The trajectory nonstability of charges in LHC due to radiation loss

Miroslav Pardy
Department of Physical Electronics
Masaryk University
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

January 31, 2022

Abstract

The quasi-classical behavior of a charged particle moving in a magnetic field is derived by the WKB approximation and wave-packet method from the Klein-Gordon equation with the Schwinger radiative term. The lifetime of the wave-packet state is calculated for a constant magnetic field. The finite lifetime of the trajectory is the proof of the nonstationary motion of charges moving in magnetic field.

1 Introduction

The problem of the influence of the bremsstrahlung on a charged particle moving in an electromagnetic field has been a subject of interest for many years (Cloetens, 1969; Grandy, 1970; Jaffe, 1972; Sen Gupta, 1972; Sorg, 1971; Shen, 1972a; Shen, 1972b; Rowe, 1975; Rowe, 1978). The natural approach to solve this problem is to derive the equation of motion involving the bremsstrahlung term and the Lorentz-Dirac equation is generally considered as the most appropriate equation describing the physical process. It was obtained on basis of classical electrodynamics by decomposing the energy momentum tensor of the retarded self field into the sum that renormalizes mass and a therm that gives radiation reaction (Dirac, 1938).

A theoretical re-derivation of this equation based on an absorber theory was provided by Wheeler and Feynman (Wheeler, et al., 1945). Nevertheless the Lorentz-Dirac equation by the different methods has certain imperfections which needs special discussions and approaches not involved in the theory. The difficulties are as follows: a) The Lorentz-Dirac equation involves the derivative of acceleration and it needs an extra-conditions in addition to the Newtonian initial condition to determine the motion. b) It gives runaway solutions which can be avoided only by pre-acceleration (Landau, et al. 1988).

In certain cases it implies that the external energy supplied to the particle goes only into kinetic energy and radiation is created from an acceleration self-energy which becomes more and more negative (Tse Chin, 1971).

The purpose of the present article is not of the re-derivation of the Lorentz-Dirac equation without imperfections but to derive by the combination of the WKB approximation and wave packet method the classical behavior of a charged radiating particle from the Klein-Gordon equation with the Schwinger radiative term. Our problem is relate to the method Censor who has considered the quantum mechanical problem of motion of particles in a dissipative system using wave-packet and eikonal representation of the wave function (Censor, 1979). The natural idea which is presented in the Censor article and which is accepted in our article is the stipulation that the group velocity must be real quantity in a dissipative system using wave-packet and eikonal representation of the wave function (Censor, 1979). We follow the author article (Pardy, 1985)

2 Formulation of the problem and solution

Our starting point is the Klein-Gordon equation with the Schwinger mass operator, or, in other words, with the radiative term (Schwinger, 1973; Tsai, 1973):

$$(\Pi^\mu \Pi_\mu + m^2 + \delta m^2)\varphi = 0, \tag{1}$$

where

$$\Pi_\mu = \frac{1}{i}\partial_\mu - eqA_\mu \tag{2}$$

with

$$\partial_\mu = \frac{\partial}{\partial x^\mu}, \quad x^\mu \equiv (ct, \mathbf{x}) \equiv (\mathbf{x^0}, \mathbf{x}) \equiv (-\mathbf{x_0}, \mathbf{x}) \tag{3}$$

and

$$q = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{4}$$

The symbol A_μ is four-vector the electromagnetic potential, q is the charge matrix in the charge space and φ is the two-component wave function and the term δm^2 is the radiative correction of the following mathematical form (Schwinger, 1973; Tsai, 1973):

$$\delta m^2 = ie^2 \int \frac{(dk)}{(2\pi)^4} \times$$

$$(2\Pi - k)^\mu \frac{1}{k^2} \frac{1}{\Pi - k)^2 + m^2} (2\Pi - k)_\mu + C.T., \tag{5}$$

where $C.T.$ are the so called contact terms (Schwinger, 1973; Tsai, 1973).

The eigenvalues of the operator (5) has been calculated for a constant magnetic field by Tsai to give

$$\delta m^2 = \kappa + i\lambda, \tag{6}$$

where (with $\hbar = 1$, n = 0,1, 2, ...) and $\kappa = \mathrm{Re}\ \delta m^2(\mathbf{E} = 0, \mathbf{H} \equiv (0, 0, H = const))$, or,

$$\kappa = \frac{\alpha}{\pi} m^2 \left\{ \left(\frac{eH}{m^2}\right) \left(\frac{4}{3} \lg \frac{m^2}{2eH} - \frac{7}{12}\right) + \right.$$

$$\left. (2n + 1)\frac{eH}{m^2} \left(\frac{8}{3} \ln \frac{m^2}{2eH} - \frac{32}{5} \lg 2 + \frac{343}{90}\right) \right\} \tag{7}$$

and $\lambda = \mathrm{Im}\ \delta m^2(\mathbf{E}=0, \mathbf{H} \equiv (0,0,H=const))$, or,

$$\lambda = -\alpha m^2 \times$$

$$\left\{\frac{4n}{3}\left(\frac{eH}{m^2}\right) + \left(\frac{eH}{m^2}\right)^3\left(-\frac{2}{15}(2n+1)^2 - \frac{4}{3}(2n+1) + \frac{22}{15}\right)\right\}. \tag{8}$$

In order to find some information concerning the classical motion of the particle, we replace the operator δm^2 in eq. (1) by its eigenvalues and then use the well known fact that the classical limit of the quantum-mechanical equations can be obtained by the WKB approximation of the form:

$$\varphi_{WKB} = e^{\frac{i}{\hbar}S}(a_0 + \hbar a_1 + \hbar^2 a_2 + ...). \tag{9}$$

It is easy to see that the zero-order approximation

$$\varphi_{(0)WKB} = a_0 e^{\frac{i}{\hbar}S}. \tag{10}$$

generates the Hamilton-Jacobi equation

$$(P^\mu + eA^\mu)(P_\mu + eA_\mu) + m^2 + \delta m^2 = 0 \tag{11}$$

with $(eq)' = -e$, where the latter incorporates the charge assignment of particle with charge e, $P^\mu = \partial^\mu S$ is the generalized momentum. However, the expression $m^2 + \delta m^2$ in the Hamilton-Jacobi equation is a complex number and it therefore makes the Hamilton-Jacobi equation meaningless in the classical sense.

To overcome this obstacle in order to get the classical information on the particle motion we will combine the zero-order WKB approximation (10) with the wave packet method.

We can obviously write in the sufficiently space-time interval the following formula (Pardy, 1985):

$$S \approx S_0 + \frac{\partial S}{\partial \mathbf{x}} + \frac{\partial S}{\partial t}, \tag{12}$$

or using the Hamilton-Jacobi equation $\mathbf{P} = \partial S/\partial \mathbf{x}$ and $-H = \partial S/\partial t$, we have

$$S \approx S_0 + \mathbf{P}\cdot\mathbf{x} - Ht, \tag{13}$$

where $\mathbf{P}$ is the generalized momentum of a particle and H is its energy. It is obvious for $|\delta m^2| \ll |m^2|$ and $A^0 = 0$, that $(\mathbf{p} = \mathbf{P} + \mathbf{A})$

$$H(A^0 = 0) = (\mathbf{p}^2 + m^2 + \delta m^2)^{1/2} \approx E + \frac{\kappa}{2E} + \frac{i}{2}\frac{\lambda}{E}, \qquad (14)$$

where $E = -(\mathbf{p}^2 + m^2)^{1/2}$ for bound states.

After insertion of eq. (14) into eq. (12) and then eq. (12) into eq. (10) we get

$$\varphi_{(0)WKB} \approx a_0 e^{\frac{i}{\hbar}[S_0 + \mathbf{P}\cdot\mathbf{x} - Et]} e^{\frac{i\lambda}{2E\hbar}t} e^{\frac{i\kappa}{2E\hbar}}. \qquad (15)$$

which means that the particle with the complex mass is in the quasi-stationary state which decays according to the decaying law $\exp \lambda/2E\hbar t$ with the decay rate

$$\gamma = -\frac{\lambda}{\hbar E}; \quad E < 0; \quad \lambda \geq 0, \qquad (16)$$

where the formula (16) is in agreement with formula (69) in monograph (Akhiezer, et al., 1969). To get further information, we use the obvious approximation:

$$H \approx H_0 + \frac{\partial H}{\partial \mathbf{P}} \cdot (\mathbf{P} - \mathbf{P}_0) + ... = H_0 + \mathbf{v} \cdot (\mathbf{P} - \mathbf{P}_0) + ..., \quad (17)$$

where we have used the Hamilton equation $\mathbf{v} = \partial H/\partial \mathbf{P}$, $\mathbf{v}$ being the velocity of of th particle with momentum $\mathbf{P}$. Then after insertion of eq (17) into eq (13) we have:

$$S \approx S_0 - H_0 t + \mathbf{P}_0 \cdot \mathbf{x} + (\mathbf{x} - \mathbf{v}t) \cdot (\mathbf{P} - \mathbf{P}_0). \qquad (18)$$

Now, we construct the wave packet solution of eq. (1) by $\mathbf{P}$-integration of eq. (10) with the exponent (18). We find

$$\varphi = a_0 \exp\left\{\frac{i}{\hbar}(S_0 - H_0 t + \mathbf{P}_0 \cdot \mathbf{x})\right\} \times$$

$$\int d\mathbf{P} g(\mathbf{P}) \exp\left\{\frac{i}{\hbar}(\mathbf{x} - \mathbf{v}t) \cdot (\mathbf{P} - \mathbf{P}_0)\right\}, \qquad (19)$$

where $g(\mathbf{P})$ is the suitable weight function which forms the envelope of the wave packet $G(\mathbf{P}_0, \mathbf{x} - \mathbf{v}t)$, or,

$$G(\mathbf{P}_0, \mathbf{x} - \mathbf{v}t) = \int d\mathbf{P} g(\mathbf{P}) \exp\left\{\frac{i}{\hbar}(\mathbf{x} - \mathbf{v}t) \cdot (\mathbf{P} - \mathbf{P}_0)\right\} \quad (20)$$

The function φ in eq. (19) describes a wave packet with a carrier wave

$$a_0 \exp\left\{\frac{i}{\hbar}(S_0 - H_0 t + \mathbf{P}_0 \cdot \mathbf{x})\right\} \quad (21)$$

and an envelope $G(\mathbf{P}_0, \mathbf{x} - \mathbf{v}t)$ which moves at constant velocity according to the law $\mathbf{x} = \mathbf{v}t$ in the small space-time interval.

We identify the motion of the envelope with their classical motion of the particle moving at the velocity $\mathbf{v}$. But at this stage of the investigation, $\mathbf{v}$ is the complex quantity and therefore it does not mean that it is the physical velocity. To avoid this obstacle in order to get the physically meaningful description of reality, we stipulate the transformation

$$\mathbf{P}_0 \to \mathbf{P}_0 + i\boldsymbol{\varepsilon}, \quad (22)$$

where $\boldsymbol{\varepsilon}$ is to be determined from eq.

$$\mathrm{Im}\, \mathbf{v} = 0. \quad (23)$$

Using

$$\mathbf{v}(\mathbf{P}_0 + i\boldsymbol{\varepsilon}, m^2 + \delta m^2) \approx \mathbf{v}(\mathbf{P}_0, m^2) + i\boldsymbol{\varepsilon}\frac{\partial \mathbf{v}}{\partial \mathbf{P}} + i\lambda\frac{\partial \mathbf{v}}{\partial m^2} + \kappa\frac{\partial \mathbf{v}}{\partial m^2}. \quad (24)$$

For $|\delta m^2| \ll |m^2|, |\boldsymbol{\varepsilon}| \ll |\mathbf{P}_0|$, we get after the application of the requirement (23) the following equation for $\boldsymbol{\varepsilon}$:

$$\frac{\partial v_i}{\partial P_j}\varepsilon_j + \lambda\frac{\partial v_i}{\partial m^2} = 0. \quad (25)$$

Then instead of equation (24) we have

$$\mathbf{v} = \mathbf{v}(\mathbf{P}_0, (m*)^2); \quad (m*)^2 = m^2 + \kappa. \quad (26)$$

The last formula $(m*)^2 = m^2 + \kappa$ can be easily interpreted in such a way that the radiation of a charged particle moving in an electromagnetic field changes its mass not only in the quantum theory but also in its the quasi-classical limit.

The transformation (22) leads to the transformation

$$\frac{i}{\hbar}\mathbf{v}\cdot\mathbf{P}_0 t \to +\frac{i}{\hbar}\mathbf{v}\cdot\mathbf{P}_0 t - \frac{2\mathbf{v}\cdot\boldsymbol{\varepsilon}}{2\hbar}t, \tag{27}$$

which necessitates the time dependence of the wave function of the form

$$\varphi = \exp -\frac{\Gamma}{2}t, \tag{28}$$

where

$$\Gamma = \gamma + \frac{2\mathbf{v}\cdot\boldsymbol{\varepsilon}}{\hbar} \tag{29}$$

and may be easy to specify the quantity Γ using the obvious relations

$$(\delta_{ij} - v_i v_j)\varepsilon_j = \frac{1}{2}\lambda(\mathbf{p}^2 + m^2)^{-1/2} v_i \tag{30}$$

$$\mathbf{v}\cdot\boldsymbol{\varepsilon} = \frac{\lambda}{2}\frac{v^2}{1-v^2}(\mathbf{p}^2 + m^2)^{-1/2}. \tag{31}$$

Then with $\mathbf{v} = \mathbf{p}(\mathbf{p}^2 + m^2)^{-1/2}$, we have

$$\Gamma = \gamma + \frac{2\mathbf{v}\cdot\boldsymbol{\varepsilon}}{\hbar} = \frac{\lambda}{\hbar}\frac{(\mathbf{p}^2 + m^2)^{1/2}}{m^2}. \tag{32}$$

The quantity Γ is here interpreted as a lifetime of a wave packet moving at velocity $\mathbf{v}$ in a magnetic field on the orbit corresponding to the quantum number n.

3 Discussion

The radiation of the accelerated particle with the nonzero charge is the key for understanding certain phenomena in modern physics and astrophysics, e.g. accelerators of particle, pulsars and so on.

We have seen that by representing the motion of the particle by the wave packet, corresponding to the solution of the Klein-Gordon equation with the Schwinger radiative bremsstrahlung term, the particle state is quasi-stationary with the decay rate given by eq. (31). The result (31) is not in contradiction with the relation (15) because the decay rate (15) corresponds to the quasi-stationary state of the wave function

$$\varphi_{(0)WKB} = a_0 e^{\frac{i}{\hbar}S}, \qquad (10)$$

while the decay rate (31) corresponds to the quasi-stationary state of the wave packet, or, in other words to different real situation.

The derived results in the present paper suggest the classical picture of the motion of the particle undergoing the radiation reaction. In the very small space-time interval a particle with mass $(m^2 + \kappa)^{1/2}$ is moving with velocity $\mathbf{v}$ and it remains at this velocity only for time Γ, Γ being the decay rate of the wave packet (31) and after time γ the particle changes its velocity. The change of velocity is caused by the complex mass of the particle which corresponds to the influence of radiation of the particle on the particle motion. The change of velocity leads to the stochastic change of trajectory which is equivalent to the Zitterbewegung of particle in magnetic field of LHC.

References

Akhiezer, A. I. and Berestetzkii, V. B. *Quantum electrodynamics*, (Nauka, Moscow, 1969)

Censor, D. (1979). Geometrical Mechanics For Particles In Dissipative Systems, Phys. Rev. D19, 1108-1111

Cloetens, W. J. M. et al. (1969). A theoretical energy paradox in the Lorentz-Dirac-Wheeler-Feynman-Rohrlich Electrodynamics - Nuovo Cim. A62, 247-254

Dirac, Paul A.M. (1938). Classical theory of radiating electrons, Proc. Roy. Soc. Lond. A167, 148-169

Grandy, W.T. (1970). Concerning a "theoretical paradox" in classical electrodynamics, Nuovo Cim. A65, 738-742

Jaffe, J. (1972). Anomalous motion of radiating particles in strong fields, Phys. Rev. D5, 2909-2911

Landau, L. D. and Lifshitz, E. M., *The Classical Theory of*

Fields, Fourth Revised English Edition, (Amsterdam, Boston, Heidelberg, London, ..., 1988).

Pardy, M. (1985). Radiative corrections to classical particle motion in a magnetic field, Phys. Rev. D 31, 325,

Rowe, E. G. P. (1975). Resolution of an Ambiguity in the Derivation of the Lorentz-Dirac Equation, Phys. Rev. D12, 1576-1587

Rowe, E. G. P. (1978). Structure of the Energy Tensor in the Classical Electrodynamics of Point Particles, Phys.Rev. D18, 3639-3654

Shen, C.S. (1972a). Magnetic bremsstrahlung in an intense magnetic field, Phys.Rev. D6 2736-2754

Shen, C. S. (1972b). Comment on the "new" equation of motion for classical charged particles - Phys. Rev. D6 3039-3040

Sen Gupta, N. D. (1972). Comments on relativistic motion with radiation reaction, Phys. Rev. D5, 1546-1547

Schwinger, J. *Particles Sources and Fields*, (Addison-Wesley, Reading, Mass., 1973)

Schwinger, J. (1973). Classical radiation of accelerated electrons - a quantum viewpoint, Phys. Rev. D7, (1973) 1696-1701

Sorg, M. (1974). Classical, Covariant Finite-size Model of the Radiating Electron Z. Naturforsch., A29, 1671

Tsai, Wu-Yang, (1973). Magnetic bremsstrahlung and modified propagation function. spin-0 charged particles in a homogeneous magnetic field, Phys. Rev. D8, 3460-3469

Tse Chin, Mo. et al. (1971). New equation of motion for classical charged particles, Phys. Rev. D4 , 3566-3571

Wheeler, J. et al. (1945). Interaction with the absorber as the mechanism of radiation, Rev. Mod. Phys. 17, 157-181