Why is the Gödel self-referential equation unsolvable?

----Tranclosed logic princiole and its inference(2)

Jincheng Zhang

Oriental Culture Research Office, Guangde County, Anhui Province China.

Email:656790205@qq.com

Abstract There exists a Gödel number for each formula of the system $\mathscr{N}$ of natural numbers. The Gödel undecidable proposition, which is also a formula of the system $\mathscr{N}$, also exists a Gödel number p; at the same time, the Gödel undecidable proposition is a self-referential proposition $\mathscr{U}(0^{(p)})$ substituted into its own Gödel number, and the self-referential proposition $\mathscr{U}(0^{(p)})$ Gödel number is also p, i.e., there is, $g(\mathscr{U}(0^{(p)})) = p$. It can be This equation has no solution.

The traditional view is that the Gödel undecidable proposition $\mathscr{U}(0^{(p)})$ is a closed formula and is a natural number proposition; we here transform the Gödel self-referential proposition into a self-referential equation and find that this equation has no solution and the Gödel undecidable proposition $\mathscr{U}(0^{(p)})$ is not a natural number proposition. $\mathscr{U}(0^{(p)})$ is an unclosed term (out-of-domain term) that evolves on the set of natural numbers and $\mathscr{U}(0^{(p)})$ is not a closed formula.

Keywords Gödel undecidable proposition, self-referential proposition, self-referential equation, unclosed term (extra-domain term).

1 Review of Gödel's construction of self-referential propositions

Let us first briefly review the process of proving **Gödel's** incompleteness theorem.

1 The set of natural numbers

$N = \{0, 1, 2, 3, \cdots, n, \cdots\}$

2 Axiomatic system of natural numbers

The "successor, addition, multiplication" on the set of natural numbers can be defined by the following set of axioms.

$(\mathscr{N}1)$ $(\forall x_1)\neg(s(x_1) = 0)$.

$(\mathscr{N}2)$ $(\forall x_1)(\forall x_2)(s(x_1) = s(x_2) \to x_1 = x_2)$.

$(\mathscr{N}3)$ $(\forall x_1)(x_1 + 0 = x_1)$.

$(\mathscr{N}4)$ $(\forall x_1)(\forall x_2)(x_1 + s(x_2) = s(x_1 + x_2))$.

$(\mathcal{N}5)$ $(\forall x_1)(x_1 \times 0 = 0)$.

$(\mathcal{N}6)$ $(\forall x_1)(\forall x_2)(x_1 \times s(x_2) = (x_1 \times x_2) + x_1)$.

$(\mathcal{N}7)$ $A(0) \to ((\forall x_1)(A(x_1) \to A(s(x_1))) \to (\forall x_1)A(x_1))$.

(for each formula $A(x_1)$, where x_1 appears freely)

In proving the incompleteness theorem, Gödel first encodes the symbols, formulas, and proofs in the formal system $\mathcal{N}$ with natural numbers. This form of encoding is called the arithmeticization of the system $\mathcal{N}$.

Gödel's method is not very complicated; he encodes the first-order arithmetic $\mathcal{N}$ by assigning a natural number to each symbol, ensemble formula, and sequence of formula proofs in $\mathcal{N}$ according to a determined rule. Such natural numbers are Gödel numbers.

2、The Gödel number of the system $\mathcal{N}$

(1)The matching number of characters, specify a Gödel number for each character(Let's say $g(x)$ is the Godel number of x).

Parentheses, commas: $g(() = 3$， $g(,) = 5$， $g()) = 7$.

Logical symbols: $g(\neg) = 9$， $g(\to) = 11$， $g(\forall) = 13$.

Variable element: $g(x_k) = 7 + 8k$， $(k = 1, 2, 3, \cdots)$.

Constant element: $g(a_k) = 9 + 8k$， $(k = 1, 2, 3, \cdots)$.

Function symbols: $g(f_k^n) = 11 + 8(2^n \times 3^k)$， $(k = 1, 2, 3, \cdots)$.

Predicate symbols: $g(A_k^n) = 13 + 8(2^n \times 3^k)$， $(k = 1, 2, 3, \cdots)$.

(2) Gödel collocation of strings

Strings $u_0, u_2, u_3, \cdots, u_k$， $g(u_0, u_1, \cdots, u_k) = 2^{g(u_0)} \cdot 3^{g(u_1)} \cdot 5^{g(u_2)} \cdot \cdots \cdot p_k^{g(u_k)}$.

(3) Gödel collocation of a finite sequence of strings

Let $s_0, s_1, s_2, \cdots, s_k$ be the string, $g(s_0, s_1, \cdots, s_k) = 2^{g(s_0)} \cdot 3^{g(s_1)} \cdot 5^{g(s_2)} \cdot \cdots \cdot p_k^{g(s_k)}$.

(where $p_1, p_2, p_3, \cdots, p_k$, i.e.: 2,3,5,7, ... denotes the kth prime number)

Each formula $A(x)$, of the system $\mathcal{N}$ under the above definition corresponds to a Gödel number $g(A(x))$.

3. Expressible definition: a k-element relation R on a set of natural numbers N is said to be expressible in $\mathcal{N}$, if there exists a formula with k free variables $\xi(x_1, x_2, \cdots, x_n)$, such that for any natural number $n_1, n_2, \cdots, n_k$,

if $R(n_1, n_2, \cdots, n_k)$ holds in N, then $\mathcal{N} \vdash \xi(0^{(n_1)}, 0^{(n_2)}, \cdots, 0^{(n_k)})$.

if $R(n_1, n_2, \cdots, n_k)$ does not hold in N, then $\mathcal{N} \vdash \neg\xi(0^{(n_1)}, 0^{(n_2)}, \cdots, 0^{(n_k)})$.

(set of natural numbers $N = \{0, 1, 2, 3, \cdots, n, \cdots\}$).

4. Expressibility theorem : recursive relations in the system $\mathcal{N}$ are expressible.

We can prove that recursive functions are expressible.

(1) zero function, the successor function is expressible.

(2) synthetic operations remain expressible.

(3) recursive operations remain expressible.

(4) the minimum number operation maintains expressibility.

Furthermore, considering that the characteristic function $C_{R(x_1, x_2, \cdots, x_k)}$ of a k-element recurrence relation $R(x_1, x_2, \cdots, x_k)$ defined on natural numbers is a recursive function, this gives us a corollary that every recurrence relation is expressible in $\mathcal{N}$.

In this way we prove the expressibility theorem.

5.The definition of the binary relation W

$W(m,n)$, m is the Gödel number of the formula $\mathcal{A}(x)$ and n is the Gödel number of the proof of the formula $\mathcal{A}(m)$ from $\mathcal{N}$.

Denoted as a set, $W = \{(m,n)\}$, $(m,n) \in W$ holds and $(m,n) \in W$ does not hold, $(m,n) \notin W$.

6. Binary relations W recursiveness

It can be shown that the binary relation $W(m,n)$ is recursive, so that $W = \{(m,n)\}$ is expressible in $\mathcal{N}$ as follows:

$(m,n) \in W \Rightarrow \mathcal{N} \vdash w(0^{(m)}, 0^{(n)})$; $(m,n) \notin W \Rightarrow \mathcal{N} \vdash \neg w(0^{(m)}, 0^{(n)})$.

7. Construction of Gödel's undecidable proposition $\mathcal{U}(0^{(p)})$

(1) Structural formula $\forall y \neg w(x,y)$.

$g(\forall y \neg w(x,y)) = p$; p is Gödel number of formula $\forall y \neg w(x,y)$.

(2) Replace all free occurrences of x in $\forall y \neg w(x,y)$ with $0^{(p)}$ to obtain $\forall y \neg w(0^{(p)}, y)$,

Denote $\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$ ----- y is the Gödel number obtained from $\mathcal{U}(0^{(p)})$.

The interpretation of $\forall y \neg w(0^{(p)}, y)$ is that " for any y, that y is the Gödel number obtained from $\mathcal{U}(0^{(p)})$ is wrong" ;

or "For any y, y is a Gödel number proved by the formula p (i.e. $\mathcal{U}(0^{(p)})$) does not hold."

Or $\forall y \neg w(0^{(p)}, y) \leftrightarrow \neg \exists y w(0^{(p)}, y)$ "There is not y, y is Gödel number proved by $\mathcal{U}(0^{(p)})$,"

that is "$\mathcal{U}(0^{(p)})$ is unprovable"; $\mathcal{U}(0^{(p)})$ narrates its own unprovability."

(3) $\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$, it is Gödel's undecidable proposition.

8. Gödel's Incompleteness Theorem

Theorem 1.1 If $\mathcal{N}$ is consistent, then $\mathcal{U}(0^{(p)})$ is not a theorem of $\mathcal{N}$, and its negation $\neg\mathcal{U}(0^{(p)})$ is not a theorem of $\mathcal{N}$. Therefore, if $\mathcal{N}$ is consistent, the system $\mathcal{N}$ is incomplete.

Proof:

（1） $(m,n) \in W \Rightarrow \mathcal{N} \vdash w(0^{(m)}, 0^{(n)})$， $(m,n) \notin W \Rightarrow \mathcal{N} \vdash \neg w(0^{(m)}, 0^{(n)})$,

（2） $\mathcal{N} \vdash \mathcal{U}(0^{(p)})$ --hypothesis，denoting the Gödel number which proved of $\mathcal{U}(0^{(p)})$ from $\mathcal{N}$ as q， then $(p,q) \in W$,（1）

（3） $\mathcal{N} \vdash w(0^{(p)}, 0^{(q)})$ --- （1），（2），

（4） $\mathcal{N} \vdash \forall y \neg w(0^{(p)}, y)$ ----------------------- （2）， $\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$,

（5） $\mathcal{N} \vdash \neg w(0^{(p)}, 0^{(q)})$ ------------------------- --------------------- （4），

（6） $\mathcal{N} \nvdash \mathcal{U}(0^{(p)})$ --- （3），（5） contradiction；

（7） $\mathcal{N} \vdash \neg\mathcal{U}(0^{(p)})$ ---hypothesis，

（8） $\mathcal{N} \vdash \neg \forall y \neg w(0^{(p)}, y) \leftrightarrow \exists y\, w(0^{(p)}, y)$ ------ （7）， $\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$, （9）（6）had proved $\mathcal{U}(0^{(p)})$ does not hold in $\mathcal{N}$, any q， $(p,q) \notin W$， -------------- （6），

（10） $\mathcal{N} \vdash \neg w(0^{(p)},0^{(q)})$ -- （1），（9），

（11） $\mathcal{N} \vdash w(0^{(p)},0^{(q)})$ ------------ （8），Let q be the Gödel number that $\mathcal{U}(0^{(p)})$ proves from $\mathcal{N}$ ，

（12） $\mathcal{N} \nvdash \neg\mathcal{U}(0^{(p)})$ -------------------------------------- （10），（11）contradiction，

(13) $\mathcal{U}(0^{(p)})$ ， $\neg\mathcal{U}(0^{(p)})$ are non-falsifiable propositions, i.e. , $\mathcal{U}(0^{(p)})$ is undecidable in the system--- （6）（12）。

The construction and proof of the above undecidable proposition $\mathcal{U}(0^{(p)})$ was given by Gödel in 1931 and can be found in the general mathematical logic literature and in [1] (some notation has been adjusted for printing convenience).

$\mathcal{N}$ contains a closed formula $\mathcal{U}(0^{(p)})$ which is true in the model N but is not a theorem of $\mathcal{N}$. The system $\mathcal{N}$ is generally considered to be incomplete. The above proof that $\mathcal{U}(0^{(p)})$ is an undecidable proposition is correct. The key is that the essence of the undecidable proposition $\mathcal{U}(0^{(p)})$ is misunderstood, and the following will prove that $\mathcal{U}(0^{(p)})$ is an unclosed term, an extradomain undecidable proposition that does not affect the completeness of the system $\mathcal{N}$.

2.Gödel self-referential equation without solution

The formula of each of system $\mathcal{N}$ is a proposition about natural numbers, and the formula of each of system $\mathcal{N}$ exists a Gödel number, Gödel undecidable proposition $\mathcal{U}(0^{(p)})$ also the formula of system $\mathcal{N}$, and also a Gödel number p ; at the same time, the Gödel undecidable proposition $\mathcal{U}(0^{(p)})$ is a self-referential proposition that substitutes its own Gödel number, and the self-referential proposition $\mathcal{U}(0^{(p)})$ Gödel number is also p that is, there is that

$$g(\mathcal{U}(0^{(p)})) = p\,.$$

But this equation has no solution.

Definition 2.1 Gödel self-referential propositions

The Gödel undecidable proposition $\mathcal{U}(0^{(p)})$,

$$\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y).$$

is also called a Gödel self-referential proposition.

In the following we analyze the undecidable proposition $\mathcal{U}(0^{(p)})$ again according to the Gödel number.

(1) The number of characters assigned, for each character, a Gödel number: (v is a character, $g(v)$ is the Gödel number of v).

Parentheses, commas: $g(() = 3$， $g(,) = 5$， $g()) = 7$.

Logical symbols: $g(\neg) = 9$， $g(\forall) = 13$.

Predicate symbols: $g(w) = 11$,

Variable element: $g(0^{(x)}) = x$.

Since, y ,x are variable elements of the formula, in the natural number system $\mathcal{N}$, y , x are natural numbers (Since $0^{(p)}$ is a systematic representation of p, which is essentially the same), we can define its Gödel numbers $g(0^{(x)}) = x$, $g(0^{(y)}) = y$.

(2) Gödel collocation of strings

Strings $v_0, v_2, v_3, \cdots, v_k$, $g(v_0, v_1, \cdots, v_k) = 2^{g(v_0)} \cdot 3^{g(v_1)} \cdot 5^{g(v_2)} \cdot \cdots \cdot p_k^{g(v_k)}$.

By construction of the Gödel undecidable proposition: p is the Gödel number of the formula $\forall y \neg w(x, y)$,

$$g(\forall y \neg w(x, y)) = p \text{ -------------- (A)}$$

Replace all free occurrences of x in $\forall y \neg w(x, y)$ with $0^{(p)}$ to obtain undecidable proposition $\forall y \neg w(0^{(p)}, y)$,

$$\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y).$$

By（A）

$$g(\forall y \neg w(x, y)) = p \Rightarrow g(\mathcal{U}(0^{(p)})) = g(\forall y \neg w(0^{(p)}, y)) = p \text{ -------------- (B)}$$

The left side of this equation is an equation containing the Gödel number p, and the right side is p.

Since $0^{(p)}$ is a systematic representation of p, which is essentially the same, we classify the inter-equation as $g(\mathcal{U}(p)) = p$.

If the equation $g(\mathcal{U}(p)) = p$ has a solution, then the equation $g(\mathcal{U}(0^{(p)})) = p$ also has a solution.

If the equation $g(\mathcal{U}(p)) = p$ has no solution, then the equation $g(\mathcal{U}(0^{(p)})) = p$ also has no solution.

To investigate whether this equation has a solution, we expand the left-hand side of the equation using the definition of the Gödel number.

$$g(\mathcal{U}(0^{(p)})) = g(\forall y \neg w(0^{(p)}, y)) = 2^{g(\forall)} \cdot 3^{g(y)} \cdot 5^{g(\neg)} \cdot 7^{g(w)} \cdot 11^{g(()} \cdot 13^{g(0^{(p)})} \cdot 17^{g(,)} \cdot 19^{g(y)} \cdot 23^{g())}.$$

$$g(\mathcal{U}(0^{(p)})) = 2^{13} \cdot 3^{g(y)} \cdot 5^{9} \cdot 7^{11} \cdot 11^{3} \cdot 13^{g(0^{(p)})} \cdot 17^{7} \cdot 19^{g(y)} \cdot 23^{5}.$$

$$g(0^{(p)}) = 2^{13} \cdot 3^{y} \cdot 5^{9} \cdot 7^{11} \cdot 11^{3} \cdot 13^{p} \cdot 17^{7} \cdot 19^{y} \cdot 23^{5} \text{--------------} \quad \text{(C)}$$

According to the idea of the proof of Gödel, by（B）, the Gödel number of equation $\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$ is also p, i.e.:

$$g(\mathcal{U}(0^{(p)})) = p \text{--------------} \quad \text{(D)}$$

Combining (C) (D) yields the equation:

$$2^{13} \cdot 3^{y} \cdot 5^{9} \cdot 7^{11} \cdot 11^{3} \cdot 13^{p} \cdot 17^{7} \cdot 19^{y} \cdot 23^{5} = p.$$

Definition 2.1 Gödel self-referential equation

Called the algebraic equation containing the Gödel numbers

$$g(\mathcal{U}(0^{(p)})) = p.$$

$$\text{or } g(\forall y \neg w(0^{(p)}, y)) = p.$$

$$\text{or } 2^{13} \cdot 3^{y} \cdot 5^{9} \cdot 7^{11} \cdot 11^{3} \cdot 13^{p} \cdot 17^{7} \cdot 19^{y} \cdot 23^{5} = p.$$

as Gödel self-referential equations.

The Gödel self-referential proposition can be transformed into the Gödel self-referential equation.

Theorem 2.1 Gödel's self-referential equation has no integer solutions

Proof: Gödel's self-referential equation

$$2^{13}\cdot 3^{y}\cdot 5^{9}\cdot 7^{11}\cdot 11^{3}\cdot 13^{p}\cdot 17^{7}\cdot 19^{y}\cdot 23^{5}=p.$$

Since $2^{13}\cdot 3^{y}\cdot 5^{9}\cdot 7^{11}\cdot 11^{3}\cdot 13^{p}\cdot 17^{7}\cdot 19^{y}\cdot 23^{5}$ is much larger than p regardless of the natural numbers of y .

It is clear that the above equation for *p* has no integer solution for p for any y.

the equation $g(\mathcal{U}(p))=p$ has no solution, then the equation $g(\mathcal{U}(0^{(p)}))=p$ also has no solution.

Or p does not exist, and neither does the Gödel undecidable proposition $\mathcal{U}(0^{(p)})$ **.**

Definition 2.2 Unclosedness of the algorithm

Let $U=\{x_1,x_2,\cdots,x_i,\cdots\}$ be the domain of definition of a certain monadic or multivariate operation $\odot$.

If $\forall a\in U，\ \forall b\in U\Rightarrow a\odot b\in U$, then, U is closed to the operation $\odot$.

If $\exists a\in U，\ \exists b\in U\Rightarrow a\odot b\notin U$, then, U is not closed for the operation $\odot$.

Example 2.1 Unclosedness of the algorithm

$N=\{0,1,2,\cdots,n,\cdots\}$,

$$\forall a\in N，\ \forall b\in N\Rightarrow a+b\in N.$$

$$\forall a\in N，\ \forall b\in N\Rightarrow a\times b\in N.$$

Therefore, N is closed for all additive operations, multiplicative operations.

$$2\in N,7\in N\Rightarrow 2-7\notin N.$$

$$2\in N,7\in N\Rightarrow 2\div 7\notin N.$$

Therefore, N is not closed for subtraction operations, nor for division operations.

Example 2.2 Unclosedness of the algorithm

Q is the set of rational numbers ,

$$\forall a\in Q，\ \forall b\in Q\Rightarrow a-b\in Q.$$

$$\forall a\in Q，\ \forall b\in Q\Rightarrow a\div b\in Q.$$

Therefore, Q is closed for subtraction operations, and for division operations.

$$2\in Q\Rightarrow \sqrt{2}\notin Q.$$

$$-3\in Q\Rightarrow \sqrt{-3}\notin Q.$$

Therefore, Q is not closed to the extraction of square root operation.

Definition 2.3 Out-of-domain terms

Let $U=\{x_1,x_2,\cdots,x_i,\cdots\}$ be a set, mapping $f:U\to U$, satisfies the solution x_P of an equation $x=f(x)$, if the equation has no solution, or element $x_P\notin U$, in the set U, the element x_P is called an extra-domain term. The essence of an extra-domain term is the unclosed term of the algorithm.

Gödel's self-referential equation

$$2^{13}\cdot 3^{y}\cdot 5^{9}\cdot 7^{11}\cdot 11^{3}\cdot 13^{p}\cdot 17^{7}\cdot 19^{y}\cdot 23^{5}=p.$$

p has no integer solution, so p is an out-of-domain term.

3. Gödel undecidable propositions $\mathcal{U}(0^{(p)})$ are out-of-domain terms

Above we transformed the Gödel self-referential proposition into Gödel self-referential equation, and found that this undecidable proposition is an arithmetic unclosed term, perhaps you may think that it is a difference of mapping methods, in fact, any mapping method, Gödel self-referential equation has no solution. In the following, we rigorously prove that the Gödel undecidable proposition $\mathcal{U}(0^{(p)})$ is an arithmetic unclosed term.

Definition 3.1 Let the set of all formulas of system $\mathcal{N}$ be $U=\{\mathcal{A}_1(x),\mathcal{A}_2(x),\cdots,\mathcal{A}_i(x),\cdots\}$, i.e., take the set of all closed formulas U of system $\mathcal{N}$ as the full set.

If $R(n_1,n_2,\cdots,n_k)$ is a recursive predicate and $\xi(0^{(n_1)},0^{(n_2)},\cdots,0^{(n_k)})$ is a formula mapped by the predicate $R(n_1,n_2,\cdots,n_k)$ onto the system $\mathcal{N}$, let $\mathcal{N}$ be the standard model of the system $\mathcal{N}$ of natural numbers.

If $R(n_1,n_2,\cdots,n_k)$ is true on N, denoted as $V(R(n_1,n_2,\cdots,n_k))=1$;

If $R(n_1,n_2,\cdots,n_k)$ is false on N, denoted as $V(R(n_1,n_2,\cdots,n_k))=0$.

The representable theorem can be written in the following form:

$V(R(n_1,n_2,\cdots,n_k))=1\Rightarrow\mathcal{N}\vdash\xi(0^{(n_1)},0^{(n_2)},\cdots,0^{(n_k)})$;

$V(R(n_1,n_2,\cdots,n_k))=0\Rightarrow\mathcal{N}\vdash\neg\xi(0^{(n_1)},0^{(n_2)},\cdots,0^{(n_k)})$.

The following proof shows that the Gödel undecidable proposition $\mathcal{U}(0^{(p)})$ is not a closed formula.

Theorem 3.1 If $\mathscr{U}(0^{(p)})$ is a closed formula, then, $W(p,q)$ is neither true nor false (there is no true or false). That is :

$$\mathscr{U}(0^{(p)}) \in U \vdash (V(W(p,q)) \neq 1) \wedge) V(W(p,q)) \neq 0).$$

Proof: The recursive predicate $\mathscr{U}(0^{(p)})$ is representable and the formula $\mathscr{U}(0^{(p)})$ on the system $\mathscr{N}$ should also have a Gödel number. The Gödel number of $\mathscr{U}(0^{(p)})$ is p. We ask whether (p,q) is in W， $W=\{(p,q)\}$， i.e., whether $W(p,q)$ holds.

According to representability theorem：

If $W(p,q)$ is satisfied ， then $\mathscr{N} \vdash w(0^{(p)},0^{(q)})$.

If $W(p,q)$ is not satisfied ， then $\mathscr{N} \vdash \neg w(0^{(p)},0^{(q)})$.

The above equation can also be expressed as follows:

$$V(W(p,q)) = 1 \Rightarrow \mathscr{N} \vdash w(0^{(p)},0^{(q)}),$$

$$V(W(p,q)) = 0 \Rightarrow \mathscr{N} \vdash \neg w(0^{(p)},0^{(q)}).$$

If $\mathscr{U}(0^{(p)})$ is a closed formula, assume $\mathscr{U}(0^{(p)}) \in U$, then one of $V(W(p,q)) = 1$, $V(W(p,q)) = 0$ must reside.

That is: $V(W(p,q)) = 1 \vee V(W(p,q)) = 0$.

（1A）If $V(W(p,q)) = 1$--hypothesis，

（2A）$\mathscr{N} \vdash w(0^{(p)},0^{(q)})$ ------------------------------（1A），Recursive representable theorem,

（3A）$\mathscr{N} \vdash \exists y w(0^{(p)},y)$ ---（2A），

（4A）$\mathscr{N} \nvdash \neg\mathscr{U}(0^{(p)})$ ------------------------------It has been proven that $\neg\mathscr{U}(0^{(p)})$ is undecidable，

（5A）$\mathscr{N} \nvdash \neg\forall y \neg w(0^{(p)},y)$ ---------------------- $\mathscr{U}(0^{(p)}) = \forall y \neg w(0^{(p)},y)$,

（6A）$\mathscr{N} \nvdash \exists y w(0^{(p)},y)$ --- （5 A），

（7A）$V(W(p,q)) \neq 1$ ----------------------------（3A）（6 A）contradiction，proof by contradiction，

（1B）If $V(W(p,q)) = 0$ ---hypothesis，

（2B）$\mathscr{N} \vdash \neg w(0^{(p)},0^{(q)})$ ----------------------------（1B），Recursive representable theorem， （3B） $\mathcal{N} \vdash \neg\exists y\, w(0^{(p)}, y)$ ---------------------------------------（2 B），

（4B） $\mathcal{N} \nvdash \mathcal{U}(0^{(p)})$ -------------------------------------It has been proven that $\mathcal{U}(0^{(p)})$ is undecidable，

（5B） $\mathcal{N} \nvdash \forall y \neg w(0^{(p)}, y)$ -------------------------$\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$,

（6B） $\mathcal{N} \nvdash \neg\exists y\, w(0^{(p)}, y)$ --（5 B），

（7B） $V(W(p,q)) \neq 0$ --------------------------（3 B）（6 B）contradiction，proof by contradiction，

（8） $(V(W(p,q)) \neq 1) \wedge (V(W(p,q)) \neq 0)$ -------------------------（7A）（7B），

If $\mathcal{U}(0^{(p)})$ is a closed formula, then $W(p,q)$ is neither true nor false (there is no true or false), which is similar to the paradox.

Theorem 3.2 Under the assumption that the evolution on U is consistent， U is not a closed formula of the system $\mathcal{N}$, i.e. $\mathcal{U}(0^{(p)}) \notin U$.

（1） $\mathcal{U}(0^{(p)}) \in U \vdash V(W(p,q)) = 1 \vee V(W(p,q)) = 0$ --------------------model definition;

（2） $\mathcal{U}(0^{(p)}) \in U \vdash \ (V(W(p,q)) \neq 1) \wedge (V((W(p,q)) \neq 0)$ ----------**theorem 3.1** above**;**

（3） $(V(W(p,q)) \neq 1) \wedge (V(W(p,q)) \neq 0) \leftrightarrow \neg(V(W(p,q)) = 1 \vee V(W(p,q)) = 0)$;

（4） $\mathcal{U}(0^{(p)}) \in U \vdash \neg(V(W(p,q)) = 1 \vee V(W(p,q)) = 0)$ ------------------（2）（3）;

（5） $\vdash \neg(\mathcal{U}(0^{(p)}) \in U)$.(i.e. $\mathcal{U}(0^{(p)}) \notin U$)---------（1）（4）contradiction，proof by contradiction;

That is, " $\mathcal{U}(0^{(p)})$ is an out-of-domain term". $\mathcal{U}(0^{(p)})$ is an unclosed term of the system $\mathcal{N}$ algorithm and does not affect the completeness of the system $\mathcal{N}$.

A common example is:

Example 3.1 Assuming the set of integers, the full set, $J = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$, $f(n) = 1 - n$, constructing a self-referential equation $n = 1 - n$,

Let $P(n)$ denote the proposition " n is even", then $\neg P(n)$ denote the proposition " n is odd" $\neg P(n)$; .

if $P(n)$: " n is even" $\Rightarrow$ “ $1-n$ is odd” $\Rightarrow n = 1-n$, “ n is odd” $\Rightarrow \neg P(n)$.

if $\neg P(n)$: “ n is odd” $\Rightarrow$ “ $1-n$ is even” $\Rightarrow n = 1-n$, “ n is even” $\Rightarrow P(n)$.

So： $P(n) \leftrightarrow \neg P(n)$.

We already know that: $n = 1 - n$， $n = \frac{1}{2}$， $\frac{1}{2} \notin J$， $\frac{1}{2}$ are unclosed terms (out-of-domain term) on the set of integers.

The above example has the following characteristics:

In this example, "n is even" and "n is not even" lead to the contradiction that $P(n)$， $\neg P(n)$ are undecidable propositions, this undecidable proposition is normal and $\frac{1}{2}$ is no longer an integer at all and is an extra-domain term.

The "Gödel undecidable proposition" in the system $\mathcal{N}$ (axiomatic system of natural numbers, hereafter) is an extraterritorial undecidable proposition in the same sense as the “undecidability in the set of integers of $P(\frac{1}{2})$” above, and the extraterritorial undecidable proposition is not related to the completeness of the system.

If Gödel's incompleteness theorem holds, the condition must be satisfied:

"p in an undecidable proposition $\mathcal{U}(0^{(p)})$ is a natural number."

Otherwise $\mathcal{U}(0^{(p)})$ is not a natural number proposition. It can be proved above that this condition is not satisfied .

Gödel's self-referential equation

$$2^{13} \cdot 3^{y} \cdot 5^{9} \cdot 7^{11} \cdot 11^{3} \cdot 13^{p} \cdot 17^{7} \cdot 19^{y} \cdot 23^{5} = p\ .$$

is no integer solution.

This also shows that $\mathcal{U}(0^{(p)}) = \forall y \neg w(0^{(p)}, y)$ is not a natural number proposition, is not a closed formula for the system $\mathcal{N}$ and that $\mathcal{U}(0^{(p)})$ is an unclosed term of the algorithm.

This paper proves that:

(1) The undecidable proposition $\mathcal{U}(0^{(p)})$ proved by Gödel back then is not false, but it is mistakenly believed that $\mathcal{U}(0^{(p)})$ is a closed formula of the system $\mathcal{N}$.

(2) The undecidable proposition $\mathcal{U}(0^{(p)})$ constructed by Gödel has no truth or falsity, is not a closed formula of the system $\mathcal{N}$, and is a unclosed term of logical algorithm.

Appendix References

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