Proton's Radius Calculated from its Mass

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Abstract

The *mass of the proton* is known to about 9 digits of accuracy, whereas the *radius of the proton* is only known to about 3 digits of accuracy. The theoretical model presented here, derives the proton's radius from its mass, thus giving a radius value equal in accuracy to that of the mass measurement used for the calculation. As a check on the validity of the theory behind the calculation, results are compared to the 2018 CODATA proton radius value and are found to be well within the error limits of that value. So, it's possible that the theoretical value of the proton's radius presented in this paper is its actual radius to 9 digits of accuracy.

1. Introduction

After a new 4% smaller measurement of the proton's radius was reported by researchers in Germany (Pohl et al) in 2010 using muonic hydrogen, it was noticed that the value reported, when converted to a circumference, was almost exactly the length of the wavelength of a photon with energy equivalent to one quarter the mass of the proton. That near equivalence is confirmed in this paper using the most recent and accurate experimental data for the mass and radius of the proton. What it may possibly signify is also discussed.

2. The Calculation

Calculating the wavelength of a photon with a given energy (or mass) is a simple exercise in elementary modern physics. Four simple equations, E=mc2, E=hf, f= c/L, and r=2πr, are needed to derive a direct relationship between the wavelength (or amplitude, or radius) of a photon and its energy (or mass equivalent). Details of the derivation are given below.

<u>Derivation of the Needed Equation</u>

$E = mc^2$	(Mass-Energy Equivalence Equation or Einstein's Equation)	E = energy
$E = hf$	(Photon Energy Equation or Planck's Energy-Freq Relation)	m = mass
$E = hc/L$	substituted c/L for f (f = c/L)	c = speed of light
$mc^2 = hc/L$	substituted mc^2 for E ($E = mc^2$)	h = Planck's constant
$L = h/mc$	rearranged	L = wavelength
$L = 2\pi r$	circumference of a circle	f = frequency
$r = h/2\pi mc$	this is the needed equation	r = radius or amplitude

The needed equation is: **r = h/2πmc**. The value of this equation (**r**) is the amplitude of the sine curve representing the undulations of the electric and magnetic fields of the photon as it propagates through space. The assumption is that if the wavelength of the photon is 2πr, then its amplitude is r.

Listed below is the most recent (2018) CODATA value for the mass of the proton in kilograms, plus the two physical constants needed for the calculation (h, c). The 2014 CODATA value for the proton's mass is also listed mainly to show it hasn't changed much in four years. The most recent and accurate proton radius measurements are also listed for comparison to the calculated radius value.

Data Needed for the Calculation and Results Evaluation

Planck's Constant
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6.626070150 E-34 Js (exact)   [2]
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Speed of Light
```
299792458 m/s       (exact)   [2]
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Mass of the Proton
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mp =   1.672 621 923 69 (53) E-27 kg   2018 CODATA    [2]
mp =   1.672 621 898    (21) E-27 kg   2014 CODATA    [2]
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Radius of the Proton
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rp = 0.8414    ±0.0019               fm  2018 CODATA    [2]
rp = 0.8409    ±0.0004               fm  PDG AVG        [1]
rp = 0.84087   ±0.00026 / ±0.00029 fm  ANTOGNINI      [1]
rp = 0.84184   ±0.00036 / ±0.00056 fm  POHL           [1]
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The proton mass used in the calculation will be the 2018 CODATA value truncated to ten digits of accuracy (1.672621923 E-27 kg), but of course divided by four. So, the mass value to input for **m** = 4.181554808 E-28 kg, which is one quarter the mass of the proton.

m = 4.181 554 808 E-28 kg (¼ mass of the proton 2018 CODATA)

$$\mathbf{r} = \frac{\mathbf{h}}{\mathbf{2\pi\, mc}}$$

r = 0.841 235 641 E-15 m (calculated radius of proton)

The result equals r=0.841 235 641 E-15 m, which will be compared to the 2018 CODATA value of the proton's radius in the next section. Since the constants used in the calculation (2, h, c, and π) are all exact, the accuracy of the result is the same as the accuracy of the proton's mass measurement that was used for m.

3. Results Evaluation

The 2018 CODATA value of the proton's radius is **0.841 4** ±0.0019 fm, and as can be seen from the table below the calculated radius value (**0.841 235 641** fm) is very close to that. It is 0.02% less than the 2018 CODATA radius value, which is just 2 parts in 10,000.

ExpRadius (fm)	Error	ThrRadius (fm)	dr	dr/ ExpRadius	dm/Error
0.**841 4**	±0.0019	0.**841 235 641**	0.000165	.02%	8.7%

So the match between the radius value derived from a photon of mass equal to **m**p/4 and the experimental value represented by the 2018 CODATA value are very close. Is this just a coincidence? It's possible, but it would be a very big coincidence if it were. It may be a clue that could lead to a better understanding of the structure of the proton. More about that in the conclusion.

4. Conclusion / Significance

What does it mean? Is the proton constructed of four photons? It's a possibility. To decide, we need to examine the *volume of a light wave* having energy equal to **m**p/4 and compare that to the *proton's volume*. Will four of them fit inside a proton?

We will use the Maxwellian model for the shape of a light wave – a sine curve – and assume the volume of a light wave equals the volume of its enclosing cylinder. The volume of the enclosing cylinder of a wavelength of light is its wavelength ($L=2\pi r$) times its base area ($A = \pi r^2$), where 'r' equals the amplitude of the sine curve, which yields: $Vcyl = (2\pi r)(\pi r^2) = 2\pi^2 r^3$. This is the same formula as the formula for the surface volume of a hypersphere of radius 'r'. $Vhss = 2\pi^2 r^3$.

So the volume needed to enclose a photon of mass **m**p/4 equals the surface volume of a hypersphere of radius 'r'. Is the surface volume of a hypersphere of radius 'r' bigger or smaller than the proton's interior 3d volume? It is much bigger. It is $(3/2)\pi$, or 4.71238898 times bigger than the proton's 3d interior volume. This strongly suggests that the four photons that may comprise the proton are not found 'inside' the proton, but 'outside' it in 4d space where the surface of the proton's hypersphere may be located. And even though the proton's hypersphere's surface is only the size of the volume of *one* quarter-proton-mass photon, photons are bosons, not fermions, so all four can occupy the same space at the same time.

If that's the case, if the proton has this structure, then since 100% of the mass of the proton is accounted for by the four quarter proton mass photons in the proton's hypersphere's surface, no mass is left to occupy the proton's 3d interior volume. This implies that there is nothing 'inside' the proton with respect to our 3d space. The proton may be said to be 'hollow' with respect to its 3d interior. If this is true then it invalidates quark theory which posits that the 3d interior of the proton is where all the matter of the proton is located (not to mention quark particles and the strong force).

In summary, the more accurate recently determined value of the proton's radius suggests the proton could possibly be constructed of four photons (light waves) occupying the surface volume of a 4-sphere with radius equal to the proton's radius. In such a model – since the entire mass of the proton is accounted for by those four photons, which have to occupy the proton's hypersphere's 3d surface rather than the proton's 3d interior to be consistent with their radii (amplitudes) and large volume – the interior 3d volume of the proton must be devoid of matter. This implies that the 2d surface of the proton is not the surface of a 3d object but the 'surface' of a 4d object. More precisely, the *2d surface of the proton* is where the proton's hypersphere's surface (where all the proton's matter is located) *intersects* our 3d space.

Note: I realize most people are unfamiliar with 4d space and have many questions about it, so here are the essentials needed to understand the preceding several paragraphs. The following is speculative, but makes sense in many ways, and some, if not all of it may turn out to be true. 3d space has zero thickness in the fourth dimensional direction, which means 4d space is next to every point in 3d space in two opposite 4d directions, so wherever a proton is in 3d space, its higher dimensional part (which contains all of its matter) can extend out into 4d space. We – our bodies – cannot access (move into) 4d space, because all the subatomic particles in our bodies are 'attached' somehow to the universe's hypersphere's surface (which is likely identical to the Higgs field). However, the photons, or quarks, that the proton and other subatomic particles are made of *can travel into 4d space* (and higher), but for some reason don't, or can't, travel far into 4d space. They stay right at the surface of the universe's hypersphere forming the hadrons of which everything is made.

4. References

[1] R.L. Workmanet al.(Particle Data Group), Prog.Theor.Exp.Phys.2022, 083C01 (2022

[2] NIST Website