Predictive signals obtained from Bayesian network and the prediction quality

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Abstract

In this paper, we will propose a method for learning signals related to a data frame D_1. The learning algorithm will be based on the biggest entropy variations of a Bayesian network. The method will make it possible to obtain an optimal Bayesian network having a high likelihood with respect to signals D_1. From the learned optimal Bayesian network, we will show what to do to infer new signals D_2 and we will also introduce the prediction quality Δ_{CR} allowing to evaluate the predictive quality of inferred signals D_2. We will then infer a large number (10000) of candidate signals D_2 and we will select the predictive signals D_2^* having the best prediction quality. Once the optimal signals D_2^* obtained, we will impose the same order of scatter (computed from the Mahalanobis) to the points of signals D_2^* as of signals D_1.

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1 Introduction

In this article, we will propose what to do to infer predictive signals D_2 that follow other learned signals related to a data frame D_1. This article is based on the results obtained in the article "Bayesian network and information theory"[5](see bibliography). The report will begin with a brief reminder of the notion of Bayesian network, Bayesian network's entropy and the probability multivariate Gaussian. Next, we will expose the BIC score expressed as a function of the Bayesian network's entropy and an algorithm that will be used in the learning of the signals related to data frame D_1. Once the signals are learned, we will obtain an optimal Bayesian network, the graph of this network will serve as a tool to infer new signals D_2. We will then explain the method that will allow to infer new signals D_2 and we will also introduce the prediction quality allowing to evaluate the predictive quality of inferred signals D_2. Among all the inferred signals D_2, we will propose a selection principle which will choose the signals predictive D_2^* having the best quality prediction value Δ_{CR}^{min}. All the points of the optimal inferred signals D_2^* will be reorganized by considering the same order of scatter as the learned signal D_1. The scatter will be modeled by a estrangement order computed from the Malahanobis distance. The article will end with an application on a data frame D_1 containing signals. The signals D_1 will be learned to deduce the predictive signals D_2^* which we will present in graphical form.

2 Bayesian network, Bayesian network's entropy and multivariate normal Gaussian

In what follows, we will consider the Bayesian network to learn signals related to a data frame to infer new signals related to a new data frame. The results are based on the paper [5]. In the article [5], we put a set of results linking the Bayesian network and information theory. We will recall below the definition of the Bayesian network with its factorization and the Bayesian network's entropy having normal Gaussian probability as the probability density.

The Bayesian network is a directed acyclic graph from which we factorise a probability density $P_X(\vec{x}|\mathcal{B})$ into conditional probabilities $P_{X_j|Pa(X_j)}(x_j, Pa(x_j))$:

$$P_X(\vec{x}|\mathcal{B}) = \prod_{X_j \in X} P_{X_j|Pa(X_j)}(x_j, Pa(x_j))$$

where we have in Gaussian case:

$$P_{X_j|Pa(X_j)}(x_j, Pa(x_j)) = (2\pi)^{-\frac{1}{2}} K^{-\frac{1}{2}}_{X_j^2|Pa(X_j)} \exp\{-\frac{(x_j - \beta_{(X_j,Pa(X_j))}.Pa(x_j) - \beta_{X_j})^2}{2.K_{X_j^2|Pa(X_j)}}\}$$

$$K_{X_j^2|Pa(X_j)} = K_{X_j^2} - K_{X_j Pa(X_j)}.K^{-1}_{Pa^2(X_j)}.K_{Pa(X_j)X_j}$$

$$\beta_{X_j} = \mu_{X_j} - K_{(X_j,Pa(X_j))}.K^{-1}_{Pa^2(X_j)}.\mu_{Pa(X_j)}$$

$$\beta_{(X_j,Pa(X_j))} = K_{(X_j,Pa(X_j))}.K^{-1}_{Pa^2(X_j)}$$

As in the article [5], we can attribute to the Bayesian network an entropy that can be computed as follows:

$$h(D|\mathcal{B}) = \sum_{X_j \in X} h(X_j|Pa(X_j)) = \sum_{X_j \in X} \frac{1}{2} \ln(2\pi e K_{X_j^2|Pa(X_j)})$$

where D corresponds to a data frame containing signals. The notion of Bayesian network's entropy will intervene in the BIC score allowing the learning of the data frame. After learning the data frame D_1 , we will have an optimal Bayesian network that will allow us to infer a new data frame D_2. **In all that follows, the data frame D_1 will be the learning data frame and D_2 will be the inferred data frame.**

When computing the Bayesian network's entropy from the variance-covariance matrix K_{X^2} of a Data, K_{X^2} must be the matrix variance-covariance of **a population**, we must therefore not forget to run the following steps with the **R software**:

$$K_{X^2} = (nrow(Data) - 1)/nrow(Data)) * var(Data)$$

3 Data frame learning and BIC score

To learn a data frame D_1 containing signals, we will use the BIC score as a function of the Bayesian network's entropy:

$$BIC(D_1|\mathcal{B}) = N.h(D_1|\mathcal{B}) + \frac{(q+n)\ln(N)}{2} = N.\sum_{X_j \in X} h(X_j|Pa(X_j)) + \frac{(q+n)\ln(N)}{2}$$

where:

$h(D_1|\mathcal{B}) = \sum_{X_j \in X} h(X_j|Pa(X_j))$ is the Bayesian network's entropy.

q is the number of edges (number of slopes) in the Bayesian network .

n is the number of nodes in the Bayesian network.

N is the number of points (number of rows) of the data frame D_1.

For the data frame learning algorithm, we must start with a Bayesian network without edges and add the edges producing the biggest entropy variations of Bayesian network without creating cycles.

The Bayesian network without edges will then become a Bayesian network with a transitive closure (chain rule).

When evaluating the BIC score expressed as a function of entropy, the minimum value of the BIC score will select the most likely Bayesian network $\tilde{\mathcal{B}}$:

$$\min_{\tilde{\mathcal{B}}} BIC(D_1|\mathcal{B})$$

4 Inference of data frame D_2 from the most likely Bayesian network $\tilde{\mathcal{B}}$

We want to infer a data frame D_2 of same size $N \times n$ as D_1 in **topological order** from the graph related to the Bayesian network. This is achieved by using OLS (Ordinary least squares) to which we add Gaussian random $\mathcal{N}$ column vectors with zero mean and a conditional variance $K_{X_j^2|Pa(X_j)}$:

$$X_j = \beta_{(X_j Pa(X_j))}.Pa(X_j) + \beta_{X_j} + \mathcal{N}(0, K_{X_j^2|Pa(X_j)})$$

Where X_j corresponds to the column vectors of the data frame D_2.

$K_{X_j^2|Pa(X_j)} = K_{X_j^2} - K_{X_j Pa(X_j)}.K^{-1}_{Pa^2(X_j)}.K_{Pa(X_j)X_j}$

$\beta_{X_j} = \mu_{X_j} - K_{(X_j,Pa(X_j))}.K^{-1}_{Pa^2(X_j)}.\mu_{Pa(X_j)}$

$\beta_{(X_j,Pa(X_j))} = K_{(X_j,Pa(X_j))}.K^{-1}_{Pa^2(X_j)}$

5 The prediction quality Δ_{CR}

The **prediction** quality Δ is the entropy deviation weighted by the size of the frames between the union of the data frame $D_1 \cup D_2$ and the data frame D_1:

$$\Delta = (N_1 + N_2)h(D_1 \cup D_2|\tilde{\mathcal{B}}) - N_1 h(D_1|\tilde{\mathcal{B}})$$

where the size N_1 and N_2 correspond respectively to the number of rows of the data frames D_1 and D_2. $\tilde{\mathcal{B}}$ is the most likely Bayesian network.

Δ corresponds to the loss of probability in logarithmic scale due to the connection of the signals D_2 to the signals D_1.

In what follows we will use the reduced centered prediction quality Δ_{CR} for 10000 candidate data frames:

$$\Delta_{CR} = \frac{(\Delta - \mu_\Delta)}{\sigma_\Delta}$$

For the minimum value Δ_{CR}^{min}, we have the most predictive signals D_2. The maximum value Δ_{CR}^{max} gives the least predictive signals D_2. Δ_{CR}^{min} attributed to signal D_2^* is the best prediction quality because the <u>union</u> $D_1 \cup D_2^*$ has a low entropy and therefore a high average probability in logarithmic scale of being obtained.

6 Scatter order compute from the Malhananobis distance

Once the data frame D_2^* has been obtained, we will order the rows of the data frame. For this, we will compute the Malhanobis distance:

$$DM(X, \mu_X, K_{X^2}) = \sqrt{(X - \mu_X)^t . K_{X^2}^{-1} . (X - \mu_X)}$$

for each row of the data frame D_1. The Malhannobis distance will then set us an increasing order of scatter. The same calculations will also be done on the data frame D_2^* and we will classify the points of D_2^* in the same order of scatter as D_1.

7 Learning and inferring signals from a Bayesian network

Consider the data frame D_1 of size 100×6 in appendix .1 that we want to learn from a Bayesian network. By applying the learning algorithm described previously in section 3, we obtain the most likely optimal Bayesian network $\tilde{\mathcal{B}}$:

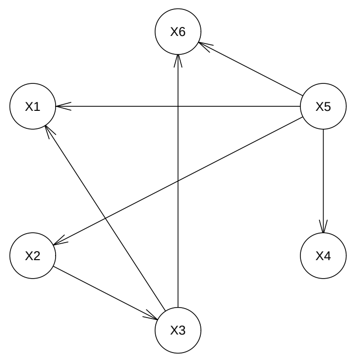


Figure 1: Bayesian network

We obtain the following entropy:

$h(D_1|\tilde{\mathcal{B}}) = 16.25305$

and the following BIC score:

$BIC(D_1|\tilde{\mathcal{B}}) = 1625.305 + \frac{(7+6)\ln(100)}{2} = 1655.239$

For the 10000 inferred data frames, we can compute the prediction quality Δ_{CR} and we can represent the prediction quality Δ_{CR} in graphical form:

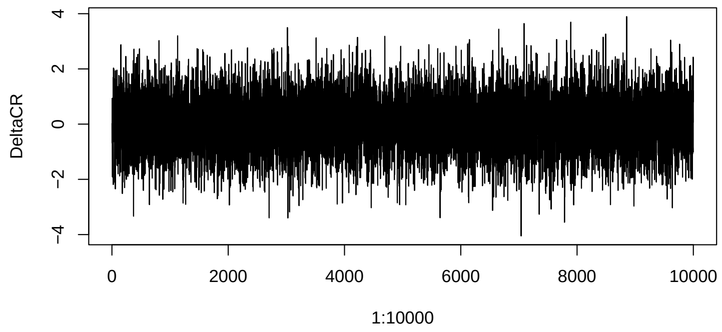


Figure 2: The prediction quality

We will represent below the prediction quality in increasing order of the values Δ_{CR}:

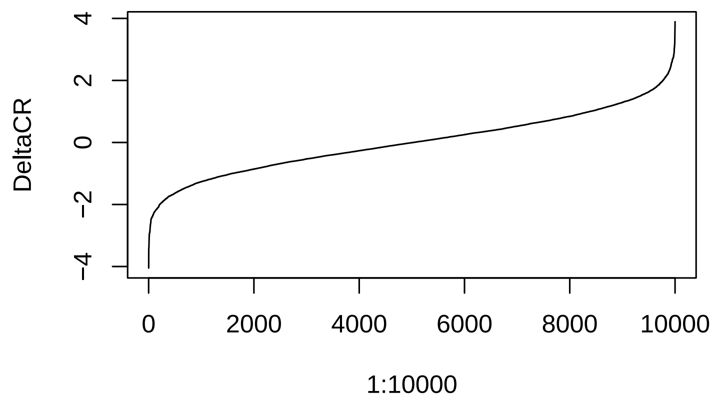
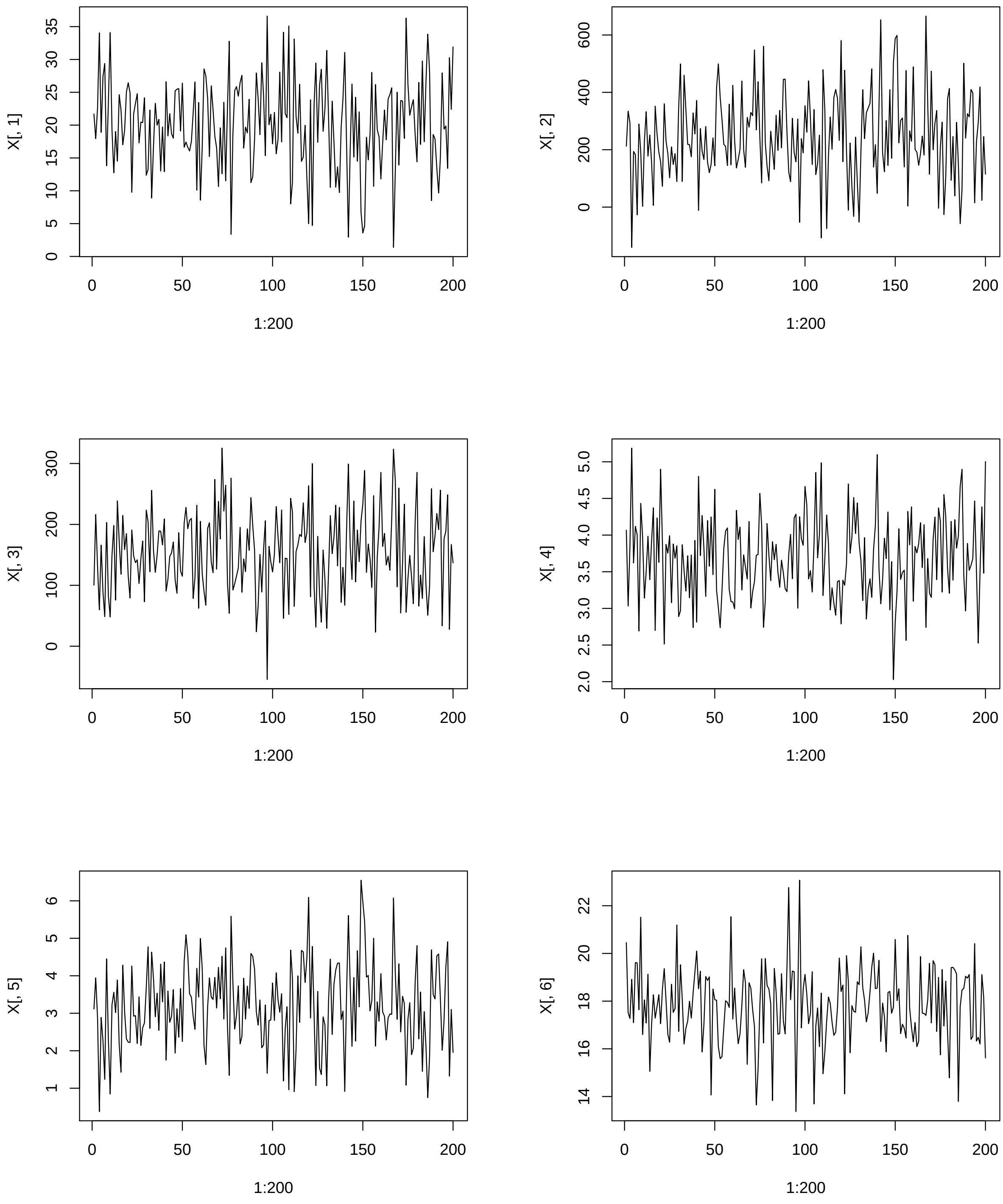


Figure 3: The prediction quality in increasing order

We obtain a maximum value $\Delta_{CR}^{max} = 3.894825$ and a minimum value $\Delta_{CR}^{min} = -4.051084$. The best prediction is given by the data frame giving the minimum value Δ_{CR}^{min}. We consider this predictive optimal signals D_2^* and we will present the signals varying over the range of values $1 \leq T \leq 100$ for the learned part and varying from $101 \leq T \leq 200$ for the inferred part. The points of the signals D_2^* will be ordered in the same order of scatter as the signals D_1.


X[, 1]
1:200
X[, 2]
1:200
X[, 3]
1:200
X[, 4]
1:200
X[, 5]
1:200
X[, 6]
1:200

8 Conclusion

In this paper, we have proposed theoretical notions allowing to learn signals contained in a data frame. From the learning we showed the steps to follow to infer predictive signals included in a new data frame.

Starting from an example of a data frame containing signals, the report ends by presenting the learned signals followed by the inferred predictive signals in graphical form.

9 Appendix

.1 Data frame D_1

T	X1	X2	X3	X4	X5	X6	DM	Scatter
1	21.697356	212.496303	100.27983	4.067217	3.1128370	20.45330	29.37503	90
2	17.933487	334.547171	216.25136	3.032607	3.9452276	17.51779	28.73959	67
3	22.593178	293.789279	131.11323	3.847017	2.8745655	17.27431	27.63717	32
4	34.049362	-140.459877	59.76671	5.185856	0.3781136	18.91340	29.44638	91
5	18.893331	193.854070	165.98525	3.619441	2.8882926	17.09582	27.37819	19
6	27.386443	183.449699	89.49365	4.120053	2.2747225	19.61326	28.93455	73
7	29.387658	-27.047273	48.51673	4.005462	1.2374293	19.60590	27.35250	18
8	13.803899	289.576913	203.10891	2.691737	4.4497561	17.63801	27.87003	39
9	23.307997	190.350364	83.14425	4.433126	2.3252469	21.52021	29.62856	92
10	34.096057	2.741246	47.94401	3.978904	0.8459643	16.59992	26.06816	3
11	19.337734	229.179303	148.90931	3.141172	3.2224283	18.05137	27.18125	12
12	12.740558	332.567200	198.15682	3.502937	3.5581512	17.08335	27.19541	13
13	19.019523	177.643152	75.71239	3.984464	3.0206339	19.12922	27.32853	16
14	14.515920	251.345140	238.63220	3.392142	3.8888359	15.05471	27.93917	41
15	24.641912	156.073251	172.47024	3.922760	2.1692936	17.03247	28.42721	57
16	22.308028	5.969799	118.17383	4.371926	1.4265646	18.26750	27.58465	30
17	17.009185	351.668352	214.58385	2.698569	4.2832532	17.28782	28.25898	53
18	19.228647	256.121885	158.85563	4.233123	3.0393819	17.76284	28.76752	69
19	25.065331	192.011334	184.91772	3.628895	2.3155048	18.26881	29.20414	82
20	26.441899	158.193829	115.22245	4.897830	2.2291997	17.05538	29.22950	84
21	24.921998	72.476092	79.17064	3.897563	2.2206522	18.50567	27.31165	15
22	9.768450	360.315682	190.86103	2.513282	4.2600833	19.36172	26.98082	8
23	21.708682	230.343087	148.22627	3.872064	2.9246166	18.02496	28.47163	62
24	23.293301	187.338921	137.30990	3.753809	2.9361905	16.60604	27.69879	33
25	24.776262	102.153614	141.87334	3.991304	2.1954443	16.27751	27.56414	29
26	17.292975	209.795454	103.01824	3.078183	3.4375820	18.70655	26.27783	4
27	20.456419	148.953697	142.97339	3.878322	2.1425225	17.52720	26.99350	10
28	20.389620	186.221825	172.58877	3.685490	2.6114183	17.69306	27.98079	43
29	24.153918	88.997109	73.04434	3.855536	2.7443201	21.19175	29.16880	81
30	12.366297	345.902899	223.52569	2.889015	3.6826036	16.73392	26.63651	5
31	13.257675	499.108686	204.25494	2.968723	4.7720203	19.52203	29.25617	86
32	22.299240	89.665515	122.32289	3.868410	2.5990239	18.06212	27.95577	42
33	8.903630	459.562948	255.89935	3.511985	4.6287205	16.20319	28.42805	58
34	16.803197	339.644711	156.85455	3.236518	3.9164802	16.86312	26.98533	9
35	23.318325	218.850544	121.29319	3.717060	2.9066304	17.16336	27.45735	23
36	19.983920	217.473115	149.56058	3.144869	3.5337949	17.99537	27.78115	34
37	20.879636	175.264826	189.29793	3.727292	2.5445839	17.28957	28.28481	54
38	13.007037	328.143139	188.78226	2.739336	4.3101301	18.32834	27.49363	26
39	19.705524	255.280329	166.26701	3.927320	3.3045594	19.21580	29.69370	93
40	12.909625	371.599198	208.89409	2.813123	4.3670589	20.09771	29.09845	76
41	26.607515	-11.397382	90.50657	4.801016	1.7515093	18.51810	29.15385	80
42	18.357485	273.627256	110.10703	3.721136	3.5930101	19.25385	28.12705	47
43	21.734258	194.486630	147.25357	4.265773	2.7584258	15.87150	27.81050	36
44	18.679990	166.121706	153.32705	3.813516	2.9196218	17.03728	27.45637	22
45	18.013215	281.022416	170.99185	3.164209	3.6305637	19.03299	28.44989	60
46	25.210935	156.325110	108.98786	4.197637	1.9370005	18.86877	28.23661	52
47	25.474886	120.080876	86.99225	3.575362	3.1195229	19.01564	28.52695	63
48	25.558855	149.638937	186.34757	4.247228	2.3641699	14.06583	28.17369	49
49	19.093939	241.095004	123.48452	3.460038	3.6587681	18.50821	27.89209	40
50	26.383655	144.062768	114.88925	4.624856	2.2478064	18.06069	29.32102	88

T	X1	X2	X3	X4	X5	X6	DM	Scatter
51	16.631879	415.350156	202.13384	3.246128	4.3677528	18.03676	29.14307	79
52	17.429027	499.253634	227.75786	3.007525	5.0938728	16.08398	29.36630	89
53	16.624258	382.803044	192.96049	2.737288	4.5662243	15.58820	27.12108	11
54	16.066352	306.918324	206.62547	3.236338	3.5222393	15.67147	26.85355	7
55	17.504180	219.025422	209.76230	3.814828	3.4294919	16.88691	29.04775	74
56	21.911348	211.557889	78.55344	4.055491	2.9154360	18.01431	27.27147	14
57	26.577185	144.955804	113.40457	4.097286	2.5707701	17.95062	28.79687	71
58	10.088415	358.589760	231.47066	3.259937	4.1975334	17.73663	28.19930	51
59	23.432039	147.405877	62.37464	3.093330	3.4331099	21.53923	28.54383	64
60	8.573387	424.640718	205.03377	3.087174	4.9942198	17.25097	27.61543	31
61	16.592986	232.340174	119.36525	2.995524	4.2073326	18.54980	27.35201	17
62	28.562558	136.617252	90.23244	4.337025	2.1406164	17.13835	28.09954	46
63	27.511746	165.838291	67.28764	3.941838	1.6319913	16.21451	25.450	1
64	24.109918	202.000099	193.57244	4.109444	3.1691412	16.63171	30.14592	98
65	15.224393	439.424883	202.70133	3.251049	3.9443693	17.80867	28.14369	48
66	25.981988	200.585319	138.01604	3.728482	3.4324030	19.31789	30.46964	99
67	22.375536	138.939767	120.75469	3.566029	3.3694704	18.77820	28.78191	70
68	18.302730	312.679313	273.84440	3.401653	3.9555278	15.35012	29.71879	94
69	16.496028	278.680945	126.55229	4.185533	3.1401651	18.77259	28.05318	45
70	10.648459	330.025619	237.83779	3.005619	4.2221736	18.54193	28.64512	66
71	19.577220	318.735572	176.17215	3.228952	3.3865012	17.64093	27.82099	38
72	12.602861	547.610755	325.18760	3.364679	4.5170861	16.94013	30.78173	100
73	23.483835	269.181617	221.92444	3.724709	2.8467341	13.64431	27.81766	37
74	11.520018	436.855756	264.40488	3.735643	4.7439170	15.15520	29.22987	85
75	22.735042	239.556194	103.32283	4.569178	2.6527874	17.80767	28.39739	56
76	32.766632	84.141232	54.03472	4.122624	1.3440861	19.77657	28.44012	59
77	3.368601	560.702187	276.03769	2.742008	5.5880158	16.24667	27.52558	27
78	18.582352	223.744131	92.24361	3.101948	3.8218877	19.78210	27.48509	25
79	25.259709	142.345346	102.57292	4.157116	2.5808681	18.65086	28.76234	68
80	25.862437	91.731591	115.25040	3.725904	2.9508191	18.51111	29.09199	75
81	24.405828	264.105590	128.24257	3.379417	3.7308016	17.89932	28.81032	72
82	26.484348	198.879063	195.17488	3.908468	2.1831227	13.83249	27.53664	28
83	27.577924	131.411099	88.60592	3.664482	2.3816511	19.36957	28.31460	55
84	16.500382	320.080511	143.27535	3.873874	3.9361777	18.36889	28.62746	65
85	19.695883	197.885483	122.45748	3.494311	2.8443474	16.61144	25.94747	2
86	18.822765	336.902803	192.74357	3.289000	3.7248344	16.63973	27.98280	44
87	23.940722	206.683049	157.44793	3.657017	3.1324352	19.14949	29.81872	95
88	11.244597	445.089964	244.00301	3.483708	4.5950475	17.16837	29.10891	77
89	12.142727	446.114532	198.29972	3.270059	4.5206776	16.62513	27.46768	24
90	17.717054	262.381335	158.13466	3.228514	4.1851491	19.39476	29.21639	83
91	27.938884	122.305509	23.85977	3.736659	3.0321913	22.76239	29.90514	96
92	23.992895	88.638377	66.26240	4.009436	2.6843940	18.06440	27.38505	20
93	18.545182	309.421665	150.63214	3.405085	3.3540838	19.26326	28.19179	50
94	29.511690	189.962882	88.99168	4.235150	2.0802414	19.23400	29.12895	78
95	24.473435	157.837949	158.95482	4.284887	2.1564811	13.37018	26.64794	6
96	15.336709	307.162666	206.02291	3.002996	3.2221951	18.15282	27.40042	21
97	36.603287	-52.933877	-54.60674	4.250180	1.3984221	23.06948	29.28035	87
98	20.055056	238.642997	164.02440	3.947206	2.8032071	16.87895	27.78737	35
99	21.648469	188.946992	138.74106	3.861113	2.8219705	18.50646	28.45320	61
100	17.156453	353.615611	121.96272	4.664249	3.8069832	19.12094	29.93572	97

.2 Data frame D_2

T	X1	X2	X3	X4	X5	X6	DM	Scatter
101	21.931961	261.306100	150.81423	4.413663	2.8088478	18.29986	28.80171	90
102	15.641855	439.984735	229.30705	3.402351	4.0796858	17.05930	27.93774	67
103	17.801376	312.942605	180.57879	3.514833	3.3616256	17.47819	27.02402	32
104	28.077265	148.532733	137.01457	3.224006	3.0267141	19.22975	28.80665	91
105	17.440781	339.878187	223.60340	3.883032	3.5210776	13.68989	26.76419	19
106	34.138838	113.291806	45.84236	4.853814	1.1950936	16.95197	28.02361	73
107	21.744179	152.202200	144.56716	3.689923	2.5911532	17.71296	26.73626	18
108	21.127887	250.556426	143.78859	4.010145	3.1741558	16.09272	27.23971	39
109	35.093088	-107.352122	52.21626	4.986353	0.9586701	18.33609	28.82812	92
110	8.002773	478.835545	242.77077	3.176895	4.6838061	14.95936	25.81205	3
111	11.325845	338.009267	219.83303	3.688146	3.9446732	15.87599	26.55169	12
112	33.108580	-74.637175	65.42240	4.273522	0.9094321	17.16619	26.59741	13
113	21.326358	178.040476	155.35346	3.885930	2.0403082	18.16930	26.63181	16
114	18.763279	313.801294	166.18747	2.979267	3.9963962	17.90784	27.25662	41
115	26.207903	201.743451	183.55082	3.280895	2.7592660	17.11350	27.72293	57
116	14.478389	381.823301	180.18822	3.071264	4.6762259	16.56754	26.99451	30
117	15.136202	409.895115	235.36804	2.906671	4.6381053	16.69438	27.68588	53
118	19.961523	378.893781	170.31716	3.366655	3.8236953	17.72568	27.96518	69
119	11.927017	232.808934	186.84221	3.379099	4.4677061	19.80736	28.51722	82
120	4.986517	580.282121	263.53654	2.787773	6.0929179	18.40696	28.52204	84
121	23.852970	158.323361	81.21380	3.384682	2.8733594	18.67204	26.62263	15
122	4.714059	477.047397	299.86474	3.315939	4.7824240	14.11091	25.98942	8
123	23.174490	196.250423	97.77232	3.607624	2.9542884	19.90687	27.83805	62
124	29.461702	-10.660006	31.28919	4.697186	1.0714152	18.85028	27.02467	33
125	17.381927	223.349976	180.33255	3.753174	3.5818757	15.83681	26.95849	29
126	26.021970	70.834398	86.32067	3.973971	1.5394510	17.81064	25.87101	4
127	28.499324	-32.453293	39.58341	4.512335	1.3655598	17.57522	26.39148	10
128	19.056801	243.896778	157.94385	4.021580	2.9045586	17.53504	27.26190	43
129	22.493150	86.587797	109.43554	4.438139	2.6921368	18.81205	28.49814	81
130	31.374273	-52.280676	29.39409	3.886678	1.0663282	18.69155	25.90628	5
131	21.320461	194.364513	122.01096	3.650344	3.3135256	20.27797	28.56751	86
132	10.514525	409.161380	214.47506	3.107237	4.4441633	18.57283	27.25676	42
133	23.650470	238.600235	151.87723	3.964452	2.4347324	18.03993	27.72710	58
134	17.437124	328.892217	179.25592	2.854177	3.7743205	17.13019	26.19258	9
135	10.561384	345.052482	231.56747	3.249953	4.1568368	17.51126	26.87804	23
136	13.650261	361.778974	131.91977	3.402690	4.3351219	18.36135	27.07369	34
137	9.728227	481.665865	227.83274	3.150947	4.3413204	19.42076	27.68591	54
138	19.702807	139.561307	72.08454	3.801340	2.8331387	20.01098	26.90924	26
139	24.048339	217.498918	129.40178	4.134105	3.0544571	18.52564	28.92014	93
140	31.070705	48.005192	67.22827	5.098372	0.9145205	18.54834	28.33240	76
141	16.830664	350.388820	193.01892	3.540472	3.5706977	19.70868	28.49123	80
142	2.933402	653.051405	299.09742	3.062405	5.6053596	16.30988	27.35337	47
143	16.306917	189.416238	164.07207	3.394193	3.7536143	17.91680	27.11857	36
144	26.248653	123.552795	109.63399	3.957235	2.1186580	17.35226	26.85639	22
145	15.137893	301.624832	238.37464	3.680099	3.9544414	15.87298	27.76215	60
146	24.235938	145.067932	105.70487	4.315574	2.2583568	18.36325	27.68351	52
147	14.499314	409.280805	190.68901	2.979686	4.6683483	18.40536	27.85284	63
148	22.017094	170.283229	138.96304	3.637595	3.1711777	17.49494	27.37484	49
149	6.734309	503.928023	205.58673	2.030357	6.5488271	17.74756	27.25648	40
150	3.575710	588.130098	233.16072	2.816435	5.9761386	20.58586	28.75643	88

T	X1	X2	X3	X4	X5	X6	DM	Scatter
151	4.632958	598.601975	288.46631	3.232081	5.4388236	18.01862	28.46535	79
152	18.157458	224.180582	121.33559	4.086431	3.9650603	18.51791	28.79048	89
153	14.705782	301.962293	167.74776	3.396155	4.0093334	16.64101	26.45779	11
154	18.667762	310.721763	143.66506	3.491294	3.0641135	17.03134	25.98369	7
155	28.022990	139.882147	96.50699	3.520983	3.3500336	16.86215	28.05803	74
156	10.679494	475.874740	247.15960	2.563840	5.0011154	16.45546	26.60282	14
157	26.171181	3.738610	23.04122	4.321621	2.1251104	20.75656	27.99968	71
158	19.279133	265.868513	146.89372	3.866439	3.3027558	17.70062	27.52608	51
159	18.208489	229.528845	195.31233	4.385412	2.7880688	16.91608	27.89633	64
160	11.802785	488.712763	285.40365	3.098212	4.0556680	16.29754	27.00925	31
161	17.342288	200.878993	163.91764	3.845273	3.0339143	17.10184	26.63561	17
162	22.292361	191.504254	185.22995	3.757232	2.9206038	16.09012	27.32947	46
163	17.758456	145.874497	133.42537	3.879682	2.2885487	16.31657	24.85334	1
164	23.955894	185.857623	147.67538	4.168476	2.8767907	19.86692	29.64877	98
165	24.654984	247.457215	124.68063	3.557734	2.9791239	17.49585	27.35657	48
166	25.683370	181.308714	208.89234	4.142059	2.9717927	17.48430	30.01343	99
167	1.379041	665.846181	323.53876	2.741004	6.0756607	17.40265	27.99233	70
168	13.093642	362.232034	271.84293	3.678794	4.0625135	18.03728	29.03171	94
169	25.014956	114.646152	97.55807	3.206542	2.8391836	19.58395	27.29503	45
170	13.932606	473.625474	259.61941	3.149794	4.3187059	17.08639	27.92767	66
171	23.753073	200.113169	54.61137	3.940088	2.5039403	19.69509	27.22002	38
172	23.650784	292.115894	135.91570	4.245670	3.4530197	19.52918	30.23576	100
173	17.977651	336.501117	233.14729	3.392075	3.2576995	16.72800	27.20062	37
174	36.297805	-3.915491	55.71104	4.370950	1.0798936	18.99607	28.52298	85
175	26.230965	207.739443	111.34219	4.193627	2.8365662	15.74778	27.70081	56
176	21.498126	295.942106	149.16114	3.223441	3.2831992	19.32337	27.75103	59
177	22.813901	-26.027650	113.81628	4.552678	1.8984568	16.95858	26.91540	27
178	23.862059	105.324480	69.78278	4.247481	2.0686871	18.83681	26.90452	25
179	18.582620	377.522559	204.17344	3.541693	3.8257162	16.68942	27.94118	68
180	14.409316	413.272451	285.46826	3.206135	4.8051182	14.78298	28.23431	75
181	26.502448	93.495446	65.97236	4.186560	2.3182290	19.40647	28.00033	72
182	17.065770	245.906921	116.76506	3.384883	3.5670251	19.41734	26.95484	28
183	29.764413	39.079872	78.42220	4.210311	1.4487808	19.31208	27.69703	55
184	17.471705	295.179608	179.95212	3.822571	3.0433610	19.13267	27.89840	65
185	26.662461	150.011832	100.12812	3.990159	2.1024685	13.79449	25.21862	2
186	33.850355	-57.859197	50.90342	4.663858	0.7512067	17.76907	27.29028	44
187	27.998693	61.028537	99.58058	4.896682	1.8563318	18.47290	29.04291	95
188	8.500233	501.265007	258.65390	3.443704	4.6924951	18.53387	28.43906	77
189	18.565342	240.657752	155.23587	2.964961	3.5021276	19.04353	26.88564	24
190	17.894126	325.452869	181.28592	3.886547	3.3835986	18.96626	28.51780	83
191	13.438490	314.752797	217.94554	3.519523	4.5288620	19.11659	29.20108	96
192	9.669251	410.158073	191.42259	3.588163	4.5822178	16.39707	26.77955	20
193	15.240436	397.722673	256.14530	3.679050	3.3509149	16.52375	27.48710	50
194	27.952913	14.896580	33.50344	4.465039	2.0152189	20.41081	28.45869	78
195	19.381359	226.370055	176.30730	3.521953	2.8095241	16.32593	25.98152	6
196	19.868761	294.389745	188.94301	2.526430	4.2304423	16.48050	26.83040	21
197	13.415532	418.898851	248.55177	3.505271	4.9065681	16.20265	28.74709	87
198	30.266598	23.481255	27.69257	4.385926	1.3239543	19.11283	27.07883	35
199	22.393340	245.771641	166.88263	3.480599	3.1017045	18.18490	27.83309	61
200	31.907894	114.971270	136.79853	5.003086	1.9551630	15.61962	29.58828	97

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