Soft Set Product Extended to HyperSoft Set and IndetermSoft Set Product Extended to IndetermHyperSoft Set (updated)

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Abstract

In this paper we define the Soft Set Product as a product of many soft sets and afterwards we extend it to the HyperSoft Set. Similarly, the IndetermSoft Product is extended to the IndetermHyperSoft Set. We also present several applications of the Soft Set Product to Fuzzy (and fuzzy-extensions) Soft Set Product and to IndetermSoft Set and IndetermHyperSoft Set.

Keywords: HyperSoft set, IndetermSoft set, IndetermHyperSoft set, product.

1 | Introduction

Using the Product of the Soft Set [1] we connect it with its generalization called HyperSoft Set [2], [3]. Then, similarly, through another Product the IndetermSoft Set [4] is connected to the IndetermHyperSoft Set [4] and we present several applications of them.

2 | Definition of Soft Set Product

Let's have $n \geq 2$ Soft Sets

$$F_1: A_1 \rightarrow P(H),$$
$$F_2: A_2 \rightarrow P(H),$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$F_n: A_n \rightarrow P(H),$$

where $a_1, a_2, \dots, a_n$ are n distinct attributes, and respectively $A_1, A_2, \dots, A_n$ their corresponding sets of attributes' values, such that

$A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$.

The Soft Set Product $F_1 \times F_2 \times \ldots \times F_n$ is defined as follows:

$$F_1 \times F_2 \times \ldots \times F_n : A_1 \times A_2 \times \ldots \times A_n \to \underbrace{P(H) \times P(x) \times \ldots \times P(H)}_{n \text{ times}}.$$

3 | The Soft Set Product transformed into a HyperSoft Set

Let's denote $F = F_1 \times F_2 \times \ldots \times F_n$.

Then we define the HyperSoft Set in the following way:

$$F : A_1 \times A_2 \times \ldots\ A_n \to P(H).$$

such that for each $(e_1, e_2, \ldots, e_n) \in A_1 \times A_2 \times \ldots\ A_n$,

$$F(e_1, e_2, \ldots, e_n) = F_1(e_1) \cap F_2(e_2) \cap \ldots \cap F_n(e_n).$$

4 | Fuzzy (and Any Fuzzy-Extension) Soft Set Product

Let's denote the fuzzy (and any fuzzy-extension degree in general) by d_0.

For example, if one uses the Fuzzy Soft Set, then $d_0 = T$ (degree of truth), $T \in [0, 1]$.

If one uses the Intuitionistic Fuzzy Set, the $d_0 = (T, F)$ or degree of truth/ falsehood, $T, F \in [0, 1]$, $0 \leq T + F \leq 1$;

If one uses the Neutrosophic Set, then $d_0 = (T, I, F)$, or degree of truth/ indeterminacy/ falsehood, one has:

$$0 \leq T + I + F \leq 3.$$

Let $H = \{h_1, h_2, h_3, h_4\}$, a set of houses.

Let the attributes a_1 = size, whose set of values is A_1 = {small, medium, big} and a_2 = location, whose set of values is A_2 = {central, peripherical}.

The soft sets

$$F_1 : A_1 \to P(H),$$

$$F_2 : A_2 \to P(H),$$

and their Product

$$F = F_1 \times F_2,$$

$$F : A_1 \times A_2 \to P(H).$$

F: {small, medium, big} $\times$ {central, peripherical} $\to P(\{h_1, h_2, h_3, h_4\})$.

F_1 (small) $= \{h_1, h_3, h_4\} \stackrel{def}{=} H_{11}$.

F_1 (medium) $= \{h_1, h_4\} \stackrel{def}{=} H_{12}$.

F_1 (big) $= \{h_2\} \stackrel{def}{=} H_{13}$.

F_2 (central) = $\{h_1\} \stackrel{def}{=}$ H_{21}.

F_2 (peripheral) = $\{h_2, h_3, h_4\} \stackrel{def}{=}$ H_{22}.

Size \ Location	Central	Peripheral
small	$\{h_1\}$	$\{h_3, h_4\}$
medium	$\{h_1\}$	$\{h_4\}$
big	∅ (no house)	$\{h_2\}$

5 | HyperSoft Set Resulted from a Product of Two Soft Sets

How to read the above table, for example the intersection between the row 1 (small) and the column 2 (peripheral) gives

F [small, peripheral] = F_1 (small) ∩ F_2 (peripheral) = $H_{11} \cap H_{22} = \{h_1, h_3, h_4\} \cap \{h_2, h_3, h_4\} = \{h_3, h_4\}$.

6 | Fuzzy and Fuzzy-Extensions Soft Set and HyperSoft Set

F_1 (small) = $H_2(d_1^0) = \{h_1, h_3, h_4\}(d_{11}^0)$.

F_2 (peripheral) = $H_2(d_2^0) = \{h_2, h_3, h_4\}(d_{22}^0)$.

F (small, peripheral) = $H_{11}(d_{11}^0) \cap H_{22}(d_{22}^0) = \{h_3, h_4\}(d_{11}^0 \wedge d_{22}^0)$.

7 | Definition of the IndetermSoft Set

Let U be a universe of discourse, H a non-empty subset of U, and P(H) the powerset of H. Let a be an attribute, and A be a set of this attribute values.

A function F: A ⟶ (H) is called an IndetermSoft Set (Function) if

I. The set A has some indeterminacy with respect to one or more attribute's values.
II. Or P(H) has some indeterminacy.
III. Or there exist at least an attribute value v ∈ A, such that F(v) = indeterminate (unclear, uncertain, or not unique).
IV. Or any two or all three of the above situations.

In other words, an IndetermSoft Set, introduced by Smarandache in [4], is a soft set that has some degree of indeterminate (unclear, uncertain, alternative, conflicting) data or procedure.

8 | Example of IndetermSoft Set and IndetermHyperSoft Set

F_1 (small) = $\{h_1, or\ h_4\}$ = {either h_1, or h_2, or h_1 and h_2} $\stackrel{def}{=}$ H_{12}.

F_2 (peripheral) = $\{\ noth_2\ \}$ = {h_1, or h_3, or h_4} $\stackrel{def}{=}$ H_{134}.

F (small, peripheral) = $H_{12} \cap H_{134}$ = {h_1, or ϕ (no house)} $\stackrel{def}{=}$ H_{10}.

The first two equations above represent two IndetermSoft Set, only the third one gets the Product of them and obtains a IndetermHyperSoft Set.

As seeing, one has indeterminacy with respect to H_{12}, H_{123}, and the final result H_{10}.

Conclusion

We presented for the first time the connection between the Product of Soft Sets and the HyperSoft Set, and afterwards the Product of IndetermSoft Set that produced the IndetermHyperSoft Set, together with several applications.

References

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