Universal NeutroAlgebra and Universal AntiAlgebra

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ABSTRACT

This paper introduces the Universal NeutroAlgebra that studies the common properties of the NeutroAlgebra structures, and the Universal AntiAlgebra that studies the common properties of the AntiAlgebraic structures.

Keywords: NeutroAlgebra, AntiAlgebra, Universal NeutroAlgebra, Universal AntiAlgebra

INTRODUCTION

In 2019 and 2020 Smarandache [1, 2, 3, 4] generalized the classical Algebraic Structures to NeutroAlgebraic Structures (or *NeutroAlgebra*) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of Partial Algebra, and to AntiAlgebraic Structures (or *AntiAlgebra*) {whose operations and axioms are totally false}.

The NeutroAlgebras & AntiAlgebras are a *new field of research*, which is inspired from our real world.

In classical algebraic structures, all axioms are 100%, and all operations are 100% well-defined,

but in real life, in many cases these restrictions are too harsh, since in our world we have things that only partially verify some laws or some operations.

Using the process of *NeutroSophication* of a classical algebraic structure we produce a NeutroAlgebra, while the process of *AntiSophication* of a classical algebraic structure produces an AntiAlgebra.

BACKGROUND

1. (Operation, NeutroOperation, AntiOperation)

01. A classical Operation ($*_m$) is an operation that is well-defined (inner-defined) for all elements of the set S, i.e. $*_m(x_1, x_2, \ldots, x_m) \in S$ for all $x_1, x_2, \ldots, x_m \in S$.

02. An AntiOperation ($*_m$) is an operation that is not well-defined (i.e. it is outer-defined) for all elements for the set S; or $*_m(x_1, x_2, \ldots, x_m) \in U \setminus S$ for all $x_1, x_2, \ldots, x_m \in S$.

03. A NeutroOperation ($*_m$) is an operation that is partially well-defined (the degree of well-defined is T), partially indeterminate (the degree of indeterminacy is I), and partially outer-defined (the degree of outer-defined is F); where $(T, I, F) \neq (1, 0, 0)$ that represents the classical Operation, and $(T, I, F) \neq (0, 0, 1)$ that represents the AntiOperation.

An operation ($*_m$) is indeterminate if there exist some elements $a_1, a_2, \ldots, a_n \in S$ such that $*_m(a_1, a_2, \ldots, a_m)$ = undefined, or unknown, or unclear, etc.

2. (Axiom, NeutroAxiom, AntiAxiom)

A1. A classical Axiom is an axiom that is true for all elements of the set S.

A2. An AntiAxiom is an axiom that is false for all elements of the set S.

A3. A NeutroAxiom is an axiom that is partially true (the degree of truth is T), partially indeterminate (the degree of indeterminacy is I), and partially false (the degree of falsehood is F), where $(T, I, F) \neq (1, 0, 0)$ that represents the classical Axiom, and $(T, I, F) \neq (0, 0, 1)$ that represents the AntiAxiom.

3. (Algebra, NeutroAlgebra, AntiAalgebra)

S1. A classical Algebra (or Algebraic Structure) is a set S endowed only with classical Operations and classical Axioms.

S2. An AntiAlgebra (or AntiAlgebraic Structure) is a set S endowed with at least one AntiOperation or one AntiAxiom

S3. A NeutroAlgebra (or NeutroAlgebraic Structure) is a set S endowed with at least one NeutroOperation or one NeutroAxiom, and no AntiOperation and no AntiAxiom.

UNIVERSAL NEUTROALGEBRA AND UNIVERSAL ANTIALGEBRA

1. A Universe of Discourse, a Set, some Operations, and some Axioms

Let's consider a non-empty set S included in a universe of discourse U, or $S \subset U$.

The set S is endowed with *n* operations, $1 \leq n \leq \infty$, $*_1$, $*_2$, …, $*_n$.

Each operation $*_i$, for $i \in \{1, 2, ..., \infty\}$, is an m_i-ary operation, where $0 \leq m_i \leq \infty$. {A o-ary operation, where "0" stands for zero (or null-ary operation), simply denotes a constant.}

Then a number of α axioms, $0 \leq \alpha \leq \infty$, is defined on S.

The axioms may take the form of identities (or equational laws), quantifications {universal quantification ($\forall$) except before an identity, existential quantification ($\exists$)}, inequalities, inequations, and other relations.

With the condition that there exist at least one m-ary operation, with $m \geq 1$, or at least one axiom.

We have taken into consideration the possibility of infinitary operations, as well as infinite number of axioms.

2. The Structures, almost all, are NeutroStructures

A classical Structure, in any field of knowledge, is composed of: a non-empty space, populated by some elements, and both (the space and all elements) are characterized by some relations among themselves, and by some attributes.

Classical Structures are mostly in theoretical, abstract, imaginary spaces.

Of course, when analysing a structure, it counts with respect to what relations and attributes we analyse it.

In our everyday life almost all structures are NeutroStructures, governed by Universal NeutroAlgebras and Universal AntiAlgebras, since they are neither perfect nor uniform, and not all elements of the structure's space have the same relations and same attributes in the same degree (not all elements behave in the same way).

Conclusions

Since our world is full of indeterminacies, uncertainties, vagueness, contradictory information almost all existing structures are NeutroStructures, since either their spaces, or their elements or their relationships between elements or between are characterized by such indeterminacies.

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