Nonintegrable Dynamics and the Naturalness Principle

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Abstract

Over the years, *naturalness* has been a key principle for guiding theory development beyond the Standard Model (BSM) and for solving the cosmological constant (CC) problem. The discovery of the Higgs boson and the exclusion of several BSM scenarios at the Large Hadron Collider (LHC) has set off an ongoing debate on the conceptual limitations of the naturalness paradigm. In contrast with the bulk of mainstream proposals on how to move beyond the paradigm, we argue here that the breakdown of naturalness follows from the nonintegrability of interacting field theory above the Standard Model scale.

Key words: Naturalness, Fine-tuning, Hierarchy problem, the Cosmological constant problem, Beyond Standard Model physics, Nonintegrability, Hamiltonian chaos.

"Integrable systems are very exceptional, and it is unfortunate that our whole intuition in mechanics, classical as well as quantum, and perhaps even statistical, is based on the experience with these special cases"

Gutzwiller M C.., Chaos in Classical and Quantum Mechanics*, Springer, 1990.*

"We need to reconsider the guiding principles that have been used for decades to address the most fundamental questions about the physical world"

Gian Giudice, https://arxiv.org/pdf/1710.07663.pdf

1. Introduction

At the time of writing, it is unclear how *effective field theory* (EFT) and the Standard Model of particle physics (SM) emerge as low-energy approximations of a complete theory including Dark Matter and Dark Energy, as well as gravity with a positive CC. During the last four decades, challenges posed to the naturalness principle were branded either as "*fine-tuning*" or "*hierarchy*" problems and invoked as primary drivers for model building beyond SM. As a result, high-energy theory has seen an overflow of proposals postulating *new objects* (elementary fields or bound states) or *hidden symmetries* that allegedly break down somewhere above the SM scale. Prime examples are Technicolor, Supersymmetry, Extra dimensions, and Composite Higgs, boldly alleging new physical phenomena beyond the LHC range. Failure to conclusively confirm either one of these proposals has sparked an ongoing debate on the value and drawbacks of the naturalness principle [1-3, 18].

Building on the view that the nonlinear dynamics of EFT takes on a leading role above the LHC range, we argue here that failure of the naturalness principle stems from *nonintegrability* and the transition of EFT to *Hamiltonian chaos.*

The paper is divided in three sections. Next couple of sections cover a brief account of naturalness in the Higgs sector and in the CC problem, respectively. The third section goes over the chaotic behavior of field theory above the SM scale and the role of *fractional dynamics* and *multifractal geometry* in the analysis of this regime.

2. Higgs boson and the hierarchy problem

The electroweak (EW) symmetry is broken by a scalar field having the following doublet structure:

$$\Phi = \begin{pmatrix} G^{+} \\ 1/\sqrt{2}\,[(H + \mathrm{v}) + iG^{0}] \end{pmatrix} \tag{1}$$

Here, G^{+} and G^{0} represent the charged and neutral Goldstone bosons generated from symmetry breaking, H is the Higgs boson and $\mathrm{v} \approx 246\, GeV$ its vacuum expectation value. The Higgs potential satisfies the requirements of renormalizability and gauge-invariance and has the form

$$V = \mu^{2}\Phi^{+}\Phi + \lambda(\Phi^{+}\Phi)^{2} \tag{2}$$

with $\lambda = O(1)$ and $\mu^{2} = O(\mathrm{v}^{2})$. A vanishing quartic coupling $(\lambda \to 0)$ represents the critical value that separates the ordinary EW phase from an unphysical phase where the Higgs field assumes unbounded values. Likewise, the coefficient μ^{2} plays the role of an order parameter whose sign describes the transition between a symmetric phase and a broken phase. Minimizing the Higgs potential yields:

$$v^2 = -(\mu^2/\lambda) \tag{3}$$

where the physical mass of the Higgs is:

$$M_H^2 = -2\lambda v^2 = 2\mu^2 \tag{4}$$

The renormalized mass squared of the Higgs scalar contains two contributions:

$$\mu^2 = \mu_0^2 + \Delta\mu^2 \tag{5}$$

in which μ_0^2 represents the ultraviolet (bare) value. This mass parameter picks up quantum corrections $\Delta\mu^2$ that depend quadratically on the ultraviolet cutoff Λ of the theory. Consider for example the contribution of radiative corrections to μ^2 from top quarks. The complete one-loop calculation of this contribution reads:

$$\Delta\mu^2 = \frac{N_c \lambda_t^2}{16\pi^2}[-2\Lambda^2 + 6M_t^2 \ln(\frac{\Lambda}{M_t}) + ...] \tag{6}$$

in which λ_t and M_t are the Yukawa coupling and mass of the top quark. If the bare Higgs mass is set near the cutoff $\mu_0^2 = O(\Lambda^2) = O(M_{Pl}^2)$, then $\Delta\mu^2 \approx$

-10^{35} GeV. This large correction must precisely cancel against μ_0^2 to protect the EW scale. This is the root cause of the hierarchy problem, which boils down to the implausible requirement that μ_0^2 and $\Delta\mu^2$ should offset each other to about 31 decimal places.

Closely related to the hierarchy problem is the question of whether the SM remains valid all the way up to the Planck scale (M_{Pl}). This question is non-trivial because it depends on how the Higgs quartic coupling λ behaves at high energies. Competing trends are at work here, namely:

1) Radiative corrections from top quarks *drop* λ at higher scales, while those from the self-interacting Higgs *grow* λ at higher scales.

2) If $\lambda(\mathrm{v})$ is too large, the Higgs loops dominate and λ diverges at some intermediate scale called the *Landau pole*. However, if $\lambda(\mathrm{v})$ is too small, the top loops dominate, λ runs negative at some intermediate scale which, in turn, makes the potential unbounded from below and destabilizes the vacuum.

3. The cosmological constant problem

The traditional computation of the vacuum energy density (VED) in QFT starts from the sum [4]

$$\rho_V = \sum_n \left\{ c_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \hbar\, \omega_n(k) \right\} \tag{7}$$

where

$$\omega_n(k) = \sqrt{m_n^2 + k^2} \tag{8}$$

Here, VED is modeled as a reservoir of free quantum harmonic oscillators in flat spacetime and (7) represents the integral of zero-point energy carried over all momenta. For large momenta $k >> m_n$, the oscillator frequency may be approximated as $\omega_n(k) \approx k$, in which case the integral (7) diverges. Inserting an ultraviolet cutoff Λ in (7) yields

$$\int_0^{\Lambda} d^3k \sqrt{m_n^2 + \Lambda^2} = \pi \left\{ \Lambda^4 + m_n^2 \Lambda^2 + \frac{m_n^4}{8} - \frac{1}{2} m_n^4 \ln\left({}^{2\Lambda}\!/\!_{m_n} \right) \right\} + O(\Lambda^{-2}) \tag{9}$$

It is apparent that (8) is quartically divergent as the UV cutoff approaches the Planck region of scales ($\Lambda = O(M_{Pl}) >> m_n$). To regularize (7), one follows the QFT renormalization prescription, according to which one starts with a bare Lagrangian and a cutoff dependent bare VED in the form

$$\rho_b = \rho_b(\Lambda) \tag{10}$$

The renormalized or effective VED is then given by

$$\rho_{eff,\mathrm{v}} = \rho_b(\Lambda) + c\Lambda^4 \tag{11}$$

where c stands for some numerical constant. Astrophysical observations from type I supernovae and from the cosmic microwave background (CMB) radiation show that

$$\rho_{eff,\mathrm{v}}^{1/4} \approx 2\times 10^{-3}\mathrm{eV} \tag{12}$$

Since experiments have confirmed that the Standard Model is valid at least up to an energy scale of $O(1\mathrm{TeV} = 10^{12}\mathrm{eV})$, one may reasonably assume that the UV cutoff can be placed around this scale ($\Lambda = O(10^{12}\mathrm{eV})$). Combined use of (11) and (12) gives

$$(2\times10^{-3}\text{eV})^4 = \rho_b(\Lambda) + c(10^{12}\text{eV})^4 \tag{13}$$

It follows that the bare value of the cosmological constant evaluated at the cutoff must be chosen so that it cancels out a contribution on the order of 10^{48}eV^4 and leaves a contribution on the order of 10^{-12}eV^4. This requires an unnatural fine-tuning of the cosmological constant on the order of 60 decimal places, which lies at the core of the CC problem.

4. Nonintegrability and chaos above the SM scale

It has been long known that, in general, interacting Hamiltonian systems are *nonintegrable* and their long-term behavior *chaotic* in the classical sense [5-7]. Also well-known is the process of *decoherence* which entails the destruction of quantum interference in open systems, systems exposed to persistent noise-like perturbations or ensembles evolving outside thermodynamic equilibrium. A plausible assumption is that, if decoherence sets in somewhere above the SM scale, *nonlinearly coupled* and *unstable* systems of quantum fields are prone to become classical and evolve towards chaos in a universal way.

The chaotic regime of interacting field theory can be characterized through *fractional dynamics* and/or the geometry of *multifractal sets* [7-10, 18]. There are several ways in which these concepts can sidestep the naturalness paradigm. In particular,

1) Mixing the ultraviolet (UV) and infrared (IR) fields in fractional dynamics follows from the nonlocal properties of *fractional differential and integral operators* [19]. Spacetime fractality vanishes in the IR limit (as $\varepsilon = 4 - D \to 0$), where dimensional regularization recovers the wide separation of fundamental scales required by the Clustering Theorem [11-12].
2) The "sum-of-squares" relationship also lends support to the *self-contained* structure of the SM and the wide separation of fundamental scales [13].
3) The UV instability of the Higgs boson solves the hierarchy problem *by default* and generates the entire gauge and flavor composition of the SM through successive period-doubling bifurcations [14-15].

4) *Neutrino oscillations* are a portal to the Cantor Dust composition of Dark Matter and account for the magnitude of the CC [16].
5) Fractional dynamics regularizes the *vacuum energy density* integral and alleviates the CC problem [17].

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