

Generalizations and Alternatives of Classical Algebraic Structures to NeutroAlgebraic Structures and AntiAlgebraic Structures

*Florentin Smarandache**

University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, USA.

ABSTRACT

In this paper we present the development from paradoxism to neutrosophy, which gave birth to neutrosophic set and logic and especially to NeutroAlgebraic Structures (or NeutroAlgebras) and AntiAlgebraic Structures (or AntiAlgebras) that are generalizations and alternatives of the classical algebraic structures.

1. From Paradoxism to Neutrosophy

1.1. Paradoxism

Paradoxism is an international movement in science and culture, founded and developed by Smarandache in 1980s, based on excessive use of antitheses, oxymoron, contradictions, and paradoxes in science, literature, and arts. During three decades (1980-2020) hundreds of authors from tens of countries around the globe contributed papers in various languages to 15 international paradoxist anthologies.

1.2. Neutrosophy

In 1995, the author extended the paradoxism (based on opposites) to a new branch of philosophy called neutrosophy (based on opposites and their neutral) that gave birth to many scientific branches, such as neutrosophic logic, neutrosophic set, neutrosophic probability and statistics, neutrosophic algebraic

structures, and so on with multiple applications in engineering, computer science, administrative work, medical research, biology, psychology, social sciences etc.

1.3. Extensions

Neutrosophy is also an extension of Dialectics (characterized by the dynamics of opposites in philosophy), and of Yin-Yang Ancient Chinese philosophy (based also on opposites: male/female, good/bad, sky/earth, etc.) that was founded and studied two and half millennia ahead of Hegel's and Marx's Dialectics.

2. From Classical Algebras to NeutroAlgebras and AntiAlgebras

2.1. Operation, NeutroOperation, and AntiOperation

When we define an operation on a given set, it does not automatically mean that the operation is well-defined. There are three possibilities:

- The operation is well-defined (or inner-defined) for all set's elements (as in classical algebraic structures; this is classical *Operation*).
- The operation is well-defined for some elements, indeterminate for other elements, and outer-defined for others elements (this is *NeutroOperation*).
- The operation is outer-defined for all set's elements (this is *AntiOperation*).

2.2. Axiom, NeutroAxiom, and AntiAxiom

Similarly for an axiom defined on a given set endowed with some operation(s). When we define an axiom on a given set, it does not automatically mean that the axiom is true for all set's elements. We have three possibilities:

- The axiom is true for all set's elements [totally true] (as in classical algebraic structures; this is classical *Axiom*).
- The axiom is true for some elements, indeterminate for other elements, and false for other elements (this is *NeutroAxiom*).
- The axiom is false for all set's elements (this is *AntiAxiom*).

Similarly for any statement, theorem, lemma, algorithm, property, etc. For example: Classical *Theorem* (which is true for all space's elements), *NeutroTheorem* (which is partially true, partially indeterminate, and partially false), and *AntiTheorem* (which is false for all space's elements).

2.3. Algebra, NeutroAlgebra, and AntiAlgebra

An algebraic structure whose all operations are well-defined and all axioms are totally true is called Classical Algebraic Structure (or Algebra). An algebraic structure that has at least one NeutroOperation or one NeutroAxiom (and no AntiOperation and no AntiAxiom) is called NeutroAlgebraic Structure (or NeutroAlgebra).

An algebraic structure that has at least one AntiOperation or AntiAxiom is called AntiAlgebraic Structure (or AntiAlgebra). Therefore, a neutrosophic triplet structure is formed:
<Algebra, NeutroAlgebra, AntiAlgebra>.

“Algebra” can be any classical algebraic structure, such as: groupoid, semigroup, monoid, group, commutative group, ring, field, vector space, BCK-Algebra, BCI-Algebra, etc.

3. Foundation of NeutroAlgebra and AntiAlgebra

The classical algebraic structures were generalized in 2019 and improved and extended in 2020 by Smarandache [1, 2, 3] to NeutroAlgebraic Structures (or NeutroAlgebras) whose operations and axioms are partially true, partially indeterminate, and partially false as extensions of partial algebra, and to AntiAlgebraic Structures (or AntiAlgebras) whose operations and axioms are totally false.

4. Foundation of NeutroStructures and AntiStructures

And in general, we extended any classical Structure, which is a space characterized by some properties, ideas, laws, shapes, hierarchy, etc., in no matter what field of knowledge, to a NeutroStructure and an AntiStructure. So, we formed a general neutrosophic triplet: Structure, NeutroStructure, and AntiStructure.

Acknowledgement

The author thanks Dr. S. A. Edalatpanah for his opinions and comments on this small paper.

References

- [1] Smarandache, F. (2020). *NeutroAlgebra is a generalization of partial algebra*. Infinite Study.
- [2] Smarandache, F. (2019). *Introduction to neutroalgebraic structures and antialgebraic structures (revisited)*. Infinite Study.
- [3] Smarandache, F. (2019). *Introduction to neutroalgebraic structures and antialgebraic structures (revisited)*. Infinite Study.