6063 & 0 \end{bmatrix}$$

(2) The distance degree of m_j is calculated as:

$$CD(m_1) = 0.8525;\ CD(m_2) = 1.3339;\ CD(m_3) = 0.7312.$$

(3) Therefore, the worst and best BPA are m_2 and m_1, respectively.

Step3: The Deng relative entropy reference comparisons for the best BPA m_1 is calculated as:

$$\begin{aligned} M_{m_1} &= (\tilde{D}_r(m_1||m_1), \tilde{D}_r(m_1||m_2), \tilde{D}_r(m_1||m_3) \\ &= (1, 24.3022, 0.1325) \end{aligned}$$

Step4: The Deng relative entropy reference comparisons for the worst BPA m_2 is calculated as:

$$\begin{aligned} M_{m_2} &= (\tilde{D}_r(m_1||m_2), \tilde{D}_r(m_2||m_2), \tilde{D}_r(m_3||m_2) \\ &= (24.3022, 1, 20.3963) \end{aligned}$$

Step5: Solved the model (34), the optimal weights is calculated as:

$$w^*(m_1) = 0.3101;\ w^*(m_2) = 0.0219;\ w^*(m_3) = 0.6680.$$

and consistency ratio is calculated as:

$$\eta_{CR} = 0.2216.$$

Step6: Discounting of BPAs and fusion results is obtained as:

According to Eq.(8), discounting of BPAs is calculated as:

$$m(A_1) = 0.0324;\ m(A_2) = 0.5983;\ m(A_3) = 0.1304$$
$$m(A_4) = 0.0402;\ m(\Psi) = 0.1987.$$

Then, the fusion resluts is obtained with the DCR:

$$m(A_1) = 0.0083;\ m(A_2) = 0.9156;\ m(A_3) = 0.0511$$
$$m(A_4) = 0.0106;\ m(\Psi) = 0.0144.$$

5.3. Discussion

To demonstrate the effectiveness of the proposed method for determining the BPA weight and the management of evidence conflicts, a comparative analysis of the weight and fusion results between the proposed method and related work, including Dempster's [1], Murphy's [30], Chen [29], Deng et al's. [36], Jiang's [32], and Xiao's [38] method are illustrated. The weights of the proposed method with the methods of Deng et al., Jiang and Xiao are shown in Figure 5. However, other weight determination models have the same purpose as our proposed method. On the one hand, the proposed method has the highest weight distribution to m_3 and the CR is calculated as $\eta_{CR} = 0.2216$, indicating a good consistency. Meanwhile, the fusion results were obtained by different conflict management methods as shown in Table 3. It is clear that their $m(\{A_2\})$ values of 0.5230 and 0.6059, respectively, then the methods of Dempster [1] and Murphy [30] cannot determine the fault type. On the other hand, compared with matrix-based can judge the fault category as "unbalanced" methods of modifying the evidence model (Deng et al. [36]; Jiang [32]; Xiao [38]) that needs to have $n^2/2 = 9/2$ comparisons, for proposed method only requires $2n-3=3$ comparisons, which reduces the complexity of comparisons. Meanwhile, the highest value $m(\{A_2\}=0.9156$ is obtained by the proposed method, which indicates the recognition of the fault with a higher recognition rate.

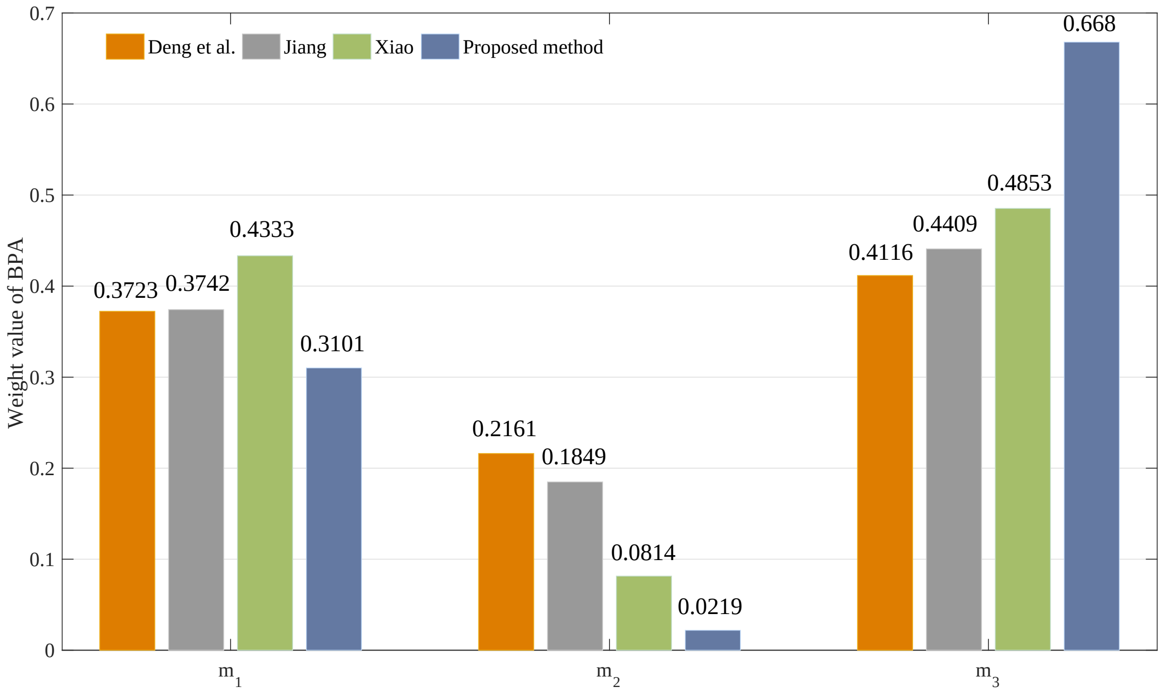

Figure 5: Comparison of the weights of BPA under different methods

Table 3: Coparison results of the proposed method with some existing methods

Methods	$m(\{A_1\})$	$m(\{A_2\})$	$m(\{A_3\})$	$m(\{A_4\})$	$m(\Psi)$	Fault type
Dempster [1]	0.0205	0.5230	0.3933	0.0309	0.0324	Cannot be determined
Murphy [30]	0.0112	0.6059	0.3508	0.0153	0.0168	Cannot be determined
Chen [29]	0.0112	0.6625	0.2944	0.0150	0.0169	Cannot be determined
Deng et al. [36]	0.0111	0.7730	0.1856	0.0139	0.0165	unbalance
Jiang [32]	0.0108	0.8063	0.1534	0.0134	0.0162	unbalance
Xiao [38]	0.0102	0.8964	0.0674	0.0113	0.0148	unbalance
Proposed method	0.0129	0.9156	0.0511	0.0106	0.0144	unbalance

6. Conclusion

To better manage the conflict evidence, a novel method of determining the optimal weights of BPA is explored and apply it to fault diagnosis in this work. Besides, the BWM method extends to the conflict management problem of evidence theory, and a concept and algorithm for determining the best and worst BPA are proposed. The properties of the preference comparison matrix of the proposed method was defined and analyzed. Then, a six-step procedure was used to derive the weights of BPAs and fusion results. Compared to other conflict management (especially matrix-based) methods, the proposed method is vector-based in conflict management that requires fewer comparisons, then the number of calculations and inaccuracy lower. Meanwhile, a consistency ratio is considered for BPA to check the reliability of comparisons, which produces more reliable results. Numerical examples are illustrated to compare the proposed method with some well-known conflict management methods to demonstrate the superiority of this novel BPA weight determination method. Furthermore, we also applied the proposed method in a fault diagnosis, and the results indicated the proposed method achieve a higher recognition rate. In summary, the proposed method has the advantages of less calculation and reliability. It can further manage the conflict between evidence, improve the accuracy of decision-making, and have great potential for expansion and application in future work.

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Conflict of interest

The authors declare that they have no conflict of interest.

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