# Introduction to SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra

F. Smarandache[1]

[1] *The University of New Mexico, Mathematics, Physics, and Natural Science Division, 705 Gurley Ave., Gallup, NM 87301, USA*

smarand@unm.edu

**Abstract**

In this paper we recall our concepts of $n^{\text{th}}$-Power Set of a Set, SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation, Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra. In general, in any field of knowledge, one actually encounters SuperHyperStructures (or more accurately $(m, n)$-SuperHyperStructures).

**Article Information**

Corresponding Author: F. Smarandache;
Received: March 2022;
Accepted: Invited paper;
Paper type: Original.

**Keywords:**

SuperHyperOperation, SuperHyperAlgebra, Neutrosophic SuperHyperAlgebra, SuperHyperStructures.

## 1 Introduction

One recalls the SuperHyperAgebra and Neutrosophic SuperHyperAlgebra introduced and developed by Smarandache [16, 18, 19] between 2016–2022.

1. **Definition of classical HyperOperations:**

Let $U$ be a universe of discourse and $H$ be a non-empty set, $H \subset U$.
A **classical Binary HyperOperation** $\circ_2^*$ is defined as follows:

$$\circ_2^* : H^2 \to P_*(H),$$

where $H$ is a discrete or continuous set, and $P_*(H)$ is the powerset of $H$ without the empty-set $\emptyset$, or $P_*(H) = P(H) \setminus \{\emptyset\}$.

A **classical $m$-ary HyperOperation** $\circ_m^*$ is defined as:

$$\circ_m^* : H^m \to P_*(H),$$

for integer $m \geq 1$. For $m = 1$ one gets a **Unary HyperOperation.**
The **classical HyperStructures** are structures endowed with classical HyperOperations.
The classical HyperOperations and classical HyperStructures were introduced by $F$. Marty [12] in 1934.

2. **Definition of the $n^{\text{th}}$-Power Set of a Set:**
The $n^{\text{th}}$-Powerset of a Set was introduced in [16, 18, 19] in the following way:
$P^n(H)$, as the $n^{\text{th}}$-Powerset of the Set $H$, for integer $n \geq 1$, is recursively defined as:
$P^2(H) = P(P(H))$, $P^3(H) = P(P^2(H)) = P(P(P(H)))$, $\cdots$,
$P^n(H) = P(P^{(n-1)}(H))$, where $P^\circ(H) \overset{\text{def}}{=} H$, and $P^1(H) \overset{\text{def}}{=} P(H)$.

The $n^{\text{th}}$-Powerset of a Set better reflects our complex reality, since a set $H$ (that may represent a group, a society, a country, a continent, etc.) of elements (such as: people, objects, and in general any items) is organized onto subsets $P(H)$, and these subsets are again organized onto subsets of subsets $P(P(H))$, and so on. That's our world.

3. **Neutrosophic HyperOperation and Neutrosophic HyperStructures [12]:**

In the classical HyperOperation and classical HyperStructures, the empty-set $\emptyset$ does not belong to the power set, or $P_*(H) = P(H) \setminus \{\emptyset\}$.
However, in the real world we encounter many situations when a HyperOperation $\circ$ is *indeterminate*, for example $a \circ b = \emptyset$ (unknown, or undefined),
or *partially indeterminate*, for example: $c \circ d = \{[0.2, 0.3], \emptyset\}$.
In our everyday life, there are many more operations and laws that have some degrees of indeterminacy (vagueness, unclearness, unknowingness, contradiction, etc.), than those that are totally determinate.
That's why in 2016 we have extended the classical HyperOperation to the Neutrosophic HyperOperation, by taking the whole power $P(H)$ (that includes the empty-set $\emptyset$ as well), instead of $P_*(H)$ (that does not include the empty-set $\emptyset$), as follows.

3.1 **Definition of Neutrosophic HyperOperation:**

Let $U$ be a universe of discourse and $H$ be a non-empty set, $H \subset U$.
A **Neutrosophic Binary HyperOperation** $\circ_2$ is defined as follows:

$$\circ_2 : H^2 \to P(H),$$

where $H$ is a discrete or continuous set, and $P(H)$ is the powerset of $H$ that includes the empty-set $\emptyset$.

A **Neutrosophic $m$-ary HyperOperation** $\circ_m$ is defined as:

$$\circ_m : H^m \to P(H),$$

for integer $m \geq 1$. Similarly, for $m = 1$ one gets a **Neutrosophic Unary HyperOperation.**

3.2 **Neutrosophic HyperStructures:**

A Neutrosophic HyperStructure is a structured endowed with Neutrosophic HyperOperations.

4. **Definition of SuperHyperOperations:**

We recall our 2016 concepts of SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra [16].

Let $P_*^n(H)$ be the $n^{\text{th}}$-powerset of the set $H$ such that none of $P(H)$, $P^2(H)$, $\cdots$, $P^n(H)$ contain the empty set $\emptyset$.

Also, let $P^n(H)$ be the $n^{\text{th}}$-powerset of the set $H$ such that at least one of the $P(H)$, $P^2(H)$,$\cdots$, $P^n(H)$ contain the empty set $\emptyset$.

The SuperHyperOperations are operations whose codomain is either $P_*^n(H)$ and in this case one has **classical-type SuperHyperOperations**, or $P^n(H)$ and in this case one has **Neutrosophic SuperHyperOperations**, for integer $n \geq 2$.

4.1 A **classical-type Binary SuperHyperOperation** $\circ^*_{(2,n)}$ is defined as follows:

$$\circ^*_{(2,n)} : H^2 \to P_*^n(H),$$

where $P_*^n(H)$ is the $n^{\text{th}}$-power set of the set $H$, with no empty-set.

4.2 **Examples of classical-type Binary SuperHyperOperation:**

1) Let $H = \{a, b\}$ be a finite discrete set; then its power set, without the empty-set $\emptyset$, is: $P(H) = \{a, b, \{a, b\}\}$, and:

$$P^2(H) = P(P(H)) = P(\{a, b, \{a, b\}\}) = \{a, b, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\},$$

$$\circ^*_{(2,2)} : H^2 \to P_*^2(H).$$

| $\circ^*_{(2,2)}$ | a | b |
|---|---|---|
| a | $\{a, \{a, b\}\}$ | $\{b, \{a, b\}\}$ |
| b | a | $\{a, b, \{a, b\}\}$ |

Table 1: Example 1 of classical-type Binary SuperHyperOperation

2) Let $H = [0, 2]$ be a continuous set.
$P(H) = P([0, 2]) = \{A \mid \ A \subseteq [0, 2],\ A = \text{subset}\}$,
$P^2(H) = P(P([0, 2]))$.
Let $c, d \in H$.

$$\circ^*_{(2,2)} : H^2 \to P_*^2(H).$$

| $\circ^*_{(2,2)}$ | c | d |
|---|---|---|
| c | $\{[0, 0.5], [1, 2]\}$ | $\{0.7, 0.9, 1.8\}$ |
| d | $\{2.5\}$ | $\{(0.3, 0.6), \{0.4, 1.9\}, 2\}$ |

Table 2: Example 2 of classical-type Binary SuperHyperOperation

4.2 **Classical-type $m$-ary SuperHyperOperation** (or a more accurate denomination **$(m, n)$-SuperHyperOperation**)

Let $U$ be a universe of discourse and a non-empty set $H$, $H \subset U$. Then:

$$\circ^*_{(m,n)} : H^m \to P^n_*(H),$$

where the integers $m, n \geq 1$,

$$H^m = \underbrace{H \times H \times \cdots \times H}_{m \text{ times}},$$

and $P^n_*(H)$ is the $n^{\text{th}}$-powerset of the set $H$ that includes the empty-set.

This SuperHyperOperation is an $m$-ary operation defined from the set $H$ to the $n^{\text{th}}$-powerset of the set $H$.

4.3 **Neutrosophic $m$-ary SuperHyperOperation** (or more accurate denomination **Neutrosophic $(m, n)$-SuperHyperOperation**):

Let $U$ be a universe of discourse and a non-empty set $H$, $H \subset U$. Then:

$$\circ_{(m,n)} : H^m \to P^n(H),$$

where the integers $m, n \geq 1$,
and $P^n(H)$ is the $n$-th powerset of the set $H$ that includes the empty-set.

5. **SuperHyperAxiom:**

A **classical-type SuperHyperAxiom** or more accurately a **$(m, n)$-SuperHyperAxiom** is an axiom based on classical-type SuperHyperOperations.

Similarly, a **Neutrosophic SuperHyperAxiom** (or Neutrosphic $(m, n)$-SuperHyperAxiom) is an axiom based on Neutrosophic SuperHyperOperations.

There are:

- **Strong SuperHyperAxioms**, when the left-hand side is equal to the right-hand side as in non-hyper axioms,

- and **Week SuperHyperAxioms**, when the intersection between the left-hand side and the right-hand side is non-empty.

For examples, one has:

• Strong SuperHyperAssociativity, when $(x \circ y) \circ z = x \circ (y \circ z)$, for all $x, y, z \in H^m$, where the law $\circ^*_{(m,n)} : H^m \to P^n_*(H)$;

• and Week SuperHyperAssociativity, when $[(x \circ y) \circ z] \cap [x \circ (y \circ z)] \neq \emptyset$, for all $x, y, z \in H^m$.

6. **SuperHyperAlgebra and SuperHyperStructure:**

A **SuperHyperAlgebra** or more accurately $(m - n)$**-SuperHyperAlgebra** is an algebra dealing with SuperHyperOperations and SuperHyperAxioms.

Again, a **Neutrosophic SuperHyperAlgebra** (or Neutrosphic $(m, n)$-SuperHyperAlgebra) is an algebra dealing with Neutrosophic SuperHyperOperations and Neutrosophic SuperHyperAxioms.

In general, we have **SuperHyperStructures** (or $(m - n)$-SuperHyperStructures), and corresponding **Neutrosophic SuperHyperStructures.**

For example, there are SuperHyperGrupoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

7. **Distinction between SuperHyperAlgebra vs. Neutrosophic SuperHyperAlgebra:**

$i$. If none of the power sets $P^k(H)$, $1 \leq k \leq n$, do not include the empty set $\emptyset$, then one has a classical-type SuperHyperAlgebra;

$ii$. If at least one power set, $P^k(H)$, $1 \leq k \leq n$, includes the empty set $\emptyset$, then one has a Neutrosophic SuperHyperAlgebra.

8. **SuperHyperGraph** (or $n$**-SuperHyperGraph):**

The SuperHyperAlgebra resembles the $n$-SuperHyperGraph [17, 18, 19], introduced by Smarandache in 2019, defined as follows:

8.1 **Definition of the n-SuperHyperGraph:**

Let $V = \{v_1, v_2, \cdots, v_m\}$, for $1 \leq m \leq \infty$, be a set of vertices, that contains Single Vertices (the classical ones), Indeterminate Vertices (unclear, vague, partially known), and Null Vertices (totally unknown, empty).

Let $P(V)$ be the power of set $V$, that includes the empty set $\emptyset$, too.
Then $P^n(V)$ be the $n$-powerset of the set $V$, defined in a recurent way, i.e.:
$P(V),\ P^2(V) = P(P(V)),\ P^3(V) = P(P^2(V)) = P(P(P(V))),\ \cdots,$
$P^n(V) = P(P^{(n-1)}(V))$, for $1 \leq n \leq \infty$, where by definition $P^0(V) \stackrel{\text{def}}{=} V$.

Then, the **$n$-SuperHyperGraph ($n$-SHG)** is an ordered pair:

$$\text{n-SHG} = (G_n, E_n),$$

where $G_n \subseteq P^n(V)$, and $E_n \subseteq P^n(V)$, for $1 \leq n \leq \infty$.
$G_n$ is the set of vertices, and $E_n$ is the set of edges.

The **set of vertices** $G_n$ contains the following types of vertices:

▪ *Singles Vertices* (the classical ones);

▪ *Indeterminate Vertices* (unclear, vagues, partially unkwnown);

▪ *Null Vertices* (totally unknown, empty);
and:

▪ *SuperVertex* (or SubsetVertex), i.e. two ore more (single, indeterminate, or null) vertices put together as a group (organization).

▪ *n-SuperVertex* that is a collection of many vertices such that at least one is a $(n-1)$-SuperVertex and all other $r$-SuperVertices into the collection, if any, have the order $r \leq n-1$.

The **set of edges** $E_n$ contains the following types of edges:

▪ Singles Edges (the classical ones);

▪ *Indeterminate Edges* (unclear, vague, partially unknown);

▪ *Null Edges* (totally unknown, empty);
and:

▪ *HyperEdge* (connecting three or more single vertices);

▪ *SuperEdge* (connecting two vertices, at least one of them being a SuperVertex);

▪ *n-SuperEdge* (connecting two vertices, at least one being an $n$-SuperVertex, and the other of order $r$-SuperVertex, with $r \leq n$);

▪ *SuperHyperEdge* (connecting three or more vertices, at least one being a SuperVertex);

▪ *n-SuperHyperEdge* (connecting three or more vertices, at least one being an $n$-SuperVertex, and the other $r$-SuperVertices with $r \leq n$;

▪ *MultiEdges* (two or more edges connecting the same two vertices);

▪ *Loop* (and edge that connects an element with itself).
and:

▪ *Directed Graph* (classical one);

▪ *Undirected Graph* (classical one);

▪ *Neutrosophic Directed Graph* (partially directed, partially undirected, partially indeterminate direction).

## 2 Conclusion

We recalled the most general form of algebras, called SuperHyperAlgebra (or more accurate denomination $(m, n)$-SuperHyperAlgebra) and the Neutrososophic SuperHyperAlgebra, and their extensions to SuperHyperStructures and respectively Neutrosophic SuperHyperAlgebra in any field of knowledge.

They are based on the $n^{\text{th}}$-Powerset of a Set, which better reflects our complex reality, since a set $H$ (that may represent a group, a society, a country, a continent, etc.) of elements (such as: people, objects, and in general any items) is organized onto subsets $P(H)$, and these subsets are again organized onto subsets of subsets $P(P(H))$, and so on. That's our world.

Hoping that this new field of SuperHyperAlgebra will inspire researchers to studying several interesting particular cases, such as the SuperHyperGroupoid, SuperHyperMonoid, SuperHyper-Semigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

## References

[1] A.A.A. Agboola, B. Davvaz, *On Neutrosophic canonical hypergroups and neutrosophic hyperrings*, Neutrosophic Sets and Systems, 2 (2014), 34–41.

[2] M. Al-Tahan, B. Davvaz, *Refined neutrosophic quadruple (po-)hypergroups and their fundamental group*, Neutrosophic Sets and Systems, 27 (2019), 138–153.

[3] M. Al-Tahan, B. Davvaz, F. Smarandache, O. Anis, *On some NeutroHyperstructures*, Symmetry, 13 (2021), 1–12.

[4] M.A. Ibrahim, A.A.A. Agboola, *Introduction to NeutroHyperGroups*, Neutrosophic Sets and Systems, 38 (2020), 15–32.

[5] M.A. Ibrahim, A.A.A. Agboola, Z.H. Ibrahim, E.O. Adeleke, *On refined neutrosophic canonical hypergroups*, Neutrosophic Sets and Systems, 45 (2021), 414–427.

[6] M.A. Ibrahim, A.A.A. Agboola, Z.H. Ibrahim, E.O. Adeleke, *On refined neutrosophic hyperrings*, Neutrosophic Sets and Systems, 45 (2021), 349–365.

[7] S. Khademan, M.M. Zahedi, R.A. Borzooei, Y.B. Jun, *Neutrosophic hyper BCK-ideals*, Neutrosophic Sets and Systems, 27 (2019), 201–217.

[8] M.A. Malik, A. Hassan, S. Broumi, F. Smarandache, *Regular bipolar single valued neutrosophic hypergraphs*, Neutrosophic Sets and Systems, 13 (2016), 84–89.

[9] D. Mandal, *Neutrosophic hyperideals of semihyperrings*, Neutrosophic Sets and Systems, 12 (2016), 105–113.

[10] N. Martin, F. Smarandache, *Concentric plithogenic hypergraph based on plithogenic hypersoft sets–A novel outlook*, Neutrosophic Sets and Systems, 33 (2020), 78–91.

[11] N. Martin, F. Smarandache, I. Pradeepa, N. Ramila Gandhi, P. Pandiammal, *Exploration of the factors causing autoimmune diseases using fuzzy cognitive maps with concentric neutrosophic hypergraphic approach*, Neutrosophic Sets and Systems, 35 (2020), 232–238.

[12] F. Marty, *Sur une généralisation de la Notion de Groupe*, 8th Congress Math. Scandinaves, Stockholm, Sweden, (1934), 45–49.

[13] S. Nawaz, M. Gulistan, S. Khan, *Weak LA-hypergroups; neutrosophy, enumeration and redox reaction*, Neutrosophic Sets and Systems, 36 (2020), 352–367.

[14] S. Rajareega, D. Preethi, J. Vimala, G. Selvachandran, F. Smarandache, *Some results on single valued neutrosophic hypergroup*, Neutrosophic Sets and Systems, 31 (2020), 80–85.

[15] A. Rezaei, F. Smarandache, S. Mirvakili, *Applications of (Neutro/Anti)sophications to Semi-HyperGroups*, Journal of Mathematics, (2021), 1–7.

[16] F. Smarandache, *SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra*, Section into the authors book Nidus Idearum. Scilogs, II: de rerum consectatione, Second Edition, (2016), 107–108.

[17] F. Smarandache, *n-SuperHyperGraph and Plithogenic n-SuperHperGraph*, in Nidus Idearum, Vol. 7, second and third editions, Pons asbl, Bruxelles, (2019), 107-113.

[18] F. Smarandache, *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*, Neutrosophic Sets and Systems, 33 (2020), 290–296.

[19] F. Smarandache, *Introduction to the n-SuperHyperGraph-the most general form of graph today*, Neutrosophic Sets and Systems, 48 (2022), 483–485.