On some Ramanujan's continued fractions: mathematical connections with MRB Constant, Higher Spin and various sectors of String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper, we analyze further Ramanujan's continued fractions. We describe the mathematical connections with MRB Constant, Higher Spin and various sectors of String Theory

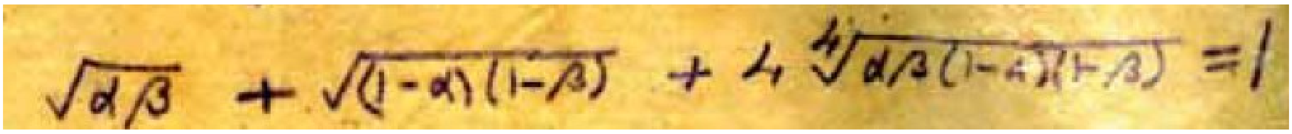
1[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

From:

Manuscript Book I - *Srinivasa Ramanujan*

We have:

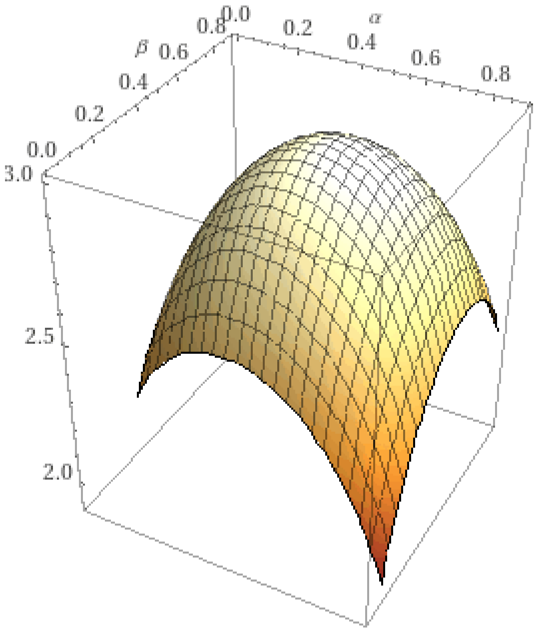


sqrt(αβ) + sqrt((1-α)(1-β)) + 4*(αβ(1-α)(1-β))^(1/4)

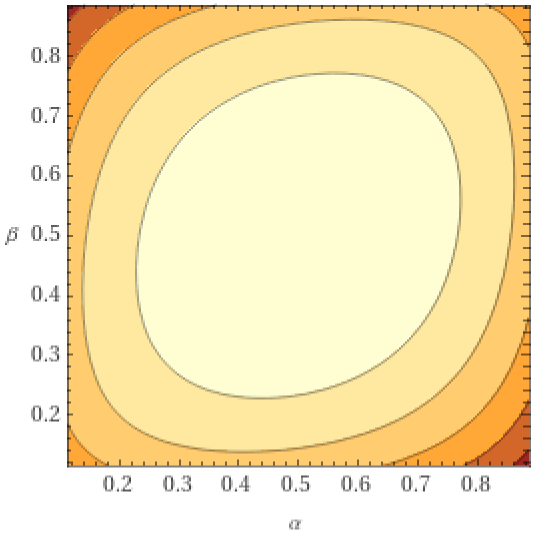
Input

$$\sqrt{\alpha\,\beta} + \sqrt{(1-\alpha)\,(1-\beta)} + 4\sqrt[4]{\alpha\,\beta\,(1-\alpha)\,(1-\beta)}$$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Alternate forms

$$\sqrt{(\alpha-1)(\beta-1)} + \sqrt{\alpha\,\beta} + 4\sqrt[4]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}$$

$$4\sqrt[4]{(\alpha-1)\,\alpha\left(\beta^2-\beta\right)} + \sqrt{(\alpha-1)(\beta-1)} + \sqrt{\alpha\,\beta}$$

Alternate form assuming α and β are positive

$$\sqrt{\alpha\,\beta} + \sqrt{1-\alpha}\,\sqrt{1-\beta}\,(-1)^{\lfloor-(\arg(1-\alpha)+\arg(1-\beta)-\pi)/(2\pi)\rfloor} + 4\sqrt[4]{1-\alpha}\,\sqrt[4]{1-\beta}\,\sqrt[4]{\alpha\,\beta}\,i^{\lfloor-(\arg(1-\alpha)+\arg(1-\beta)-\pi)/(2\pi)\rfloor}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Real roots

$\alpha = 0, \quad \beta = 1$

$\alpha = 1, \quad \beta = 0$

Integer roots

$\alpha = 0\,, \quad \beta = 1$

$\alpha = 1\,, \quad \beta = 0$

Roots for the variable β

$$\beta = \frac{96\,\alpha^2 - 56\sqrt{3}\,\sqrt{\alpha^4 - 2\,\alpha^3 + \alpha^2} - 95\,\alpha - 1}{192\,\alpha^2 - 192\,\alpha - 1}$$

$$\beta = \frac{96\,\alpha^2 + 56\sqrt{3}\,\sqrt{\alpha^4 - 2\,\alpha^3 + \alpha^2} - 95\,\alpha - 1}{192\,\alpha^2 - 192\,\alpha - 1}$$

$$\beta = \frac{160\,\alpha^2 - 72\sqrt{5}\,\sqrt{\alpha^4 - 2\,\alpha^3 + \alpha^2} - 159\,\alpha - 1}{320\,\alpha^2 - 320\,\alpha - 1}$$

$$\beta = \frac{160\,\alpha^2 + 72\sqrt{5}\,\sqrt{\alpha^4 - 2\,\alpha^3 + \alpha^2} - 159\,\alpha - 1}{320\,\alpha^2 - 320\,\alpha - 1}$$

Series expansion at α=0

$$\sqrt{1-\beta} + 4\sqrt[4]{\alpha}\,\sqrt[4]{-(\beta-1)\,\beta} + \sqrt{\alpha}\,\sqrt{\beta} - \frac{1}{2}\,\alpha\,\sqrt{1-\beta} - \alpha^{5/4}\,\sqrt[4]{-(\beta-1)\,\beta} - \frac{1}{8}\,\alpha^2\,\sqrt{1-\beta} - \frac{3}{8}\,\alpha^{9/4}\,\sqrt[4]{-(\beta-1)\,\beta} - \frac{1}{16}\,\alpha^3\,\sqrt{1-\beta} - \frac{7}{32}\,\alpha^{13/4}\,\sqrt[4]{-(\beta-1)\,\beta} - \frac{5}{128}\,\alpha^4\,\sqrt{1-\beta} - \frac{77}{512}\,\alpha^{17/4}\,\sqrt[4]{-(\beta-1)\,\beta} + O\left(\alpha^5\right)$$

(Puiseux series)

Series expansion at α=∞

$$\frac{\sqrt{\alpha}\left(4\sqrt[4]{\alpha^2(\beta-1)\beta}+\sqrt{\alpha(\beta-1)}+\sqrt{\alpha\beta}\right)}{\sqrt{\alpha}}+\frac{\sqrt{\frac{1}{\alpha}}\left(-2\sqrt[4]{\alpha^2(\beta-1)\beta}-\sqrt{\alpha(\beta-1)}\right)}{2\sqrt{\alpha}}+\frac{\left(\frac{1}{\alpha}\right)^{3/2}\left(-3\sqrt[4]{\alpha^2(\beta-1)\beta}-\sqrt{\alpha(\beta-1)}\right)}{8\sqrt{\alpha}}+\frac{\left(\frac{1}{\alpha}\right)^{5/2}\left(-7\sqrt[4]{\alpha^2(\beta-1)\beta}-2\sqrt{\alpha(\beta-1)}\right)}{32\sqrt{\alpha}}+O\left(\left(\frac{1}{\alpha}\right)^3\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(\sqrt{\alpha\beta}+\sqrt{(1-\alpha)(1-\beta)}+4\sqrt[4]{\alpha\beta(1-\alpha)(1-\beta)}\right)=\frac{1}{2}\left(\frac{\beta-1}{\sqrt{(\alpha-1)(\beta-1)}}+\frac{2(2\alpha-1)\beta(\beta-1)}{((\alpha-1)\alpha(\beta-1)\beta)^{3/4}}+\frac{\beta}{\sqrt{\alpha\beta}}\right)$$

Indefinite integral

$$\int\left(\sqrt{\alpha\beta}+\sqrt{(1-\alpha)(1-\beta)}+4\sqrt[4]{\alpha\beta(1-\alpha)(1-\beta)}\right)d\alpha=\frac{2}{15}\left(5(\alpha-1)\sqrt{(\alpha-1)(\beta-1)}+5\alpha\sqrt{\alpha\beta}+\frac{24(\alpha-1)\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};1-\alpha\right)}{\sqrt[4]{\alpha}}\right)+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

Local maximum

$$\max\left\{\sqrt{\alpha\,\beta} + \sqrt{(1-\alpha)(1-\beta)} + 4\sqrt[4]{\alpha\,\beta\,(1-\alpha)(1-\beta)}\right\} = 3 \text{ at } (\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

From:

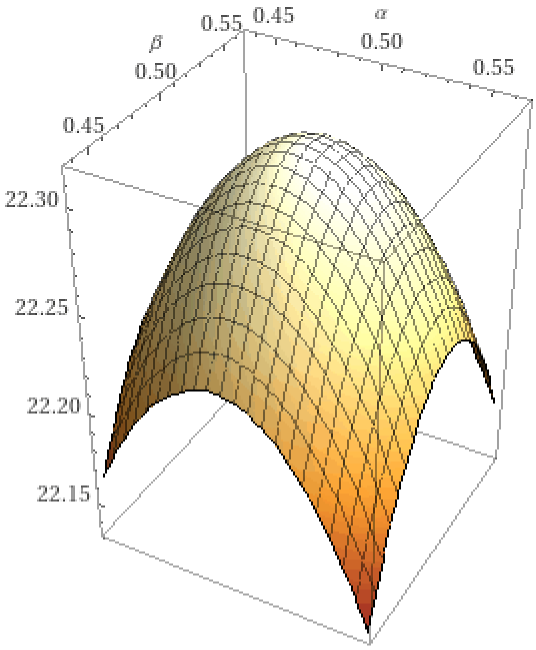
$$\sqrt{\alpha\beta} + \sqrt{(1-\alpha)(1-\beta)} + 20\sqrt[4]{\alpha\beta(1-\alpha)(1-\beta)} + 8\sqrt{2}\sqrt[8]{\alpha\beta(1-\alpha)(1-\beta)}\left\{\sqrt[4]{\alpha\beta} + \sqrt[4]{(1-\alpha)(1-\beta)}\right\} = 1$$

sqrt(αβ) + sqrt((1-α)(1-β)) + 20*(αβ(1-α)(1-β))^(1/4) + 8sqrt2 * (αβ(1-α)(1-β))^(1/8) * [(αβ)^(1/4)+((1-α)(1-β))^(1/4)]

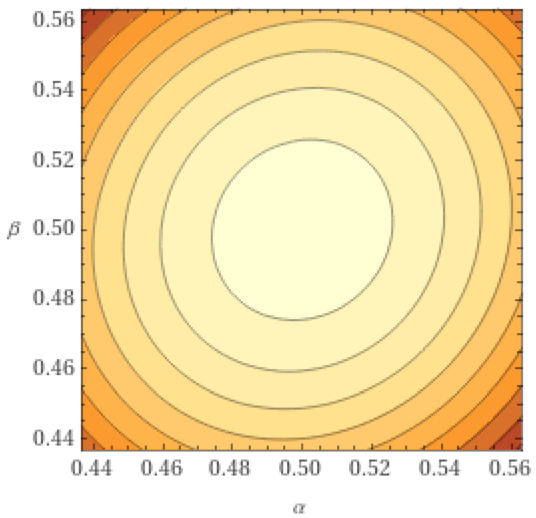
Input

$$\sqrt{\alpha\,\beta} + \sqrt{(1-\alpha)(1-\beta)} + 20\sqrt[4]{\alpha\,\beta\,(1-\alpha)(1-\beta)} + 8\sqrt{2}\,\sqrt[8]{\alpha\,\beta\,(1-\alpha)(1-\beta)}\left(\sqrt[4]{\alpha\,\beta} + \sqrt[4]{(1-\alpha)(1-\beta)}\right)$$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Alternate forms

$$8\sqrt{2}\,\sqrt[8]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\left(\sqrt[4]{(\alpha-1)(\beta-1)}+\sqrt[4]{\alpha\,\beta}\right)+\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\,\beta}+20\sqrt[4]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}$$

$$8\sqrt{2}\sqrt[8]{(\alpha-1)\,\alpha\left(\beta^2-\beta\right)}\sqrt[4]{(\alpha-1)(\beta-1)}+20\sqrt[4]{(\alpha-1)\,\alpha\left(\beta^2-\beta\right)}+ 8\sqrt{2}\sqrt[4]{\alpha\,\beta}\sqrt[8]{(\alpha-1)\,\alpha\left(\beta^2-\beta\right)}+\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\,\beta}$$

Expanded forms

$$8\sqrt{2}\sqrt[8]{\alpha^2\,\beta^2-\alpha^2\,\beta-\alpha\,\beta^2+\alpha\,\beta}\sqrt[4]{\alpha\,\beta}+20\sqrt[4]{\alpha^2\,\beta^2-\alpha^2\,\beta-\alpha\,\beta^2+\alpha\,\beta}+ 8\sqrt{2}\sqrt[4]{\alpha\,\beta-\alpha-\beta+1}\sqrt[8]{\alpha^2\,\beta^2-\alpha^2\,\beta-\alpha\,\beta^2+\alpha\,\beta}+\sqrt{\alpha\,\beta}+\sqrt{\alpha\,\beta-\alpha-\beta+1}$$

$$\sqrt{(1-\alpha)(1-\beta)}+8\sqrt{2}\sqrt[8]{(1-\alpha)\,\alpha\,(1-\beta)\,\beta}\sqrt[4]{(1-\alpha)(1-\beta)}+ \sqrt{\alpha\,\beta}+20\sqrt[4]{(1-\alpha)\,\alpha\,(1-\beta)\,\beta}+8\sqrt{2}\sqrt[4]{\alpha\,\beta}\sqrt[8]{(1-\alpha)\,\alpha\,(1-\beta)\,\beta}$$

Alternate form assuming α and β are positive

$$\sqrt{\alpha\,\beta}+\sqrt{1-\alpha}\sqrt{1-\beta}\,(-1)^{\lfloor-(\arg(1-\alpha)+\arg(1-\beta)-\pi)/(2\pi)\rfloor}+ 8\sqrt{2}\sqrt[8]{1-\alpha}\sqrt[8]{1-\beta}\sqrt[8]{\alpha\,\beta}\,e^{\frac{1}{4}i\pi\left\lfloor-\frac{\arg(1-\alpha)+\arg(1-\beta)-\pi}{2\pi}\right\rfloor} \left(\sqrt[4]{\alpha\,\beta}+\sqrt[4]{1-\alpha}\sqrt[4]{1-\beta}\,i^{\lfloor-(\arg(1-\alpha)+\arg(1-\beta)-\pi)/(2\pi)\rfloor}\right)+ 20\sqrt[4]{1-\alpha}\sqrt[4]{1-\beta}\sqrt[4]{\alpha\,\beta}\,i^{\lfloor-(\arg(1-\alpha)+\arg(1-\beta)-\pi)/(2\pi)\rfloor}$$

$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function

Series expansion at α=0

$$\sqrt{1-\beta} + 8\sqrt{2}\,\sqrt[8]{\alpha}\,\sqrt[4]{1-\beta}\,\sqrt[8]{-(\beta-1)\beta} + 20\sqrt[4]{\alpha}\,\sqrt[4]{-(\beta-1)\beta} + 8\sqrt{2}\,\alpha^{3/8}\sqrt[4]{\beta}\,\sqrt[8]{-(\beta-1)\beta} + \sqrt{\alpha}\,\sqrt{\beta} - \frac{1}{2}\alpha\sqrt{1-\beta} - 3\alpha^{9/8}\left(\sqrt{2}\,\sqrt[4]{1-\beta}\,\sqrt[8]{-(\beta-1)\beta}\right) - 5\alpha^{5/4}\sqrt[4]{-(\beta-1)\beta} - \sqrt{2}\,\alpha^{11/8}\sqrt[4]{\beta}\,\sqrt[8]{-(\beta-1)\beta} - \frac{1}{8}\alpha^2\sqrt{1-\beta} - \frac{15\alpha^{17/8}\left(\sqrt[4]{1-\beta}\,\sqrt[8]{-(\beta-1)\beta}\right)}{8\sqrt{2}} - \frac{15}{8}\alpha^{9/4}\sqrt[4]{-(\beta-1)\beta} - \frac{7\alpha^{19/8}\left(\sqrt[4]{\beta}\,\sqrt[8]{-(\beta-1)\beta}\right)}{8\sqrt{2}} - \frac{1}{16}\alpha^3\sqrt{1-\beta} - \frac{65\alpha^{25/8}\left(\sqrt[4]{1-\beta}\,\sqrt[8]{-(\beta-1)\beta}\right)}{64\sqrt{2}} - \frac{35}{32}\alpha^{13/4}\sqrt[4]{-(\beta-1)\beta} - \frac{35\alpha^{27/8}\left(\sqrt[4]{\beta}\,\sqrt[8]{-(\beta-1)\beta}\right)}{64\sqrt{2}} - \frac{5}{128}\alpha^4\sqrt{1-\beta} - \frac{1365\alpha^{33/8}\left(\sqrt[4]{1-\beta}\,\sqrt[8]{-(\beta-1)\beta}\right)}{2048\sqrt{2}} - \frac{385}{512}\alpha^{17/4}\sqrt[4]{-(\beta-1)\beta} - \frac{805\alpha^{35/8}\left(\sqrt[4]{\beta}\,\sqrt[8]{-(\beta-1)\beta}\right)}{2048\sqrt{2}} + O\left(\alpha^5\right)$$

(Puiseux series)

Derivative

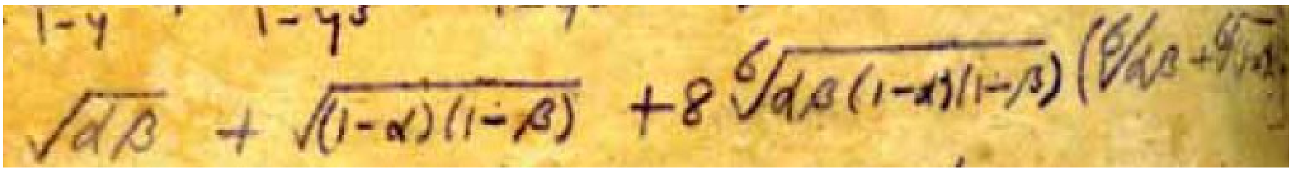
$$\frac{\partial}{\partial\alpha}\left(\sqrt{\alpha\beta} + \sqrt{(1-\alpha)(1-\beta)} + 20\sqrt[4]{\alpha\beta(1-\alpha)(1-\beta)} + 8\sqrt{2}\,\sqrt[8]{\alpha\beta(1-\alpha)(1-\beta)}\left(\sqrt[4]{\alpha\beta} + \sqrt[4]{(1-\alpha)(1-\beta)}\right)\right) = \frac{\sqrt{2}\,(2\alpha-1)\beta(\beta-1)\left(\sqrt[4]{(\alpha-1)(\beta-1)} + \sqrt[4]{\alpha\beta}\right)}{((\alpha-1)\alpha(\beta-1)\beta)^{7/8}} + \frac{\beta-1}{2\sqrt{(\alpha-1)(\beta-1)}} + \frac{5(2\alpha-1)\beta(\beta-1)}{((\alpha-1)\alpha(\beta-1)\beta)^{3/4}} + 2\sqrt{2}\,\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\left(\frac{\beta-1}{((\alpha-1)(\beta-1))^{3/4}} + \frac{\beta}{(\alpha\beta)^{3/4}}\right) + \frac{\beta}{2\sqrt{\alpha\beta}}$$

Indefinite integral

$$\int\left(\sqrt{\alpha\,\beta}+\sqrt{(1-\alpha)(1-\beta)}+20\sqrt[4]{\alpha\,\beta(1-\alpha)(1-\beta)}+8\sqrt{2}\sqrt[8]{\alpha\,\beta(1-\alpha)(1-\beta)}\left(\sqrt[4]{\alpha\,\beta}+\sqrt[4]{(1-\alpha)(1-\beta)}\right)\right)d\alpha=$$
$$\frac{2}{3}(\alpha-1)\sqrt{(\alpha-1)(\beta-1)}+\frac{2}{3}\alpha\sqrt{\alpha\,\beta}+\frac{16(\alpha-1)\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};1-\alpha\right)}{\sqrt[4]{\alpha}}+$$
$$\frac{64\sqrt{2}\,\alpha\sqrt[4]{\alpha(\beta-1)-\beta+1}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{3}{8},\frac{9}{8};\frac{17}{8};-\frac{\alpha(\beta-1)}{1-\beta}\right)}{9\left(\frac{\alpha(\beta-1)}{1-\beta}+1\right)^{3/8}}+$$
$$\frac{64\sqrt{2}\,\alpha\sqrt[4]{\alpha\,\beta}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{8},\frac{11}{8};\frac{19}{8};\alpha\right)}{11\sqrt[8]{1-\alpha}}+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

From:

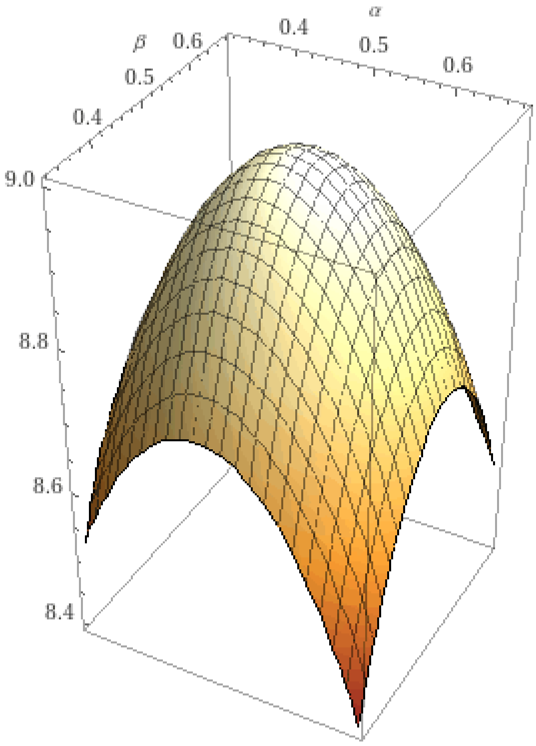


sqrt(αβ) + sqrt((1-α)(1-β)) + 8*(αβ(1-α)(1-β))^(1/6) * [(αβ)^(1/6)+((1-α)(1-β))^(1/6)]

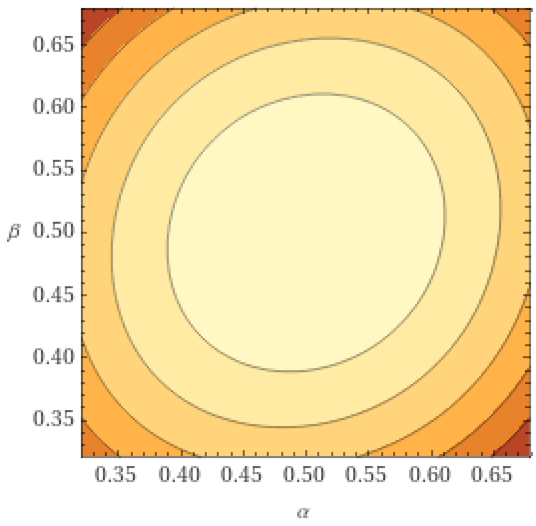
Input

$$\sqrt{\alpha\,\beta}+\sqrt{(1-\alpha)(1-\beta)}+8\sqrt[6]{\alpha\,\beta(1-\alpha)(1-\beta)}\left(\sqrt[6]{\alpha\,\beta}+\sqrt[6]{(1-\alpha)(1-\beta)}\right)$$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Alternate forms

$$8\sqrt[6]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\,\beta}\right)+\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\,\beta}$$

$$\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\,\beta}\right)\left(8\sqrt[6]{(\alpha-1)\,\alpha\left(\beta^2-\beta\right)}+\sqrt[3]{(\alpha-1)(\beta-1)}-\sqrt[6]{\alpha\,\beta}\,\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[3]{\alpha\,\beta}\right)$$

Expanded form

$$\sqrt{(1-\alpha)(1-\beta)}+8\sqrt[6]{(1-\alpha)\,\alpha\,(1-\beta)\,\beta}\,\sqrt[6]{(1-\alpha)(1-\beta)}+\sqrt{\alpha\,\beta}+8\sqrt[6]{\alpha\,\beta}\,\sqrt[6]{(1-\alpha)\,\alpha\,(1-\beta)\,\beta}$$

Alternate form assuming α and β are positive

$$\sqrt{\alpha\,\beta}+\sqrt{1-\alpha}\,\sqrt{1-\beta}\,(-1)^{\lfloor-(\arg(1-\alpha)+\arg(1-\beta)-\pi)/(2\pi)\rfloor}+8\sqrt[3]{1-\alpha}\,\sqrt[3]{1-\beta}\,\sqrt[6]{\alpha\,\beta}\,e^{\frac{2}{3}i\pi\left\lfloor-\frac{\arg(1-\alpha)+\arg(1-\beta)-\pi}{2\pi}\right\rfloor}+8\sqrt[6]{1-\alpha}\,\sqrt[6]{1-\beta}\,\sqrt[3]{\alpha\,\beta}\,e^{\frac{1}{3}i\pi\left\lfloor-\frac{\arg(1-\alpha)+\arg(1-\beta)-\pi}{2\pi}\right\rfloor}$$

$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function

Series expansion at α=0

$$\sqrt{1-\beta}+8\sqrt[6]{\alpha}\,\sqrt[6]{1-\beta}\,\sqrt[6]{-(\beta-1)\,\beta}+8\sqrt[3]{\alpha}\,\sqrt[6]{\beta}\,\sqrt[6]{-(\beta-1)\,\beta}+\sqrt{\alpha}\,\sqrt{\beta}-\frac{1}{2}\alpha\sqrt{1-\beta}-\frac{8}{3}\alpha^{7/6}\left(\sqrt[6]{1-\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{4}{3}\alpha^{4/3}\left(\sqrt[6]{\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{1}{8}\alpha^2\sqrt{1-\beta}-\frac{8}{9}\alpha^{13/6}\left(\sqrt[6]{1-\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{5}{9}\alpha^{7/3}\left(\sqrt[6]{\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{1}{16}\alpha^3\sqrt{1-\beta}-\frac{40}{81}\alpha^{19/6}\left(\sqrt[6]{1-\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{55}{162}\alpha^{10/3}\left(\sqrt[6]{\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{5}{128}\alpha^4\sqrt{1-\beta}-\frac{80}{243}\alpha^{25/6}\left(\sqrt[6]{1-\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)-\frac{935\,\alpha^{13/3}\left(\sqrt[6]{\beta}\,\sqrt[6]{-(\beta-1)\,\beta}\right)}{3888}+O\left(\alpha^5\right)$$

(Puiseux series)

Series expansion at α=∞

$$\frac{\sqrt{\alpha}\left(8\sqrt[6]{\alpha^2(\beta-1)\beta}\ \sqrt[6]{\alpha(\beta-1)}+8\sqrt[6]{\alpha\beta}\ \sqrt[6]{\alpha^2(\beta-1)\beta}+\sqrt{\alpha(\beta-1)}+\sqrt{\alpha\beta}\right)}{\sqrt{\alpha}}+\frac{\sqrt{\frac{1}{\alpha}}\left(-16\sqrt[6]{\alpha^2(\beta-1)\beta}\ \sqrt[6]{\alpha(\beta-1)}-8\sqrt[6]{\alpha\beta}\ \sqrt[6]{\alpha^2(\beta-1)\beta}-3\sqrt{\alpha(\beta-1)}\right)}{6\sqrt{\alpha}}+O\left(\left(\frac{1}{\alpha}\right)^1\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(\sqrt{\alpha\beta}+\sqrt{(1-\alpha)(1-\beta)}+8\sqrt[6]{\alpha\beta(1-\alpha)(1-\beta)}\left(\sqrt[6]{\alpha\beta}+\sqrt[6]{(1-\alpha)(1-\beta)}\right)\right)=\frac{1}{6}\left(\frac{8(2\alpha-1)\beta(\beta-1)\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\beta}\right)}{((\alpha-1)\alpha(\beta-1)\beta)^{5/6}}+\frac{3(\beta-1)}{\sqrt{(\alpha-1)(\beta-1)}}+8\sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\left(\frac{\beta-1}{((\alpha-1)(\beta-1))^{5/6}}+\frac{\beta}{(\alpha\beta)^{5/6}}\right)+\frac{3\beta}{\sqrt{\alpha\beta}}\right)$$

Indefinite integral

$$\int\left(\sqrt{\alpha\beta}+\sqrt{(1-\alpha)(1-\beta)}+8\sqrt[6]{\alpha\beta(1-\alpha)(1-\beta)}\left(\sqrt[6]{\alpha\beta}+\sqrt[6]{(1-\alpha)(1-\beta)}\right)\right)d\alpha=\frac{2}{3}(\alpha-1)\sqrt{(\alpha-1)(\beta-1)}+\frac{2}{3}\alpha\sqrt{\alpha\beta}+\frac{48\alpha\sqrt[6]{\alpha(\beta-1)-\beta+1}\ \sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\ {}_2F_1\left(-\frac{1}{3},\frac{7}{6};\frac{13}{6};-\frac{\alpha(\beta-1)}{1-\beta}\right)}{7\sqrt[3]{\frac{\alpha(\beta-1)}{1-\beta}+1}}+\frac{6\alpha\sqrt[6]{\alpha\beta}\ \sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\ {}_2F_1\left(-\frac{1}{6},\frac{4}{3};\frac{7}{3};\alpha\right)}{\sqrt[6]{1-\alpha}}+\text{constant}$$

${}_2F_1(a, b; c; x)$ is the hypergeometric function

From the sum of

((√((α-1)(β-1))+√(αβ)+4((α-1)α(β-1)β)^(1/4)))+((8√(2)((α-1)α(β-1)β)^(1/8)(((α-1)(β-1))^(1/4)+(αβ)^(1/4))+√((α-1)(β-1))+√(αβ)+20((α-1)α(β-1)β)^(1/4)))

Input

$$\left(\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\beta}+4\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}\right)+\left(8\sqrt{2}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[4]{(\alpha-1)(\beta-1)}+\sqrt[4]{\alpha\beta}\right)+\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\beta}+20\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}\right)$$

Exact result

$$8\sqrt{2}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[4]{(\alpha-1)(\beta-1)}+\sqrt[4]{\alpha\beta}\right)+2\sqrt{(\alpha-1)(\beta-1)}+2\sqrt{\alpha\beta}+24\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}$$

((8((α-1)α(β-1)β)^(1/6)(((α-1)(β-1))^(1/6)+(αβ)^(1/6))+√((α-1)(β-1))+√(αβ)))

$$8\sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\beta}\right)+\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\beta}$$

we obtain:

8√(2)((α-1)α(β-1)β)^(1/8)(((α-1)(β-1))^(1/4)+(αβ)^(1/4))+2√((α-1)(β-1))+2√(αβ)+24((α-1)α(β-1)β)^(1/4)+((8((α-1)α(β-1)β)^(1/6)(((α-1)(β-1))^(1/6)+(αβ)^(1/6))+√((α-1)(β-1))+√(αβ)))

Input

$$8\sqrt{2}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[4]{(\alpha-1)(\beta-1)}+\sqrt[4]{\alpha\beta}\right)+2\sqrt{(\alpha-1)(\beta-1)}+2\sqrt{\alpha\beta}+24\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}+\left(8\sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\beta}\right)+\sqrt{(\alpha-1)(\beta-1)}+\sqrt{\alpha\beta}\right)$$

Exact result

$$8\sqrt[6]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\,\beta}\right)+ 8\sqrt{2}\,\sqrt[8]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\left(\sqrt[4]{(\alpha-1)(\beta-1)}+\sqrt[4]{\alpha\,\beta}\right)+ 3\sqrt{(\alpha-1)(\beta-1)}+3\sqrt{\alpha\,\beta}+24\sqrt[4]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}$$

Expanded form

$$3\sqrt{(\alpha-1)(\beta-1)}+8\sqrt{2}\,\sqrt[8]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\sqrt[4]{(\alpha-1)(\beta-1)}+ 8\sqrt[6]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\sqrt[6]{(\alpha-1)(\beta-1)}+3\sqrt{\alpha\,\beta}+24\sqrt[4]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}+ 8\sqrt[6]{\alpha\,\beta}\,\sqrt[6]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}+8\sqrt{2}\,\sqrt[4]{\alpha\,\beta}\,\sqrt[8]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}$$

Alternate forms assuming α and β are positive

$$3\left(\sqrt{\alpha\,\beta}+\sqrt{\alpha-1}\,\sqrt{\beta-1}\,(-1)^{\lfloor-(\arg(\alpha-1)+\arg(\beta-1)-\pi)/(2\pi)\rfloor}+ 8\sqrt[4]{\alpha-1}\,\sqrt[4]{\beta-1}\,\sqrt[4]{\alpha\,\beta}\,i^{\lfloor-(\arg(\alpha-1)+\arg(\beta-1)-\pi)/(2\pi)\rfloor}\right)+ 8\sqrt{2}\,\sqrt[8]{\alpha-1}\,\sqrt[8]{\beta-1}\,\sqrt[8]{\alpha\,\beta}\,e^{\frac{1}{4}i\pi\left\lfloor-\frac{\arg(\alpha-1)+\arg(\beta-1)-\pi}{2\pi}\right\rfloor} \left(\sqrt[4]{\alpha\,\beta}+\sqrt[4]{\alpha-1}\,\sqrt[4]{\beta-1}\,i^{\lfloor-(\arg(\alpha-1)+\arg(\beta-1)-\pi)/(2\pi)\rfloor}\right)+ 8\sqrt[6]{\alpha-1}\,\sqrt[6]{\beta-1}\,\sqrt[3]{\alpha\,\beta}\,e^{\frac{1}{3}i\pi\left\lfloor-\frac{\arg(\alpha-1)+\arg(\beta-1)-\pi}{2\pi}\right\rfloor}+ 8\sqrt[3]{\alpha-1}\,\sqrt[3]{\beta-1}\,\sqrt[6]{\alpha\,\beta}\,e^{\frac{2}{3}i\pi\left\lfloor-\frac{\arg(\alpha-1)+\arg(\beta-1)-\pi}{2\pi}\right\rfloor}$$

$$3\sqrt{\alpha\,\beta}+8\sqrt[6]{\alpha-1}\,\sqrt[6]{\beta-1}\,\sqrt[6]{\alpha\,\beta}\,\exp\left(\frac{1}{3}i\pi\left\lfloor-\frac{\arg(\alpha-1)}{2\pi}-\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right) \left(\sqrt[6]{\alpha\,\beta}+\sqrt[6]{\alpha-1}\,\sqrt[6]{\beta-1}\,\exp\left(\frac{1}{3}i\pi\left\lfloor-\frac{\arg(\alpha-1)}{2\pi}-\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right)\right)+ 8\sqrt{2}\,\sqrt[8]{\alpha-1}\,\sqrt[8]{\beta-1}\,\sqrt[8]{\alpha\,\beta}\,\exp\left(\frac{1}{4}i\pi\left\lfloor-\frac{\arg(\alpha-1)}{2\pi}-\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right) \left(\sqrt[4]{\alpha\,\beta}+\sqrt[4]{\alpha-1}\,\sqrt[4]{\beta-1}\,\exp\left(\frac{1}{2}i\pi\left\lfloor-\frac{\arg(\alpha-1)}{2\pi}-\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right)\right)+ 3\sqrt{\alpha-1}\,\sqrt{\beta-1}\,\exp\left(i\pi\left\lfloor-\frac{\arg(\alpha-1)}{2\pi}-\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right)+ 24\sqrt[4]{\alpha-1}\,\sqrt[4]{\beta-1}\,\sqrt[4]{\alpha\,\beta}\,\exp\left(\frac{1}{2}i\pi\left\lfloor-\frac{\arg(\alpha-1)}{2\pi}-\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right)$$

$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function

Series expansion at α=0

$$3\sqrt{1-\beta}+8\sqrt{2}\sqrt[8]{\alpha}\sqrt[4]{1-\beta}\sqrt[8]{-(\beta-1)\beta}+8\sqrt[6]{\alpha}\sqrt[6]{1-\beta}\sqrt[6]{-(\beta-1)\beta}+24\sqrt[4]{\alpha}\sqrt[4]{-(\beta-1)\beta}+8\sqrt[3]{\alpha}\sqrt[6]{\beta}\sqrt[6]{-(\beta-1)\beta}+8\sqrt{2}\,\alpha^{3/8}\sqrt[4]{\beta}\sqrt[8]{-(\beta-1)\beta}+3\sqrt{\alpha}\sqrt{\beta}-\frac{3}{2}\alpha\sqrt{1-\beta}-3\alpha^{9/8}\left(\sqrt{2}\sqrt[4]{1-\beta}\sqrt[8]{-(\beta-1)\beta}\right)-\frac{8}{3}\alpha^{7/6}\left(\sqrt[6]{1-\beta}\sqrt[6]{-(\beta-1)\beta}\right)-6\alpha^{5/4}\sqrt[4]{-(\beta-1)\beta}-\frac{4}{3}\alpha^{4/3}\left(\sqrt[6]{\beta}\sqrt[6]{-(\beta-1)\beta}\right)-\sqrt{2}\,\alpha^{11/8}\sqrt[4]{\beta}\sqrt[8]{-(\beta-1)\beta}-\frac{3}{8}\alpha^{2}\sqrt{1-\beta}-\frac{15\alpha^{17/8}\left(\sqrt[4]{1-\beta}\sqrt[8]{-(\beta-1)\beta}\right)}{8\sqrt{2}}-\frac{8}{9}\alpha^{13/6}\left(\sqrt[6]{1-\beta}\sqrt[6]{-(\beta-1)\beta}\right)-\frac{9}{4}\alpha^{9/4}\sqrt[4]{-(\beta-1)\beta}-\frac{5}{9}\alpha^{7/3}\left(\sqrt[6]{\beta}\sqrt[6]{-(\beta-1)\beta}\right)-\frac{7\alpha^{19/8}\left(\sqrt[4]{\beta}\sqrt[8]{-(\beta-1)\beta}\right)}{8\sqrt{2}}-\frac{3}{16}\alpha^{3}\sqrt{1-\beta}-\frac{65\alpha^{25/8}\left(\sqrt[4]{1-\beta}\sqrt[8]{-(\beta-1)\beta}\right)}{64\sqrt{2}}-\frac{40}{81}\alpha^{19/6}\left(\sqrt[6]{1-\beta}\sqrt[6]{-(\beta-1)\beta}\right)-\frac{21}{16}\alpha^{13/4}\sqrt[4]{-(\beta-1)\beta}-\frac{55}{162}\alpha^{10/3}\left(\sqrt[6]{\beta}\sqrt[6]{-(\beta-1)\beta}\right)-\frac{35\alpha^{27/8}\left(\sqrt[4]{\beta}\sqrt[8]{-(\beta-1)\beta}\right)}{64\sqrt{2}}-\frac{15}{128}\alpha^{4}\sqrt{1-\beta}-\frac{1365\alpha^{33/8}\left(\sqrt[4]{1-\beta}\sqrt[8]{-(\beta-1)\beta}\right)}{2048\sqrt{2}}-\frac{80}{243}\alpha^{25/6}\left(\sqrt[6]{1-\beta}\sqrt[6]{-(\beta-1)\beta}\right)-\frac{231}{256}\alpha^{17/4}\sqrt[4]{-(\beta-1)\beta}-\frac{935\alpha^{13/3}\left(\sqrt[6]{\beta}\sqrt[6]{-(\beta-1)\beta}\right)}{3888}-\frac{805\alpha^{35/8}\left(\sqrt[4]{\beta}\sqrt[8]{-(\beta-1)\beta}\right)}{2048\sqrt{2}}+O\left(\alpha^{5}\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial \alpha}\Bigg(8\sqrt{2}\,\sqrt[8]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\left(\sqrt[4]{(\alpha-1)\,(\beta-1)}+\sqrt[4]{\alpha\,\beta}\right)+ 2\sqrt{(\alpha-1)\,(\beta-1)}+2\sqrt{\alpha\,\beta}+24\sqrt[4]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}+ \left(8\sqrt[6]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\,\left(\sqrt[6]{(\alpha-1)\,(\beta-1)}+\sqrt[6]{\alpha\,\beta}\right)+ \sqrt{(\alpha-1)\,(\beta-1)}+\sqrt{\alpha\,\beta}\right)\Bigg)=$$

$$\frac{4\,(2\,\alpha-1)\,\beta\,(\beta-1)\left(\sqrt[6]{(\alpha-1)\,(\beta-1)}+\sqrt[6]{\alpha\,\beta}\right)}{3\,((\alpha-1)\,\alpha\,(\beta-1)\,\beta)^{5/6}}+ \frac{\sqrt{2}\,(2\,\alpha-1)\,\beta\,(\beta-1)\left(\sqrt[4]{(\alpha-1)\,(\beta-1)}+\sqrt[4]{\alpha\,\beta}\right)}{((\alpha-1)\,\alpha\,(\beta-1)\,\beta)^{7/8}}+ \frac{3\,(\beta-1)}{2\sqrt{(\alpha-1)\,(\beta-1)}}+\frac{6\,(2\,\alpha-1)\,\beta\,(\beta-1)}{((\alpha-1)\,\alpha\,(\beta-1)\,\beta)^{3/4}}+ \frac{4}{3}\,\sqrt[6]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\left(\frac{\beta-1}{((\alpha-1)\,(\beta-1))^{5/6}}+\frac{\beta}{(\alpha\,\beta)^{5/6}}\right)+ 2\sqrt{2}\,\sqrt[8]{(\alpha-1)\,\alpha\,(\beta-1)\,\beta}\left(\frac{\beta-1}{((\alpha-1)\,(\beta-1))^{3/4}}+\frac{\beta}{(\alpha\,\beta)^{3/4}}\right)+\frac{3\,\beta}{2\sqrt{\alpha\,\beta}}$$

Indefinite integral

$$\int\Big(3\sqrt{(-1+\alpha)(-1+\beta)}+3\sqrt{\alpha\beta}+24\sqrt[4]{(-1+\alpha)\alpha(-1+\beta)\beta}+8\sqrt[6]{(-1+\alpha)\alpha(-1+\beta)\beta}\left(\sqrt[6]{(-1+\alpha)(-1+\beta)}+\sqrt[6]{\alpha\beta}\right)+8\sqrt{2}\sqrt[8]{(-1+\alpha)\alpha(-1+\beta)\beta}\left(\sqrt[4]{(-1+\alpha)(-1+\beta)}+\sqrt[4]{\alpha\beta}\right)\Big)d\alpha=$$

$$2(\alpha-1)\sqrt{(\alpha-1)(\beta-1)}+2\alpha\sqrt{\alpha\beta}+\frac{96(\alpha-1)\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};1-\alpha\right)}{5\sqrt[4]{\alpha}}+\frac{64\sqrt{2}\,\alpha\sqrt[4]{\alpha(\beta-1)-\beta+1}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{3}{8},\frac{9}{8};\frac{17}{8};-\frac{\alpha(\beta-1)}{1-\beta}\right)}{9\left(\frac{\alpha(\beta-1)}{1-\beta}+1\right)^{3/8}}+\frac{48\alpha\sqrt[6]{\alpha(\beta-1)-\beta+1}\sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{3},\frac{7}{6};\frac{13}{6};-\frac{\alpha(\beta-1)}{1-\beta}\right)}{7\sqrt[3]{\frac{\alpha(\beta-1)}{1-\beta}+1}}+\frac{6\alpha\sqrt[6]{\alpha\beta}\sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{6},\frac{4}{3};\frac{7}{3};\alpha\right)}{\sqrt[6]{1-\alpha}}+\frac{64\sqrt{2}\,\alpha\sqrt[4]{\alpha\beta}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\;{}_2F_1\left(-\frac{1}{8},\frac{11}{8};\frac{19}{8};\alpha\right)}{11\sqrt[8]{1-\alpha}}+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

From the exact result

$$8\sqrt[6]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[6]{(\alpha-1)(\beta-1)}+\sqrt[6]{\alpha\beta}\right)+8\sqrt{2}\sqrt[8]{(\alpha-1)\alpha(\beta-1)\beta}\left(\sqrt[4]{(\alpha-1)(\beta-1)}+\sqrt[4]{\alpha\beta}\right)+3\sqrt{(\alpha-1)(\beta-1)}+3\sqrt{\alpha\beta}+24\sqrt[4]{(\alpha-1)\alpha(\beta-1)\beta}$$

we obtain:

8 ((α - 1) α (β - 1) β)^(1/6) (((α - 1) (β - 1))^(1/6) + (α β)^(1/6)) + 8 sqrt(2) ((α - 1) α (β - 1) β)^(1/8) (((α - 1) (β - 1))^(1/4) + (α β)^(1/4)) + 3 sqrt((α - 1) (β - 1)) + 3 sqrt(α β) + 24 ((α - 1) α (β - 1) β)^(1/4)

for α = 8 and β = 16 , we obtain:

8 ((8-1) 8(16-1)16)^(1/6) (((8-1)(16-1))^(1/6)+(8*16)^(1/6))+8√(2)((8-1)8(16-1)16)^(1/8)(((8-1)(16-1))^(1/4)+(8*16)^(1/4))+3√((8-1)(16-1))+3√(8*16)+24((8-1)8(16-1)16)^(1/4)

Input

$$8\sqrt[6]{(8-1)\times 8(16-1)\times 16}\left(\sqrt[6]{(8-1)(16-1)}+\sqrt[6]{8\times 16}\right)+8\sqrt{2}\sqrt[8]{(8-1)\times 8(16-1)\times 16}\left(\sqrt[4]{(8-1)(16-1)}+\sqrt[4]{8\times 16}\right)+3\sqrt{(8-1)(16-1)}+3\sqrt{8\times 16}+24\sqrt[4]{(8-1)\times 8(16-1)\times 16}$$

Result

$$24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)$$

Decimal approximation

739.09744601752870146783716018224441669783369732849198807395093463
…

739.097446…. (we note that 739 – 11 = 728 = 9^3 -1 = Ramanujan taxicab number)

Alternate forms

$$32\sqrt[3]{2}\sqrt[6]{105}+16\sqrt[6]{2}\sqrt[3]{105}+64\sqrt[8]{210}+16\times 210^{3/8}+3\sqrt{105}+24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}$$

$$24\sqrt{2}+32\sqrt[3]{2}\sqrt[6]{105}+48\times 2^{3/4}\sqrt[4]{105}+16\sqrt[6]{2}\sqrt[3]{105}+3\sqrt{105}+2^{3/8}\left(32\times 2^{3/4}\sqrt[8]{105}+16\times 105^{3/8}\right)$$

$$24\sqrt{2}+32\sqrt[3]{2}\sqrt[6]{105}+48\times 2^{3/4}\sqrt[4]{105}+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)+\sqrt[6]{105}\left(3\sqrt[3]{105}+16\sqrt[6]{210}\right)$$

From the result

$$24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)$$

we obtain, after some calculations:

ln (ζ(64^-8) ((24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + sinh((0.8/(64Pi))^8)^2(16 210^(1/6) (2 2^(1/6) + 105^(1/6)))x+cosh((0.8/(64Pi))^8)^2(16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4)))))y)

Input

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + \sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)\left(16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right)\right)x + \cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)\left(16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\right)y\right)$$

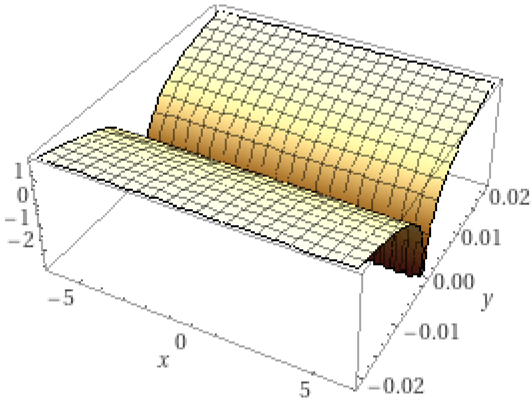
$\zeta(s)$ is the Riemann zeta function
$\sinh(x)$ is the hyperbolic sine function
$\cosh(x)$ is the hyperbolic cosine function
$\log(x)$ is the natural logarithm

Result

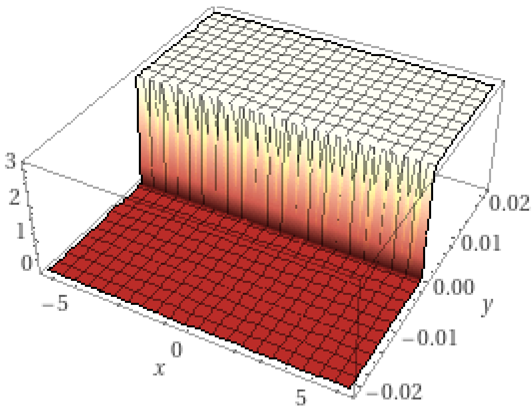
$$\log\left(\left(6.79894\times 10^{-37}\,x + 566.8\right)y\,\zeta\left(\frac{1}{281474976710656}\right)\right)$$

3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

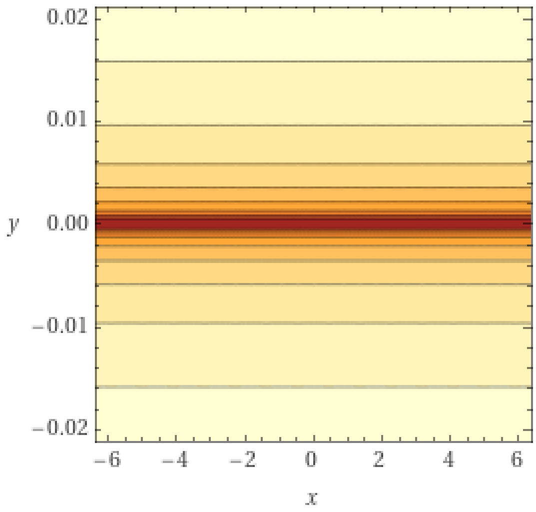


Imaginary part

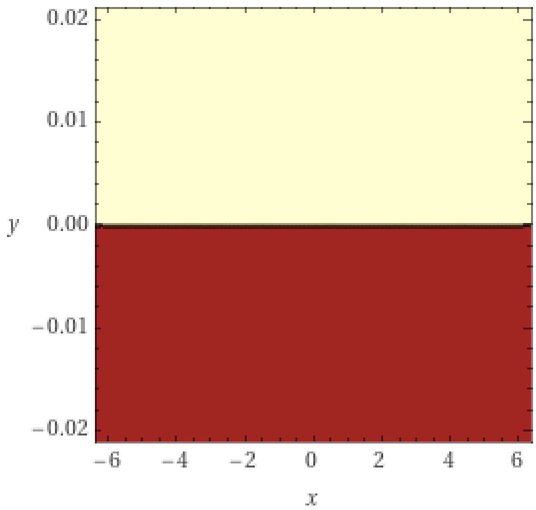


Contour plots

Real part



Imaginary part



Alternate form

$\log((-3.39947\times 10^{-37}\, x - 283.4)\, y)$

Alternate forms assuming x and y are positive

$$\log\left(3.39947\times 10^{-37}\, x + 283.4\right) + \log(y) + i\,\pi$$

$$\log\left(6.79894\times 10^{-37}\, x + 566.8\right) + \log(y) + \log\left(-\zeta\left(\frac{1}{281\,474\,976\,710\,656}\right)\right) + i\,\pi$$

Root

$$7.66466\times 10^{6}\, x + 6.38972\times 10^{45} \neq 0\,,\quad y \approx -\frac{2.25466\times 10^{43}}{7.66466\times 10^{6}\, x + 6.38972\times 10^{45}}$$

Root for the variable y

$$y \approx \frac{1}{-3.39947\times 10^{-37}\, x - 283.4}$$

Series expansion at x=0

$$\log(-283.4\, y) + 1.19953\times 10^{-39}\, x - 7.19437\times 10^{-79}\, x^2 + 5.75325\times 10^{-118}\, x^3 - 5.1759\times 10^{-157}\, x^4 + 4.96692\times 10^{-196}\, x^5 + O\left(x^6\right)$$

(Taylor series)

Series expansion at x=∞

$$\log\left(-3.39947\times 10^{-37}\, x\, y\right) + \frac{8.33659\times 10^{38}}{x} - \frac{3.47494\times 10^{77}}{x^2} + \frac{1.93128\times 10^{116}}{x^3} - \frac{1.20752\times 10^{155}}{x^4} + \frac{8.05328\times 10^{193}}{x^5} + O\left(\left(\frac{1}{x}\right)^6\right)$$

(generalized Puiseux series)

Partial derivatives

$$\frac{\partial}{\partial x}\left(\log\left(\left(6.79894\times 10^{-37}\,x+566.8\right)y\,\zeta\left(\frac{1}{281\,474\,976\,710\,656}\right)\right)\right)=\frac{6.79894\times 10^{-37}}{6.79894\times 10^{-37}\,x+566.8}$$

$$\frac{\partial}{\partial y}\left(\log\left(\left(6.79894\times 10^{-37}\,x+566.8\right)y\,\zeta\left(\frac{1}{281\,474\,976\,710\,656}\right)\right)\right)=\frac{1}{y}$$

Indefinite integral

$$\int\log\left(\left(566.8+6.79894\times 10^{-37}\,x\right)y\,\zeta\left(\frac{1}{281\,474\,976\,710\,656}\right)\right)dx=\left(x+8.33659\times 10^{38}\right)\log\left(\left(-3.39947\times 10^{-37}\,x-283.4\right)y\right)-x+\text{constant}$$

(assuming a complex-valued logarithm)

Limit

$$\lim_{x\to\pm\infty}\log\left(\left(566.8+6.79894\times 10^{-37}\,x\right)y\,\zeta\left(\frac{1}{281\,474\,976\,710\,656}\right)\right)=\log(-283.4\,y)$$

Alternative representations

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\,\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\,\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log_e\left(y\left(48\times 2^{3/4}\sqrt[4]{105}+16\times 2^{3/8}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\sqrt[8]{105}\cosh^2\left(\left(\frac{0.8}{64\,\pi}\right)^8\right)+16\,x\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\sqrt[6]{210}\sinh^2\left(\left(\frac{0.8}{64\,\pi}\right)^8\right)+24\sqrt{2}+3\sqrt{105}\right)\zeta\left(\frac{1}{64^8}\right)\right)$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=\log(a)\log_a\left(y\left(48\times 2^{3/4}\sqrt[4]{105}+16\times 2^{3/8}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\sqrt[8]{105}\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)+16x\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\sqrt[6]{210}\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)+24\sqrt{2}+3\sqrt{105}\right)\zeta\left(\frac{1}{64^8}\right)\right)$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=\log\left(y\left(48\times 2^{3/4}\sqrt[4]{105}+16\times 2^{3/8}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\sqrt[8]{105}\cos^2\left(-i\left(\frac{0.8}{64\pi}\right)^8\right)+16x\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\sqrt[6]{210}\left(\frac{1}{2}\left(-e^{-(0.8/(64\pi))^8}+e^{(0.8/(64\pi))^8}\right)\right)^2+24\sqrt{2}+3\sqrt{105}\right)\zeta\left(\frac{1}{64^8},1\right)\right)$$

$\log_b(x)$ is the base- b logarithm

$\zeta(s, a)$ is the generalized Riemann zeta function

Series representations

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1-283.4\,y-3.39947\times 10^{-37}\,x\,y\right)^k}{k}$$

for $\left|1+283.4\,y+3.39947\times 10^{-37}\,x\,y\right|<1$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log(-1-283.4\,y-3.39947\times 10^{-37}\,x\,y)-\sum_{k=1}^{\infty}\frac{(-1)^k(-1-283.4\,y-3.39947\times 10^{-37}\,x\,y)^{-k}}{k}$$
$$\text{for } |1+283.4\,y+3.39947\times 10^{-37}\,x\,y|>1$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log\left(-1.\,(566.8+6.79894\times 10^{-37}\,x)\,y\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}(-1)^k(1+k)^{281\,474\,976\,710\,655/281\,474\,976\,710\,656}\binom{n}{k}}{1+n}\right)$$

$|z|$ is the absolute value of z

$\binom{n}{m}$ is the binomial coefficient

Integral representations

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=\int_1^{(-283.4-3.39947\times 10^{-37}\,x)\,y}\frac{1}{t}\,dt$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log\left((-283.4-3.39947\times 10^{-37}\,x)\,y\int_0^\infty \sqrt[281474976710656]{t}\,\operatorname{sech}^2(t)\,dt\right)$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log\left((-2.01368\times 10^{-12}-2.41547\times 10^{-51}\,x)\,y\int_0^\infty \frac{1}{(1+e^t)\,t^{281474976710655/281474976710656}}\,dt\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Functional equations

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log\left((-283.4-3.39947\times 10^{-37}\,x)\,y\right)$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+\left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)\right)+\cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)y\right)=$$
$$\log\left((-283.4-3.39947\times 10^{-37}\,x)\,y\right)$$

$$\log\left(\zeta\left(\frac{1}{64^8}\right)\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + \left(\sinh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)x\right)16\left(\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right)\right) + \cosh^2\left(\left(\frac{0.8}{64\pi}\right)^8\right)16\left(2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\right)y\right) = n\log\left(\sqrt[n]{(-283.4 - 3.39947\times 10^{-37}x)\,y}\right)$$

for $(-283.4 - 3.39947\times 10^{-37}x)\,y \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

From:

$$\log\left((6.79894\times 10^{-37}x + 566.8)\,y\,\zeta\left(\frac{1}{281474976710656}\right)\right)$$

we obtain, after some calculations:

3d plot log((cosh(6.79894×10^-37) x + tanh(566.8)) cosh(ζ(1/281474976710656))y)

Input interpretation

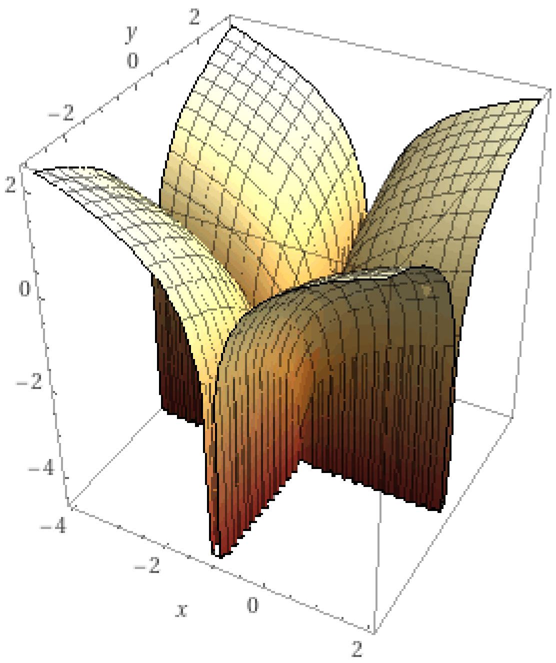
3D plot	$\log\left((\cosh(6.79894\times 10^{-37})\,x + \tanh(566.8))\cosh\left(\zeta\left(\frac{1}{281474976710656}\right)\right)y\right)$

$\cosh(x)$ is the hyperbolic cosine function
$\tanh(x)$ is the hyperbolic tangent function
$\zeta(s)$ is the Riemann zeta function
$\log(x)$ is the natural logarithm

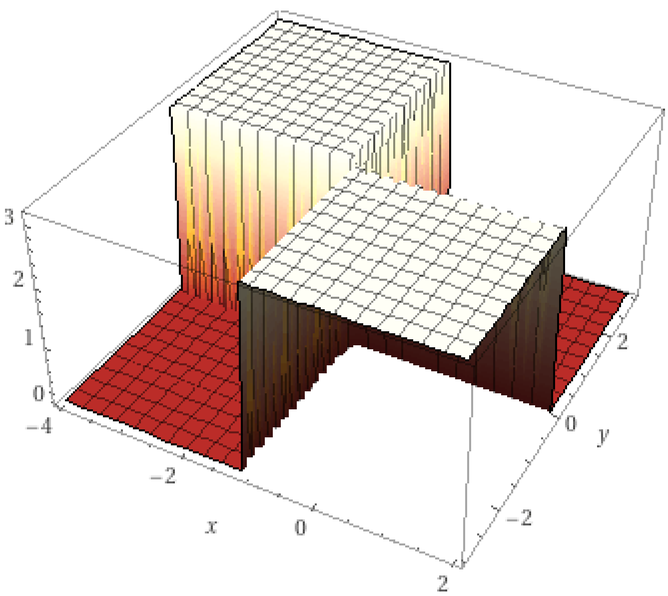
3D plots

Real part

(figures that can be related to a D-branes/Instantons and a fractals)

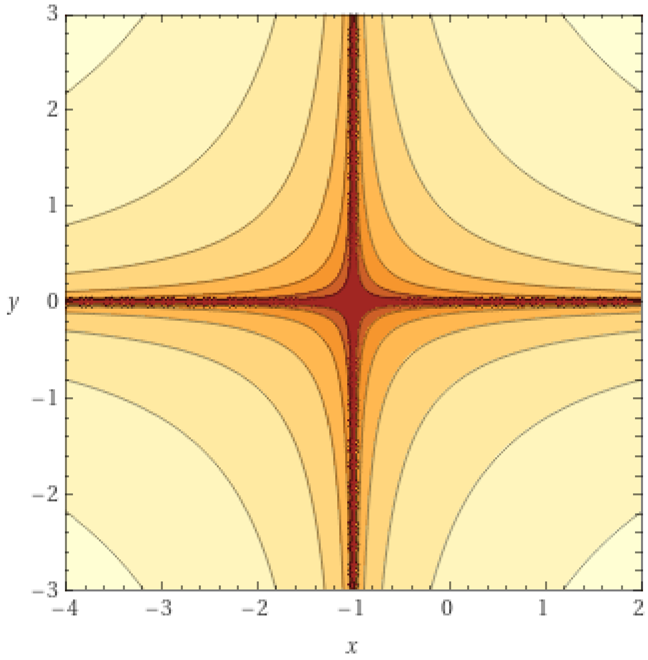


Imaginary part

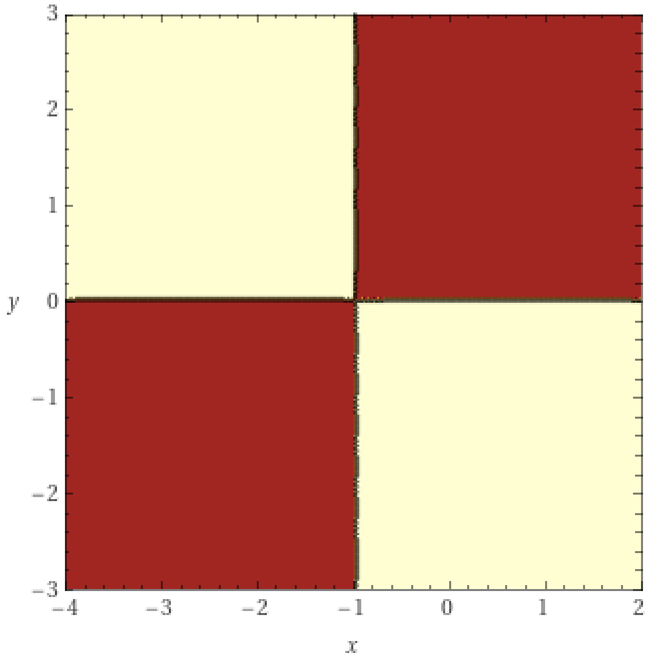


Contour plots

Real part



Imaginary part



From the same formula, we obtain:

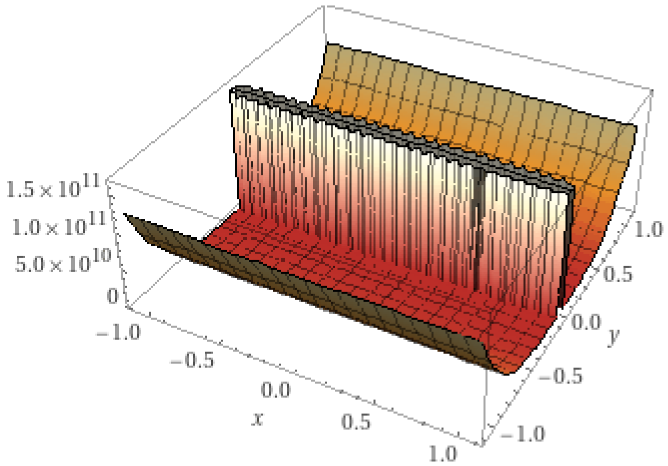
3d plot zeta(24) cosh^4(log(6.79894×10^-37 x + 566.8) + log(y) +cosh^-2(log(-ζ(1/281474976710656)) + i π))

Input interpretation

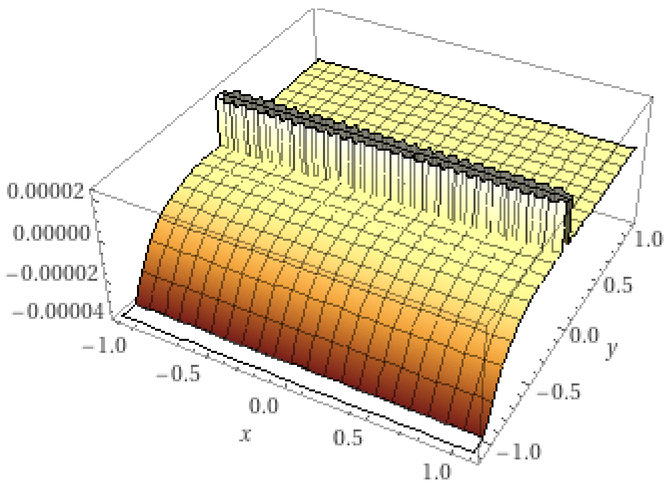
3D plot	$\zeta(24)\cosh^4\left(\log(6.79894\times10^{-37}\,x+566.8)+\log(y)+\frac{1}{\cosh^2\left(\log\left(-\zeta\left(\frac{1}{281474976710656}\right)\right)+i\,\pi\right)}\right)$

$\zeta(s)$ is the Riemann zeta function
$\log(x)$ is the natural logarithm
$\cosh(x)$ is the hyperbolic cosine function
i is the imaginary unit

3D plots
Real part **(figures that can be related to the D-branes/Instantons)**

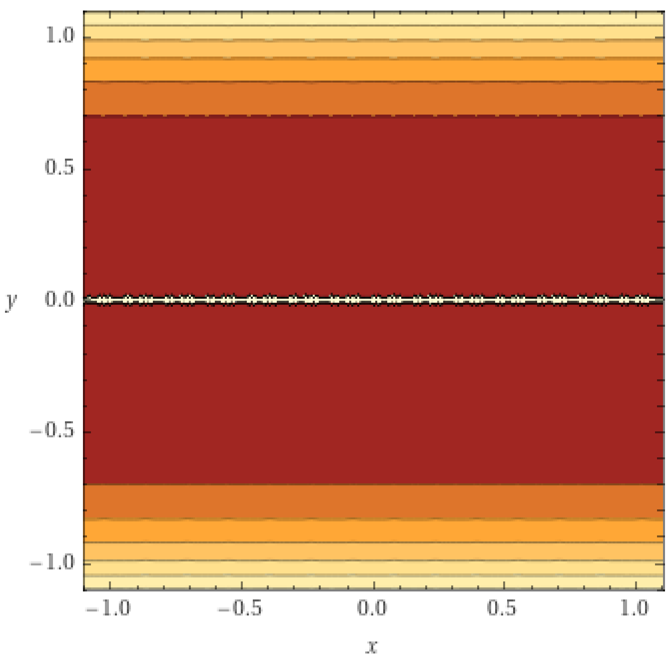


Imaginary part

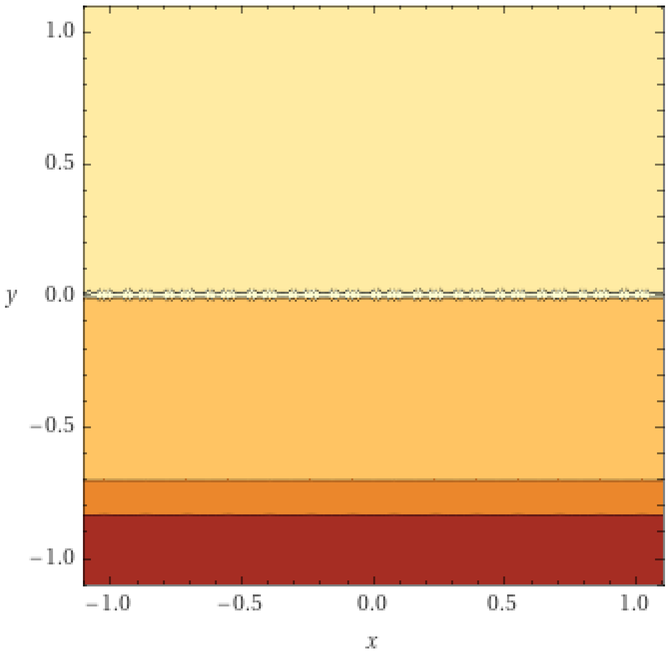


Contour plots

Real part



Imaginary part



From

$$8\sqrt[6]{(8-1)\times 8(16-1)\times 16}\left(\sqrt[6]{(8-1)(16-1)}+\sqrt[6]{8\times 16}\right)+8\sqrt{2}\sqrt[8]{(8-1)\times 8(16-1)\times 16}\left(\sqrt[4]{(8-1)(16-1)}+\sqrt[4]{8\times 16}\right)+3\sqrt{(8-1)(16-1)}+3\sqrt{8\times 16}+24\sqrt[4]{(8-1)\times 8(16-1)\times 16}$$

$$24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)$$

= 739.097446…. (we note that 739 – 11 = 728 = 9^3 -1 = Ramanujan taxicab number)

we obtain also:

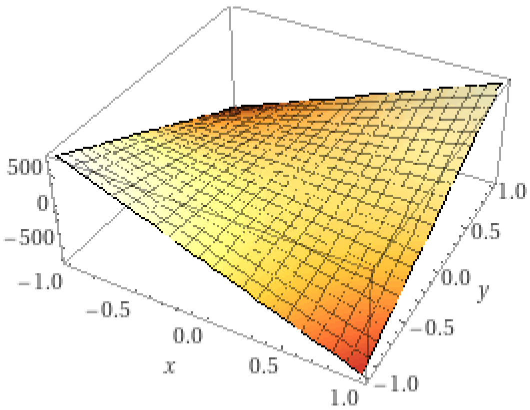
(24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + 16 210^(1/6) (2 2^(1/6) + 105^(1/6)) + 16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4)))dxdy

Indefinite integral

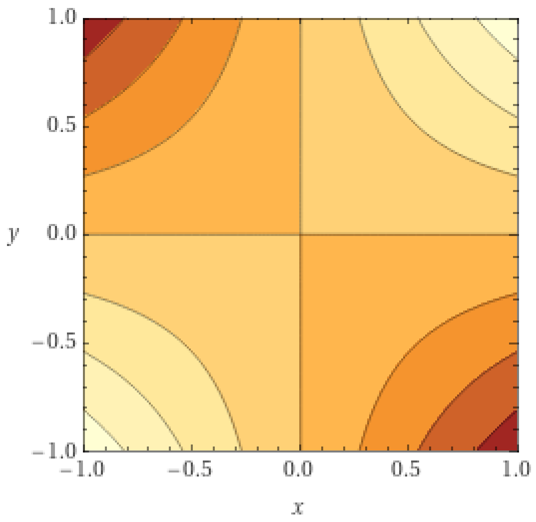
$$\int\int\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)dx\,dy =$$

$$c_1 x + c_2 + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)x\,y + 16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)x\,y + 3\sqrt{105}\,x\,y + 48\times 2^{3/4}\sqrt[4]{105}\,x\,y + 24\sqrt{2}\,x\,y$$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right) dy\, dx = \left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\pi R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right) dx\, dy = 4\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right) L^2$$

Dividing the result of the two integrals by

$$24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)$$

we obtain:

((24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + 16 210^(1/6) (2 2^(1/6) + 105^(1/6)) + 16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4))) π)/(739.0974460175287)

Input interpretation

$$\frac{1}{739.0974460175287}\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\pi$$

Result

3.141592653589793…

3.141592653…. = π

Series representations

$$\left(\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\pi\right) / 739.09744601752870000 = 0.912485182916869322\,\pi + \sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\,\pi\left(-\frac{1}{2}\right)_k\sqrt{z_0}\left(0.03247203752268251\,(2-z_0)^k + 0.004059004690335313\,(105-z_0)^k\right) z_0^{-k} \quad \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\left(\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\pi\right) / 739.09744601752870000 = 0.912485182916869322\,\pi + \sum_{k=0}^{\infty} \frac{1}{k!}(-1)^k \pi x^{-k} \left(0.03247203752268251\,(2-x)^k \exp\left(i\pi\left\lfloor \frac{\arg(2-x)}{2\pi}\right\rfloor\right) + 0.004059004690335313\,(105-x)^k \exp\left(i\pi\left\lfloor \frac{\arg(105-x)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k \sqrt{x} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\left(\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\pi\right) / 739.09744601752870000 = 0.91248518291686932189\,\pi + \sum_{k=0}^{\infty} \frac{1}{k!}\, 0.032472037522682506\,(-1)^k \pi \left(-\frac{1}{2}\right)_k z_0^{1/2-k} \left(1.000000000000000\,(2-z_0)^k \left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} + 0.1250000000000000\,(105-z_0)^k \left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(105-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor \arg(105-z_0)/(2\pi)\rfloor}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

1/6(((24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + 16 210^(1/6) (2 2^(1/6) + 105^(1/6)) + 16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4))) π)/(739.0974460175287))^2

Input interpretation

$$\frac{1}{6}\left(\left(\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\pi\right)/739.0974460175287\right)^2$$

Result

1.644934066848226…

1.644934066…. = ζ(2) = $\pi^2/6$ (trace of the instanton shape)

Series representations

$$\frac{1}{6}\left(\left(\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\pi\right)/739.09744601752870000\right)^2 = 0.000175738870145750106\,\pi^2 \left(28.10064450927898438+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k\left(1.000000000000000\,(2-z_0)^k+0.1250000000000000\,(105-z_0)^k\right)z_0^{-k}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{6}\left(\left(\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\pi\right)/739.09744601752870000\right)^2 = 0.000175738870145750106\,\pi^2\left(28.10064450927898438+\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k x^{-k}\left(1.000000000000000\,(2-x)^k\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)+0.1250000000000000\,(105-x)^k\exp\left(i\pi\left\lfloor\frac{\arg(105-x)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k\sqrt{x}\right)^2 \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{1}{6}\left(\left(\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\pi\right)/739.09744601752870000\right)^2 = 3.0510220511414948897\times 10^{-7}\,\pi^2\left(48\times 2^{3/4}\sqrt[4]{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)+\sum_{k=0}^{\infty}\frac{1}{k!}3(-1)^k\left(-\frac{1}{2}\right)_k z_0^{1/2-k}\left(8(2-z_0)^k\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}+(105-z_0)^k\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(105-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(105-z_0)/(2\pi)\rfloor}\right)\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

1+1/16(((24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + 16 210^(1/6) (2 2^(1/6) + 105^(1/6)) + 16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4))) π)/(739.0974460175287))^2+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$1 + \frac{1}{16}\left(\left(\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\pi\right)/739.0974460175287\right)^2 + C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Result

1.617973070449617…

1.61797307…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

And again:

((4 (24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + 16 210^(1/6) (2 2^(1/6) + 105^(1/6)) + 16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4))))/(739.0974460175287))^6

Input interpretation

$$\left(\left(4\left(24\sqrt{2} + 48\times 2^{3/4}\sqrt[4]{105} + 3\sqrt{105} + 16\sqrt[6]{210}\left(2\sqrt[6]{2} + \sqrt[6]{105}\right) + 16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4} + \sqrt[4]{105}\right)\right)\right)/739.0974460175287\right)^6$$

Result

4096.00000000000…

$4096 = 64^2$

27sqrt(((4 (24 sqrt(2) + 48 2^(3/4) 105^(1/4) + 3 sqrt(105) + 16 210^(1/6) (2 2^(1/6) + 105^(1/6)) + 16 2^(3/8) 105^(1/8) (2 2^(3/4) + 105^(1/4))))/(739.0974460175287))^6)+1

Input interpretation

$$27\sqrt{\left(\left(4\left(24\sqrt{2}+48\times 2^{3/4}\sqrt[4]{105}+3\sqrt{105}+16\sqrt[6]{210}\left(2\sqrt[6]{2}+\sqrt[6]{105}\right)+16\times 2^{3/8}\sqrt[8]{105}\left(2\times 2^{3/4}+\sqrt[4]{105}\right)\right)\right)/739.0974460175287\right)^6+1}$$

Result

1729.000000000000…

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

An Introduction to Higher-Spin Fields - *Augusto Sagnotti*-Scuola Normale Superiore, Pisa - Eotvos Superstring Workshop, Budapest, Sept. 2007

We have:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi)^2 + s\,\partial\cdot\varphi\,C + s(s-1)\,\partial\cdot C\,D + \frac{s(s-1)}{2}(\partial_\mu D)^2 - \frac{s}{2}C^2,$$

1/2(δ*φ)^2 + s*δ * φ*C + s(s-1)*δ*C*D + (s(s-1))/2 * (δ*D)^2 – s/2 * C^2

Input

$$-\frac{1}{2}(\delta\,\phi)^2 + s\,\delta\,\phi\,C + s\,(s-1)\,\delta\,C\,D + \left(\frac{1}{2}\,(s\,(s-1))\right)(\delta\,D)^2 - \frac{s}{2}\,C^2$$

ϕ is the golden ratio

Solutions

$$s = \frac{1}{2\left(C\,\delta\,D + \frac{\delta^2\,D^2}{2}\right)}\left(\left(\frac{C^2}{2} - C\,\delta\,\phi + C\,\delta\,D + \frac{\delta^2\,D^2}{2}\right) \pm \sqrt{\left(-\frac{C^2}{2} + C\,\delta\,\phi - C\,\delta\,D - \frac{\delta^2\,D^2}{2}\right)^2 + 2\,\delta^2\,\phi^2\left(C\,\delta\,D + \frac{\delta^2\,D^2}{2}\right)}\right) \quad (\delta\,D\,(2\,C + \delta\,D) \neq 0)$$

Geometric figure
parabola

Alternate forms

$$C\left(-\frac{C\,s}{2} + D\,s\,(\delta\,s - \delta) + \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)\delta\,s\right) + \left(-\frac{3}{4} - \frac{\sqrt{5}}{4}\right)\delta^2 + D^2\,s\left(\frac{\delta^2\,s}{2} - \frac{\delta^2}{2}\right)$$

$$D\left(C\,\delta\,(s-1)\,s + \delta^2\,D\left(\frac{s}{2} - \frac{1}{2}\right)s\right) + C\left(\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)\delta\,s - \frac{C\,s}{2}\right) + \left(-\frac{3}{4} - \frac{\sqrt{5}}{4}\right)\delta^2$$

$$-\frac{C^2\,s}{2} + C\left(\delta\,D\left(s^2 - s\right) + \delta\,s\,\phi\right) - \frac{\delta^2\,\phi^2}{2} + \delta^2\,D^2\left(\frac{s^2}{2} - \frac{s}{2}\right)$$

Expanded forms

$$-\frac{C^2 s}{2} + C\,\delta\,D\,s^2 - C\,\delta\,D\,s + \frac{1}{2}\sqrt{5}\,C\,\delta\,s + \frac{C\,\delta\,s}{2} + \frac{1}{4}\left(-3-\sqrt{5}\right)\delta^2 + \frac{1}{2}\delta^2 D^2 s^2 - \frac{1}{2}\delta^2 D^2 s$$

$$-\frac{C^2 s}{2} + C\,\delta\,D\,s^2 - C\,\delta\,D\,s + C\,\delta\,s\,\phi - \frac{\delta^2\phi^2}{2} + \frac{1}{2}\delta^2 D^2 s^2 - \frac{1}{2}\delta^2 D^2 s$$

Derivative

$$\frac{\partial}{\partial s}\left(-\frac{1}{2}(\delta\,\phi)^2 + s\,\delta\,\phi\,C + s\,(s-1)\,\delta\,C\,D + \frac{1}{2}(s\,(s-1))\,(\delta\,D)^2 - \frac{s\,C^2}{2}\right) = -\frac{C^2}{2} + C\,\delta\,(D\,(2\,s-1)+\phi) + \frac{1}{2}\delta^2 D^2 (2\,s-1)$$

Indefinite integral

$$\int\left(-\frac{1}{2}(\delta\,\phi)^2 + s\,\delta\,\phi\,C + s\,(s-1)\,\delta\,C\,D + \frac{1}{2}(s\,(s-1))\,(\delta\,D)^2 - \frac{s\,C^2}{2}\right)ds = \frac{1}{12}s\left(-3\,C^2 s + 2\,C\,\delta\,s\,(D\,(2\,s-3)+3\,\phi) + \delta^2\left(D^2 s\,(2\,s-3) - 6\,\phi^2\right)\right) + \text{constant}$$

From

$$-\frac{1}{2}(\delta\,\phi)^2 + s\,\delta\,\phi\,C + s\,(s-1)\,\delta\,C\,D + \left(\frac{1}{2}(s\,(s-1))\right)(\delta\,D)^2 - \frac{s}{2}C^2$$

first equation

for $s = 0.8\ /\ \pi^2$ and $\delta = 2$, we obtain:

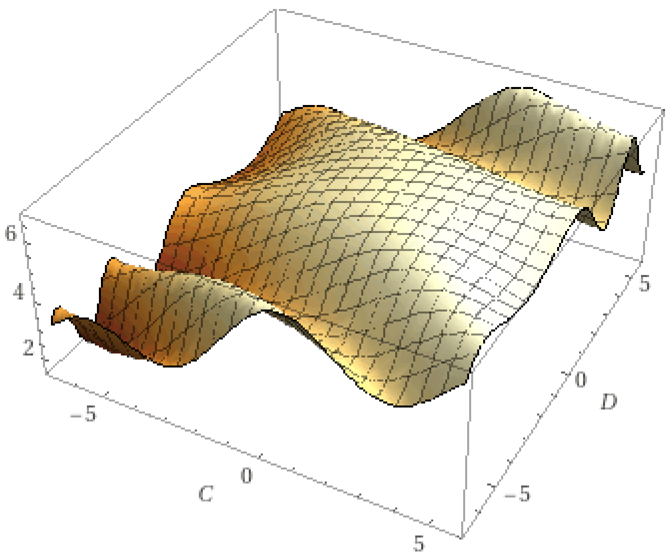
3d Plot zeta(24) (1/2(2*φ)^2 + (0.8π^-2)*2 * φ*C + cos(0.8π^-2(0.8π^-2-1)*2*C*D + (0.8π^-2(0.8π^-2-1))/2 * (2*D)^2) – sin((0.8π^-2)/2 * C^2))

Input interpretation

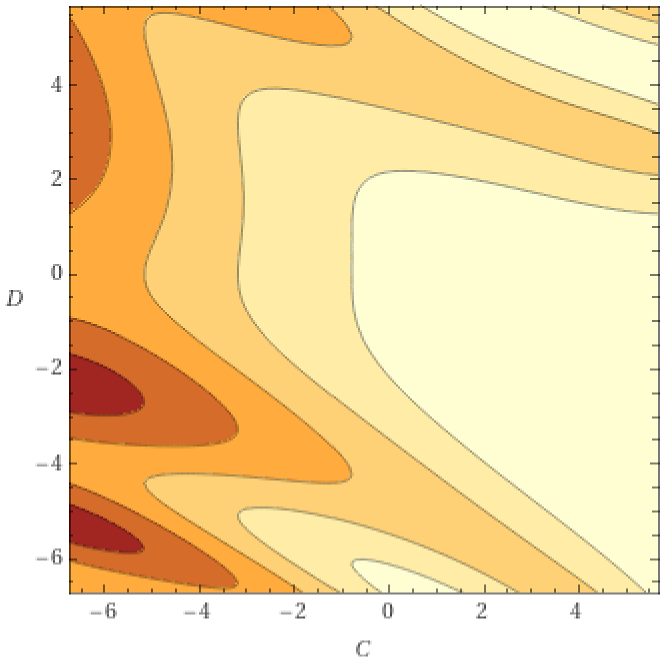
3D plot	$\zeta(24)\left(\frac{1}{2}(2\phi)^2+\frac{0.8}{\pi^2}\times 2\phi C+\cos\left(\frac{0.8\left(\left(\frac{0.8}{\pi^2}-1\right)\times 2CD\right)}{\pi^2}+\left(\frac{1}{2}\times\frac{0.8\left(\frac{0.8}{\pi^2}-1\right)}{\pi^2}\right)(2D)^2\right)-\sin\left(\frac{\frac{0.8}{\pi^2}}{2}C^2\right)\right)$

$\zeta(s)$ is the Riemann zeta function
ϕ is the golden ratio

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



and also:

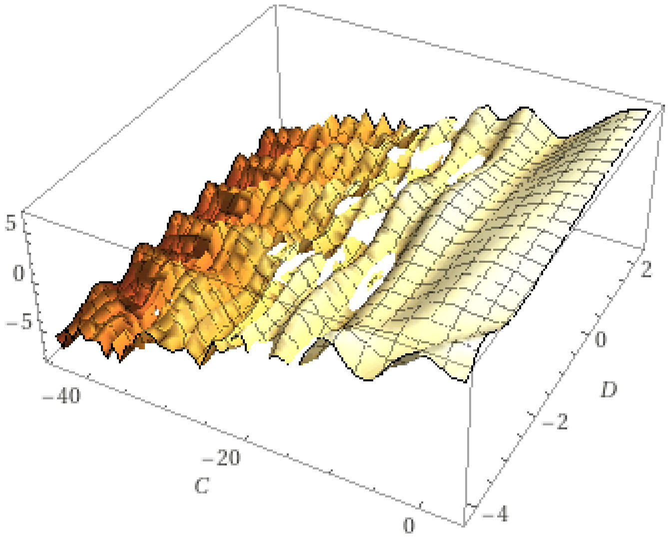
3d Plot zeta(24) (1/2(2*φ)^2 + (0.8π^-2)*2 * φ*C + cos(0.8π^-2(0.8π^-2-1)*2*C*D + cosh((0.8π^-2(0.8π^-2-1))/2 * (2*D)^2)) – sin((0.8π^-2)/2 * C^2))

Input interpretation

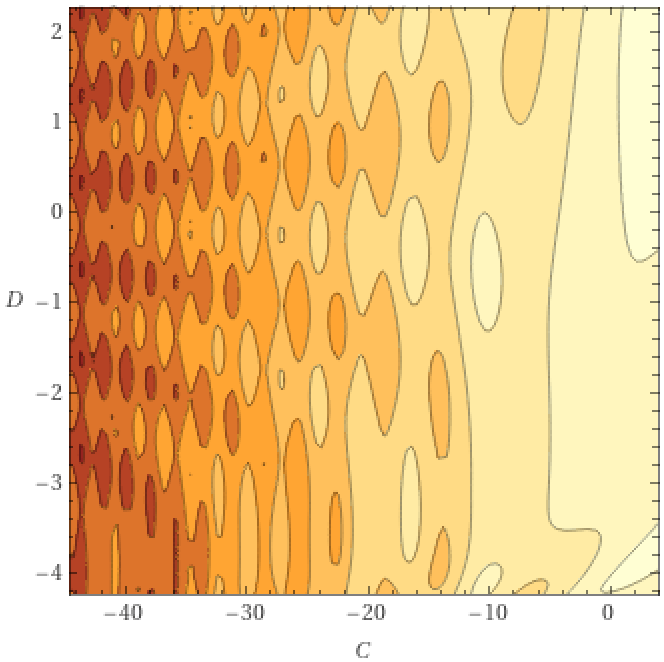
3D plot	$\zeta(24)\left(\frac{1}{2}(2\phi)^2 + \frac{0.8}{\pi^2}\times 2\phi C + \cos\left(\frac{0.8\left(\left(\frac{0.8}{\pi^2}-1\right)\times 2 C D\right)}{\pi^2}\right) + \cosh\left(\left(\frac{1}{2}\times\frac{0.8\left(\frac{0.8}{\pi^2}-1\right)}{\pi^2}\right)(2D)^2\right) - \sin\left(\frac{\frac{0.8}{\pi^2}}{2}C^2\right)\right)$

$\zeta(s)$ is the Riemann zeta function
$\cosh(x)$ is the hyperbolic cosine function
ϕ is the golden ratio

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



And again, after some calculations, we obtain:

3d Plot zeta(24) (sinh(1/2(2*φ)^2 + (0.8π^-2)*2 * φ*C) + cosh(0.8π^-2(0.8π^-2-1)*2*C*D + cos((0.8π^-2(0.8π^-2-1))/2 * (2*D)^2)) – sin((0.8π^-2)/2 * C^2))

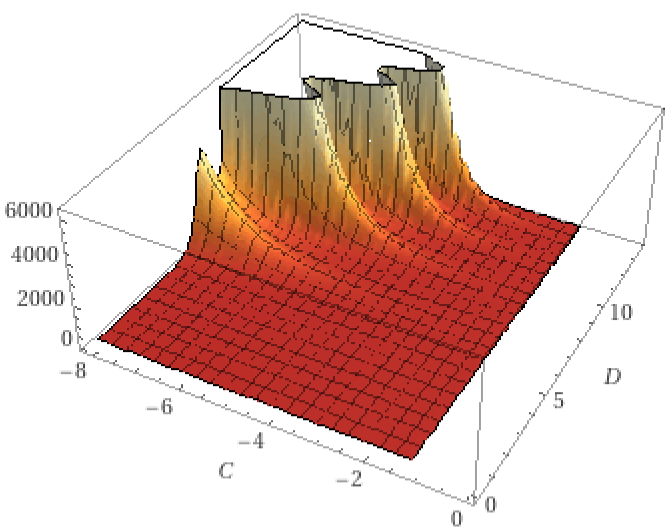
Input interpretation

3D plot	$\zeta(24)\left(\sinh\left(\frac{1}{2}(2\phi)^2+\frac{0.8}{\pi^2}\times 2\phi C\right)+\cosh\left(\frac{0.8\left(\left(\frac{0.8}{\pi^2}-1\right)\times 2CD\right)}{\pi^2}\right)+\cos\left(\left(\frac{1}{2}\times\frac{0.8\left(\frac{0.8}{\pi^2}-1\right)}{\pi^2}\right)(2D)^2\right)-\sin\left(\frac{\frac{0.8}{\pi^2}}{2}C^2\right)\right)$

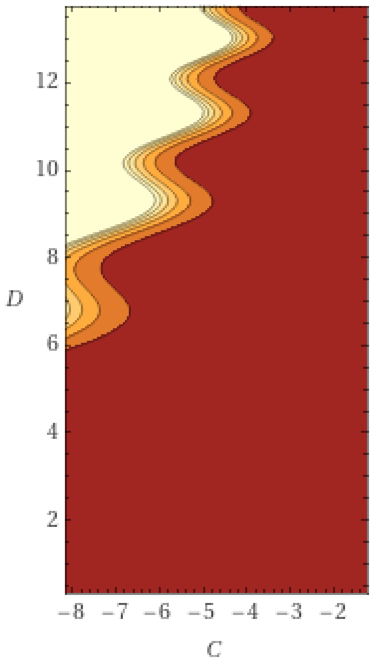
$\zeta(s)$ is the Riemann zeta function
$\sinh(x)$ is the hyperbolic sine function
$\cosh(x)$ is the hyperbolic cosine function
ϕ is the golden ratio

3D plot

(figure that can be related to a D-brane/Instanton and to a sector of the Riemann zeta function landscape)



Contour plot



From

$$s = \frac{1}{2\left(C\,\delta\,D + \frac{\delta^2 D^2}{2}\right)}\left(\left(\frac{C^2}{2} - C\,\delta\,\phi + C\,\delta\,D + \frac{\delta^2 D^2}{2}\right) \pm \sqrt{\left(-\frac{C^2}{2} + C\,\delta\,\phi - C\,\delta\,D - \frac{\delta^2 D^2}{2}\right)^2 + 2\,\delta^2\,\phi^2\left(C\,\delta\,D + \frac{\delta^2 D^2}{2}\right)}\right) \quad (\delta\, D\,(2\,C + \delta\,D) \neq 0)$$

we obtain:

((C^2/2 - C δ φ + C δ D + (δ^2 D^2)/2) + √((-C^2/2 + C δ φ - C δ D - (δ^2 D^2)/2)^2 + 2 δ^2 φ^2 (C δ D + (δ^2 D^2)/2)))/(2 (C δ D + (δ^2 D^2)/2))

Input

$$\frac{1}{2\left(C\,\delta\,D+\frac{1}{2}\left(\delta^2\,D^2\right)\right)}\left(\left(\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{1}{2}\left(\delta^2\,D^2\right)\right)+\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{1}{2}\left(\delta^2\,D^2\right)\right)^2+2\,\delta^2\,\phi\left(C\,\delta\,D+\frac{1}{2}\left(\delta^2\,D^2\right)\right)^2}\right)$$

$\phi(n)$ is the Euler totient function

ϕ is the golden ratio

Exact result

$$\frac{1}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}\left(\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2\,D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}+\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)$$

Alternate forms

$$\frac{1}{2\,\delta\,D\,(2\,C+\delta\,D)}\left(2\sqrt{\frac{1}{4}\left(C^2+2\,C\,\delta\,(D-\phi)+\delta^2\,D^2\right)^2+2\,\delta^2\,\phi\left(\frac{1}{2}\,D\,\delta\,(2\,C+D\,\delta)\right)^2}+C^2-C\,\delta\left(-2\,D+\sqrt{5}+1\right)+\delta^2\,D^2\right)$$

$$\frac{1}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}\left(\sqrt{\left(-\frac{C^2}{2}+\frac{1}{2}\left(1+\sqrt{5}\right)C\,\delta-C\,\delta\,D-\frac{\delta^2\,D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}+\frac{C^2}{2}+\frac{1}{2}\left(-1-\sqrt{5}\right)C\,\delta+C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)$$

Alternate form assuming C, D, and δ are positive

$$\frac{C^2}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2\,D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}-\frac{C\,\delta\,\phi}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{C\,\delta\,D}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{\delta^2\,D^2}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}$$

Expanded form

$$\frac{C^2}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{\sqrt{\left(-\frac{C^2}{2}+\frac{1}{2}\left(1+\sqrt{5}\right)C\,\delta-C\,\delta\,D-\frac{\delta^2\,D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{C\,\delta\,D}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}-\frac{\sqrt{5}\,C\,\delta}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}-\frac{C\,\delta}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{\delta^2\,D^2}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}$$

Performing the following calculation

((C^2/2 - C δ φ + C δ D + (δ^2 D^2)/2) + √((-C^2/2 + C δ φ - C δ D - (δ^2 D^2)/2)^2 + 2 δ^2 φ^2 (C δ D + (δ^2 D^2)/2)))/(2 (C δ D + (δ^2 D^2)/2))dxdy

we obtain:

Indefinite integral

$$\int\int \frac{1}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}\left(\left(\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)+\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2\,D^2}{2}\right)^2+2\,\delta^2\,\phi\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)^2}\right)dx\,dy=$$

$$\frac{x\,y\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2\,D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{C^2\,x\,y}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}-\frac{C\,\delta\,x\,y\,\phi}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{C\,\delta\,D\,x\,y}{2\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+\frac{\delta^2\,D^2\,x\,y}{4\left(C\,\delta\,D+\frac{\delta^2\,D^2}{2}\right)}+c_1\,x+c_2$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \frac{1}{2\left(C\,\delta\,D+\frac{\delta^2 D^2}{2}\right)} \left(\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2 D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}+\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{\delta^2 D^2}{2}\right)dy\,dx = \frac{1}{2\left(C\,\delta\,D+\frac{\delta^2 D^2}{2}\right)}\,\pi\,R^2\left(\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2 D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}+\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{\delta^2 D^2}{2}\right)$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} \frac{1}{2\left(C\,D\,\delta+\frac{D^2\,\delta^2}{2}\right)} \left(\frac{C^2}{2}+C\,D\,\delta-C\,\phi\,\delta+\frac{D^2\,\delta^2}{2}+\sqrt{\left(-\frac{C^2}{2}-C\,D\,\delta+C\,\phi\,\delta-\frac{D^2\,\delta^2}{2}\right)^2+2\,\delta^2\,\phi\left(C\,D\,\delta+\frac{D^2\,\delta^2}{2}\right)^2}\right)dx\,dy = \frac{1}{C\,\delta\,D+\frac{\delta^2 D^2}{2}}\,2\,L^2\left(\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{\delta^2 D^2}{2}\right)^2+2\,\delta^2\,\phi\left(\frac{D^2\,\delta^2}{2}+C\,D\,\delta\right)^2}+\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{\delta^2 D^2}{2}\right)$$

From:

$$\frac{1}{2\left(C\,\delta\,D+\frac{1}{2}\left(\delta^2\,D^2\right)\right)}\left(\left(\frac{C^2}{2}-C\,\delta\,\phi+C\,\delta\,D+\frac{1}{2}\left(\delta^2\,D^2\right)\right)+\sqrt{\left(-\frac{C^2}{2}+C\,\delta\,\phi-C\,\delta\,D-\frac{1}{2}\left(\delta^2\,D^2\right)\right)^2+2\,\delta^2\,\phi\left(C\,\delta\,D+\frac{1}{2}\left(\delta^2\,D^2\right)\right)^2}\right)$$

simplifying for C = 64, D = 8, δ = 2 :

((64^2/2 - 64 2 φ + 64 2 8 + (2^2 8^2)/2) + √((-64^2/2 + 64 2 φ - 64 2 8 - (2^2 8^2)/2)^2 + 2 2^2 φ^2 (64 2 8 + (2^2 8^2)/2)))/(2 (64 2 8 + (2^2 8^2)/2))

Input

$$\left(\left(\frac{64^2}{2}-64\times2\,\phi+64\times2\times8+\frac{1}{2}\left(2^2\times8^2\right)\right)+\sqrt{\left(\left(-\frac{64^2}{2}+64\times2\,\phi-64\times2\times8-\frac{1}{2}\left(2^2\times8^2\right)\right)^2+2\times2^2\,\phi\left(64\times2\times8+\frac{1}{2}\left(2^2\times8^2\right)\right)^2\right)}\right)\Big/\left(2\left(64\times2\times8+\frac{1}{2}\left(2^2\times8^2\right)\right)\right)$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio

Result

$$\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200}{2304}\approx 2.68089$$

2.68089

Alternate forms

$$\frac{1}{18}\left(-\phi+\sqrt{-50\,\phi+\phi^2+697}+25\right)$$

$$\frac{1}{36}\left(49-\sqrt{5}+\sqrt{2694-98\sqrt{5}}\right)$$

$$\frac{1}{36}\left(49-\sqrt{5}+\sqrt{2\left(1347-49\sqrt{5}\right)}\right)$$

$$-\frac{\phi}{18}+\frac{\sqrt{(128\,\phi-3200)^2+1\,179\,648}}{2304}+\frac{25}{18}$$

Expanded forms

$$\frac{\sqrt{11\,034\,624-401\,408\sqrt{5}}}{2304}+\frac{1}{36}\left(49-\sqrt{5}\right)$$

$$\frac{49}{36}-\frac{\sqrt{5}}{36}+\frac{\sqrt{1\,179\,648+\left(64\left(1+\sqrt{5}\right)-3200\right)^2}}{2304}$$

and:

(((64^2/2 - 64 2 φ + 64 2 8 + (2^2 8^2)/2) + √((-64^2/2 + 64 2 φ - 64 2 8 - (2^2 8^2)/2)^2 + 2 2^2 φ^2 (64 2 8 + (2^2 8^2)/2)))/(2 (64 2 8 + (2^2 8^2)/2)))dxdy

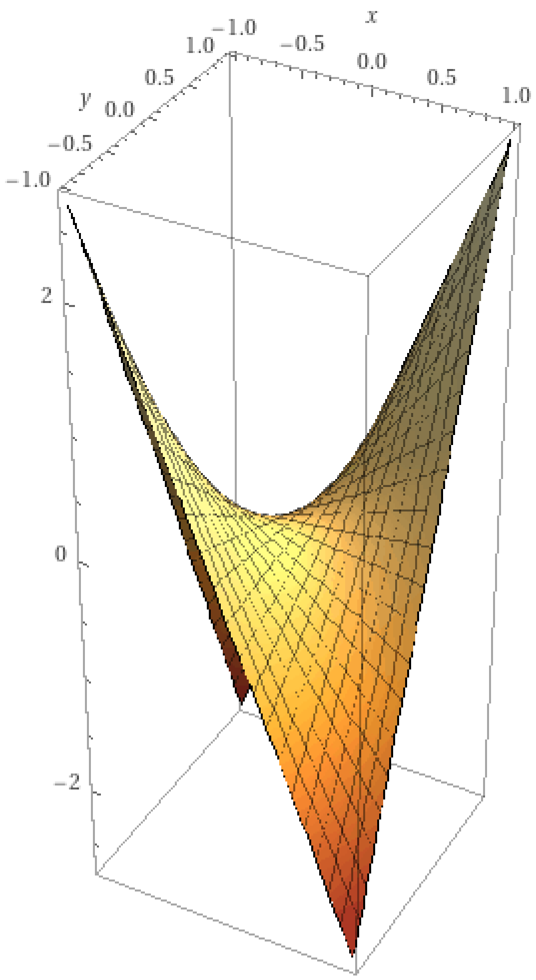
Indefinite integral

$$\int\int\left(\left(\left(\frac{64^2}{2}-64\times 2\,\phi+64\times 2\times 8+\frac{2^2\times 8^2}{2}\right)+\sqrt{\left(\left(-\frac{64^2}{2}+64\times 2\,\phi-64\times 2\times 8-\frac{2^2\times 8^2}{2}\right)^2+2\times 2^2\,\phi\left(64\times 2\times 8+\frac{2^2\times 8^2}{2}\right)^2\right)}\right)\Big/\left(2\left(64\times 2\times 8+\frac{2^2\times 8^2}{2}\right)\right)\right)dx\,dy=c_1\,x+c_2+\frac{x\,y\,\sqrt{(128\,\phi-3200)^2+1\,179\,648}}{2304}-\frac{x\,y\,\phi}{18}+\frac{25\,x\,y}{18}$$

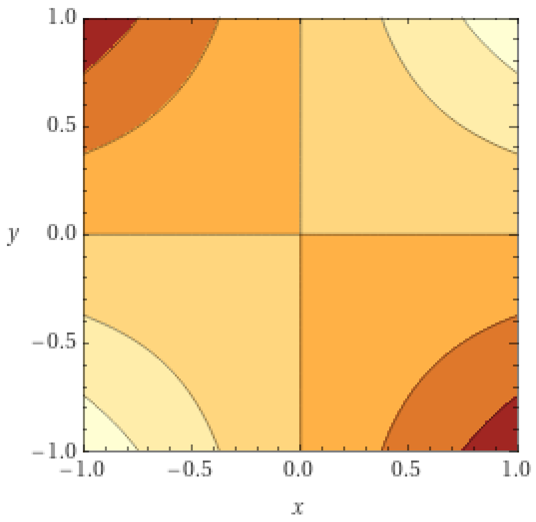
$\phi(n)$ is the Euler totient function

ϕ is the golden ratio

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \frac{-128\,\phi + \sqrt{(128\,\phi - 3200)^2 + 1\,179\,648} + 3200}{2304}\,dy\,dx =$$
$$\frac{\pi R^2 \left(-128\,\phi + \sqrt{(128\,\phi - 3200)^2 + 1\,179\,648} + 3200\right)}{2304}$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} \frac{3200 - 128\,\phi + \sqrt{1\,179\,648 + (-3200 + 128\,\phi)^2}}{2304}\,dx\,dy =$$
$$\frac{1}{576} L^2 \left(-128\,\phi + \sqrt{(128\,\phi - 3200)^2 + 1\,179\,648} + 3200\right)$$

Dividing the two integral results by

$$\frac{-128\,\phi + \sqrt{(128\,\phi - 3200)^2 + 1\,179\,648} + 3200}{2304} \approx 2.68089$$

we obtain:

((π(-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200))/2304) 1/ ((-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)/2304)

Input

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}\times\frac{1}{\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200}{2304}}$$

ϕ is the golden ratio

Exact result

π

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923 …

3.141592653….

Property

π is a transcendental number

Series representations

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)2304}{2304}}=4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}$$

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)2304}{2304}} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)2304}{2304}} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)2304}{2304}} = 4\int_0^1 \sqrt{1-t^2}\,dt$$

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)2304}{2304}} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}\,+3200\right)2304}{2304}} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

1/6(((π(-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200))/2304) 1/ ((-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)/2304))^2

Input

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}\times\frac{1}{\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200}{2304}}\right)^2$$

ϕ is the golden ratio

Exact result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.6449340668482264364724151666460251892189499012067984377355582293
...

1.644934066…. = ζ(2)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{2304\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}}\right)^2=\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{2304\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}}\right)^2=-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{2304\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}}\right)^2=\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^2}$$

Integral representations

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{2304\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}}\right)^2=\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{2304\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}}\right)^2=\frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{\frac{2304\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}}\right)^2=\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

1+1/16(((π(-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200))/2304) 1/ ((-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)/2304))^2+(MRB const)^(1-1/(4π)+π)

Input

$$1+\frac{1}{16}\left(\frac{\pi\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)}{2304}\times\frac{1}{\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200}{2304}}\right)^2+C_{\text{MRB}}{}^{1-1/(4\,\pi)+\pi}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}{}^{1-1/(4\,\pi)+\pi}+1+\frac{\pi^2}{16}$$

Decimal approximation

1.6179730704496170410388008481138568890480655109784256217769273353

...

1.6179730704…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$\frac{1}{16}\left(16\,C_{\text{MRB}}{}^{1-1/(4\,\pi)+\pi}+16+\pi^2\right)$$

$$\frac{1}{16}\,C_{\text{MRB}}{}^{-1/(4\,\pi)}\left(16\sqrt[4\pi]{C_{\text{MRB}}}+16\,C_{\text{MRB}}{}^{1+\pi}+\pi^2\sqrt[4\pi]{C_{\text{MRB}}}\right)$$

And again:

((((1/576 (-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)))1/ ((-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)/2304)))^6

Input

$$\left(\left(\frac{1}{576}\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)\right)\times\frac{1}{\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200}{2304}}\right)^6$$

ϕ is the golden ratio

Exact result

4096

4096 = 64^2

27sqrt(((((1/576 (-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)))1/ ((-128 φ + sqrt((128 φ - 3200)^2 + 1179648) + 3200)/2304)))^6)+1

Input

$$27\sqrt{\left(\left(\frac{1}{576}\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200\right)\right)\times\frac{1}{\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1\,179\,648}+3200}{2304}}\right)^6}+1$$

ϕ is the golden ratio

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$27\sqrt{\left(\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1179648}+3200}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1179648}+3200\right)576}{2304}}\right)^6}+1=1+27\sqrt{4095}\sum_{k=0}^{\infty}4095^{-k}\binom{\frac{1}{2}}{k}$$

$$27\sqrt{\left(\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1179648}+3200}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1179648}+3200\right)576}{2304}}\right)^6}+1=1+27\sqrt{4095}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4095}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$27\sqrt{\left(\frac{-128\,\phi+\sqrt{(128\,\phi-3200)^2+1179648}+3200}{\frac{\left(-128\,\phi+\sqrt{(128\,\phi-3200)^2+1179648}+3200\right)576}{2304}}\right)^6}+1=1+\frac{27\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4095^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)
$\Gamma(x)$ is the gamma function
$\operatorname{Res}_{z=z_0} f$ is a complex residue

Now, we have:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 + \frac{s}{2}(\partial \cdot \varphi)^2 + s(s-1)\,\partial \cdot \partial \cdot \varphi\, D + s(s-1)\,(\partial_\mu D)^2 + \frac{s(s-1)(s-2)}{2}(\partial \cdot D)^2$$

-1/2*(δ*φ)^2 + s/2*(δ*φ)^2 + s(s-1)*δ*δ*φ*D + s(s-1)*(δ*D)^2+ (s(s-1)(s-2))/2 * (δ*D)^2

Input

$$-\frac{1}{2}(\delta\,\phi)^2 + \frac{s}{2}(\delta\,\phi)^2 + s\,(s-1)\,\delta\,\delta\,\phi\,D + s\,(s-1)\,(\delta\,D)^2 + \left(\frac{1}{2}(s\,(s-1)\,(s-2))\right)(\delta\,D)^2$$

ϕ is the golden ratio

Exact result

$$-\frac{\delta^2\,\phi^2}{2} + \delta^2\,D^2\,(s-1)\,s + \frac{1}{2}\,\delta^2\,D^2\,(s-2)\,(s-1)\,s + \delta^2\,D\,(s-1)\,s\,\phi + \frac{1}{2}\,\delta^2\,s\,\phi^2$$

second equation

Alternate forms

$$\left(-\frac{3}{4} - \frac{\sqrt{5}}{4}\right)\delta^2 + s\left(s\left(\frac{1}{2}\,\delta^2\,D^2\,s + \delta^2\left(-\frac{D}{2} + \frac{\sqrt{5}}{2} + \frac{1}{2}\right)D\right) + \delta^2\left(\left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right)D + \frac{\sqrt{5}}{4} + \frac{3}{4}\right)\right)$$

$$D\left(\delta^2 D\left(\frac{s}{2}-\frac{1}{2}\right)s^2+\delta^2 s\left(\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)s-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)+\delta^2\left(\left(\frac{3}{4}+\frac{\sqrt{5}}{4}\right)s-\frac{\sqrt{5}}{4}-\frac{3}{4}\right)$$

$$\frac{\delta^2 (D\,s+\phi)^3}{2\,D}-\frac{\left(\sqrt{5}\,\delta^2+\delta^2+2\,\delta^2 D\right)(D\,s+\phi)^2}{4\,D}$$

Expanded form

$$-\frac{\delta^2\phi^2}{2}+\frac{1}{2}\delta^2 D^2 s^3-\frac{1}{2}\delta^2 D^2 s^2+\delta^2 D\,s^2\phi-\delta^2 D\,s\,\phi+\frac{1}{2}\delta^2 s\,\phi^2$$

Root

$$D\neq 0\,,\quad s=\frac{-1-\sqrt{5}}{2\,D}$$

Roots

$$s=1$$

$$\delta=0$$

Derivative

$$\frac{\partial}{\partial s}\left(-\frac{1}{2}(\delta\,\phi)^2+\frac{1}{2}s\,(\delta\,\phi)^2+s\,(s-1)\,\delta\,\delta\,\phi\,D+s\,(s-1)\,(\delta\,D)^2+\frac{1}{2}(s\,(s-1)\,(s-2))\,(\delta\,D)^2\right)=\frac{1}{2}\delta^2\left(D^2 s\,(3\,s-2)+2\,D\,(2\,s-1)\,\phi+\phi^2\right)$$

Indefinite integral

$$\int \left(-\frac{1}{2}\phi^2 \delta^2 + \frac{1}{2}\phi^2 s \delta^2 + D^2 (-1+s) s \delta^2 + D \phi (-1+s) s \delta^2 + \frac{1}{2} D^2 (-2+s)(-1+s) s \delta^2\right) ds = \frac{1}{24}\delta^2 s \left(D^2 (3s-4) s^2 + 4 D (2s-3) s \phi + 6 (s-2) \phi^2\right) + \text{constant}$$

From the result of the above indefinite integral, we obtain:

((1/24 s (6 φ^2 (-2 + s) + 4 D φ s (-3 + 2 s) + D^2 s^2 (-4 + 3 s)) δ^2))

Input

$$\frac{1}{24} s \left(6\phi(-2+s)^2 + 4 D \phi s (-3+2s) + D^2 s^2 (-4+3s)\right)\delta^2$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio

Alternate forms

$$\frac{1}{24}\delta^2 s \left(3 D^2 s^3 - 4 D^2 s^2 + 8 D s^2 \phi - 12 D s \phi + 6 \phi(s-2)^2\right)$$

$$\frac{1}{24}\delta^2 s \left(D^2 (3s-4) s^2 + 2\left(1+\sqrt{5}\right) D (2s-3) s + 6\phi(s-2)^2\right)$$

$$\delta^2 D^2 \left(\frac{s^4}{8} - \frac{s^3}{6}\right) + \delta^2 D \left(\frac{s^3 \phi}{3} - \frac{s^2 \phi}{2}\right) + \frac{1}{4}\delta^2 s \phi(s-2)^2$$

Alternate form assuming D, s, and δ are positive

$$\frac{1}{24}\delta^2 D^2 (3s-4) s^3 + \frac{1}{6}\delta^2 D (2s-3) s^2 \phi + \frac{1}{4}\delta^2 s \phi(s-2)^2$$

Expanded forms

$$\frac{1}{8}\delta^2 D^2 s^4 - \frac{1}{6}\delta^2 D^2 s^3 + \frac{1}{6}\sqrt{5}\,\delta^2 D s^3 + \frac{1}{6}\delta^2 D s^3 + \frac{1}{4}\left(-1-\sqrt{5}\right)\delta^2 D s^2 + \frac{1}{4}\delta^2 s\,\phi(s-2)^2$$

$$\frac{1}{8}\delta^2 D^2 s^4 - \frac{1}{6}\delta^2 D^2 s^3 + \frac{1}{3}\delta^2 D s^3 \phi - \frac{1}{2}\delta^2 D s^2 \phi + \frac{1}{4}\delta^2 s\,\phi(s-2)^2$$

Performing the following calculation

((1/24 s (6 φ^2 (-2 + s) + 4 D φ s (-3 + 2 s) + D^2 s^2 (-4 + 3 s)) δ^2))dxdy

we obtain:

Indefinite integral

$$\int\int \frac{1}{24} s\left(6\,\phi(-2+s)^2 + 4\,D\,\phi\,s\,(-3+2\,s) + D^2 s^2 (-4+3\,s)\right)\delta^2\,dx\,dy = c_1 x + c_2 + \frac{1}{24}\delta^2 D^2 (3\,s-4)\,s^3\,x\,y + \frac{1}{6}\delta^2 D\,(2\,s-3)\,s^2\,x\,y\,\phi + \frac{1}{4}\delta^2 s\,x\,y\,\phi(s-2)^2$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \frac{1}{24}\delta^2 s\left(D^2 (3\,s-4)\,s^2 + 4\,D\,(2\,s-3)\,s\,\phi + 6\,\phi(s-2)^2\right)dy\,dx = \frac{1}{24}\pi\,\delta^2 R^2 s\left(D^2 (3\,s-4)\,s^2 + 4\,D\,(2\,s-3)\,s\,\phi + 6\,\phi(s-2)^2\right)$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\frac{1}{24}\,s\,\delta^2\left(4\,D\,\phi\,s\,(-3+2\,s)+D^2\,s^2\,(-4+3\,s)+6\,\phi(-2+s)^2\right)dx\,dy = \frac{1}{6}\,\delta^2\,L^2\,s\left(D^2\,(3\,s-4)\,s^2+4\,D\,(2\,s-3)\,s\,\phi+6\,\phi(s-2)^2\right)$$

Dividing the result of the two above integrals by

$$\frac{1}{24}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)D\,(2\,s-3)\,s+6\,\phi(s-2)^2\right)$$

we obtain:

(1/24 π δ^2 s (D^2 (3 s - 4) s^2 + 4 D (2 s - 3) s φ + 6 φ(s - 2)^2))/(1/24 δ^2 s (D^2 (3 s - 4) s^2 + 2 (1 + sqrt(5)) D (2 s - 3) s + 6 φ(s – 2)^2))

Input

$$\frac{\frac{1}{24}\,\pi\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+4\times\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)}{\frac{1}{24}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\times\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)}$$

$\phi(n)$ is the Euler totient function

ϕ is the golden ratio

Result

$$\frac{\pi\left(D^2\,(3\,s-4)\,s^2+6\,\phi(s-2)^2+8\,s\,\phi\right)}{D^2\,(3\,s-4)\,s^2+6\,\phi(s-2)^2+4\left(1+\sqrt{5}\right)s}\approx 3.14159$$

3.14159 ≈ π

Alternate form assuming D and s are positive

$$\frac{8 \pi s \phi}{D^2 (3 s - 4) s^2 + 6 \phi (s - 2)^2 + 4 \left(1 + \sqrt{5}\right) s} + \frac{\pi D^2 (3 s - 4) s^2}{D^2 (3 s - 4) s^2 + 6 \phi (s - 2)^2 + 4 \left(1 + \sqrt{5}\right) s} + \frac{6 \pi \phi (s - 2)^2}{D^2 (3 s - 4) s^2 + 6 \phi (s - 2)^2 + 4 \left(1 + \sqrt{5}\right) s}$$

Expanded form

π

Series representations

$$\frac{\pi \left(\delta^2 \left(s \left(D^2 (3 s - 4) s^2 + 4 \frac{\partial (2 s - 3)}{\partial s} s \phi + 6 \phi (s - 2)^2\right)\right)\right)}{\frac{1}{24} \left(\delta^2 s \left(D^2 (3 s - 4) s^2 + 2 \left(1 + \sqrt{5}\right) \frac{\partial (2 s - 3)}{\partial s} s + 6 \phi (s - 2)^2\right)\right) 24} = \pi$$

$$\frac{\pi \left(\delta^2 \left(s \left(D^2 (3 s - 4) s^2 + 4 \frac{\partial (2 s - 3)}{\partial s} s \phi + 6 \phi (s - 2)^2\right)\right)\right)}{\frac{1}{24} \left(\delta^2 s \left(D^2 (3 s - 4) s^2 + 2 \left(1 + \sqrt{5}\right) \frac{\partial (2 s - 3)}{\partial s} s + 6 \phi (s - 2)^2\right)\right) 24} = \pi \quad \text{for } -2 + s \in \mathbb{P}$$

$$\frac{\pi \left(\delta^2 \left(s \left(D^2 (3 s - 4) s^2 + 4 \frac{\partial (2 s - 3)}{\partial s} s \phi + 6 \phi (s - 2)^2\right)\right)\right)}{\frac{1}{24} \left(\delta^2 s \left(D^2 (3 s - 4) s^2 + 2 \left(1 + \sqrt{5}\right) \frac{\partial (2 s - 3)}{\partial s} s + 6 \phi (s - 2)^2\right)\right) 24} = \pi$$
$\text{for } (s \in \mathbb{Z} \text{ and } s \geq 2)$

$\mathbb{P}$ is the set of prime numbers
$\mathbb{Z}$ is the set of integers

and:

1/6((1/24 π δ^2 s (D^2 (3 s - 4) s^2 + 4 D (2 s - 3) s φ + 6 φ(s - 2)^2))/(1/24 δ^2 s (D^2 (3 s - 4) s^2 + 2 (1 + sqrt(5)) D (2 s - 3) s + 6 φ(s – 2)^2)))^2

Input

$$\frac{1}{6}\left(\frac{\frac{1}{24}\pi\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+4\times\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)}{\frac{1}{24}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\times\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)}\right)^2$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio

Result

$$\frac{\pi^2\left(D^2\,(3\,s-4)\,s^2+6\,\phi(s-2)^2+8\,s\,\phi\right)^2}{6\left(D^2\,(3\,s-4)\,s^2+6\,\phi(s-2)^2+4\left(1+\sqrt{5}\right)s\right)^2}\approx 1.64493$$

1.64493 ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape)

Expanded form

$$\frac{\pi^2}{6}$$

Series representations

$$\frac{1}{6}\left(\frac{\pi\,\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{24}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)}\right)^2=\frac{\pi^2}{6}$$

$$\frac{1}{6}\left(\frac{\pi\,\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{24}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)}\right)^2=\frac{\pi^2}{6}$$

for $-2+s\in\mathbb{P}$

$$\frac{1}{6}\left(\frac{\pi\,\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{24}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)}\right)^2=\frac{\pi^2}{6}$$

for ($s\in\mathbb{Z}$ and $s\geq 2$)

$\mathbb{P}$ is the set of prime numbers

1+1/16((1/24 π δ^2 s (D^2 (3 s - 4) s^2 + 4 D (2 s - 3) s φ + 6 φ(s - 2)^2))/(1/24 δ^2 s (D^2 (3 s - 4) s^2 + 2 (1 + sqrt(5)) D (2 s - 3) s + 6 φ(s - 2)^2)))^2+(MRB const)^(1-1/(4π)+π)

Input

$$1 + \frac{1}{16}\left(\frac{\frac{1}{24}\pi\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2 + 4\times\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi + 6\,\phi(s-2)^2\right)}{\frac{1}{24}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2 + 2\left(1+\sqrt{5}\right)\times\frac{\partial(2\,s-3)}{\partial s}\,s + 6\,\phi(s-2)^2\right)}\right)^2 + C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio
C_{MRB} is the MRB constant

Result

$$\frac{\pi^2\left(D^2\,(3\,s-4)\,s^2 + 6\,\phi(s-2)^2 + 8\,s\,\phi\right)^2}{16\left(D^2\,(3\,s-4)\,s^2 + 6\,\phi(s-2)^2 + 4\left(1+\sqrt{5}\right)s\right)^2} + C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi} + 1 \approx 1.61797$$

1.61797 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$\mathrm{Factor}\left[\frac{\pi^2\left(D^2\,(3\,s-4)\,s^2 + 6\,\phi(s-2)^2 + 8\,s\,\phi\right)^2}{16\left(D^2\,(3\,s-4)\,s^2 + 6\,\phi(s-2)^2 + 4\left(1+\sqrt{5}\right)s\right)^2} + C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi} + 1,\ \mathrm{Extension} \to C_{\mathrm{MRB}}{}^{-1/(4\pi)}\right]$$

$$\frac{1}{16}\,C_{\mathrm{MRB}}{}^{-1/(4\pi)}\left(16\sqrt[4\pi]{C_{\mathrm{MRB}}} + 16\,C_{\mathrm{MRB}}{}^{1+\pi} + \pi^2\sqrt[4\pi]{C_{\mathrm{MRB}}}\right)$$

$$\frac{1}{16}\left(16\,C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi} + 16 + \pi^2\right)$$

Expanded form

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi}+1+\frac{\pi^2}{16}$$

And again:

((1/6 δ^2 s (D^2 (3 s - 4) s^2 + 4 D (2 s - 3) s φ + 6 φ(s - 2)^2))/(1/24 δ^2 s (D^2 (3 s - 4) s^2 + 2 (1 + sqrt(5)) D (2 s - 3) s + 6 φ(s – 2)^2)))^6

Input

$$\left(\frac{\frac{1}{6}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+4\times\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)}{\frac{1}{24}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\times\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)}\right)^6$$

$\phi(n)$ is the Euler totient function

ϕ is the golden ratio

Result

4096

4096 = 64^2

Series representations

$$\left(\frac{\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{1}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)6}\right)^6=4096$$

$$\left(\frac{\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{1}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)6}\right)^6=4096$$

for $-2+s\in\mathbb{P}$

$$\left(\frac{\delta^2 \left(s \left(D^2 (3 s - 4) s^2 + 4 \frac{\partial(2 s-3)}{\partial s} s \phi + 6 \phi(s-2)^2\right)\right)}{\frac{1}{24} \left(\delta^2 s \left(D^2 (3 s - 4) s^2 + 2 \left(1 + \sqrt{5}\right) \frac{\partial(2 s-3)}{\partial s} s + 6 \phi(s-2)^2\right)\right) 6}\right)^6 = 4096$$

for $(s \in \mathbb{Z}$ and $s \geq 2)$

Integral representation

$$(1+z)^a = \frac{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(s) \Gamma(-a-s)}{z^s} \, ds}{(2 \pi i) \Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

i is the imaginary unit

Symbolic integer derivatives

$$\frac{\partial^n z}{\partial z^n} = \delta_n z + \delta_{n-1} \quad \text{for } (n \in \mathbb{Z} \text{ and } n \geq 0)$$

$$\frac{\partial^n}{\partial z^n} \frac{1}{z} = (-1)^n n! \, z^{-1-n} \quad \text{for } (n \in \mathbb{Z} \text{ and } n \geq 0)$$

$$\frac{\partial^n z^a}{\partial z^n} = (a-n+1)_n \, z^{a-n} \quad \text{for } (n \in \mathbb{Z} \text{ and } n \geq 0)$$

δ_{n_1,n_2} is the Kronecker delta function

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

27sqrt(((1/6 δ^2 s (D^2 (3 s - 4) s^2 + 4 D (2 s - 3) s φ + 6 φ(s - 2)^2))/(1/24 δ^2 s (D^2 (3 s - 4) s^2 + 2 (1 + sqrt(5)) D (2 s - 3) s + 6 φ(s – 2)^2)))^6)+1

Input

$$27\sqrt{\left(\frac{\frac{1}{6}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+4\times\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)}{\frac{1}{24}\,\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\times\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)}\right)^6}+1$$

$\phi(n)$ is the Euler totient function
ϕ is the golden ratio

Result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$27\sqrt{\left(\frac{\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{1}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)6}\right)^6}+1=1729$$

$$27\sqrt{\left(\frac{\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{1}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)6}\right)^6}+1=1729$$

for $-2+s\in\mathbb{P}$

$$27\sqrt{\left(\frac{\delta^2\left(s\left(D^2\,(3\,s-4)\,s^2+4\,\frac{\partial(2\,s-3)}{\partial s}\,s\,\phi+6\,\phi(s-2)^2\right)\right)}{\frac{1}{24}\left(\delta^2\,s\left(D^2\,(3\,s-4)\,s^2+2\left(1+\sqrt{5}\right)\frac{\partial(2\,s-3)}{\partial s}\,s+6\,\phi(s-2)^2\right)\right)6}\right)^6}+1=1729$$

for ($s\in\mathbb{Z}$ and $s\geq 2$)

$\mathbb{P}$ is the set of prime numbers
$\mathbb{Z}$ is the set of integers

From

$$-\frac{\delta^2 \phi^2}{2} + \delta^2 D^2 (s-1) s + \frac{1}{2} \delta^2 D^2 (s-2)(s-1) s + \delta^2 D (s-1) s \phi + \frac{1}{2} \delta^2 s \phi^2$$

second equation

after some calculations, we obtain:

3D plot	$\zeta(24)\left(-\frac{1}{2}(2^2 \phi^2) + 2^2 y^2 \left(\frac{0.2}{\pi} - 1\right) \times \frac{0.2}{\pi} + \sinh\left(\frac{1}{2} \times 2^2 y^2 \left(\frac{0.2}{\pi} - 2\right)\left(\frac{0.2}{\pi} - 1\right) \times \frac{0.2}{\pi}\right) + \tanh\left(2^2 y \left(\frac{0.2}{\pi} - 1\right)\left(\frac{0.2}{\pi} \phi\right) + \frac{1}{2} \times 2^2 \left(\frac{0.2}{\pi} \phi^2\right)\right)\right)$

Indeed:

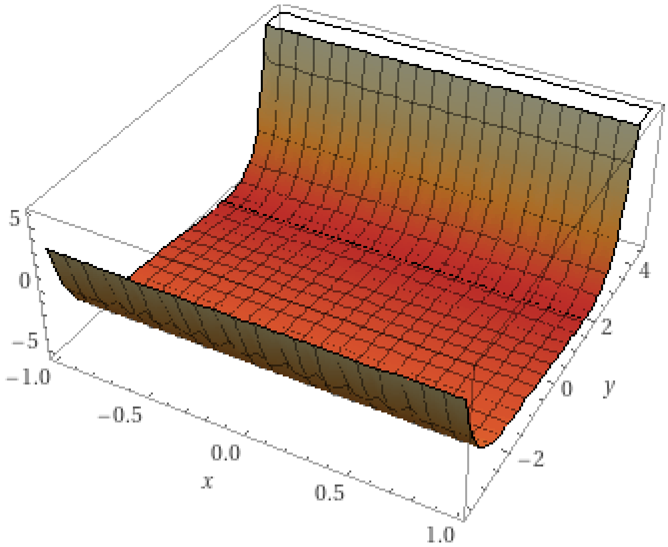
3d Plot of zeta(24)(-(2^2 φ^2)/2+2^2 (y)^2(0.2/Pi-1)0.2/Pi+sinh(1/2 2^2(y)^2(0.2/Pi-2)(0.2/Pi-1)0.2/Pi)+tanh(2^2 (y)(0.2/Pi-1)0.2/Pi*φ+1/2 2^2 0.2/Pi*φ^2))

Input interpretation

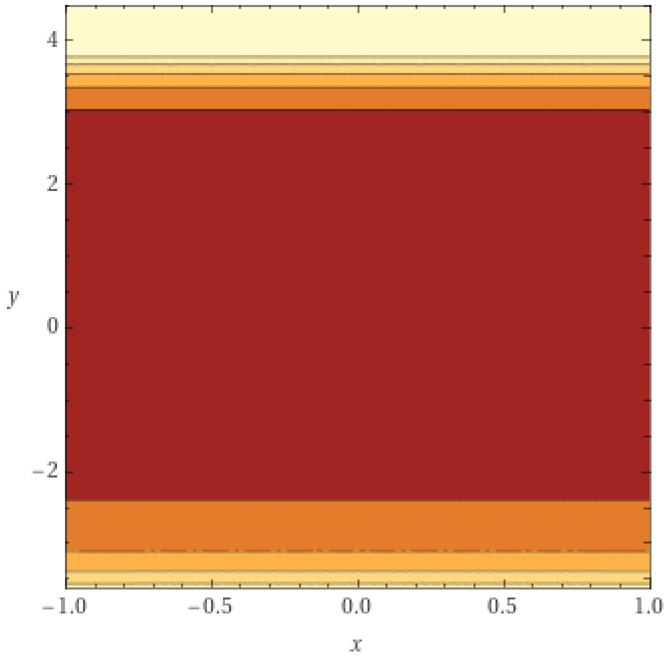
3D plot	$\zeta(24)\left(-\frac{1}{2}(2^2 \phi^2) + 2^2 y^2 \left(\frac{0.2}{\pi} - 1\right) \times \frac{0.2}{\pi} + \sinh\left(\frac{1}{2} \times 2^2 y^2 \left(\frac{0.2}{\pi} - 2\right)\left(\frac{0.2}{\pi} - 1\right) \times \frac{0.2}{\pi}\right) + \tanh\left(2^2 y \left(\frac{0.2}{\pi} - 1\right)\left(\frac{0.2}{\pi} \phi\right) + \frac{1}{2} \times 2^2 \left(\frac{0.2}{\pi} \phi^2\right)\right)\right)$

$\tanh(x)$ is the hyperbolic tangent function
ϕ is the golden ratio

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



From the result of the first equation

$$-\frac{1}{2}(\delta\,\phi)^2 + s\,\delta\,\phi\,C + s\,(s-1)\,\delta\,C\,D + \left(\frac{1}{2}(s\,(s-1))\right)(\delta\,D)^2 - \frac{s}{2}C^2$$

for δ = 2 and s = 0.8/π[2]

3D plot	$\zeta(24)\left(\sinh\left(\frac{1}{2}(2\,\phi)^2 + \frac{0.8}{\pi^2}\times 2\,\phi\,C\right) + \cosh\left(\frac{0.8\left(\left(\frac{0.8}{\pi^2}-1\right)\times 2\,C\,D\right)}{\pi^2} + \cos\left(\left(\frac{1}{2}\times\frac{0.8\left(\frac{0.8}{\pi^2}-1\right)}{\pi^2}\right)(2\,D)^2\right)\right) - \sin\left(\frac{\frac{0.8}{\pi^2}}{2}C^2\right)\right)$

we obtain:

$$\zeta(24)\left(\sinh\left(\frac{1}{2}(2\,\phi)^2 + \frac{0.8}{\pi^2}\times 2\,\phi\,C\right) + \cosh\left(\frac{0.8\left(\left(\frac{0.8}{\pi^2}-1\right)\times 2\,C\,D\right)}{\pi^2} + \cos\left(\left(\frac{1}{2}\times\frac{0.8\left(\frac{0.8}{\pi^2}-1\right)}{\pi^2}\right)(2\,D)^2\right)\right) - \sin\left(\frac{\frac{0.8}{\pi^2}}{2}C^2\right)\right)$$

Result

$$\left(236\,364\,091\,\pi^{24}\left(-\sin\left(0.0405285\,C^2\right) + \cosh\left(0.148973\,C\,D - \cos\left(0.148973\,D^2\right)\right) + \sinh\left(0.262306\,C + 2\,\phi^2\right)\right)\right) / 201\,919\,571\,963\,756\,521\,875$$

((236364091 π^24 (-sin(0.0405285 C^2) + cosh(0.148973 C D - cos(0.148973 D^2)) + sinh(0.262306 C + 2 φ^2)))/201919571963756521875)

Input interpretation

$$(236364091\,\pi^{24}\,(-\sin(0.0405285\,C^2) + \cosh(0.148973\,C\,D - \cos(0.148973\,D^2)) + \sinh(0.262306\,C + 2\,\phi^2)))/201919571963756521875$$

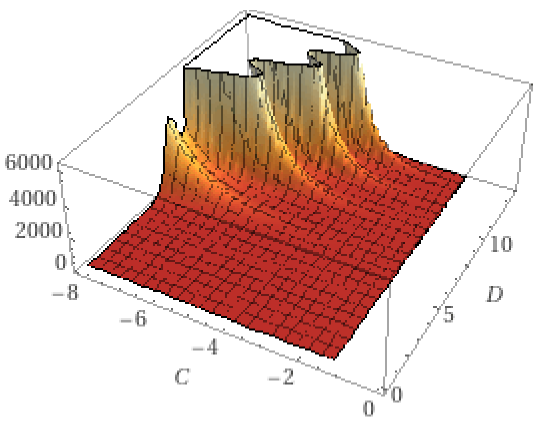
$\cosh(x)$ is the hyperbolic cosine function
$\sinh(x)$ is the hyperbolic sine function
ϕ is the golden ratio

Result

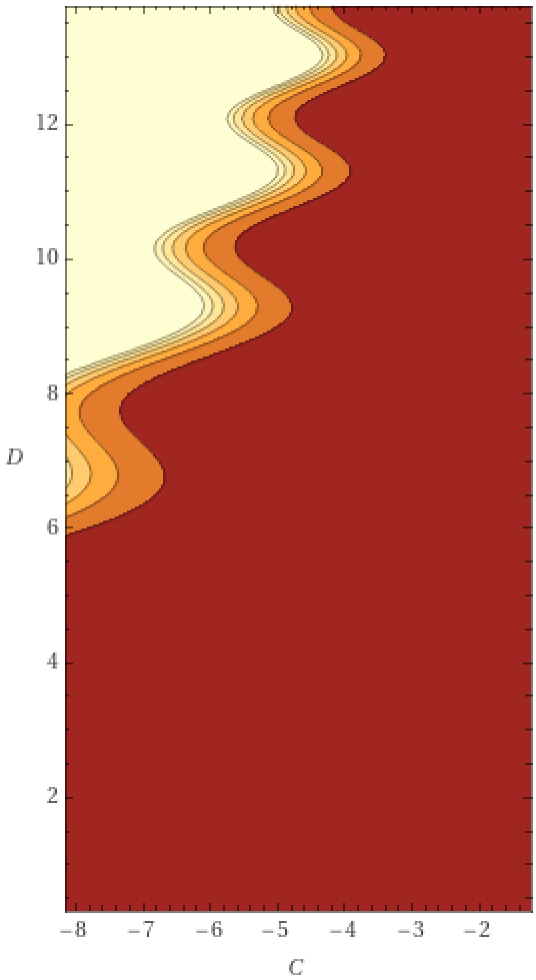
$$(236364091\,\pi^{24}\,(-\sin(0.0405285\,C^2) + \cosh(0.148973\,C\,D - \cos(0.148973\,D^2)) + \sinh(0.262306\,C + 2\,\phi^2)))/201919571963756521875$$

3D plot

(figure that can be related to a D-brane/Instanton and to a sector of the Riemann zeta function landscape)



Contour plot



Alternate forms

$$\left(236\,364\,091\,\pi^{24}\left(-\sin\left(0.0405285\,C^2\right)+\cosh\left(0.148973\,C\,D-\cos\left(0.148973\,D^2\right)\right)+\sinh\left(0.262306\,C+\sqrt{5}+3\right)\right)\right)/\,201\,919\,571\,963\,756\,521\,875$$

$$\left(236\,364\,091\,\pi^{24}\left(-2\sin\left(0.0405285\,C^2\right)+2\cosh\left(0.148973\,C\,D-\cos\left(0.148973\,D^2\right)\right)-e^{-0.262306\,C-5.23607}+e^{0.262306\,C+5.23607}\right)\right)/\,403\,839\,143\,927\,513\,043\,750$$

$$\left(236\,364\,091\left(-\pi^{24}\sin\left((0.0405285+0\,i)\,C^2\right)+\pi^{24}\cosh\left(0.148973\,C\,D-\cos\left((0.148973+0\,i)\,D^2\right)\right)+\pi^{24}\sinh\left(0.262306\,C+\sqrt{5}+3\right)\right)\right)/\,201\,919\,571\,963\,756\,521\,875$$

Expanded form

$$-\frac{236364091\,\pi^{24}\sin(0.0405285\,C^2)}{201919571963756521875}+\frac{236364091\,\pi^{24}\cosh(0.148973\,C\,D-\cos(0.148973\,D^2))}{201919571963756521875}+\frac{236364091\,\pi^{24}\sinh(0.262306\,C+2\,\phi^2)}{201919571963756521875}$$

Alternate form assuming C and D are real

$$-1.\sin(0.0405285\,C^2)+1.\cosh(0.148973\,C\,D-\cos(0.148973\,D^2)+0)+1.\sinh(0.262306\,C+2\,\phi^2)+0$$

Series expansion at C=0

$$1.\,(\cosh(\cos(0.148973\,D^2))+93.9622)+C\,(24.6482-0.148973\,D\sinh(\cos(0.148973\,D^2)))+C^2\,(0.0110965\,D^2\cosh(\cos(0.148973\,D^2))+3.19198)+C^3\,(0.282651-0.000551025\,D^3\sinh(\cos(0.148973\,D^2)))+C^4\,(0.000020522\,D^4\cosh(\cos(0.148973\,D^2))+0.0185342)+O(C^5)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial C}\left(\left(236364091\,\pi^{24}\left(-\sin(0.0405285\,C^2)+\cosh(0.148973\,C\,D-\cos(0.148973\,D^2))+\sinh(0.262306\,C+2\,\phi^2)\right)\right)/201919571963756521875\right)=-0.081057\,C\cos(0.0405285\,C^2)+0.148973\,D\sinh(0.148973\,C\,D-\cos(0.148973\,D^2))+0.262306\cosh(0.262306\,C+5.23607)$$

Indefinite integral

$$\int ((236364091\,\pi^{24}\,(\cosh(0.148973\,C\,D - \cos(0.148973\,D^2)) - \sin(0.0405285\,C^2) + \sinh(0.262306\,C + 2\,\phi^2)))/201919571963756521875)\,dC = \frac{1}{D}(6.71263\sinh(0.148973\,C\,D)\cosh(\cos(0.148973\,D^2)) - 6.71263\cosh(0.148973\,C\,D)\sinh(\cos(0.148973\,D^2)) + 358.216\,D\sinh(0.262306\,C) + 358.236\,D\cosh(0.262306\,C)) - 6.22558\,S(0.160628\,C) + \text{constant}$$

$S(x)$ is the Fresnel S integral

From the result of the second equation

3D plot	$\zeta(24)\left(-\frac{1}{2}(2^2\,\phi^2) + 2^2\,y^2\left(\frac{0.2}{\pi} - 1\right)\times\frac{0.2}{\pi} + \sinh\left(\frac{1}{2}\times 2^2\,y^2\left(\frac{0.2}{\pi} - 2\right)\left(\frac{0.2}{\pi} - 1\right)\times\frac{0.2}{\pi}\right) + \tanh\left(2^2\,y\left(\frac{0.2}{\pi} - 1\right)\left(\frac{0.2}{\pi}\,\phi\right) + \frac{1}{2}\times 2^2\left(\frac{0.2}{\pi}\,\phi^2\right)\right)\right)$

we obtain:

Input

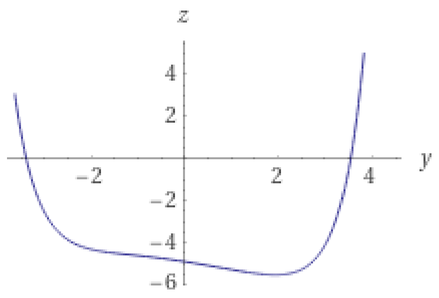
$$\zeta(24)\left(-\frac{1}{2}(2^2\,\phi^2) + 2^2\,y^2\left(\frac{0.2}{\pi} - 1\right)\times\frac{0.2}{\pi} + \sinh\left(\frac{1}{2}\times 2^2\,y^2\left(\frac{0.2}{\pi} - 2\right)\left(\frac{0.2}{\pi} - 1\right)\times\frac{0.2}{\pi}\right) + \tanh\left(2^2\,y\left(\frac{0.2}{\pi} - 1\right)\left(\frac{0.2}{\pi}\,\phi\right) + \frac{1}{2}\times 2^2\left(\frac{0.2}{\pi}\,\phi^2\right)\right)\right)$$

$\zeta(s)$ is the Riemann zeta function
$\sinh(x)$ is the hyperbolic sine function
$\tanh(x)$ is the hyperbolic tangent function
ϕ is the golden ratio

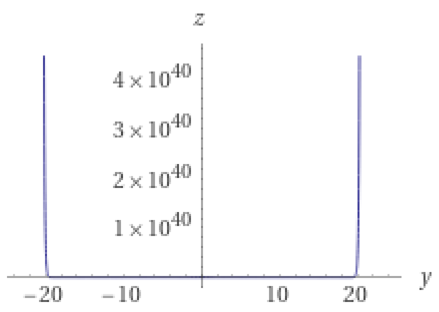
Result

$$(236364091\,\pi^{24}\,(\tanh(0.333338 - 0.385798\,y) - 0.238437\,y^2 + \sinh(0.230847\,y^2) - 2\,\phi^2))/201919571963756521875$$

Plots **(figures that can be related to the open strings)**



(y from −3.6 to 4.5)



(y from −24 to 24.8)

Alternate forms

$$1.\tanh(0.333338 - 0.385798\,y) - 0.238437\,y^2 + 1.\sinh(0.230847\,y^2) - 5.23607$$

$$-1.17059\times 10^{-12}\,(-8.54274\times 10^{11}\tanh(0.333338 - 0.385798\,y) + 2.0369\times 10^{11}\,y^2 - 8.54274\times 10^{11}\sinh(0.230847\,y^2) + 4.47303\times 10^{12})$$

$$-1.(-\tanh(0.333338 - 0.385798\,y) + 0.238437\,y^2 - \sinh(0.230847\,y^2) + 5.23607)$$

Expanded forms

$$\frac{236\,364\,091\,\pi^{24}\tanh(0.333338-0.385798\,y)}{201\,919\,571\,963\,756\,521\,875}-0.238437\,y^2+\frac{236\,364\,091\,\pi^{24}\sinh\left(0.230847\,y^2\right)}{201\,919\,571\,963\,756\,521\,875}-\frac{472\,728\,182\,\pi^{24}\,\phi^2}{201\,919\,571\,963\,756\,521\,875}$$

$$\frac{236\,364\,091\,\pi^{24}\tanh(0.333338-0.385798\,y)}{201\,919\,571\,963\,756\,521\,875}-0.238437\,y^2+\frac{236\,364\,091\,\pi^{24}\sinh\left(0.230847\,y^2\right)}{201\,919\,571\,963\,756\,521\,875}-\frac{236\,364\,091\,\pi^{24}}{40\,383\,914\,392\,751\,304\,375\,\sqrt{5}}-\frac{236\,364\,091\,\pi^{24}}{67\,306\,523\,987\,918\,840\,625}$$

Alternate form assuming y is real

$$\frac{236\,364\,091\,\pi^{24}\sinh(2\,(0.333338-0.385798\,y))}{201\,919\,571\,963\,756\,521\,875\,(\cosh(2\,(0.333338-0.385798\,y))+1)}-0.238437\,y^2+1.\sinh\left(0.230847\,y^2\right)-5.23607$$

$\cosh(x)$ is the hyperbolic cosine function

Roots

$y=-3.38855$

$y=3.54031$

Series expansion at y=0

$$-4.91455-0.345917\,y-0.0504975\,y^2+0.0118398\,y^3+0.00359743\,y^4+O\left(y^5\right)$$

(Taylor series)

Derivative

$$\frac{d}{dy}\left(\zeta(24)\left(-\frac{1}{2}\left(2^2\,\phi^2\right)+\frac{2^2\,y^2\left(\frac{0.2}{\pi}-1\right)\times 0.2}{\pi}+\sinh\left(\frac{2^2\,y^2\left(\frac{0.2}{\pi}-2\right)\left(\frac{0.2}{\pi}-1\right)\times 0.2}{2\,\pi}\right)+\tanh\left(\frac{2^2\,y\left(\frac{0.2}{\pi}-1\right)(0.2\,\phi)}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{2\,\pi}\right)\right)\right)=$$

$$-0.385798\,\text{sech}^2(0.333338-0.385798\,y)+0.461694\,y\cosh\left(0.230847\,y^2\right)-0.476873\,y$$

sech(x) is the hyperbolic secant function

Indefinite integral

$$\int\frac{236364091\,\pi^{24}\left(-2\,\phi^2-0.238437\,y^2+\sinh\left(0.230847\,y^2\right)+\tanh(0.333338-0.385798\,y)\right)}{201919571963756521875}\,dy=$$

$$1.29601\log\left(1-\tanh^2((0.333338+0\,i)-(0.385798+0\,i)\,y)\right)-0.922259\,\text{erf}(0.480465\,y)+0.922259\,\text{erfi}(0.480465\,y)-0.0794788\,y^3-5.23607\,y+\text{constant}$$

(assuming a complex-valued logarithm)

erf(x) is the error function
erfi(x) is the imaginary error function
log(x) is the natural logarithm

Alternative representations

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\,\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2\,y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\,\pi}\right)+\tanh\left(\frac{\left(2^2\,\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=$$

$$\left(-1-i\cos\left(\frac{\pi}{2}-\frac{0.4\,i\left(-2+\frac{0.2}{\pi}\right)\left(-1+\frac{0.2}{\pi}\right)y^2}{\pi}\right)-2\,\phi^2+\frac{0.8\left(-1+\frac{0.2}{\pi}\right)y^2}{\pi}+\frac{2}{1+\exp\left(-2\left(\frac{0.4\,\phi^2}{\pi}+\frac{0.8\,\phi\,y\left(-1+\frac{0.2}{\pi}\right)}{\pi}\right)\right)}\right)\zeta(24,1)$$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\,\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2\,y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\,\pi}\right)+\tanh\left(\frac{\left(2^2\,\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=$$

$$\left(i\cot\left(\frac{\pi}{2}+i\left(\frac{0.4\,\phi^2}{\pi}+\frac{0.8\,\phi\,y\left(-1+\frac{0.2}{\pi}\right)}{\pi}\right)\right)+\frac{1}{2}\left(-e^{-\left(0.4\left(-2+\frac{0.2}{\pi}\right)\left(-1+\frac{0.2}{\pi}\right)y^2\right)/\pi}+e^{\left(0.4\left(-2+\frac{0.2}{\pi}\right)\left(-1+\frac{0.2}{\pi}\right)y^2\right)/\pi}\right)-2\,\phi^2+\frac{0.8\left(-1+\frac{0.2}{\pi}\right)y^2}{\pi}\right)\zeta(24,1)$$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\,\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2\,y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\,\pi}\right)+\tanh\left(\frac{\left(2^2\,\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=$$

$$\frac{1}{-1+2^{24}}\left(-1-i\cos\left(\frac{\pi}{2}-\frac{0.4\,i\left(-2+\frac{0.2}{\pi}\right)\left(-1+\frac{0.2}{\pi}\right)y^2}{\pi}\right)-2\,\phi^2+\frac{0.8\left(-1+\frac{0.2}{\pi}\right)y^2}{\pi}+\frac{2}{1+\exp\left(-2\left(\frac{0.4\,\phi^2}{\pi}+\frac{0.8\,\phi\,y\left(-1+\frac{0.2}{\pi}\right)}{\pi}\right)\right)}\right)\zeta\left(24,\frac{1}{2}\right)$$

$\zeta(s, a)$ is the generalized Riemann zeta function

i is the imaginary unit

$\cot(x)$ is the cotangent function

Series representations

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\, y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\, 2}\right)\right)=-6.23607-0.238437\,y^2+\sum_{k=0}^{\infty}\left(2.\,(-1)^k e^{2(1+k)(0.333338-0.385798\,y)}+\frac{0.230847\,e^{-2.932k}\left(y^2\right)^{1+2k}}{(1+2k)!}\right)$$

for $\mathrm{Re}(y)>0.864022$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\, y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\, 2}\right)\right)=-0.238437\left(26.154+y^2+8.38798\sum_{k=1}^{\infty}(-1)^k q^{2k}-4.19399\sum_{k=0}^{\infty}\frac{0.230847^{1+2k}\left(y^2\right)^{1+2k}}{(1+2k)!}\right)$$

for $q=1.39562\,e^{-0.385798\,y}$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\, y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\, 2}\right)\right)=-0.238437\left(26.154+y^2+8.38798\sum_{k=1}^{\infty}(-1)^k q^{2k}-(4.19399\,i)\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+0.230847\,y^2\right)^{2k}}{(2k)!}\right)$$

for $q=1.39562\,e^{-0.385798\,y}$

$n!$ is the factorial function

$\mathrm{Re}(z)$ is the real part of z

Integral representations

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=$$
$$-5.23607-0.238437\,y^2+\int_0^1\left(0.230847\,y^2\cosh\left(0.230847\,t\,y^2\right)+(0.333338-0.385798\,y)\,\mathrm{sech}^2(t\,(0.333338-0.385798\,y))\right)dt$$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=(0.0325603+0\,i)$$
$$\left((-160.811+0\,i)-(7.32291+0\,i)\,y^2-i\,y^2\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{s+\left(0.0133226\,y^4\right)/s}}{s^{3/2}}\,ds+30.7122+0\,i\int_0^{0.333338-0.385798\,y}\mathrm{sech}^2(t)\,dt\right)\quad\text{for }\gamma>0$$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=$$
$$0.230847\left(-22.682-1.03288\,y^2-2.75776\,i\int_0^\infty\frac{-1+t^{0.21221\,i-(0.245607\,i)\,y}}{-1+t^2}\,dt+y^2\int_0^1\cosh\left(0.230847\,t\,y^2\right)dt\right)\quad\text{for }-\frac{\pi}{2}<-0.385798\,\mathrm{Im}(y)<0$$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=(-0.0325603\,i)\left(-160.811\,i-(7.32291\,i)\,y^2+(1+0\,i)\,y^2\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{s+\left(0.0133226\,y^4\right)/s}}{s^{3/2}}\,ds+19.552+0\,i\int_0^\infty\frac{-1+t^{0.21221\,i-(0.245607\,i)\,y}}{-1+t^2}\,dt\right)$$

for ($\gamma > 0$ and $\mathrm{Im}(y) > 0$ and $\mathrm{Im}(y) < 4.07155$)

$\mathrm{Im}(z)$ is the imaginary part of z

Functional equations

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=\left(-5.23607-0.238437\,y^2+2.\tanh(0.166669-0.192899\,y)+\left(-5.23607-0.238437\,y^2\right)\tanh^2(0.166669-0.192899\,y)+\sinh\left(0.230847\,y^2\right)\left(1.+1.\tanh^2(0.166669-0.192899\,y)\right)\right)/\left(1+\tanh^2(0.166669-0.192899\,y)\right)$$

$$\zeta(24)\left(-\frac{1}{2}\left(2^2\phi^2\right)+\frac{2^2\times 0.2\,y^2\left(\frac{0.2}{\pi}-1\right)}{\pi}+\sinh\left(\frac{\left(2^2 y^2\right)\left(\frac{0.2}{\pi}-2\right)\left(\left(\frac{0.2}{\pi}-1\right)0.2\right)}{2\pi}\right)+\tanh\left(\frac{\left(2^2\phi\,y\left(\frac{0.2}{\pi}-1\right)\right)0.2}{\pi}+\frac{2^2\left(0.2\,\phi^2\right)}{\pi\,2}\right)\right)=\left(236\,364\,091\,\pi^{24}\left(-2\,\phi^2-0.238437\,y^2+\sinh\left(0.230847\,y^2\right)+\frac{2\tanh\left(\frac{1}{2}\,(0.333338-0.385798\,y)\right)}{1+\tanh^2\left(\frac{1}{2}\,(0.333338-0.385798\,y)\right)}\right)\right)/\,201\,919\,571\,963\,756\,521\,875$$

From

$$(236364091\,\pi^{24}\,(-\sin(0.0405285\,C^2) + \cosh(0.148973\,C\,D - \cos(0.148973\,D^2)) + \sinh(0.262306\,C + 2\,\phi^2)))/201919571963756521875$$

for C = D = 1 :

(236364091 π^24 (-sin(0.0405285) + cosh(0.148973 1 - cos(0.148973)) + sinh(0.262306 1 + 2 φ^2)))/201919571963756521875

Input interpretation

$$(236364091\,\pi^{24}\,(-\sin(0.0405285) + \cosh(0.148973\times 1 - \cos(0.148973)) + \sinh(0.262306\times 1 + 2\,\phi^2)))/201919571963756521875$$

cosh(x) is the hyperbolic cosine function
sinh(x) is the hyperbolic sine function
ϕ is the golden ratio

Result

123.4786…

123.4786….

Alternative representations

$$(236364091\,(\pi^{24}\,(-\sin(0.0405285) + \cosh(0.148973\times 1 - \cos(0.148973)) + \sinh(0.262306\times 1 + 2\,\phi^2))))/201919571963756521875 = \left(236364091\left(\cos\left(0.0405285 + \frac{\pi}{2}\right) + \frac{1}{2}\left(e^{0.148973-\cos(0.148973)} + e^{-0.148973+\cos(0.148973)}\right) + \frac{1}{2}\left(-e^{-0.262306-2\phi^2} + e^{0.262306+2\phi^2}\right)\right)\pi^{24}\right)/201919571963756521875$$

$$\left(236364091\left(\pi^{24}\left(-\sin(0.0405285)+\cosh(0.148973\times 1-\cos(0.148973))+\sinh\left(0.262306\times 1+2\phi^2\right)\right)\right)\right)/201919571963756521875=\left(236364091\left(-\cos\left(-0.0405285+\frac{\pi}{2}\right)+\frac{1}{2}\left(e^{0.148973-\cos(0.148973)}+e^{-0.148973+\cos(0.148973)}\right)+\frac{1}{2}\left(-e^{-0.262306-2\phi^2}+e^{0.262306+2\phi^2}\right)\right)\pi^{24}\right)/201919571963756521875$$

$$\left(236364091\left(\pi^{24}\left(-\sin(0.0405285)+\cosh(0.148973\times 1-\cos(0.148973))+\sinh\left(0.262306\times 1+2\phi^2\right)\right)\right)\right)/201919571963756521875=\left(236364091\left(-\cos\left(-0.0405285+\frac{\pi}{2}\right)-i\cos\left(\frac{\pi}{2}-i\left(0.262306+2\phi^2\right)\right)+\frac{1}{2}\left(e^{0.148973-\cos(0.148973)}+e^{-0.148973+\cos(0.148973)}\right)\right)\pi^{24}\right)/201919571963756521875$$

i is the imaginary unit

Series representations

$$\left(236364091\left(\pi^{24}\left(-\sin(0.0405285)+\cosh(0.148973\times 1-\cos(0.148973))+\sinh\left(0.262306\times 1+2\phi^2\right)\right)\right)\right)/201919571963756521875=\sum_{k=0}^{\infty}\pi^{24}\left(-2.34117\times 10^{-12}(-1)^k J_{1+2k}(0.0405285)+\frac{1.17059\times 10^{-12}(0.148973-\cos(0.148973))^{2k}}{(2k)!}+\frac{\left(3.07052\times 10^{-13}+2.34117\times 10^{-12}\phi^2\right)\left(0.262306+2\phi^2\right)^{2k}}{(1+2k)!}\right)$$

$$\left(236364091\left(\pi^{24}\left(-\sin(0.0405285)+\cosh(0.148973\times 1-\cos(0.148973))+\sinh\left(0.262306\times 1+2\phi^2\right)\right)\right)\right)/201919571963756521875=\sum_{k=0}^{\infty}\left(236364091\pi^{24}\left((0.148973-\cos(0.148973))^{2k}+2I_{1+2k}\left(0.262306+2\phi^2\right)(2k)!-2(-1)^k J_{1+2k}(0.0405285)(2k)!\right)\right)/(201919571963756521875(2k)!)$$

$$(236\,364\,091\,(\pi^{24}\,(-\sin(0.0405285) + \cosh(0.148973\times 1 - \cos(0.148973)) + \sinh(0.262306\times 1 + 2\,\phi^2))))/201\,919\,571\,963\,756\,521\,875 = \sum_{k=0}^{\infty}\pi^{24}\Big(2.34117\times 10^{-12}\,I_{1+2k}(0.262306 + 2\,\phi^2) - 2.34117\times 10^{-12}\,(-1)^k\,J_{1+2k}(0.0405285) + \frac{1}{(1+2k)!}\,i\left(0.148973 - \frac{i\pi}{2} - \cos(0.148973)\right)^{2k}(1.74386\times 10^{-13} - 5.85293\times 10^{-13}\,i\,\pi - 1.17059\times 10^{-12}\cos(0.148973))\Big)$$

$J_n(z)$ is the Bessel function of the first kind
$n!$ is the factorial function
$I_n(z)$ is the modified Bessel function of the first kind

Integral representations

$$(236\,364\,091\,(\pi^{24}\,(-\sin(0.0405285) + \cosh(0.148973\times 1 - \cos(0.148973)) + \sinh(0.262306\times 1 + 2\,\phi^2))))/201\,919\,571\,963\,756\,521\,875 = \int_0^1 \pi^{24}\,((3.07052\times 10^{-13} + 2.34117\times 10^{-12}\,\phi^2)\cosh(2\,(0.131153 + \phi^2)\,t) - 4.74421\times 10^{-14}\cos(0.0405285\,t) + (1.74386\times 10^{-13} - 5.85293\times 10^{-13}\,i\,\pi - 1.17059\times 10^{-12}\cos(0.148973))\,\sinh(i\,\pi\,(0.5 - 0.5\,t) + t\,(0.148973 - \cos(0.148973))))\,dt$$

$$(236\,364\,091\,(\pi^{24}\,(-\sin(0.0405285) + \cosh(0.148973\times 1 - \cos(0.148973)) + \sinh(0.262306\times 1 + 2\,\phi^2))))/201\,919\,571\,963\,756\,521\,875 = \left(236\,364\,091\,\pi^{24}\left(1 + \int_0^1 (2\,(0.131153 + \phi^2)\cosh(2\,(0.131153 + \phi^2)\,t) - 0.0405285\cos(0.0405285\,t) - (-0.148973 + \cos(0.148973))\,\sinh(-(t\,(-0.148973 + \cos(0.148973)))))\,dt\right)\right)/201\,919\,571\,963\,756\,521\,875$$

$$(236364091\,(\pi^{24}\,(-\sin(0.0405285)+\cosh(0.148973\times 1-\cos(0.148973))+\sinh(0.262306\times 1+2\,\phi^2))))/201919571963756521875 = 1.17059\times 10^{-12}\left(\pi^{24}\int_{\frac{i\pi}{2}}^{0.148973-\cos(0.148973)}\sinh(t)\,dt + 8.54273\times 10^{11}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{1}{i\,s^{3/2}}\,e^{-0.00041064/s+s}\left(-1.18605\times 10^{-14}+e^{(0.0176117+0.262306\,\phi^2+\phi^4)/s}\left(7.67629\times 10^{-14}+5.85293\times 10^{-13}\,\phi^2\right)\right)\pi^{23}\sqrt{\pi}\,ds\right) \text{ for } \gamma > 0$$

From

$$(236364091\,\pi^{24}\,(\tanh(0.333338-0.385798\,y)-0.238437\,y^2+\sinh(0.230847\,y^2)-2\,\phi^2))/201919571963756521875$$

for y = 1 :

(236364091 π^24 (tanh(0.333338 - 0.385798) - 0.238437 + sinh(0.230847) - 2 φ^2))/201919571963756521875

Input interpretation

$$\frac{236364091\,\pi^{24}\,(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\,\phi^2)}{201919571963756521875}$$

tanh(x) is the hyperbolic tangent function

sinh(x) is the hyperbolic sine function

Result

−5.294014…

-5.294014….

Alternative representations

$$\frac{236\,364\,091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\,\phi^2\right)\right)}{201\,919\,571\,963\,756\,521\,875} = \frac{236\,364\,091\,\pi^{24}\left(-1.23844+\frac{1}{2}\left(-\frac{1}{e^{0.230847}}+e^{0.230847}\right)-2\,\phi^2+\frac{2}{1+e^{0.10492}}\right)}{201\,919\,571\,963\,756\,521\,875}$$

$$\frac{236\,364\,091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\,\phi^2\right)\right)}{201\,919\,571\,963\,756\,521\,875} = \frac{236\,364\,091\,\pi^{24}\left(-1.23844+i\cos\left(0.230847\,i+\frac{\pi}{2}\right)-2\,\phi^2+\frac{2}{1+e^{0.10492}}\right)}{201\,919\,571\,963\,756\,521\,875}$$

$$\frac{236\,364\,091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\,\phi^2\right)\right)}{201\,919\,571\,963\,756\,521\,875} = \frac{236\,364\,091\,\pi^{24}\left(-1.23844-i\cos\left(-0.230847\,i+\frac{\pi}{2}\right)-2\,\phi^2+\frac{2}{1+e^{0.10492}}\right)}{201\,919\,571\,963\,756\,521\,875}$$

i is the imaginary unit

Series representations

$$\frac{236\,364\,091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\,\phi^2\right)\right)}{201\,919\,571\,963\,756\,521\,875} = -2.34117\times10^{-12}\,\pi^{24}\left(0.619219+\phi^2+\sum_{k=1}^{\infty}(-1)^k\,q^{2k}-0.5\sum_{k=0}^{\infty}\frac{0.230847^{1+2k}}{(1+2k)!}\right) \text{ for } q=0.948892$$

$$\frac{236\,364\,091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\,\phi^2\right)\right)}{201\,919\,571\,963\,756\,521\,875} = -2.34117\times10^{-12}\,\pi^{24}\left(0.619219+\phi^2+\sum_{k=1}^{\infty}(-1)^k\,q^{2k}-\sum_{k=0}^{\infty}I_{1+2k}(0.230847)\right) \text{ for } q=0.948892$$

$$\frac{236364091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\phi^2\right)\right)}{201919571963756521875}$$
$$=-2.34117\times10^{-12}\,\pi^{24}\left(0.619219+\phi^2+\sum_{k=1}^{\infty}(-1)^k q^{2k}-0.5\,i\sum_{k=0}^{\infty}\frac{\left(0.230847-\frac{i\pi}{2}\right)^{2k}}{(2k)!}\right)$$
for $q=0.948892$

$n!$ is the factorial function
$I_n(z)$ is the modified Bessel function of the first kind

Integral representations

$$\frac{236364091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\phi^2\right)\right)}{201919571963756521875}$$
$$=-2.79111\times10^{-13}\,\pi^{24}-2.34117\times10^{-12}\,\phi^2\,\pi^{24}+\int_0^1\pi^{24}\left(2.70226\times10^{-13}\cosh(0.230847\,t)-6.14089\times10^{-14}\operatorname{sech}^2(0-0.05246\,t)\right)dt$$

$$\frac{236364091\left(\pi^{24}\left(\tanh(0.333338-0.385798)-0.238437+\sinh(0.230847)-2\phi^2\right)\right)}{201919571963756521875}$$
$$=-\frac{1}{i}2.34117\times10^{-12}\,\pi^{23}\left(0.119219\,i\,\pi+\phi^2\,i\,\pi-0.5\,i\,\pi\int_0^{-0.05246}\operatorname{sech}^2(t)\,dt-0.0288559\sqrt{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{0.0133226/s+s}}{s^{3/2}}\,ds\right)$$ for $\gamma>0$

$\cosh(x)$ is the hyperbolic cosine function
$\operatorname{sech}(x)$ is the hyperbolic secant function

From the previous two results, after some calculations, we obtain:

3d Plot of zeta(24) cosh(((123.4786)/(-5.294014))x^(-((Pi^4)/12)))

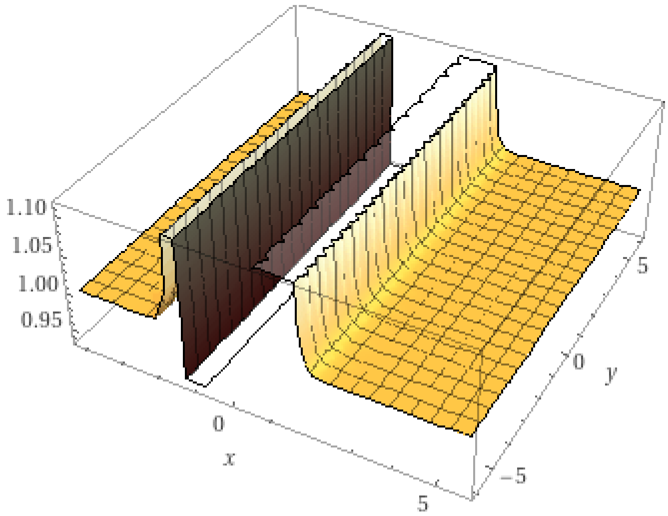
Input interpretation

3D plot	$\zeta(24)\cosh\left(-\frac{123.4786}{5.294014}\,x^{-\pi^4/12}\right)$

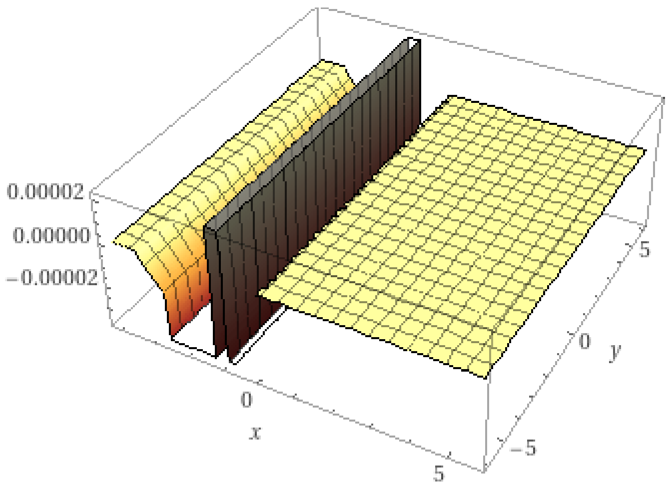
$\zeta(s)$ is the Riemann zeta function
$\cosh(x)$ is the hyperbolic cosine function

3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

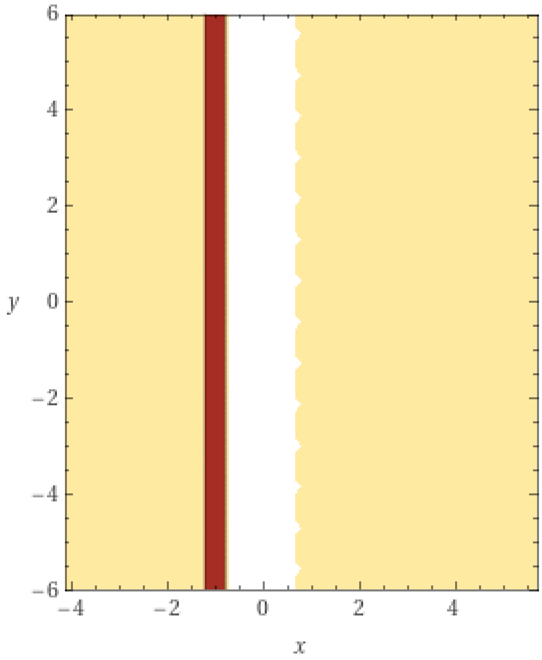


Imaginary part

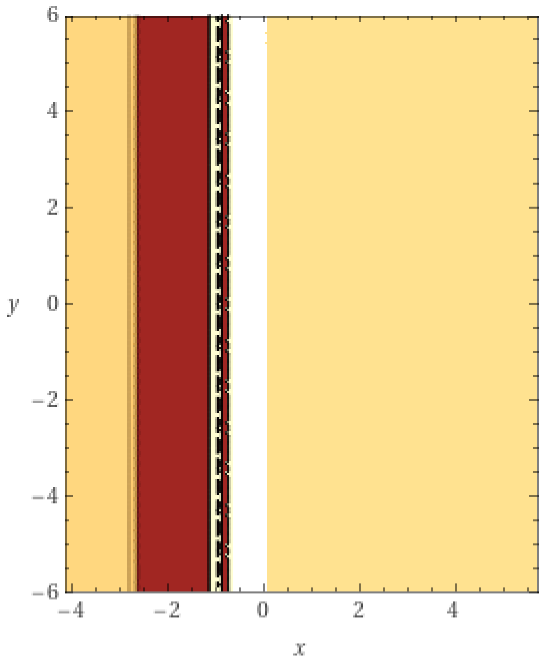


Contour plots

Real part



Imaginary part



Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

References

Manuscript Book I - *Srinivasa Ramanujan*

An Introduction to Higher-Spin Fields - *Augusto Sagnotti*-Scuola Normale Superiore, Pisa - Eotvos Superstring Workshop, Budapest, Sept. 2007

The Geometry of the MRB constant *by Marvin Ray Burns*
https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant