Introduction to Plithogenic Logic as generalization of MultiVariate Logic

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Abstract: A Plithogenic Logical proposition *P* is a proposition that is characterized by many degrees of truth-values with respect to many corresponding attribute-values (or random variables) that characterize *P*. Each degree of truth-value may be classical, fuzzy, intuitionistic fuzzy, neutrosophic, or other fuzzy extension type logic. At the end, a cumulative truth of *P* is computed.

Keywords: neutrosophic logic, plithogenic logic, plithogenic multi-variate analysis, cumulative truth, plithogenic logic application

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1. Introduction

We recall the Plithogenic Logic and explain it in detail by showing a practical application.

A Plithogenic Logical proposition *P* is a proposition that is characterized by many degrees of truth-values with respect to many corresponding attribute-values (or random variables) that characterize *P*.

It is a pluri-logic.

We denote it by $P(V_1, V_2, \ldots, V_n)$, for $n \geq 1$, where $V_1, V_2, \ldots, V_n$ are the random variables that determine, each of them in some degree, the truth-value of *P*.

The variables may be independent one by one, or may have some degree of dependence among some of them. The degrees of independence and dependence of variables determine the plithogenic logic conjunctive operator to be used in the computing of the cumulative truth of *P*.

The random variables may be: classical, fuzzy, intuitionistic fuzzy, indeterminate, neutrosophic, and other types of fuzzy extensions.

$P(V_1) = t_1$ or the truth-value of the proposition *P* with respect to the random variable V_1.

$P(V_2) = t_2$ or the truth-value of the proposition *P* with respect to the random variable V_2.

…

And so on, $P(V_n) = t_n$ or the truth-value of the proposition *P* with respect to the random variable V_n.

The variables $V_1, V_2, \ldots, V_n$ are described by various types of probability distributions, $P(V_1), P(V_2), \ldots, P(V_n)$. The whole proposition *P* is, therefore, characterized by *n* probability distributions, or n sub-truth-values. By combining all of them, we get a *cumulative truth-value* of the logical proposition *P*.

Plithogenic Logic / Set / Probability and Statistics were introduced by Smarandache [1] in 2017 and he further on (2018 -2020) developed them [2-6].

They were applied in many fields by various authors [7-27].

The Plithogenic MultiVariate Analysis used in the Set theory, Probability, and Statistics is now used in Logic, giving birth to the Plithogenic Logic.

Plithogenic MultiVariate Analysis is a generalization of the classical MultiVariate Analysis.

2. Classification of the Plithogenic Logics

Depending on the real-values of $t_1, t_2, \ldots, t_n$, we have:

2.1. Plithogenic Boolean (or Classical) Logic

It occurs when the degrees of truths $t_1, t_2, \ldots, t_n \in \{0,1\}$, where 0 = false, and 1 = true.

2.2. Plithogenic Fuzzy Logic

When the degrees of truths $t_1, t_2, \ldots, t_n$ are included in [0,1], and at least one of them is included in (0, 1), in order to distinguish it from the previous Plithogenic Boolean Logic.

Herein, we have:

2.2.1. Single-Valued Plithogenic Fuzzy Logic, if the degrees of truths $t_1, t_2, \ldots, t_n$ are single (crisp) numbers in [0, 1].

2.2.2. Subset-Valued (such as Interval-Valued, Hesitant-Valued, etc.) Plithogenic Fuzzy Logic, when the degrees of truths $t_1, t_2, \ldots, t_n$ are subsets (intervals, hesitant subsets, etc.) of [0, 1].

2.3. Plithogenic Intuitionistic Fuzzy Logic

When $P(V_j) = (t_j, f_j)$, when t_j, f_j are included in [0,1], $1 \leq j \leq n$, where t_j is the degree of truth, and f_j is the degree of falsehood of the proposition P, with respect to the variable V_j.

In the same way, we have:

2.3.1. Single-Valued Plithogenic Intuitionistic Fuzzy Logic, when all degrees of truths and falsehoods are single-valued (crisp) numbers in [0, 1].

2.3.2. Subset-Valued Plithogenic Intuitionistic Fuzzy Logic, when all degrees of truths and falsehoods are subset-values included in [0, 1].

2.4. Plithogenic Indeterminate Logic

When the probability distributions of the random variables $V_1, V_2, \ldots, V_n$ are indeterminate (neutrosophic) functions, i.e. functions with vague or unclear arguments and/or values.

2.5. Plithogenic Neutrosophic Logic

When $P(V_j) = (t_j, i_j, f_j)$, with t_j, i_j, f_j included in [0,1], $1 \leq j \leq n$, where t_j, i_j, f_j are the degrees of truth, indeterminacy, and falsehood respectively of the proposition P with respect to the random variable V_j.

Similarly, we have:

2.5.1. Single-Valued Plithogenic Neutrosophic Logic, when all degrees of truths, indeterminacies, and falsehoods are single-valued (crisp) numbers in [0, 1].

2.5.2. Subset-Valued Plithogenic Neutrosophic Logic, when all degrees of truths, indeterminacies, and falsehoods are subset-values included in [0, 1].

2.6. Plithogenic (other fuzzy extensions) Logic

Where other fuzzy extensions are, as of today: Pythagorean Fuzzy, Picture Fuzzy, Fermatean Fuzzy, Spherical Fuzzy, q-Rung Orthopair Fuzzy, Refined Neutrosophic Logic, and refined any other fuzzy-extension logic, etc.

2.7. Plithogenic Hybrid Logic

When $P(V_1), P(V_2), \ldots, P(V_n)$ are mixed types of the above probability distributions.

3. Applications

3.1. Pluri-Truth Variables

In our everyday life, we rarely have a "simple truth", we mostly deal with "complex truths". For example:

- You like somebody for something, but dislike him for another thing;
- Or, you like somebody for something in a degree, and for another thing in a different degree;
- Similarly, for hating somebody for something in a degree, and for another thing in a different degree.

An egalitarian society (or system) does not exist in fact in our real world. It is too rigid. The individuals are different, and act differently.

Therefore, in our world, we deal with a "*plitho-logic*" (*plitho* means, in Greek, *many, pluri-*) or "complex logic". And this is best characterized by the *Plithogenic Logic*.

3.2. Types of Random Truth-Variables

The truth depends on many parameters (random variables), not only on a single one, and at the end we need to compute the *cumulative truth* (truth of all truths).

The random variables may be classical (with crisp/exact values), but often in our world they are vague, unclear, only partially known, with indeterminate data.

3.3. Weights of the Truth-Variables

Some truth may weight more than another truth.

For example, you may like somebody for something more than you dislike him/her for another thing.

Or the opposite, you may dislike somebody for something more than you like him/her for another thing.

3.4. Degrees of Subjectivity of the Truth Variables

In the soft sciences, such as: sociology, political science, psychology, linguistics, etc., or in the culture, literature, art, theatre, dance, there exists a significant *degree of subjectivity* in measuring the truth. *It is not beautiful what is beautiful, but it is beautiful what I like myself*, says a Romanian proverb.

4. Generalizations

Plithogenic Logic is a generalization of all previous logics: Boolean, Fuzzy, Intuitionistic Fuzzy, Neutrosophic Logic, and all other fuzzy-extension logics.

It is a MultiVariate Logic, whose truth variables may be in any type of the above logics.

5. Example

Let us consider an ordinary proposition P, defined as below:

$$P = \textit{John loves his city}$$

and let's calculate its complex truth-value.

Of course, lots of attributes (truth-variables) may characterize a city (some of them unknown, other partially known, or approximately known). A complete spectrum of attributes to study is unreachable.

For the sake of simplicity, we consider the below five propositions as 100% independent two by two.

In this example we only chose a few variables V_j, for $1 \le j \le 5$:

V_1 : low / high percentage of COVID-19 virus infected inhabitants;

V_2 : nonviolent / violent;

V_3 : crowded / uncrowded;

V_4 : clean / dirty;

V_5 : quiet / noisy,

A more accurate representation of the proposition P is $P(V_1, V_2, V_3, V_4, V_5)$.

With respect to each variable V_j, the $P(V_j)$ included in [0, 1] has, in general, different truth-values, for $1 \leq j \leq 5$.

Suppose John prefer his city to have (or to be): low percentage of COVID-19 infected inhabitants, non-violent, uncrowded, clean, and quiet.

$P(V_j)$ is the degree in which John loves the city with respect to the way the variable V_j characterizes it.

5.1. Plithogenic Boolean (Classical) Logic

$P(V_1) = 1$
$P(V_2) = 0$
$P(V_3) = 1$
$P(V_4) = 0$
$P(V_5) = 1$

Therefore $P(V_1, V_2, V_3, V_4, V_5) = (1, 0, 1, 0, 1)$, or John loves his city in the following ways:

- in a degree of 100% with respect to variable V_1;
- in a degree of 0% with respect to variable V_2;
- in a degree of 100% with respect to variable V_3;
- in a degree of 0% with respect to variable V_4;
- in a degree of 100% with respect to variable V_5.

The cumulative truth-value will be, in the classical way, the classical conjunction ($\wedge_c$), where c stands for classical:

$$1 \wedge_c 0 \wedge_c 1 \wedge_c 0 \wedge_c 1 = 0,$$

or John likes his city in a cumulative classical degree of 0%!

The classical logic is rough, therefore more refined logics give a better accuracy, as follows.

5.2. Plithogenic Fuzzy Logic

The 100% or 0% truth-variables may not exactly fit John's preferences, but they may be close. For example:

$$P(V_1, V_2, V_3, V_4, V_5) = (0.95, 0.15, 0.80, 0.25, 0.85),$$

which means that John loves his city:

- in a degree of 95% with respect to variable V_1;
- in a degree of 15% with respect to variable V_2;
- in a degree of 80% with respect to variable V_3;
- in a degree of 25% with respect to variable V_4;
- in a degree of 85% with respect to variable V_5.

Using the fuzzy conjunction ($\wedge_F$) min operator, we get:

$$0.95 \wedge_F 0.15 \wedge_F 0.80 \wedge_F 0.25 \wedge_F 0.85 = \min\{0.95, 0.15, 0.80, 0.25, 0.85\} = 0.15$$

or John likes his city in a cumulative fuzzy degree of 15%.

5.3. Plithogenic Intuitionistic Fuzzy Logic

$$P(V_1, V_2, V_3, V_4, V_5) = ((0.80, 0.20), (0.15, 0.70), (0.92, 0.05), (0.10, 0.75), (0.83, 0.07)),$$

which means that John loves his city in a degree of 80%, and dislikes it a degree of 20%, and so on with respect to the other variables.

Using the intuitionistic fuzzy conjunction ($\wedge_{IF}$) min/max operator in order to get the cummulative truth-value, one has:

$$(0.80, 0.20) \wedge_{IF} (0.15, 0.70) \wedge_{IF} (0.92, 0.05) \wedge_{IF} (0.10, 0.75) \wedge_{IF} (0.83, 0.07) =$$
$$= (\min\{0.80, 0.15, 0.92, 0.10, 0.83\}, \max\{0.20, 0.70, 0.05, 0.75, 0.07\}) = (0.10, 0.75),$$

or John likes and dislikes his city in a cumulative intuitionistic fuzzy degree of 10%, and respectively 75%.

5.4. Plithogenic Indeterminate Logic

$$P(V_1, V_2, V_3, V_4, V_5) = (0.80 \text{ or } 0.90, [0.10, 0.15], [0.60, \text{unknown}], > 0.13, 0.79),$$

which means that John loves his city:

- in a degree of 80% or 90% (he is not sure about) with respect to variable V_1;
- in a degree between 10% or 15% with respect to variable V_2;
- in a degree of 60% or greater with respect to variable V_3;
- in a degree greater than 13% (i.e. in the interval (0.13, 1]) with respect to variable V_4;
- in a degree of 79% with respect to variable V_5.

Therefore, the variables provide indeterminate (unclear, vague) values.

Applying the indeterminate conjunction ($\wedge_I$) min operator, we get:

$$\min\{(0.80 \text{ or } 0.90), [0.10, 0.15], [0.60, \text{unknown}), (0.13, 1], 0.79\} = 0.10.$$

5.5. Plithogenic Neutrosophic Logic

$$P(V_1, V_2, V_3, V_4, V_5) = ((0.86, 0.12, 0.54), (0.18, 0.44, 0.72), (0.90, 0.05, 0.05), (0.09, 0.14, 0.82), (0.82, 0.09, 0.14)),$$

which means that John loves the city in a degree of 86%, the degree of indeterminate love is 12%, and the degree of dislike is 54% with respect to the variable V_1, and similarly with respect to the other variables.

Again, using the neutrosophic conjunction ($\wedge_N$) min/max/max operator in order to obtain the cumulative truth-value, one gets:

$$(0.86, 0.12, 0.54) \wedge_N (0.18, 0.44, 0.72) \wedge_N (0.90, 0.05, 0.05) \wedge_N (0.09, 0.14, 0.82) \wedge_N (0.82, 0.09, 0.14) = (\min\{0.86, 0.18, 0.90, 0.09, 0.82\}, \max\{0.12, 0.44, 0.05, 0.14, 0.09\}, \max\{0.54, 0.72, 0.05, 0.82, 0.14\}) = (0.09, 0.44, 0.82),$$

or John loves, is not sure (indeterminate), and dislikes his city with a cumulative neutrosophic degree of 9%, 44%, and 82% respectively.

5. Future Research

To construct the *plithogenic aggregation operators* (such as: intersection, union, negation, implication, etc.) of the variables $V_1, V_2, \ldots, V_n$ all together (cumulative aggregation), in the cases when variables V_i and V_j have some degree of dependence d_{ij} and degree of independence $1- d_{ij}$, with $d_{ij} \in [0, 1]$, for all $i, j \in \{1, 2, \ldots, n\}$, and $n \geq 2$.

6. Conclusions

We showed in this paper that the Plithogenic Logic is the largest possible logic of today. Since we live in a world full of indeterminacy and conflicting data, we have to deal, instead of a *simple truth* with a *complex truth*, where the last one is a cumulative truth resulted from the plithogenic aggregation of many truth-value random variables that characterize an item (or event).

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

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(updated version)