Practical Applications of the Independent Neutrosophic Components and of the Neutrosophic Offset Components

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Abstract: The newly introduced theories, proposed as extensions of the fuzzy theory, such as the Neutrosophic, Pythagorean, Spherical, Picture, Cubic theories, and their numerous hybrid forms, are criticized by the authors of [1]. In this paper we respond to their critics with respect to the neutrosophic theories and show that the DST, that they want to replace the A-IFS with, has many flaws.

Their misunderstanding, with respect to the partial and total independence of the neutrosophic components, is that in the framework of the neutrosophic theories we deal with a *MultiVariate Truth-Value* (truth upon many independent random variables) as in our real-life world, not with a *UniVariate Truth-Value* (truth upon only one random variable) as they believe.

About the membership degrees outside of the interval [0, 1], which are now in the arXiv and HAL mainstream, it is normal that somebody who over-works (works overtime) to have an over-membership (i.e., membership degree above 1) to be distinguished from those who do not work overtime (whose membership degree is between 0 and 1). And, similarly, a negative employee (that who does only damages to the company) to have a negative membership (i.e., membership degree below 0) in order to distinguish him from the positive employees (those whose membership degree is above 0). There are elementary practical applications in this paper that allow us to think out of box (in this case the box is the interval [0, 1]).

Keywords: Neutrosophy; Neutrosophic Components; Neutrosophic Offset Components; TriVariate Truth-Value; MultiVariate Truth-Value; UniVariate Truth-Value.

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1. Independence and Dependence of the Neutrosophic Components

The introduction should briefly place the study in a broad context and highlight why it is important. It should define the purpose of the work and its significance. The current state of the research field should be reviewed carefully and key publications cited. Please highlight controversial and diverging hypotheses when necessary. Finally, briefly mention the main aim of the work and highlight the principal conclusions. As far as possible, please keep the introduction comprehensible to scientists outside your particular field of research. References should be numbered in order of appearance and indicated by a numeral or numerals in square brackets, e.g., [1] or [2,3], or [4–6]. See the end of the document for further details on references.

1.1 TriVariate Truth-Value

Neutrosophy [15], as new branch of philosophy, started from the practical principle that everything (E) should be evaluated from *three independent points of view (or sources of information, or* *random variables)*: two opposite ones (positive and negative), and a third one the neutral in between them, for a fear evaluation. Thus, a neutrosophic triplet has been constructed, <positive, neutral, negative>, for studying especially contrary philosophical concepts, ideas, and schools. Therefore, one deals with a TriVariate Truth-Value because one uses three independent random variables (sources of information): one that presents the degree of positive side of E, another that presents the degree of negative side of E, and a third one that presents the degree of neutral (indeterminate) side of E.

That's what happens in our everyday life, and the most known one is in the court of law (defender, persecutor, jury). Also, everything has good, bad, and common features.

{Surely, more generally, everything may be evaluated from n points of view (n random variables, or n sources of information), for any integer $2 \le n \le \infty$, as such dealing with a *MultiVariate Truth-Value*, where the random variables may have degrees of positiveness, or negativeness, or neutrality (indeterminacy), but this case falls under the Refined Neutrosophic Logic [13], or under the Plithogenic Logic as generalization of MultiValued Logic [14], or under the Plithogenic Probability & Statistics as generalizations of MultiVariate Probability & Statistics [30], which are different stories.}

For example, in general you are evaluated by a friend in a positive way, by an enemy in a negative way, and by a neutral person in a neutral way.

Surely, in the Refined Neutrosophic Set and Logic and Probability , you may be evaluated by many friends in positive ways, and by many enemies in negative ways, and by many neutral persons in neutral ways. That's life, as in neutrosophy.

This ThreeVariate way of thinking (neutrosophic evaluation) was transferred to the scientific disciplines that resulted from neutrosophy:

Neutrosophic Set (degree of membership, degree of indeterminate-membership, degree of nonmembership);

Neutrosophic Logic (degree of truth (T), degree of indeterminate-truth (I), degree of falsehood (F));

Neutrosophic Probability (chance of an event to occur, indeterminate-chance of the event to occur or not, chance of the event not to occur); etc.

For simplicity, we preferred to use the descriptive notation (T, I, F) for all neutrosophic triplets.

Let's consider the single-valued neutrosophic components, where all $T, I, F \in [0,1]$.

Depending on each application, in the neutrosophic theories one may encounter three (or more) possibilities:

a. UniVariate Truth-Value, when only one source assigns values to the neutrosophic components, and thus the neutrosophic components are totally dependent as in the other fuzzy theories, whence $0 \le T + I + F \le 1$.
b. BiVariate Truth-Value, when two independent sources assign values to the neutrosophic components, for example one source assigns values to two neutrosophic components (let's assume to T and F, thus $0 \le T + F \le 1$) and the second one to the other neutrosophic component (which is I, thus $0 \le I \le 1$), and therefore the neutrosophic components are partially dependent and partially independent {or T and F are totally dependent of each other, while I is totally independent from both of them}, whence $0 \le T + I + F \le 2$.
c. TriVariate Truth-Value, when three independent sources assign values to the neutrosophic components, each source to one distinct neutrosophic component, thus $0 \le T + I + F \le 3$) and all three neutrosophic components are totally independent.
d. TriVariate Truth-Value, when the three sources are partially dependent and partially independent. For example, John's work is evaluated by three sources: a friend, an enemy, and a neutral person, which communicate with each other and arrive to some agreement about John's work that is interpreted as *degree of dependence* (d) between these three sources,

and to some disagreement about John's work that is interpreted as *degree of independence* (*i*) between the three sources, where $d, i \in [0,1], d + i = 1$.

e. <u>MultiVariate Truth-Value</u>, in general, for Refined Neutrosophic Set/Logic/Probability [13], and for Plithogenic Logic/Probability/Statistics [14, 30].

1.2 "Unfortunately, this fact [*independence of components – our note*] is not usually taken into account in the works, where NST was applied."

Their assertion is untrue, the independence of components was used in most of the neutrosophic applications.

The independence of the neutrosophic components comes from the unrestricted summation T + I + F that can get any value between 0 and 3. The independence comes from the fact that if a neutrosophic component gets a value, it does not affect in no way the other two neutrosophic components' values. Not restricting the value of the sum T + I + F means from the start the existence of degrees of independence and dependence between the components.

In many neutrosophic applications that presented numerical examples, looking at the neutrosophic triplets (T, I, F), you would see: some whose sum is < 1, others whose sum is > 1, and others whose sum is = 1. For example (0.1, 0.3, 0.5), or (0.9, 0.8, 0.6), or (0.7, 0.1, 0.2), etc.

Also, in all neutrosophic papers the neutrosophic operators were employed, which means that the Indeterminacy (I) was used independently from T and F into the operators' formulas, which is not the case for the previous classical, fuzzy (especially A-IFS) set and logic, and probability theories.

Unlike in other previous theories (for example in DST), no normalization is done in the neutrosophic theories, therefore, after aggregation, the resulted neutrosophic components sum may be any number between 0 and 3.

Yet, the situation is more complex, since the neutrosophic theories comprises all possibilities of the neutrosophic components, i.e.: to be totally independent, partially independent and partially dependent, and totally dependent. Not only the case of the totally independent components - as they have written in their equation (6).

1.3. In their paper [1], their equation (6):

"$0 \le T + I + F \le 3$ for the completely independent components"

is partially wrong.

The correct one is only:

"$0 \le T + I + F \le 3$"

which means that the summation T + I + F can be any number in [0, 3], with $T, I, F \in [0,1]$,

and consequently, it comprises all possibilities, i.e. the components may be:

either totally independent, or partially independent and partially dependent, or totally dependent.

The independence and dependence of the components depend on each application and on the experts. Practical examples will follow below.

It is obvious that if $T, I, F \in [0,1]$, then of course $0 \le T + I + F \le 3$, but we emphasized this double inequality to make sure the readers would not take for granted that $0 \le T + I + F \le 1$ as in the previous classical, fuzzy set and logic, and probability theories. Therefore $0 \le T + I + F \le 3$ is no restriction at all!

1.4. "We have deep doubts about the validity of this hypothesis of the components mutual independence from its practical applicability point of view" (p. 3).

Ironically, just the practical applications have inspired us to consider the independence of the components, and very simple ones, as these authors will see below.

Their misunderstanding is that these authors are considering only the UniVariate Truth-Value {truth that depends on a single parameter (or point of view, or random variable), which enforces the sum of the neutrosophic components to be up to 1, and they are totally dependent}. But, in our everyday life, we almost always deal with a MultiVariate Truth-Value {truth that depends on many independent parameters (or random variables, or sources of information), and the neutrosophic components may be: partially dependent and partially independent, or they may be totally independent}.

Practical Examples will follow below.

In general,

$$\textit{UniVariate Truth-Value} \neq \textit{MultiVariate Truth-Value}.$$

Complete Independence of the neutrosophic components means that there are different (and independent) sources of information that provide estimations on each of T, I, and F respectively.

This happens in our everyday life: an item (person, object, event, action, proposition, theory, etc.) is evaluated from many points of view (or many random variables).

1.5 "According to the independence hypothesis, the event $T = 1$, $F = 1$ and $I = 1$ is allowed in the NST and in this case, the constraint (6) is fulfilled. Suppose T, F and I are the degrees of truth, false and indeterminacy, respectively (this is the notation used in the NST). Thus, if we deal with a complete truth ($T = 1$), then in compliance with the formal logic and common sense, the measure of false is 0 ($F = 0$) without any indeterminacy ($I = 0$)."

{We used the notations T, I, F because they are more descriptive for the Truth, Indeterminacy (or Neutrality), and Falsehood respectively, instead of the Greek letters μ, π, ν that are not descriptive and were used in their paper [1].}

Here it is their confusion, these authors consider only the UniVariate Truth-Value of a proposition.

As we showed before, from a point of view a proposition may be true, from other point of view it may be false, or may be neutral (or indeterminate).

When these authors talk about "common sense" they are automatically / stereotypically referring to a single source of information that provides information about all three neutrosophic components of a proposition (therefore the components are all totally dependent). When a single source provides information about an event, it knows and adjusts the sum of the components to be 1. See the below practical examples.

1.6 "It is interesting that the events $T = 1$, $F = 1$, $I = 1$ and $T = 0$, $F = 0$, $I = 0$ are interpreted in [9] as a paradox, and its definition is treated as a merit of the NST. In our opinion, generally, it seems to be more reliable to use theories, which have no paradoxes" (p. 4).

We agree to these authors with the fact that the theories that have paradoxes are not reliable, but the Neutrosophic Logic was not designed for the theories with paradoxes.

We only proved that a proposition (not theory) P, which is a paradox (totally true and false in the same time, and totally uncertain as well), can be represented in the Neutrosophic Logic as P(1, 1, 1), while in other classical or fuzzy and fuzzy extension set, logic or probability theories the proposition P cannot be represented, since the sum of the components is not allowed to be greater than 1.

2. Practical Examples of Independent or Dependent Neutrosophic Components

Let's see several practical examples, as these authors have required:

2.1 Practical Example 1

The following event E takes place:

E = {There is a street protest in Minneapolis}.

a. From the point of view of the Human Rights Activists the protest is positive, because people have the right to express their view, and consequently the CNN television station (reflecting the *left politics*) joys it. Let's say T(positiveness) = 0.8.

b. But, from the point view of the Police, the protest is negative, since the protesters are violent and destroy and burn houses and injure people; then the Fox News television station (reflecting the *right politics*) presents the negative side of the protests: violence, destruction, arson, chaos. Let's say F(negativeness) = 0.9.

c. Let's consider an unbiased (neutral) Media that reports on the event. This is the neutral source, it evaluates the event in general as, for example, I(positiveness and negativeness) = 0.4.

As seen, T + I + F > 1, and the three neutrosophic components T, I, and F are totally independently assessed, since the Human Right Activists, the Police, and Media are three different and independent entities.

The authors wrote: "Therefore, we can say, e.g., that the high degree of truth is obligatory accompanied by the low degrees of false and indeterminacy." (p. 3).

This is true ONLY for the UniVariate Truth-Value of the Classical and Fuzzy Logic. This is false for the MultiVariate Truth-Value of the Neutrosophic Logic as we previously proved with several elementary practical examples.

To contradict these authors, let's assume, in this practical example, that the Human Rights Activists reassess their evaluation of the event, and they reassign T(positiveness) = 0.7. But this has nothing to do with the Police or Media to reassess their evaluations of F(negativeness) and I(positiveness and negativeness) respectively. Since all three sources, and thus the T, I, F, are totally independent. If a neutrosophic component increases or decreases, it may have no effect on the other neutrosophic components.

This is a TriVariate Truth-Value, since the event E is evaluated by three independent parameters (from three different points of view): Human Rights Activists, Police, and Media.

As seen, it's not fair to analyze something from only one point of view (from only one parameter).

This is a TriVariate Truth-Value.

2.2 Practical Example 2

A murderer John Doe is being tried in the court of law for having committed a crime. There are three player parts in the court:

the Persecutor team, which presents the suspect in a negative way, for example F(Doe) = 0.9;

the Defense team, that presents the suspect in a positive way, for example T(Doe) = 0.4;

and the Jury, that is neutral, where $I(Doe) \in [0,1]$.

Herein, the Persecutor and the Defense are totally independent sources (since they are opposite). Therefore, T and F are totally independent.

But the Jury is dependent on the evidences provided by both the Persecutor and the Defense.

Therefore, the neutrosophic component I is totally dependent on both T and F.

Let's assume I = 0 means not guilty, I = 1 means guilty, while $I \in (0,1)$ means a hung-jury (i.e. some jurors say he is guilty, while others say he is not guilty) or unable to reach a verdict.

This is a TriVariate Truth-Value.

2.3 Practical Example 3 that refutes their assertation

Proposition: G = George is a good student.

George is evaluated by three different independent professors.

The math prof: George is excellent in mathematics and he gets only A's. Hence T(G) = 1.

The sport prof: George is the worst athlete in the team since he cannot run, cannot play baseball. Hence F(G) = 1.

The literature prof: I am totally uncertain about George's ability to write a literary composition since he never turned in any of them. Hence I(G) = 1.

Therefore we got G(1, 1, 1).

This is a TriVariate Truth-Value.

2.4 Example 4 that refutes their assertation

A paradox is a proposition that is true and false at the same time (hence T = F = 1), and completely unclear/indeterminate (hence I = 1).

2.5 Example 5 from mathematics that refutes their assertation

Assume the proposition M is "1 + 1 = 10".

If the base of numeration is 2, then proposition M is true: T(M) = 1.

If the base of numeration is 10, the proposition M is false: F(M) = 1.

This is a proposition that is totally true and totally false, without being a paradox.

Herein one has a BiVariate Truth-Value (i.e. with respect to two parameters: Base 2, and respectively Base 10).

If the base of numeration is unknown (let's denote it by *b*), then the truth-value of M is also unknown (indeterminate): I(M) = 1.

Now one has a TriVariate Truth-Value (i.e. with respect to three parameters: Base 2, Base 5, and unknown Base *b*).

2.6 Example 6 of independent and dependent neutrosophic components

There will be a football match between Poland and Belarus. For each country there are three possibilities: to win, to draw, or to lose. Therefore, as in neutrosophic theories.

a) Totally independent neutrosophic components

Asking a Polish person what is Poland's chance to win, he may say T(Poland) = 0.8.

But a Belarusian person may say that Belarus will win, let's say F(Poland) = 0.7.

Another person, from another country (Romania), may answer that it is a chance of a tie game: I(Poland) = 0.4.

It is supposed that the three sources, the Polish, Belarusian, and Romanian persons do not communicate nor know the evaluations of the others. They are totally independent and consequently are the components T, I, F.

Herein there is a TriVariate Truth-Value.

b) Totally Dependent Neutrosophic Components

Let's assume that a Polish mathematician evaluates all three possibilities of Poland. Being a mathematician, he knows that the sum of the component has to be 1, as in the classical and fuzzy set theories, logic, or probability.

He then may say: T = 0.7, F = 0.1, I = 0.2.

The neutrosophic components are totally dependent, since all three depend on a single source.

Herein there is a UniVariate Truth-Value.

c) Partially Dependent and Partially Independent Neutrosophic Components

Another situation. Assume that a scientist George has to evaluate both chances of Poland, to win or to lose.

If he choses T = 0.6, for example, he knows that 0 ≤ F ≤ 1 – 0.6 = 0.4. Suppose he takes F = 0.3.

A second source Marcel has to evaluate the possibility of tie-game, without nothing anything about George's. Let's suppose that he says: I = 0.8.

In this case, T and F are totally dependent of each other, while I is totally independent from both T and F. Herein $0 \le T + I + F \le 2$.

Herein there is a BiVariate Truth-Value.

3. Neutrosophic Overset/Underset/Offset

"In our opinion, the most daring theory was proposed in [*18]. This theory allows negative and greater than 1 values of membership degrees. There are some basic definitions introduced in [*18], but here, we analyze only the most general one:

Definition 4. For T(x), I(x) and F(x) being the degrees of truth, indeterminacy and false, respectively, a Single-Valued Neutrosophic Offset A is defined as follows:

A = {(x, <T(x), I(x), F(x)>), x U},

such that there exist some elements in A that have at least one neutrosophic component that is > 1, and at least another neutrosophic component that is < 0.

For example: A = {(x1, <1.3, 0.3, 0.2>), (x2, <0.1, 0.4, -0.8>)}, since T(x1) = 1.3 > 1 and F(x2) = -0.8 < 0." (p. 6)

{We took the liberty of updating the reference citation to be adjusted to our paper. Instead of [16] as in these authors' reference, we wrote [*18]. See more papers on Neutrosophic Overset/Underset/Offset: [27-29].}

These neutrosophic overset (degree > 1), neutrosophic underset (degree < 0), and neutrosophic offset (some degree > 1 and other degree < 0) were well understood by the prestigious Cornell University arXiv (New York City) mainstream Archives that approved our book:

https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf

and by the mainstream French Hal Archives as well:

https://hal.archives-ouvertes.fr/hal-01340830 .

These concepts were inspired from our real life [*18, 27, 28, 29].

The authors continue with the below citation from our book:

"There is a crucial example in [*18], which clarifies the author's reasoning that we critically analyze: "In a given company a full-time employer works 40 h per week. Let's consider the last week period. Helen worked part-time, only 30 h, and the other 10 h she was absent without payment; hence, her membership degree was 30/40 = 0.75 < 1.

John worked full-time, 40 h, so he had the membership degree 40/40 = 1, with respect to this company. But George worked overtime 5 h, so his membership degree was (40+5)/40 = 45/40 = 1.125 > 1. Thus, we need to make distinction between employees who work overtime, and those who work full-time or part-time. That's why we need to associate a degree of membership strictly greater than 1 to the overtime workers." (p. 6)

The above was our practical example.

The authors reject it:

"The crucial drawback of this reasoning is the lack of the clear definition of fuzzy classes, which memberships are estimated. We can see here two distinct fuzzy classes: the class of employees working at least no more than 40 h a week and the class of employees that works more than 40 h. The first class is presented by the membership function rising from 0 to 1 in the interval [0, 40] of worked hours and equal to 0 if the sum of worked hours is greater than 40. The second class is defined by the membership function increasing from 0 to 1 in the interval of worked hours from 40 to Hmax, where Hmax is the maximal allowed by government (and trade unions) value of worked hours. We can see that such an obvious reasoning does not allow membership degrees greater than 1. The incorrect reasoning of the author of [*18] is also based on the implicit mechanical conjunction of two different classes with not intersected supports. Of course, such a conjunction can be made, but the resulting fuzzy class and the corresponding membership function should have a new sense reflecting a synthetic nature of a new class. In the considered case, we can introduce the class of "hard working employees" with the membership function rising from 0 to 1 in the interval [0, Hmax]."

There are people who invent theories and then try to squeeze the reality into them.

But, we did the opposite, we started from the real-world problems (over-work, negative work) and tried to make the theories that model / approximate the reality as accurate as possible. Late on, we improved our models little by little.

First, we do not work with fuzzy classes, but with a neutrosophic approach.

Also, we see no reason to make two classes where the membership, in both of them, starts from 0 and ends to 1. What about if one gets the same value, for example the membership degree T = 0.3 in both classes [or in the three classes, as they added one more similar class for the negative membership]? It's a confusion. On the other hand, these two classes cannot catch the employees with negative membership (those who produce damages to the company, T < 0).

These authors belong to the category of people that try to squeeze the reality (the membership degree of overtime workers which overpasses 1, or T > 1) to the narrow classical set theory, where the membership degree has to be T ≤ 1. The classical set theory is not written in stone, so we may enlarge it if the reality requires it.

When Zadeh founded in 1965 the Fuzzy Set and allowed the membership degree to be any number between 0 and 1 (not only 0 or 1 as in classical Set Theory) he was criticized at that time by several scientists (as he told me in 2003 at an international conference at the University of Berkeley, California, where we met). But he prevailed, because in the real world there exist many partial memberships.

About the membership degrees that are outside of the interval [0, 1], it is normal that somebody who over-works (works overtime) to have an over-membership (i.e., membership degree above 1) to be distinguished from those who do not work overtime (whose membership degree is between 0 and 1).

Our example of negative employee who deserves a negative membership (T < 0), is cited by these authors:

"Let us turn to the example: "Yet, Richard, who was also hired as a full-time, not only did not come to work last week at all (0 worked hours), but he produced, by accidentally starting a devastating fire, much damage to the company, which was estimated at a value half of his salary (i.e., as he would have gotten for working 20 h that week). Therefore, his membership degree has to be less that Jane's (since Jane produced no damage). Whence, Richard's degree of membership, with respect to this company, was -20/40 = -0.50 < 0." " (p. 6)

The authors continue:

"As we are analyzing only the last week, we can see that Richard does not belong to any of the classes described above. It is a member of a practically unlimited class of those who do not work for a given company. We can significantly narrow this class by considering only those people who, by their actions or inaction, cause damage to the company (the most harmful are the top managers of competing firms). This way we can estimate the maximum damage Dmax (it does not matter in money or equivalent worked hours), which can be inflicted on the company by an external detractor. Thus, the class of external (nonworking for the company) people who bring company damages can be presented by the membership function varying from 0 to 1 in the interval of damages [0, Dmax]. There is no place for any negative membership degree."

These authors did not read/understand exactly: Richard is indeed a full-time employee, he works for the company, as we have written into our book: "Richard, who was also hired as a full-time" it is certainly an employee. The authors make a false statement for Richard as "nonworking for the company".

Even so, it is not clear, why did they make a third class varying from 0 to 1 for the negative employees? As such, we'd like to return the ancient Occam's wisdom back to themselves: "Entities should not be multiplied unnecessarily."

If you have a negative person in your group, for example, which creates only problems to the group, you cannot assign him a membership degree equals to zero (as for people that do neither positive nor negative things to the group), but you should assign him a negative membership degree. It is very logical this way.

A negative employee (that who does only damages to the company) has to have a negative membership (i.e., membership degree below 0) in order to distinguish him from the positive employees (those whose membership degree is above 0).

We see no reason to complicate the problem by creating three classes of membership degrees in order to avoid membership degree values greater than 1 or less than 0, instead of keeping a single class, but enlarging it to the left-hand side of 0 and respectively to the right-hand side of 1.

Because neutrosophic set has 3 components, they would need 9 classes, not talking of the refined neutrosophic set, that may have any number $2 \leq n \leq \infty$ of refined neutrosophic components, therefore they would need 3n classes! Better they should think out of box (in this case the box is the interval [0, 1]).

4. Applicability

The authors wrote: "there is no need for such somewhat artificial and heuristic theories as the Neutrosophic, Pythagorean and Spherical sets and their derivatives" (p. 5).

We disagree. The neutrosophic theories are not artificial, they started from our real-world practicability, where there are so many neutrosophic triplets (<A>, <neutA>, <antiA), where <A> is an item (concept, proposition, idea, etc.), formed by two opposites <A> and <antiA>, together with their neutrality (indeterminacy) <neutA>.

For examples: (friend, neutral, enemy), (positive particle, neutral particle, negative particle), (masculine, transgender, feminine), (true, indeterminate, false), (win, tie-game, defeat), (yes, uncertain, no), (take a decision, pending, not taking a decision), etc.

The neutrosophic theories have many applications [25] in various fields such as: Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Genetics, Biomedical, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Chemistry, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences, etc. and there were published over 2,000 papers, books, conference presentations, MSc and PhD theses by researchers from 82 countries around the world.

With respect to what the neutrosophic theories brought new, we invite these authors to read our 2019 paper, so we do not repeat the things [26], whose weblink is provided.

Rather, these authors' transformation/substitution of the Atanassov-Intuitionistic Fuzzy Set (A-IFS) into the Dempster-Shafer Theory (DST) framework is artificial, since their transformation is not quite equivalent with the A-IFS, while practically their transformation is useless because of the very large intervals they use that supposed to catch the solution.

5. Publications

They say that the "caution of editors and reviewers of solid old journals is not caused by their conservatism at all, but by the desire to see, in addition to formal definitions of these theories and numerous theorems, the solution of real methodological and practical problems" (p. 1).

In general, in any field of knowledge, when a Theory1 is generalized by the Theory2, the proponents of Theory1 are reluctant to publish and even to admit Theory2, and the first reason is the rivalry between theories, the conservatism is only an excuse. But each theory has its flavor.

The authors are less informed, since in the last years there have been books published by prestigious publishing houses such as Springer [19, 21], Elsevier [20], IGI Global [22-24] (we cite the last ones), etc. and many high rank journals by Springer, Elsevier, IOS Press, Tayler & Francis, MDPI, Hindawi, Emerald Publishing, IGI Global, World Academy of Science Engineering and Technology, IEEE, Wiley, etc. have published papers on the neutrosophic environment, such as: Complex & Intelligent Systems, Cognitive Computation, Artificial Intelligence Review, International Journal of Fuzzy Systems, Evolving Systems, Complex & Intelligent Systems, Soft Computing, Journal of Machine Learning & Cybernetics, Multiple-Valued Logic, Design Automation for Embedded Systems, Granular Computing, Neural Computing and Applications, Journal of Systems Architecture, Applied Soft Computing, Measurement, Symmetry, Mathematics, Information, Axioms, Entropy, Computational and Applied Mathematics, BMC Medical Research Methodology, International Journal of Aerospace and Mechanical, Cognitive Systems Research, Theoretical and Applied Climatology, Journal of Metrology Society of India, Journal of King Saud University – Science, Journal of Intelligent & Fuzzy Systems, IEEE Access, Expert Systems, etc.

Further on, they will see in this paper many solutions using the neutrosophic theories to practical problems.

6. Critics of the DST

These authors [1] want to destroy the fuzzy extension theories just to promote the Dempster-Shafer Theory (DST) that they support, but from the beginning they are going on an uncertain way, since DST is a flawed theory which gives many counter-intuitive results [2-8; weblinks provided; download the papers and respond to the DST problems], as we'll show below.

They assert that all fuzzy extension theories can be substituted by the DST, which is not true.

6.1 The DST fails in the Zadeh's Counter-Example

Zadeh's Counter-Example [2], as know by all fusion community, is the following:

Two doctors examine a patient and agree that he suffers from either meningitis (M), contusion (C), or brain tumor (T). Thus Θ = {M, C, T} is the frame of discernment. Assume that the doctors agree in their low expectation of a tumor, but disagree in likely cause and provide the following diagnosis:

$m_1(M) = 0.99$, $m_1(T) = 0.01$, and $m_2(C) = 0.99$ $m_2(T) = 0.01$, where $m_1(.)$ represents the diagnoses provided by the first doctor, while $m_2(.)$ the diagnoses by the second doctor. If we combine the two basic belief functions using the DST (first doing the conjunctive rule, then the Dempster's rule of combination), one gets the unexpected conclusion:

$$m(T) = \frac{0.0001}{1 - 0.0099 - 0.0099 - 0.9801} = 1$$

which means that the patient suffers <u>with certainty</u> from brain tumor, which is wrong.

Zadeh [2] has clearly written down: "there is a serious flaw in Dempster's rule which restricts its use in many applications".

Similarly, P. M. Williams questioned the validity of Dempster's Rule [31].

6.2 The A-IFS gives a better solution to Zadeh's Counter-Example than DST

After criticizing Atanassov's Intuitionistic Fuzzy Set (A-IFS), the authors proposed "redefining the A-IFS in the framework of the more general Dempster–Shafer theory of evidence (DST)" (p. 2).

Okay, then let's set and analyze the Zadeh's Counter-Example in the frame of the A-IFS, and we show that A-IFS gives better result than DST.

Let:

$D_1 = \{M(0.99, 0), C(0, 0), T(0.01, 0)\}$,

$D_2 = \{M(0, 0), C(0.99, 0), T(0.01, 0)\}$,

where D_1 represents the diagnoses provided by the first doctor, i.e.

M(0.99, 0) means that the degree of membership (truth) of the patient to have meningitis is 0.99, and the degree of nonmembership (falsehood) of the patient not to have meningitis is 0;

And similarly for the other diseases.

And where D_2 represents the diagnoses provided by the second doctor.

Let's use the A-IFS min/max intersection operator ($\wedge_{A-IFS}$) for the two doctors' diagnoses:

$$D_1 \wedge_{A-IFS} D_2 = \{(0.99,\ 0) \wedge_{A-IFS} (0,\ 0), (0,\ 0) \wedge_{A-IFS} (0.99,\ 0), (0.01,\ 0) \wedge_{A-IFS} (0.01,\ 0)\} =$$

$$\{(\min\{0.99, 0\}, \max\{0, 0\}), (\min\{0, 0.99\}, \max\{0, 0\}), (\min\{0.01, 0.01\}, \max\{0, 0\})\} =$$

$$\{(0,0), (0,0), (0.01, 0)\} \equiv \{M(0,0), C(0,0), T(0.01,0)\}.$$

A-IFS shows that the chance of the patient of having tumor is 0.01, which is more realistic with respect to the chance of tumor of the patient, than DST's.

More counter-examples to the Dempster's rule have been published in the literature [3-8].

After these failures of the DST, new theories have been proposed, such as TBM, DSmT [9], etc. and many quantitative and qualitative fusion rules [10-12] in order to overcome the Dempster's rule counter-intuitive results.

7. Conversion from A-IFS to DST

The authors [1] propose the conversion from the framework of the A-IFS to the DST in the following way (pp. 7-8).

Let *U* be a universe of discourse, and:

$B_{A-IFS} = \{(x, <T(x), F(x)>), T(x), F(x) \in [0,1], 0 \le T(x) + F(x) \le 1, x \in U\}$ be a non-empty subset of it, that is called an Atanassov-Intuitionistic Fuzzy Set (A-IFS).

Let's $x(T(x), F(x))$ be a generic element that belongs to B_{A-IFS}, with $T(x), F(x) \in [0,1], 0 \le T(x) + F(x) \le 1,$ whence the indeterminacy (hesitancy) is $I(x) = 1 - T(x) - F(x) \in [0,1].$

From the fusion theory, and especially from Dempster-Shafer Theory, the Basic Believe Assignment (*bba*), denoted by *m(.)*, is defined as:

$m : 2^{B_{A-IFS}} \to [0,1]$, such that $m(\phi) = 0,$ where ϕ is the empty-set, and $\sum_{x \in 2^{B_{A-IFS}}} m(x) = 1.$

And the Believe Function *Bel* and the Plausible Function *Pl* are defined as follows:

$$Bel : 2^{B_{A-IFS}} \to [0,1], Bel(x) = \sum_{y \in 2^{B_{A-IFS}}, y \subseteq x} m(y),$$

$$Pl : 2^{B_{A-IFS}} \to [0,1], Pl(x) = \sum_{y \in 2^{B_{A-IFS}}, y \cap x \ne \phi} m(y).$$

Afterwards, they approximate the above B_{A-IFS} to an interval-valued fuzzy set (IVFS), denoted as $C_{IVFS} = \{(x, [Bel(x), Pl(x)]), x \in 2^{B_{A-IFS}}\} = \{(x, [T(x), T(x) + I(x)]); T(x), I(x) \in [0,1], T(x) + I(x) \le 1; x \in 2^{B_{A-IFS}}\}$ which is not equal to B_{A-IFS}. Their approach is similar to that of a Vague Set.

The interval $[Bel(x), Pl(x)] \triangleq BI(x)$ was called Believe Interval (BI).

Mathematically, this is beautiful, but practically it is useless. When converting from an approach to another one, it is supposed to diminish the indeterminacy (hesitancy) and get better results. But, it is not the case. The higher is indeterminacy (I) the larger is the believe interval that suppose to catch the solution.

As counter-examples, let's consider the following A-IFS triplets (their components' sums are equal to 1):

(T, I, F) = (0.2, 0.5, 0.3) produces the BI = [0.2, 0.7];

(T, I, F) = (0.3, 0.6, 0.1) produces the BI = [0.3, 0.9];

(T, I, F) = (0.2, 0.8, 0.0) produces the BI = [0.2, 1], etc.

There are pretty large intervals to deal with, that make the result vaguer. To say that the solution lies inside of the interval, for example [0.2, 1], means almost nothing towards solving the problem whose solution is always between 0 and 1.

Another drawback is the fact that computing with intervals is more complicated than computing with crisp numbers.

8. Differences between A-IFS and NST

"The conceptual difference between the NST and the A - IFS is the introduction of the hypothesis of complete independence of the components" (p. 3).

By NST they meant Neutrosophic Set Theory.

This is not the only difference, another big distinction is with respect to the construction of the neutrosophic operators (negation, intersection, union, implication, equivalence, etc.), since within the frame of neutrosophic environment the Indeterminacy (I) is getting full consideration and "I" is involved in the neutrosophic operators' formulas, while in the A-IFS operators the indeterminacy (called hesitancy) is completely ignored and does not appear in none of their operators' formulas.

Even for the case when the sum of the neutrosophic components is equal to 1, as occurs for the A-IFS components, the results after applying the neutrosophic operators are different from those obtained by the A-IFS operators.

A simple example is below, for the neutrosophic conjunction ($\wedge_{NS}$) vs. A-IFS conjunction ($\wedge_{A-IFS}$).

Let's denote by $\wedge_{FS}$ the fuzzy set t-norm, and by $\vee_{FS}$ the fuzzy set co-norm.

Let (T_1, I_1, F_1) and (T_2, I_2, F_2) be two neutrosophic triplets, where $T_1, I_1, F_1, T_2, I_2, F_2 \in [0,1]$, and there is no restriction on the sums of the two neutrosophic triplets.

Then, the neutrosophic conjunction is:

$$(T_1, I_1, F_1) \wedge_{NS} (T_2, I_2, F_2) = (T_1 \wedge_{FS} T_2, I_1 \vee_{FS} I_2, F_1 \vee_{FS} F_2),$$

where we clearly see that the indeterminacy/hesitancy (I) is involved in the above formula on the right-hand side: $I_1 \vee_{FS} I_2$.

But, for the A-IFS conjunction formula the indeterminacy/hesitancy is completely ignored, which makes the operator less accurate. If $T_1 + F_1 \leq 1$, $T_2 + F_2 \leq 1$, and $T_1 + I_1 + F_1 = 1, T_2 + I_2 + F_2 = 1$, in order to comply with the A-IFS constrains, one gets:

$$(T_1, F_1) \wedge_{A-IFS} (T_2, F_2) = (T_1 \wedge_{FS} T_2, F_1 \vee_{FS} F_2),$$

unfortunately, no indeterminacy/hesitancy (*I*) is involved into the formula.

Even when the sum pf the neutrosophic components is 1, as in A-IFS, the results of the neutrosophic and respectively A-IFS operators are different. Let's see this numerical example:

$$(0.6, 0.1, 0.3) \wedge_{NS} (0.5, 0.4, 0.1) = (\min\{0.6, 0.5\}, \max\{0.1, 0.4\}, \max\{0.3, 0.1\}) = (0.5, 0.4, 0.3)$$

while

$$(0.6, 0.3) \wedge_{A-IFS} (0.5, 0.1) = (\min\{0.6, 0.5\}, \max\{0.3, 0.1\}) = (0.5, 0.3)$$

whence the indeterminacy/hesitancy = 1 – 0.5 – 0.3 = 0.2 $\neq$ 0.4.

In this case these authors agree with us:

"In the case of mutually dependent components, the main constraint $0 \leq T + F + I \leq 1$ in the NST seems to be more fruitful than that in the A - IFS ($T + F + I = 1$). This was quickly discovered and the so-called Picture fuzzy sets theory (PFS) was proposed" (p. 4).

Thanks to the indeterminacy (I), that plays an important role in the neutrosophic environment and in the real world that is full of indeterminate (vague, unclear, conflicting, incomplete, etc.) data, more fields were developed within the field of neutrosophy, such as: Neutrosophic Algebraic Structures (based on neutrosophic numbers of the form *a* + *bI*, where *I* = literal indeterminacy, and *a*, *b* are real or complex numbers), Neutrosophic Statistics (using classical statistical procedures and inference methods but on indeterminate data), Neutrosophic Probability (chance of an event to occur, indeterminate-chance of the event to occurring or not, and chance of the event not to occur), etc.

Therefore, there are many distinctions between the neutrosophic theories and the A-IFS.

9. Conclusions (authors also should add some future directions points related to her/his research)

Many practical applications have been given in this paper about the independence and dependence of the neutrosophic components in our every-day life.

The misunderstanding of some authors, with respect to the partial and total independence of the neutrosophic components, is that in the framework of the neutrosophic theories we deal with a *MultiVariate Truth-Value* (truth upon many independent random variables) as in our real-life world, not with a *UniVariate Truth-Value* (truth upon only one random variable) as they believe.

Similarly with respect to the degrees of memberships greater than 1 or less than 0, which are now mainstream subjects. The neutrosophic theories were inspired from the practical applications..

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