Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set

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Abstract. In this paper, we generalize the soft set to the hypersoft set by transforming the function F into a multi-attribute function. Then we introduce the hybrids of Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic Hypersoft Set.

Keywords: Plithogeny; Plithogenic Set; Soft Set; Hypersoft Set; Plithogenic Hypersoft Set; Multi-argument Function.

1 Introduction

We generalize the soft set to the **hypersoft set** by transforming the function *F* into a **multi-argument function**.

Then we make the distinction between the types of Universes of Discourse: **crisp**, **fuzzy**, **intuitionistic fuzzy**, **neutrosophic**, and respectively **plithogenic**.

Similarly, we show that a hypersoft set can be **crisp**, **fuzzy**, **intuitionistic fuzzy**, **neutrosophic**, or **plithogenic**.

A detailed numerical example is presented for all types.

2 Definition of Soft Set [1]

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$, and A a set of attributes. Then, the pair $(F, \mathcal{U})$, where

$$F: A \longrightarrow \mathcal{P}(\mathcal{U}) \quad (1)$$

is called a **Soft Set** over $\mathcal{U}$.

3 Definition of Hypersoft Set

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$.

Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$.

Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where:

$$F: A_1 \times A_2 \times \ldots \times A_n \longrightarrow \mathcal{P}(\mathcal{U}) \quad (2)$$

is called a **Hypersoft Set** over $\mathcal{U}$.

4 Particular case

For $n = 2$, we obtain the Γ–Soft Set [2].

5 Types of Universes of Discourses

5.1. A **Universe of Discourse** $\mathcal{U}_C$ is called **Crisp** if $\forall x \in \mathcal{U}_C$, *x* belongs 100% to $\mathcal{U}_C$, or *x*'s membership (T_x) with respect to $\mathcal{U}_C$ is 1. Let's denote it *x*(1).

5.2. A **Universe of Discourse** $\mathcal{U}_F$ is called **Fuzzy** if $\forall x \in \mathcal{U}_c$, *x* partially belongs to $\mathcal{U}_F$, or $T_x \subseteq [0, 1]$, where T_x may be a subset, an interval, a hesitant set, a single-value, etc. Let's denote it by $x(T_x)$.

5.3. A **Universe of Discourse** $\mathcal{U}_{IF}$ is called **Intuitionistic Fuzzy** if $\forall x \in \mathcal{U}_{IF}$, x partially belongs (T_x) and partially doesn't belong (F_x) to $\mathcal{U}_{IF}$, or $T_x, F_x \subseteq [0, 1]$, where T_x and F_x may be subsets, intervals, hesitant sets, single-values, etc. Let's denote it by $x(T_x, F_x)$.

5.4. A **Universe of Discourse** $\mathcal{U}_N$ is called **Neutrosophic** if $\forall x \in \mathcal{U}_N$, *x* partially belongs (T_x), partially its membership is indeterminate (I_x), and partially it doesn't belong (F_x) to $\mathcal{U}_N$, where $T_x, I_x, F_x \subseteq [0, 1]$, may be subsets, intervals, hesitant sets, single-values, etc. Let's denote it by $x(T_x, I_x, F_x)$.

5.5. A **Universe of Discourse** $\mathcal{U}_P$ **over a set** ***V*** **of attributes' values**, where $V = \{v_1, v_2, \ldots, v_n\}, n \geq 1$, is called Plithogenic, if $\forall x \in \mathcal{U}_P$, *x* belongs to $\mathcal{U}_P$ in the degree $d_x^0(v_i)$ with respect to the attribute value v_i, for all

$i \in \{1, 2, \ldots, n\}$. Since the degree of membership $d_x^0(v_i)$ may be crisp, fuzzy, intuitionistic fuzzy, or neutrosophic, the Plithogenic Universe of Discourse can be Crisp, Fuzzy, Intuitionistic Fuzzy, or respectively Neutrosophic.

Consequently, a Hypersoft Set over a Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic / or Plithogenic Universe of Discourse is respectively called **Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic /** or **Plithogenic Hypersoft Set**.

6 Numerical Example

Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$ and a set $\mathcal{M} = \{x_1, x_3\} \subset \mathcal{U}$.

Let the attributes be: $a_1 =$ size, $a_2 =$ color, $a_3 =$ gender, $a_4 =$ nationality, and their attributes' values respectively:

Size = A_1 ={small, medium, tall},
Color = A_2 ={white, yellow, red, black},
Gender = A_3 ={male, female},
Nationality = A_4 ={American, French, Spanish, Italian, Chinese}.

Let the function be:

$F: A_1 \times A_2 \times A_3 \times A_4 \longrightarrow \mathcal{P}(\mathcal{U})$. (3)

Let's assume:

$F(\{\text{tall, white, female, Italian}\}) = \{x_1, x_3\}$.

With respect to the set $\mathcal{M}$, one has:

6.1 Crisp Hypersoft Set

$F(\{\text{tall, white, female, Italian}\}) = \{x_1(1), x_3(1)\}$, (4)

which means that, with respect to the attributes' values $\{\text{tall, white, female, Italian}\}$ *all together*, x_1 belongs 100% to the set $\mathcal{M}$; similarly x_3.

6.2 Fuzzy Hypersoft Set

$F(\{\text{tall, white, female, Italian}\}) = \{x_1(0.6), x_3(0.7)\}$, (5)

which means that, with respect to the attributes' values $\{\text{tall, white, female, Italian}\}$ *all together*, x_1 belongs 60% to the set $\mathcal{M}$; similarly, x_3 belongs 70% to the set $\mathcal{M}$.

6.3 Intuitionistic Fuzzy Hypersoft Set

$F(\{\text{tall, white, female, Italian}\}) = \{x_1(0.6, 0.1), x_3(0.7, 0.2)\}$, (6)

which means that, with respect to the attributes' values $\{\text{tall, white, female, Italian}\}$ *all together*, x_1 belongs 60% and 10% it does not belong to the set $\mathcal{M}$; similarly, x_3 belongs 70% and 20% it does not belong to the set $\mathcal{M}$.

6.4 Neutrosophic Hypersoft Set

$F(\{\text{tall, white, female, Italian}\}) = \{x_1(0.6, 0.2, 0.1), x_3(0.7, 0.3, 0.2)\}$, (7)

which means that, with respect to the attributes' values $\{\text{tall, white, female, Italian}\}$ all together, x_1 belongs 60% and its indeterminate-belongness is 20% and it doesn't belong 10% to the set $\mathcal{M}$; similarly, x_3 belongs 70% and its indeterminate-belongness is 30% and it doesn't belong 20%.

6.5 Plithogenic Hypersoft Set

$$F(\{\text{tall, white, female, Italian}\}) = \left\{\begin{matrix} x_1\left(d_{x_1}^0(\text{tall}), d_{x_1}^0(\text{white}), d_{x_1}^0(\text{female}), d_{x_1}^0(\text{Italian})\right), \\ x_2\left(d_{x_2}^0(\text{tall}), d_{x_2}^0(\text{white}), d_{x_2}^0(\text{female}), d_{x_2}^0(\text{Italian})\right) \end{matrix}\right\}, \quad (8)$$

where $d_{x_1}^0(\alpha)$ means the degree of appurtenance of element x_1 to the set $\mathcal{M}$ with respect to the attribute value α; and similarly $d_{x_2}^0(\alpha)$ means the degree of appurtenance of element x_2 to the set $\mathcal{M}$ with respect to the attribute value α; where $\alpha \in \{\text{tall, white, female, Italian}\}$.

Unlike the Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic Hypersoft Sets [where the degree of appurtenance of an element *x* to the set $\mathcal{M}$ is *with respect to all attribute values* ***tall, white, female, Italian*** *together* (as a whole), therefore a degree of appurtenance *with respect to a set of attribute values*], the Plithogenic Hypersoft Set is a **refinement** of Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic Hypersoft Sets [since the degree of appurtenance of an element *x* to the set $\mathcal{M}$ is with respect to each single attribute value].

But the Plithogenic Hypersoft St is also **combined** with each of the above, since the degree of degree of appurtenance of an element *x* to the set $\mathcal{M}$ with respect to each single attribute value may be: crisp, fuzzy, intuitionistic fuzzy, or neutrosophic.

7 Classification of Plithogenic Hypersoft Sets

7.1 Plithogenic Crisp Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element *x* to the set $\mathcal{M}$, with respect to each attribute value, is **crisp**:

$d_x^0(\alpha) = 0$ (nonappurtenance), or 1 (appurtenance).

In our example:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(1,1,1,1), x_3(1,1,1,1)\}. \quad (9)$$

7.2 Plithogenic Fuzzy Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element *x* to the set $\mathcal{M}$, with respect to each attribute value, is **fuzzy**:

$d_x^0(\alpha) \in \mathcal{P}([0,1])$, power set of $[0,1]$,

where $d_x^0(\cdot)$ may be a subset, an interval, a hesitant set, a single-valued number, etc.

In our example, for a single-valued number:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(0.4, 0.7, 0.6, 0.5), x_3(0.8, 0.2, 0.7, 0.7)\}. \quad (10)$$

7.3 Plithogenic Intuitionistic Fuzzy Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element *x* to the set $\mathcal{M}$, with respect to each attribute value, is **intuitionistic fuzzy**:

$d_x^0(\alpha) \in \mathcal{P}([0,1]^2)$, power set of $[0,1]^2$,

where similarly $d_x^0(\alpha)$ may be: a Cartesian product of subsets, of intervals, of hesitant sets, of single-valued numbers, etc.

In our example, for single-valued numbers:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \begin{Bmatrix} x_1(0.4,0.3)(0.7,0.2)(0.6,0.0)(0.5,0.1) \\ x_3(0.8,0.1)(0.2,0.5)(0.7,0.0)(0.7,0.4) \end{Bmatrix}. \quad (11)$$

7.4 Plithogenic Neutrosophic Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element *x* to the set $\mathcal{M}$, with respect to each attribute value, is **neutrosophic**:

$d_x^0(\alpha) \in \mathcal{P}([0,1]^3)$, power set of $[0,1]^3$,

where $d_x^0(\alpha)$ may be: a triple Cartesian product of subsets, of intervals, of hesitant sets, of single-valued numbers, etc.

In our example, for single-valued numbers:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \begin{Bmatrix} x\_1\ [(0.4,0.1,0.3)(0.7,0.0,0.2)(0.6,0.3,0.0)(0.5,0.2,0.1)] \\ x\_3\ [(0.8,0.1,0.1)(0.2,0.4,0.5)(0.7,0.1,0.0)(0.7,0.5,0.4)] \end{Bmatrix}. \quad (12)$$

Conclusion & Future Research

For all types of plithogenic hypersoft sets, the aggregation operators (union, intersection, complement, inclusion, equality) have to be defined and their properties found.

Applications in various engineering, technical, medical, social science, administrative, decision making and so on, fields of knowledge of these types of plithogenic hypersoft sets should be investigated.

References

[1] D. Molodtsov (1999). Soft Set Theory First Results. *Computer Math. Applic*. 37, 19-31.

[2] T. Srinivasa Rao, B. Srinivasa Kumar, S. Hanumanth Rao. A Study on Neutrosophic Soft Set in Decision Making Problem. *Journal of Engineering and Applied Sciences*, Asian Research Publishing Network (ARPN), vol. 13, no. 7, April 2018.

[3] Florentin Smarandache. *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics*. Brussels: Pons Editions, 2017.

[4] Florentin Smarandache. Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited. *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166. https://doi.org/10.5281/zenodo.1408740.