The Structure of The Proton and the Calculation of Its G-factor

(Some Recent Results from Ether Electrodynamics)

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Abstract: Some recent results of a more rigorous electrodynamics than Special Relativity or Quantum Electrodynamics are summarized with some pedagogy. The results include the correct explanation of line radiation, the correct interpretation of the g-factor, the introduction of the precessional mass, the dismissal of the half-quantum, the deduction of the proton structure from first principles, and a ”T-shirt” calculation of the proton g-factor.

1 General Background

Upon examination of the modern physics literature relating to g-factors of certain particles such as the electron, one notices certain discrepancies. For the electron, a calculation using the current-loop model of magnetic moment results in,

$$\mu = IA = \frac{e}{2\pi r} v\pi r^2 = \frac{evr}{2} = \frac{evr}{2}\frac{m_e}{m_e} = \frac{em_e vr}{2m_e} = \frac{eL}{2m_e} \tag{1}$$

where I is the current, A is the loop area, e is the electronic charge, v is the equatorial velocity of rotation of the electron, r is the radius of the electron, m_e is the mass of the electron, and L is the spin angular momentum of the electron. This, when inserted into Larmor’s precession equation for μ,

$$\Omega = \frac{\vec{\tau}}{L\sin\theta} = \frac{\vec{\mu}\times\vec{B}}{L\sin\theta} = \frac{|\vec{\mu}||\vec{B}|\sin\theta}{L\sin\theta} = \frac{\mu B}{L} \tag{2}$$

where Ω is the angular precession frequency, $\vec{\tau}$ is the torque vector, and $\vec{B}$ is the magnetic field vector, gives,

$$\Omega = \frac{eB}{2m_e}. \tag{3}$$

Early actual measurements of the spin g-factor of the free electron, g_e, seemed to indicate the value was about 2. As well, the Dirac equation gave a value of exactly 2. In other words the electron seemed to precess about twice as fast as (3) indicates.[1]

Now we will examine the (g-2) experiments. In Combley[2] Eq. (3.1) we have the cyclotron frequency for a non-relativistic particle (all equations will be converted to SI units and the particle treated in Combley will be changed from muon to electron),

$$\vec{\omega_c} = \frac{e\vec{B}}{m_e}, \tag{4}$$

while Combley (3.2) becomes,

$$\vec{\omega_L} = \frac{2\mu\vec{B}}{\hbar} = g\left(\frac{e\vec{B}}{2m_e}\right) = (1 + a_e)\left(\frac{e\vec{B}}{m_e}\right), \tag{5}$$

where $\vec{\omega_L}$ is the Larmor frequency, the spin angular momentum of the electron is $\hbar/2$, and a_e is the electron anomaly. Combley (3.3) defines the anomalous

[1] "...Kronig immediately thought of a self-rotating electron, i.e., an electron rotating about its own axis with an angular momentum of self-rotation of 1/2 and a g-factor of $g_0 = 2$." Sin-itiro Tomonaga. Translated by Takeshi Oka. *The Story of Spin.* U. Chicago P., Chicago. 1997. see pg. 33.

[2] Combley, F. et. al., The CERN muon (g-2) Experiments, CERN-EP/80-96, 9 June 1980, `https://lib-extopc.kek.jp/preprints/PDF/1980/8006/8006282.pdf`

precession frequency $\vec{\omega_a}$ in terms of the spin-precession (Larmor) frequency,

$$\vec{\omega_a} = \vec{\omega_L} - \vec{\omega_c} = a_e \left(\frac{e}{m} \right) \vec{B}. \tag{6}$$

At relativistic speeds (the speeds in the g-2 experiment) we must use Combley (3.4) for the cyclotron frequency,

$$\vec{\omega_c} = \frac{e\vec{B}}{\gamma m_e}, \tag{7}$$

where γ is the relativistic factor,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{8}$$

Next, we introduce the Thomas Precession factor $\vec{\omega_T}$, Combley (3.5),

$$\vec{\omega_T} = \left(1 - \frac{1}{\gamma} \right) \frac{e\vec{B}}{m} \tag{9}$$

The Thomas Precession is the rotation frequency of the *frame* of the particle and is in the opposite direction to the precession of the particle as in Combley (3.6),

$$\vec{\omega_S} = \vec{\omega_L} - \vec{\omega_T} = (1 + a_e) \left(\frac{e\vec{B}}{m_e} \right) - \left(1 - \frac{1}{\gamma} \right) \left(\frac{e\vec{B}}{m_e} \right) = \left(a_e + \frac{1}{\gamma} \right) \left(\frac{e\vec{B}}{m_e} \right). \tag{10}$$

There are several problems with (10):

1. **Philosophical error.** The Thomas Precession is Special Relativistic and violates the laws of physics, in particular the conservation of energy.

This is caused by time dilation which is a time transformation. The energy of periodic emissions in one frame does not match the energy of periodic emissions in the other due to a frequency transformation.[3]

2. **Philosophical error.** The Thomas Precession violates mathematical law, specifically the transitive property of arithmetic. For example take an observer in frame $\mathcal{O}$ and a particle in frame $\mathcal{P}$ which is moving relatively to $\mathcal{O}$ at velocity $c/2$. The Thomas Precession $\vec{\omega_T}(\mathcal{OP}) = \frac{2-\sqrt{3}}{2}\frac{e\vec{B}}{m_e}$ by (8) and (9). Now take another frame $\mathcal{P}'$ moving relatively to $\mathcal{P}$ in the same direction from $\mathcal{O}$ at velocity $c/2$. The Thomas Precession $\vec{\omega_T}(\mathcal{PP}')$ is also $\frac{2-\sqrt{3}}{2}\frac{e\vec{B}}{m_e}$. The Thomas Precession of $\mathcal{OP} + \mathcal{PP}' = 0.267949\ \frac{e\vec{B}}{m_e}$. If we calculate the Thomas Precession for $\mathcal{OP}'$ which, according to the velocity addition equation has a composed velocity of $\sqrt{1 - 6.17284 \times 10^{-35}}\ c$ and which has a gamma factor 1.27279×10^{17} we obtain a Thomas Precession $\vec{\omega_T}(\mathcal{PP}')$ nearly equal to $\frac{e\vec{B}}{m_e}$. Intuition dictates against this.

3. **Logic error.** The magnetic field is not transformed Relativistically from the lab frame to the particle frame as it should be,[4]

$$B'_\perp = \gamma(B_\perp - \frac{1}{c^2}\ v \times E) \qquad (11)$$

[3] In Joseph Catania, *Ether Electrodynamics & Other Modern Lectures in Classical Physics, Unpublished* line radiation from precessing particles is established.

[4] Wikipedia. 2022. Classical electromagnetism and special relativity. Last edited on 26 January 2022, at 17:02 (UTC). https://en.wikipedia.org/wiki/Classical_electromagnetism_and_special_relativity.

where E is the electric field, which in the particle frame is zero, so:

$$B'_{\perp} = \gamma B_{\perp}. \tag{12}$$

> Since γ in one version of (g-2) is 29.3 this would greatly affect the Larmor precession frequency so it would appear that Special Relativity does not apply.[5]

Upon examining (4) and (5) the quantities e, B, and m_e appear but μ (the magnetic moment) does not appear because it is canceled from (2). Thus, the anomaly can only be due to e, B, and/or m_e and since e and B do not show anomalies the anomaly must be in m_e. Therefore the epithet ”anomalous magnetic moment” should be viewed as a misnomer. Thus, also, all attempts to relegate the g-factor to various distributions of mass vs. charge over the surface of the electron are completely without basis.

2 Line Radiation & the Electron G-factor

It is well known that Nuclear Magnetic Resonance (NMR) radiation is line radiation caused by precession of the nuclear spin axis in a homogeneous applied magnetic field. The angle of the nuclear magnetic moment vector $\vec{\mu}$ is immaterial to the precession frequency Ω as seen in (2) due to cancellation of the angular information θ. In the case of magnetogyric (also called gyromagnetic) radiation, electrons travel helically down magnetic field lines producing synchrotron radiation with superimposed line radiation. The

[5] see Combley Eq. (7.3)

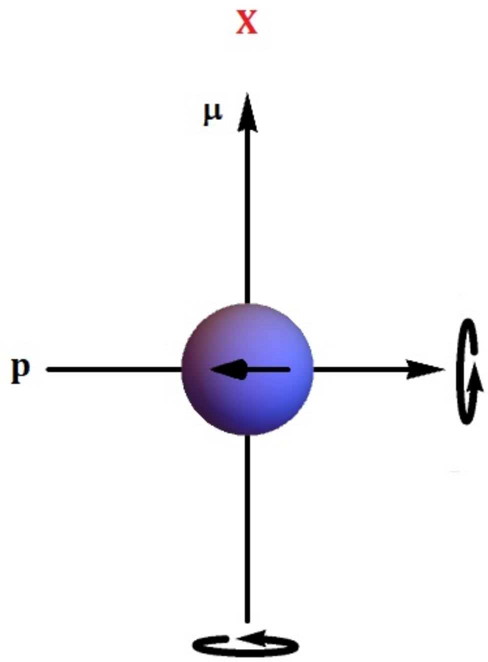

Figure 1: **The electron.** μ labels the magnetic moment vector (the spin angular momentum vector would be in the opposite direction since the electron is a negatively charged particle). p labels the precession axis. The arrow on the body of the electron labels the spin direction. The red X denotes the position of an observer looking down on the electron.

motion of the electrons can be decomposed into a uniform translation and a circular motion. The translation produces no radiation while the circular motion produce synchrotron (broadband) radiation. The line radiation can be deduced to be caused by the precession of the electron spin axis around the magnetic field lines. From the above we can deduce that *atomic line radiation* is also caused by the spin axis precession of charged particles.

For a spinning particle it is clear that the spin velocity gets higher as one approaches the equator. With reference to Fig. 1, an observer looking down from the red X at the top of the electron, when the pole of the spin axis is directly beneath him, will see a surface velocity of zero for the point on the electron directly beneath him (in this case the north spin-pole). As the electron precesses, i.e. rotates around the perpendicular precession axis, the surface velocity observed increases sinusoidally until the spin-equator of the electron is beneath him at which time it is a maximum. Upon further rotation the velocity decreases until the south spin-pole is beneath the observer and the velocity is again zero. The sinusoidal velocity effect holds true, not only for the great circle containing both poles but for any circle coaxial with the precession axis which lies on the surface of the electron. This sinusoidal velocity will create a temporally and spatially sinusoidal and radially inverse-square law electric field due to an effect called Source Drag.[6] It is also true that the radiation not in the plane of the great circle containing the poles will be circularly polarized as it is in atomic line radiation.

The cause of line radiation given by Bohr (an electron jumping instantaneously from one Bohr orbital to another which then emits radiation with a frequency equal to $\frac{\Delta E}{h}$) should be rejected. Certainly electromagnetic radiation has been indisputably shown to have wave nature as Bohr himself believed.[7] Therefore the energy of a wave would need to be a function of the intensity and time of emission and would be emitted continuously in the case of line radiation.

As for the g-factor of the electron it can be shown that the potential energy of the electric field of the electron correlates to the inertia or

[6]see Joseph Catania, *Ether Electrodynamics & Other Modern Lectures in Classical Physics, Unpublished* for a full treatment.

[7]"...[Bohr] denied equal status to the wave and particle pictures, stressing the primacy of the classical wave picture of light and of the classical particle picture of the electron." Stachel J. (2009) Bohr and the Photon. In: Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle. The Western Ontario Series in Philosophy of Science, vol 73. Springer, Dordrecht. `https://doi.org/10.1007/978-1-4020-9107-0_5`

mass.[7] Thus, if the translational mass of a stationary electron is m_e the precessional mass will be $\frac{m_e}{2}$ due to the fact that for a stationary electron the electric field is given, in essence, by the inverse-square law field but for the precessing electron the inverse-square law field is *sinusoidally modulated.* If one integrates the energy density (sine-squared) function over a quarter-circle one gets:

$$\int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}, \tag{13}$$

whereas the *average* of this over the interval $[0, \frac{\pi}{2}]$ is:

$$\frac{\frac{\pi}{4}}{\frac{\pi}{2}} = \frac{1}{2}, \tag{14}$$

(as is well known from electronic engineering). If the average field of a precessing electron is one-half the field of a stationary one the mass will be one-half.[8] (It appears this may also be looked at from the torque point of view.) Revisiting (5), repeated here for convenience, we have,

$$\vec{\omega_L} = \frac{2\mu\vec{B}}{\hbar} = g\left(\frac{e\vec{B}}{2m_e}\right) = (1+a_e)\left(\frac{e\vec{B}}{m_e}\right). \tag{15}$$

When $g = 2$ the central term is,

$$\vec{\omega_L} = g\left(\frac{e\vec{B}}{2m_e}\right) = 2\left(\frac{e\vec{B}}{2m_e}\right) = \left(\frac{e\vec{B}}{2\frac{m_e}{2}}\right), \tag{16}$$

which is equivalent (rightmost term) to what we obtain by substituting the precessional mass, $\frac{m_e}{2}$, for the translational mass, m_e, without using the g-

[7] *Ether Electrodynamics & Other Modern Lectures in Classical Physics, Unpublished*
[8] Ibid.

factor. Thus the g-factor can be seen as related to the mass or inertia of the particle and is no longer necessary.

Another place where the precessional mass comes in handy is in explaining the spin angular momentum of the electron,

$$\frac{\hbar}{2} = m_e v r. \tag{17}$$

The appearance of $\frac{\hbar}{2}$ is odd because it seems as if we have the unlikely situation of half of a quantum.[9] But this is easily resolved upon replacement of m_e with $\frac{m_e}{2}$, the denominator of 2 really belonging to the mass, not the quantum. It also solves the problem of the two-valuedness of the eigenvalues of the z-component of the spin operator, S_z, with eigenvalues $\pm\frac{\hbar}{2}$ where its eigenfunction,

$$S_z = e^{\pm i\frac{\phi}{2}} \tag{18}$$

does not come back to its original value after rotation by 360° but requires 720°.[10] If we interpret the g-factor as related to the mass instead of the spin the problems related to the spin g-factor as compared to the orbital g-factor (which is unity) disappear.

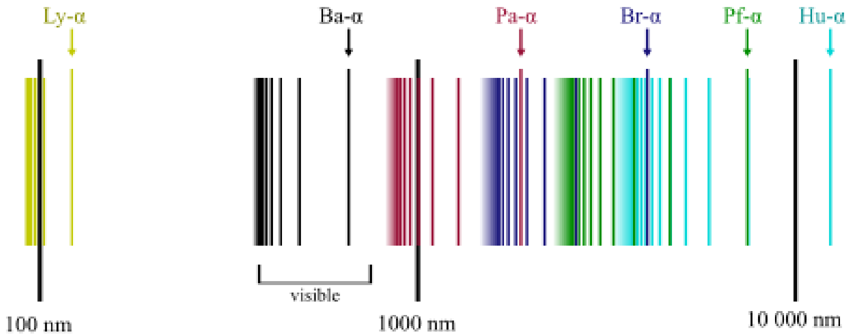

Figure 2: Hydrogen spectrum. Hydrogen spectrum.svg, created by OrangeDog under CC BY-SA 3.0, no modifications made, `https://creativecommons.org/licenses/by-sa/3.0/` structure

3 The Structure of the Proton

We start by examining the line spectrum of hydrogen (see Fig. 2) which consists of a set of discrete frequencies. We determined in **§2** that atomic line radiation is caused by the precession of the spin axis of charged particles. Since there are two particles in atomic hydrogen (or rather there are two particles involved in creating the emission spectrum[11] of hydrogen—the proton and the electron), we might expect two different spectra. The second spectrum should look like the first, only frequency shifted, relative to the first, because we expect the proton to precess at a different Larmor frequency than the electron. Turning to the Larmor precession equation, which we repeat

[9] "Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton." Dirac, Paul Adrien Maurice. 1928. The quantum theory of the electron. Proc. R. Soc. Lond. A 117:610–624.`http://doi.org/10.1098/rspa.1928.0023`

[10] Sin-itiro Tomonaga. Translated by Takeshi Oka. *The Story of Spin.* U. Chicago P., Chicago. 1997. see pp. 46-48.

[11] Wikipedia.2022. *Emission Spectrum.* Last edited on 29 January 2022, at 10:31 (UTC).`https://en.wikipedia.org/wiki/Emission_spectrum#cite_note-1`

here with augmentation,

$$\Omega_e = \frac{\vec{\tau}}{L_e \sin\theta} = \frac{\vec{\mu_e} \times \vec{B_p}}{L_e \sin\theta} = \frac{|\vec{\mu_e}||\vec{B_p}|}{L_e} = \frac{\mu_e B_p}{L_e} \approx \frac{\mu_e \frac{2\mu_p}{r^3}}{L_e}, \tag{19}$$

where the subscript index e indicates electron, the subscript index p indicates proton and in the last term the magnetic field has been expanded to $\approx \frac{2\mu_p}{r^3}$. Eq. (19) gives the Larmor frequency of an electron with magnetic moment μ_e in the magnetic field of the proton B_p. We now give a similar equation for the Larmor frequency of the proton in the magnetic field of the electron,

$$\Omega_p = \frac{\vec{\tau}}{L_p \sin\theta} = \frac{\vec{\mu_p} \times \vec{B_e}}{L_p \sin\theta} = \frac{|\vec{\mu_p}||\vec{B_e}|}{L_p} = \frac{\mu_p B_e}{L_p} \approx \frac{\mu_p \frac{2\mu_e}{r^3}}{L_p}. \tag{20}$$

We compare the last terms of (19) and (20):

$$\frac{\mu_e \frac{2\mu_p}{r^3}}{L_e}, \frac{\mu_p \frac{2\mu_e}{r^3}}{L_p}. \tag{21}$$

Note that the numerators in (21) are equal by the commutativity of multiplication while the denominators are not equal. L_e and L_p are the spin angular momenta of the electron and the proton respectively and since they are two totally different particles we expect their values to be different, but the fact that the spectrum is single tends to argue against a difference in the spin angular momenta. Looking up some constants (all units SI) we have for the electron magnetic moment[12] 9.284764×10^{-24} J/T, proton

[12]Wikipedia. 2022. *Electron magnetic moment*. Last edited on 24 February 2022, at 07:39 (UTC). `https://en.wikipedia.org/wiki/Electron_magnetic_moment`

magnetic moment[13] 4.410606×10^{-26} J/T, electron g-factor 2.00, proton g-factor[14] 5.5857, proton-to-electron mass ratio[15] 1836.152 . From (20),

$$\Omega_p = g_p \frac{eB_e}{2m_p}; \qquad \Omega_e = g_e \frac{eB_p}{2m_e}; \qquad \frac{\Omega_p}{\Omega_e} = \frac{g_p}{g_e}\frac{B_e}{B_p}\frac{m_e}{m_p} = 1, \tag{22}$$

since

$$\frac{\mu_e}{\mu_p} = 658.21, \quad \frac{g_p}{g_e} = 2.79285, \quad \frac{m_e}{m_p} = \frac{1}{1836.152}, \tag{23}$$

with $B_p \propto \mu_p$ and $B_e \propto \mu_e$.

The last equation of (22) shows that the proton and electron have the same precession frequency inside the hydrogen atom. If the proton precesses at the same frequency as the electron it must be made of particles that precess at that frequency, namely the electron or positron. Since the charge of the proton is +1 the simplest composition would be two positrons and one electron.

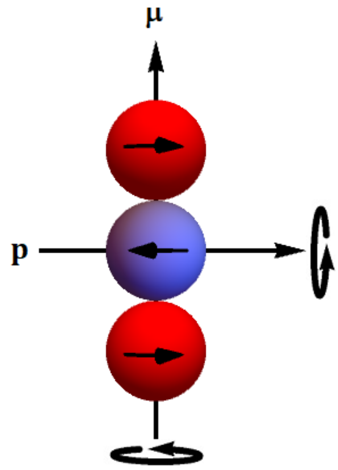

Figure 3: The proton structure. The positrons (red) spin opposite to the electron (blue).

4 The G-factor of the Proton

(Note: Due to an error, this section and the pertinent calculation are being revamped although it seems the coincidence with the proton g-factor cannot be accidental.)

[13] Wikipedia. 2022. *Proton magnetic moment.* Last edited on 19 October 2021, at 13:26 (UTC). `https://en.wikipedia.org/wiki/Proton_magnetic_moment`

[14] Ibid

[15] Wikipedia. 2022. *Proton-to-electron mass ratio.* Last edited on 31 October 2021, at 14:21 (UTC). `https://en.wikipedia.org/wiki/Proton-to-electron_mass_ratio`

With a calculation similar to that performed for the g-factor of the electron we will attempt to understand the g-factor of the proton. We need to understand how the field lines surrounding the proton interact with the proton. We concentrate on a surface-octant of the topmost positron in the proton as seen in Fig. 4.

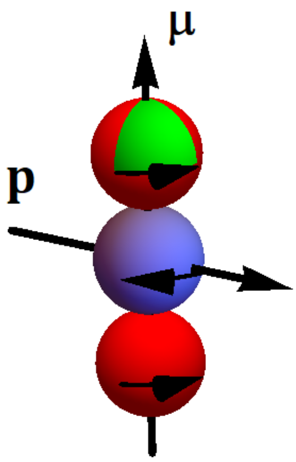

Figure 4: A proton surface-octant. The green area has intervals $\phi = [0, \frac{\pi}{2}], \theta = [0, \frac{\pi}{2}]$.

The velocities of the points in the octant around the precession axis, p, are given by $k\sqrt{(2+\sin^{-1}x\cos\phi)^2+x^2}$, i.e. k times the radius from p, where k is a proportionality constant which can be set equal to unity for particles of radius unity, x is distance from the center of the precession axis along the precession axis and ϕ is the angle from the top of the circles coaxial to p which lie on the surface of the particle. The angle the Galilean transformed incoming field lines make with respect to the surface tangents of the surface-octant is given by,

$$\tan^{-1}\frac{x}{2+\sin^{-1}x\cos\phi} - \tan^{-1}\frac{\sin^{-1}x\cos\phi}{x}. \tag{24}$$

The projection of these lines, one set upon the other, when integrated over an octant give,

$$\int_0^1 \int_0^{\frac{\pi}{2}} \cos\left(\tan^{-1}\frac{x}{2+\sin^{-1}x\cos\phi} - \tan^{-1}\frac{\sin^{-1}x\cos\phi}{x}\right) d\phi\, dx = 1.397036 \tag{25}$$

which, when multiplied by 4 gives,

$$\frac{1.397036 \times 4}{g_p} = \frac{5.58814}{5.5857} = 1.00044, \tag{26}$$

where g_p is the g-factor of the proton and which agrees numerically rather precisely. A factor of two is explained by the sine-squared nature of the electric field (a comparison with the electron's g-factor of 2), while the other factor of two awaits clarification.