

Mass Spectrum of the Xi Baryons

D.G. Grossman
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Abstract

6.00 S7h = 1314.878 versus 1314.86 PDG FIT for the neutral Xi baryon. All subatomic particle masses can be expressed as multiples of hypersphere surface volumes times 'h' – Planck's constant's coefficient. In the case of the Xi baryons, the hypersphere surface volume needed is that of a 7-sphere (S7 for short). The purpose of this paper is to show that the masses of all Xi baryons can be expressed as simply defined multiples of S7h, by matching experimental Xi masses with their theoretical values in the mass spectrums generated by S7h that are presented in the paper.

Contents

1. Introduction
2. Determining the Xi Baryon's Quark Content
3. How to Calculate the Factoring Unit from the Quark Content
4. How to Read the Mass Spectrums
5. Mass Spectrums of Individual Xi Baryons' Experimental Mass Data
 - 5.1 Xi⁰
 - 5.2 Xi⁻
 - 5.3 Xi(1530)⁰, Xi(1530)⁻
 - 5.4 Xi(1620), Xi(1690)⁻
 - 5.5 Xi(1820)
 - 5.6 Xi(1950)
 - 5.7 Xi(2030)
 - 5.8 Xi(2120)
 - 5.9 Xi(2250)
 - 5.10 Xi(2370)
 - 5.11 Xi(2500)
6. Large Block Mass Spectrum of the Xi Baryons
7. Summary
8. References

1. Introduction

Baryons are subatomic particles composed of three quarks. According to Quark Theory, the three quarks inside baryons orbit one another in 3d space under the influence of a central force called the Strong Force. The mathematics for predicting baryon masses using the Quark Model is very complicated and uncertain since two key elements necessary for making accurate predictions are missing, namely the exact masses of the quarks, and an exact mathematical expression for the Strong Force. In this paper, quarks are treated as hypersphere surface volumes of mass of various dimensions, instead of point particles. The highest mathematics involved are the formulae for the hypersphere surface volumes.

It is hoped that the study of hypersphere surface volume factoring of particle masses will *result* in a comprehensive theory of particle structure, including discovering the complex mathematics that is likely associated with physics in higher dimensions.

2. Determining the Xi Baryon's Quark Content

The quark content of the Xi baryons is widely quoted as being **uss** or **dss**, depending on charge (neutral or negative). There is good reason to believe that that is incorrect. The correct quark content of the Xi baryons is most likely **ddd**, as the following derivation will show. The derivation below is based on assuming the following quark to hypersphere surface volume *translations* are correct.

<u>Quark</u>	<u>Hypersphere Surface Volume</u>
u	$S_2 = 2 \pi r$
d	$S_3 = 4 \pi r^2$
s	$S_4 = 2 \pi^2 r^3$

Substituting the correct hypersphere surface volume formulae for **uss**, and **ddd**, and multiplying them together with 'h' - Planck's constant's coefficient, and setting $r=1$, one gets:

$$\mathbf{uss}h = (2 \pi r)(2 \pi^2 r^3)(2 \pi^2 r^3)h = (8 \pi^5 r^7)h = \mathbf{16221.663} \text{ MeV}/c^2$$

$$\mathbf{ddd}h = (4 \pi r^2)(4 \pi r^2)(4 \pi r^2)h = (64 \pi^3 r^6)h = \mathbf{13148.784} \text{ MeV}/c^2$$

PDG's FIT for the neutral Xi baryon is **1314.86** MeV/c², which is almost exactly **1/10th** the value of **dddh**, shown above. Is this just a coincidence? No. It shows that the quark content of the Xi baryons is **ddd**, not **uss**. It could also be **udd**, since they differ by only a factor of 2r, but **ddd** will be assumed throughout this paper. Many other factorings, as shown in this paper, affirm the conclusion that **ddd** (or udd) is the correct quark content of the Xi baryons. (For most Xi's anyway. Xi(1820) and Xi(1950) seem to have a different quark content.)

3. Calculating the Factoring Unit from the Quark Content

To find the factoring unit of a subatomic particle, its quark content must be translated to the corresponding hypersphere surface volume formulae, then multiplied together along with 'h' - Planck's constant's coefficient. In this paper, the quark content of Xi baryons is assumed to be **ddd**, even though they are widely reported as having a quark content of **uss** or **dss**. (see *Determining the Xi Baryon's Quark Content*, above.)

The mass of the 'd' quark corresponds to 3-sphere surface volume (call it S3). So, to get the *factoring unit* of the **ddd** baryon (in units of MeV/c²) multiply the corresponding quark content surface volume formulae together along with 'h' - the coefficient of Planck's constant.

$$\text{Factoring unit of } \mathbf{ddd} = S3 S3 S3 h$$

$$\text{Factoring unit of } \mathbf{ddd} = (4 \pi r^2) (4 \pi r^2) (4 \pi r^2) h$$

$$\text{Factoring unit of } \mathbf{ddd} = 64 \pi^3 r^6 h$$

The surface volume of a 7-sphere is: **S7** = (16/15) $\pi^3 r^6$. The factoring unit of **ddd** is 60 times bigger, but except for that constant of multiplication, they are the same. Throughout this paper **S7h** will be used instead of **S3S3S3h** for factoring Xi baryon masses, because it is more concise (there is less to type) and it is a smaller factoring unit, both of which are advantageous. Factoring results - the theoretical masses found - will be exactly the same regardless of which of the two factoring units is used. Only the expressions found for the results will be different. They will differ by a factor of 60, so, to convert from one to the other just divide or multiply by 60.

To get the value of the factoring unit that is used in this paper, set r=1, then multiply S7 by the coefficient of Planck's constant (h = 6.62607015).

$$\mathbf{S7 h} = \pi^3 1^6 h$$

$$\mathbf{S7 h} = 219.1464153 \text{ MeV}/c^2$$

This is the *factoring unit* that was used to construct the mass spectrums in this paper.

3. How to Read the Mass Spectrums

- Col 1: **n** The first column of the mass spectrum holds 'n', the *multiplier* of **S6h**.
- Col 2: **nS6h** The second column shows the *theoretical mass*, which results from multiplying 'n' with **S6h**.
- Col 3: **ExpMass** The third column holds 'ExpMass', the *experimental mass*. It is placed next to the theoretical mass in the mass spectrum that most closely matches it.
- Col 4: **Error** The fourth column holds the *error of the experimental mass*. It is plus or minus.
- Col 5: **dm** This is the difference between the experimental and theoretical masses.
- Col 6: **dm/Error** This is **dm** divided by the **Error**, and the result is shown as a percentage. It shows the difference between theoretical and experimental mass as a percentage of error size.

Mass Spectrum of Ξ^0 Data

n	$6 + \frac{n}{4096} S7h$	ExpMass	Error	
-28	1313.380	1313.4	1.8	
-27	1313.434			
-26	1313.487			
-25	1313.541			
-24	1313.594			
-23	1313.648			
-22	1313.701			
-21	1313.755			
-20	1313.808			
-19	1313.862			
-18	1313.915			
-17	1313.969			
-16	1314.022			
-15	1314.076			
-14	1314.129			
-13	1314.183			
-12	1314.236			
-11	1314.290			
-10	1314.343			
-9	1314.397			
-8	1314.450			
-7	1314.504			
-6	1314.557			
-5	1314.611			
-4	1314.664			
-3	1314.718			
-2	1314.771			
-1	1314.825	1314.82	0.06/0.20	
6.0000 S7h	0	1314.878	1314.86	0.20 PDG's FIT
1	1314.932			
2	1314.985			
3	1315.039			
4	1315.093			
5	1315.146			
6	1315.200	1315.2	0.92	
7	1315.253			
8	1315.307			
9	1315.360			
10	1315.414			
11	1315.467			
12	1315.521			
13	1315.574			
14	1315.628			

6.0000 S7h = 1314.878 Theoretical Mass
 1314.86 PDG's FIT

Mass Spectrum of Ξ^- Data

n	$6 + \frac{n}{4096} S7h$	ExpMass	Error	
108	1320.656			
109	1320.710			
110	1320.763			
111	1320.817			
112	1320.870			
113	1320.924			
114	1320.977			
115	1321.031			
116	1321.085	1321.1	0.3	
116.666	1321.120	1321.12	0.41	
117	1321.138			
118	1321.192			
119	1321.245			
120	1321.299	1321.3	0.4	
121	1321.352			
122	1321.406	1321.4	1.1	
123	1321.459	1321.46	0.34	
124	1321.513			
125	1321.566			
126	1321.620			
127	1321.673	1321.67	0.52	
6.03125 S7h	128 1321.727	1321.71	0.07	PDG's FIT
129	1321.780			
130	1321.834			
131	1321.887	1321.87	0.51	
132	1321.941			
133	1321.994			
134	1322.048			
135	1322.101			
136	1322.155			
137	1322.208			
138	1322.262			
139	1322.315			
140	1322.369			
141	1322.422			
142	1322.476			
143	1322.529			
144	1322.583			
145	1322.636			
146	1322.690			
147	1322.743			
148	1322.797			

6.03125 S7h = 1321.727 Theoretical Mass
 1321.71 PDG's FIT

Mass Spectrum of $\text{Xi}(1530)^0$, $\text{Xi}(1530)^-$ Data

n	$7 + \frac{n}{4096}$ S7h	ExpMass	Error	Baryon
-128	1527.176	1527	6	$\text{X}(1530)^0$
-100	1528.674	1528.7	1.1	$\text{X}(1530)^0$
-76	1529.959	1530	1	$\text{X}(1530)^0$
-51	1531.296	1531.3	0.6	$\text{X}(1530)^0$
-50	1531.350			
-49	1531.403	1531.4	0.8	$\text{Xi}(1530)^0$
-48	1531.457			
-47	1531.510			
-46	1531.564			
-45	1531.617	1531.6	0.4	$\text{X}(1530)^0$
-44	1531.671			
-43	1531.724			
-42	1531.778			
-41	1531.831			
-40	1531.885			
-39	1531.938			
-38	1531.992	1532.0	0.4	$\text{X}(1530)^0$
-37	1532.045			
-36	1532.099	1532.1	0.4	$\text{X}(1530)^0$
-35	1532.152			
-34	1532.206	1532.2	0.7	$\text{X}(1530)^0$
-33	1532.25			
-32	1532.313	1532.3	0.7	$\text{X}(1530)^0$
-31	1532.366			
-30	1532.420			
-20	1532.955			
-19	1533.008	1533	1	$\text{X}(1530)^0$
-18	1533.062			
-17	1533.115			
-16	1533.169			
-15	1533.222			
-14	1533.276			
-13	1533.329			
-12	1533.383			
-11	1533.436			
-10	1533.490			
-9	1533.543			
-8	1533.597	1533.6	1.4	$\text{X}(1530)^0$
-7	1533.650			
-6	1533.704			
-5	1533.757			
-4	1533.811			
-3	1533.864			
-2	1533.918			
-1	1533.971			
7.0000 S7h	0	1534.025		
1	1534.078			
2	1534.132			
3	1534.185			
4	1534.239			
5	1534.292			
6	1534.345			
7	1534.399	1534.4	1.1	$\text{X}(1530)^-$
8	1534.452			
9	1534.506	1534.5	1.2	$\text{X}(1530)^-$
10	1534.559			
11	1534.613			
12	1534.666			
13	1534.720	1534.7	1.1	$\text{X}(1530)^-$
18	1534.987	1535	4	$\text{X}(1530)^0$
24	1535.308	1535.3	2.0	$\text{X}(1530)^-$
32	1535.736	1535.7	3.2	$\text{X}(1530)^-$
41	1536.218	1536.2	1.6	$\text{X}(1530)^-$
112	1540.017	1540	3	$\text{X}(1530)^-$

Commentary on Xi(1530)'s Mass Spectrum

The interesting thing about this mass spectrum is that 12 of the 13 experimental mass data points for the *neutral X(1530)* are less than **7.000 S7h**, while all 7 of the *negative X(1530)* experimental mass data points are greater than **7.000 S7h**.

Both sets of data are very close to 7.000 S7h.

Most *neutral X(1530)* data points are within 0.25% of **7.000 S7h**.

Most *negative X(1530)* data points are within 0.15% of **7.000 S7h**.

Mass Spectrum of Xi(1620), Xi(1690)⁻ Data

$$\text{Res} = (32/7^4) \quad \mathbf{s7h} = 2.920735231 \text{ MeV}/c^2$$

	n	$\frac{\mathbf{n}}{7^4} \mathbf{S7h}$	ExpMass	Error	Baryon
	544 (32)	1588.880			
	545 (32)	1591.801			
	546 (32)	1594.721			
	547 (32)	1597.642			
137 (128)	= 548 (32)	1600.563			
	549 (32)	1603.484			
	550 (32)	1606.404	1606	6	$\Xi(1620)$
	551 (32)	1609.325			
138 (128)	= 552 (32)	1612.246			
	553 (32)	1615.167			
	554 (32)	1618.087			
	555 (32)	1621.008			
139 (128)	= 556 (32)	1623.929	1624	3	$\Xi(1620)$
	557 (32)	1626.850			
	558 (32)	1629.770			
	559 (32)	1632.691	1633	12	$\Xi(1620)$
140 (128)	= 560 (32)	1635.612			
	561 (32)	1638.532			
	562 (32)	1641.453			
	563 (32)	1644.374			
141 (128)	= 564 (32)	1647.295			
	565 (32)	1650.215			
	566 (32)	1653.136			
	567 (32)	1656.057			
142 (128)	= 568 (32)	1658.978			
	569 (32)	1661.898			
	570 (32)	1664.819			
	571 (32)	1667.740			
143 (128)	= 572 (32)	1670.661			
	573 (32)	1673.581			
	574 (32)	1676.502			
	575 (32)	1679.423			
9 (2048)	= 576 (32)	1682.343			
	577 (32)	1685.264			
	578 (32)	1688.185			
	579 (32)	1691.106	1691.1	1.9/2.0	$\Xi(1690)^-$
145 (128)	= 580 (32)	1694.026	1694	6	$\Xi(1690)^-$
	581 (32)	1696.947			
	582 (32)	1699.867	1700	10	$\Xi(1690)^-$
	583 (32)	1702.788			
	584 (32)	1705.709			

Mass Spectrum of $\Xi(1820)$ Data

n	$\frac{n}{77}$	S13h	ExpMass	Error	dm	dm/Error
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This baryon will be treated in another paper, because it appears not to factor with **S7h** or **S6h**, therefore does not have **ddd** or **udd** quark content, and therefore it is not a Xi baryon as defined in this paper. It appears to have possibly **sss** or **ccc** quark content.

Mass Spectrum of $\Xi(1950)$ Data

n	$\frac{n}{81}$	S13h	ExpMass	Error	dm	dm/Error
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This baryon will be treated in another paper, because it appears not to factor with **S7h** or **S6h**, therefore does not have **ddd** or **udd** quark content, and therefore it is not a Xi baryon as defined in this paper. It appears to have possibly **sss** or **ccc** quark content.

Mass Spectrum of Xi(2030) Data (7³ divisor)

$$\text{Res} = (1/343) \mathbf{s7h} = 0.638910832 \text{ MeV}/c^2$$

n	$\frac{n}{7^3} \mathbf{S7h}$	ExpMass	Error	dm	dm/Error	
	3156	2016.403				
	3157	2017.041				
	3158	2017.680				
	3159	2018.319				
395 (8) =	3160	2018.958	2019	7	0.042	0.6%
	3161	2019.597				
	3162	2020.236				
	3163	2020.875				
	3164	2021.514				
	3165	2022.153	2022	7	0.153	2.2%
	3166	2022.792				
	3167	2023.431				
99 (32) =	3168	2024.070	2024	2	0.070	3.5%
	3169	2024.708				
	3170	2025.347				
	3171	2025.986				
	3172	2026.625				
	3173	2027.264				
	3174	2027.903				
	3175	2028.542				
397 (8) =	3176	2029.181				
	3177	2029.820				
	3178	2030.459				
	3179	2031.098				
	3180	2031.736				
	3181	2032.375				
	3182	2033.014				
	3183	2033.653				
398 (8) =	3184	2034.292				
	3185	2034.931				
	3186	2035.570				
	3187	2036.209				
	3188	2036.848				
	3189	2037.487				
	3190	2038.126				
	3191	2038.764				
399 (8) =	3192	2039.403				
	3193	2040.042				
	3194	2040.681				
	3195	2041.320				
	3196	2041.959				
	3197	2042.598				
	3198	2043.237				
	3199	2043.876				
100 (32) =	3200	2044.515	2044	8	0.515	6.4%
	3201	2045.154				
	3202	2045.792				
	3203	2046.431				
	3204	2047.070				

Mass Spectrum of Xi(2030) Data (7⁴ divisor)

$$\text{Res} = (32/2401) \quad \mathbf{s7h} = 2.920735231 \text{ MeV}/c^2$$

n	$\frac{n}{7^4}$ S7h	ExpMass	Error	dm	dm/Error
684 (32)	1997.783				
685 (32)	2000.704				
686 (32)	2003.624				
687 (32)	2006.545				
688 (32)	2009.466				
689 (32)	2012.387				
690 (32)	2015.307				
691 (32)	2018.228				
692 (32)	2021.149				
693 (32)	2024.070	2024	2	0.070	3.5%
694 (32)	2026.990				
695 (32)	2029.911	2030	10	0.089	0.9%
696 (32)	2032.832				
697 (32)	2035.752				
698 (32)	2038.673				
699 (32)	2041.594				
700 (32)	2044.515	2044	8	0.515	6.4%
701 (32)	2047.435				
702 (32)	2050.356				
703 (32)	2053.277				
11 (2048) = 704 (32)	2056.198	2058	17	1.802	3.5%
705 (32)	2059.118				
706 (32)	2062.039				
707 (32)	2064.960				
708 (32)	2067.881				
709 (32)	2070.801				
710 (32)	2073.722				
711 (32)	2076.643				
712 (32)	2079.563				
713 (32)	2082.484				
714 (32)	2085.405				
715 (32)	2088.326				
716 (32)	2091.246				
717 (32)	2094.167				
718 (32)	2097.088				
719 (32)	2100.009				
720 (32)	2102.929				
721 (32)	2105.850				
722 (32)	2108.771				
723 (32)	2111.692				
724 (32)	2114.612				

Mass Spectrum of Xi(2250) Data

$$\text{Res} = (32/2401) \quad \mathbf{s7h} = 2.920735231 \text{ MeV}/c^2$$

n	$\frac{\mathbf{n} \cdot \mathbf{s7h}}{7^4}$	ExpMass	Error	dm	dm/Error
748 (32)	2184.710				
749 (32)	2187.631				
749.5 (32)	2189.091	2189	7	0.091	1.3%
750 (32)	2190.551				
751 (32)	2193.472				
752 (32)	2196.393				
753 (32)	2199.314				
754 (32)	2202.234				
755 (32)	2205.155				
756 (32)	2208.076				
757 (32)	2210.997				
758 (32)	2213.917	2214	5	0.083	1.7%
759 (32)	2216.838				
760 (32)	2219.759				
761 (32)	2222.680				
762 (32)	2225.600				
763 (32)	2228.521				
764 (32)	2231.442				
765 (32)	2234.362				
766 (32)	2237.283				
767 (32)	2240.204				
12 (2048) = 768 (32)	2243.125	2244	52	0.875	1.7%
769 (32)	2246.045				
770 (32)	2248.966				
771 (32)	2251.887				
772 (32)	2254.808				
773 (32)	2257.728				
774 (32)	2260.649				
775 (32)	2263.570				
776 (32)	2266.491				
777 (32)	2269.411				
778 (32)	2272.332				
779 (32)	2275.253				
780 (32)	2278.173				
781 (32)	2281.094				
782 (32)	2284.015				
783 (32)	2286.936				
784 (32)	2289.856				
785 (32)	2292.777				
786 (32)	2295.698	2295	15	0.698	4.6%
787 (32)	2298.619				
788 (32)	2301.539				
789 (32)	2304.460				
790 (32)	2307.381				

Mass Spectrum of Xi(2500) Data

$$\text{Res} = (32/2401) \quad \mathbf{s7h} = 2.920735231 \text{ MeV}/c^2$$

n	$\frac{n}{7^4}$ S7h	ExpMass	Error	dm	dm/Error
814 (32)	2377.478				
815 (32)	2380.399				
816 (32)	2383.320				
817 (32)	2386.241				
818 (32)	2389.161				
819 (32)	2392.082				
820 (32)	2395.003				
821 (32)	2397.924				
822 (32)	2400.844				
823 (32)	2403.765				
824 (32)	2406.686				
825 (32)	2409.607				
826 (32)	2412.527				
827 (32)	2415.448				
828 (32)	2418.369				
829 (32)	2421.290				
830 (32)	2424.210				
831 (32)	2427.131				
13 (2048) =	832 (32) 2430.052	2430	20	0.052	0.3%
	833 (32) 2432.972				
	834 (32) 2435.893				
	835 (32) 2438.814				
	836 (32) 2441.735				
	837 (32) 2444.655				
	838 (32) 2447.576				
	839 (32) 2450.497				
	840 (32) 2453.418				
	841 (32) 2456.338				
	842 (32) 2459.259				
	843 (32) 2462.180				
	844 (32) 2465.101				
	845 (32) 2468.021				
	846 (32) 2470.942				
	847 (32) 2473.863				
	848 (32) 2476.783				
	849 (32) 2479.704				
	850 (32) 2482.625				
	851 (32) 2485.546				
	852 (32) 2488.466				
	853 (32) 2491.387				
	854 (32) 2494.308				
	855 (32) 2497.228				
	856 (32) 2500.149	2500	10	0.149	1.5%
	857 (32) 2503.070				
	858 (32) 2505.990	2505	10	0.990	9.9%
	859 (32) 2508.911				
	860 (32) 2511.832				

6. Large Block Mass Spectrum of the Xi Baryons

$$\text{Res} = (2048 / 7^4) S7h = 186.9270548 \text{ MeV}/c^2$$

Large Block	n	$\frac{n}{7^4} S7h$	ExpMass	Error	Baryon
9 (2048)	= 143 (128)	1670.660	1694	6	Ξ(1690)
	= 144 (128)	1682.343			
	= 145 (128)	1694.026			
10 (2048)	= 159 (128)	1857.588	1870	9	Ξ(1820)
	= 160 (128)	1869.271			
	= 161 (128)	1880.953			
11 (2048)	= 175 (128)	2044.515	2044	8	Ξ(2030)
	= 176 (128)	2056.198			
	= 177 (128)	2067.881			
12 (2048)	= 191 (128)	2231.442	2244	52	Ξ(2250)
	= 192 (128)	2243.125			
	= 193 (128)	2254.808			
13 (2048)	= 207 (128)	2418.369	2430	20	Ξ(2500)
	= 208 (128)	2430.052			
	= 209 (128)	2441.735			

Commentary

Four Xi baryons have experimental masses that fall exactly on large factor blocks. One Xi baryon has mass data that falls *very close* to a large factor block - the Xi(1690).

7. Summary

The first four Xi baryons - Ξ^0 , Ξ^- , $\Xi(1530)^0$, and $\Xi(1530)^-$ - all seem to factor with power of two denominator fractions. They might have quark content **udd** instead of **ddd**, because all the N and Δ baryons, which have quark content **udd**, factor that way also. The rest of the Xi baryons seem to factor with power of seven denominator fractions times S7h, so they probably have quark content **ddd**.

The $\Xi(1820)$ and $\Xi(1950)$ don't seem to factor with S7h or S6h. They seem to factor with S13h. If that's the case, that would mean they have a quark content of **sss** or **ccc**, so they are not treated in this paper.

Two other Xi baryons - the $\Xi(2120)$ and $\Xi(2370)$ - are also not treated. More experimental mass data of good accuracy is needed to reach a firm conclusion about how they factor.

8. References

[1] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)