Correct definition of the set of natural numbers

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Abstract: It was proved that a set that already contains all natural numbers does not exist in real mathematical world because the concept of all natural numbers does not exist in the world. The correct definition of the set of natural numbers is a set that contains infinite natural numbers 1,2,3.... but can never contain all natural numbers. There are many sets of infinite natural numbers with different sizes. Thus, it is true that the subsets of N cannot correspond one-to-one with N, but it is false that the subsets cannot correspond one-to-one with any set of natural numbers.

Key words: mathematic fundation; infinite set; sets of natural numbers

People usually define the set of natural numbers as a set that already contains all natural numbers.

Of course, it would be very convenient if such a set of deterministically invariant natural numbers existed.

However, this philosophical definition obviously contradicts the infinity of the set of natural numbers: if there is a set that already contains all the natural numbers, it means that the natural numbers can no longer be added, which will inevitably lead to many contradictions! For example, let $N_2=\{x|x=2*n-1,n\in N\}\cup\{x|x=2*n,n\in N\}=\{1,3,5...,2,4,6.. ....\}=\{1, 2, 3...\}$, N_2 has twice as many elements as N: Both numbers of {1,3,5...,} and (2,4,6.. ....} are the same as that of N, denoted by ∞, so the number of {1,3,5...,2,4,6.. ....} is $\infty + \infty = 2\infty$.

Note that calculation rules for cardinality cannot be used here, because we are counting the number of elements and not the cardinality here.

Clearly, the natural number set N_2 is not the same set as N because of their different sizes.

Assuming that N is a set that already contains all natural numbers, then the elements of N_2 are also natural numbers. The set N_2-N is a non-empty and the natural numbers in the set cannot be included in N, contradiction!

Mathematics is strict, and every detail must be carefully scrutinized. It must not be sloppy, careless, or allow any self-contradictions and counter-examples. Therefore, the above counter-example has proved that there is no set that already contains all the natural numbers.

We cannot replace strict mathematical derivation with philosophical speculations that we take for granted, let alone treat fantasies and wishes as facts for convenience! For example, a set of natural numbers that already contains all natural numbers and has been completed and determined is of course very convenient to handle, but such a set does not exist at all.

Moreover, the above-mentioned multiplication process of the elements of the set of natural numbers from N to N_2 can be carried out indefinitely. Suppose it is carried out ∞ times to form a set of natural numbers,denoted by N_L, with the number of elements 2^{∞} times that of N.

Therefore, the correct definition of the set of natural numbers is a set that contains infinite natural numbers 1,2,3.... but can never contain all natural numbers. This is because natural numbers can be increased forever, and this process of growth can never be ended, so there is no concept of all natural numbers. That is to say, we can always only get partial natural numbers. Nevertheless, parts of the natural numbers can also be infinite, and parts also have relatively large parts and relatively small parts, which perfectly explains the infinite sets of natural numbers that can have different sizes. For example, N, N_2 N_L and so on. They are actually infinite sets of natural numbers of different sizes.

Many mathematical theories are based on the assumption that the set of natural numbers is unique.These need to be re-examined. Taking Cantor's theorem[1] as an example, although the subsets of N cannot be numbered by N itself, it can be numbered by N_L, a larger set of natural numbers.

Thus, it is true that the subsets of N cannot correspond one-to-one with N, but it is false that the subsets cannot correspond one-to-one with any set of natural numbers.

[1] Cantor. The theoretical basis of transfinite numbers, second edition, Commercial Press(in Chinese), 2016