

IP(IN)=IN

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Abstract

In this page I will prove that $IP(IN)=IN$ using the Zermelo's natural Numbers construction.

Definition Zermelo's natural Numbers

Set $0 = \{\}$ the empty set

Define $S(a) = \{a\}$ for every set a . $S(a)$ is the successor of a , and S is called the successor function.

Example

$$0 = \{\}$$

$$1 = \{0\} = \{\{\}\}$$

$$2 = \{1\} = \{\{\{\}\}\}$$

...

$$n = \{n-1\} = \{\{\dots\}\}$$

Definition the power set

The power set of a set S is the set of all subsets of S , including the empty set and S itself.

The power set is Denotated $IP(S)$

Example

$$IP(\{1,2,3\}) = \{\{\}, \{0\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$$

Theorem

$$IP(IN) = IN$$

Proof

$$IP(IN) = \{\{\}, \{0\}, \{1\}, \{2\}, \{3\}, \dots, \{1,2\}, \{2,3\}, \{1,3\}, \dots, \{1,2,3\}, \dots, IN\}$$

But $\{1\}$ is in $\{2\}$ that is in $\{3\}$ etc.

$$\text{Then } \{1,3\} = \{1\} \cup \{3\} = \{3\}$$

$$\text{So } IP(IN) = \{0, 1, 2, 3, 4, \dots, 3, 4, 4, \dots, 4, \dots, IN\} = \{0, 1, 2, 3, 4, \dots, IN\} = IN$$