Respiratory viruses' epidemic model considering immunity loss

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Abstract

A previous work on Covid-19 forecast miserably failed to preview the epidemic evolution with the massive vaccination done during 2021. This paper aims to workaround its weak point, which was to not consider immunity loss in its model. The set of SIR equations was reviewed including immunity loss, β profile was recalculated and the model was tuned using real data of 2021. This way was achieved a good conformance between the simulation and data, roughly within the calculated uncertainty of 25%. The simulation for 2022 presented Omicron peak but switched in time. The probable explanation for that is an unbalance in β profile in the beginning of 2022, resulting in a bigger peak in January and in consequence a smaller one latter, due to more immune people. It was explored the hypothesis of different immunity losses for natural and vaccine immunities. This case showed a theoretical profile similar to the real data observed. As a limit case theoretical study, was verified that the epidemic evolution in several years more similar to real data was the case in that the vaccination didn't avoid transmission or avoid as little as 20%. Simulation showed, as expected, that if β is below some limit the epidemic vanishes. Data showed that Covid-19 seems to be naturally vanishing by itself, meaning that no measures so far were effective. New approaches are speculated to provide a better performance on epidemic combat based on ventilation and air sterilization using GUV. Suggestions on how to test those approaches are presented.

Introduction

In a previous paper [1], it was presented the dynamics of the Covid 19 pandemic considering perfect immunity, natural or acquired by vaccination. Later data showed this is not a valid premise and that immunity loss is not negligible at all. This can be seen in Figure 1 that presents the number of cases in Florida for 2020 and 2021, in which the orange line is the real data and the red line is the simulation considering vaccination without immunity loss. The peaks of spring and summer of 2021 that are observed in the real data were completely missed by the simulation, which considered that around the middle of June the number of cases would fall to zero. This way, the aim of the present paper is to show that immunity loss has a main role in some kinds of respiratory virus epidemic dynamic. If the immunity provided by the mass vaccination done worldwide in 2021 was really effective, the epidemic would have ended as calculations showed in that paper. This does not mean that the vaccine is not efficient. It is, but its immunity or even natural immunity decays with time and this can be mathematically observed.
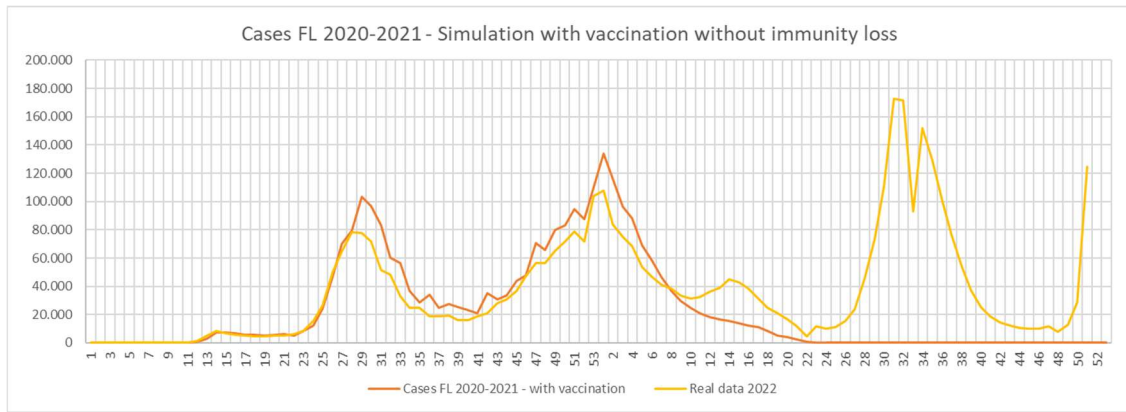
Figure 1

For this study, the states of Brazil were removed because during the year of 2022 the access to their data weren't always available, being impossible to keep the data consistency over all the period from 2020 to 2022. This way, to keep the study covering tropical, temperate and monsoon regions was kept Florida, as a tropical region, India as a monsoon region and added Germany for temperate region, in the place of New York State. This was done because New York data does not include New York City, which could make it less representative.

All the model and calculations were ready by the beginning of 2022, when was planned to publish this paper. With the advent of Omicron variant and its huge wave was more convenient wait some months to observe its evolution and the conformity of the model to it.

Presentation of the mathematical model

Figure 2 shows the block diagram of the SIR Model with vaccination and immunity loss adopted. It is an improved model of the one presented at [1]. It was included the immunity loss path and because of that the Recovered equation might have been revised to provide a precise calculation of R to allow the calculation of the immunity loss. In that previous paper the aim was to calculate the weekly cases without any immunity loss and R wasn't really necessary. This is not the case anymore.
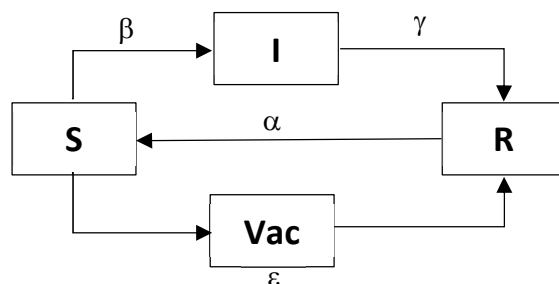

Figure 2

Meaning of the variables:

- S – Susceptible
- I – Infected
- R – Recovered
- α – Immunity loss rate

- β – Transmission rate
- γ – Removal rate
- Vac – Vaccinated
- ε – Vaccine efficacy
- N – Population
- C – Accumulated Cases

Classical SIR model equations for S, I and R are presented below. The factor for quantity of vaccinated people who is subtracted from susceptible ones and added to recovered ones is ε.dVac/dt, where ε is the efficacy of the vaccine, normally above 90%, and dVac/dt is the quantity of people vaccinated during the integration interval. In this study the integration interval is one week. The quantity of people who lose immunity and is added in the group of susceptible and subtracted from the group of recovered is αR, where α is the immunity loss rate, which has its value evaluated in this study.

$$\frac{dS}{dt} = -\frac{\beta SI}{N} - \varepsilon \frac{dVac}{dt} + \alpha R$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I + \varepsilon \frac{dVac}{dt} - \alpha R$$

Naturally, one can write that:

$$N = S + I + R$$

The variation of the accumulated cases is equal the variation of S, but not including the vaccinated and who lose immunity. Those will be considered in the next integration interval. So:

$$\frac{dC}{dt} = \frac{\beta SI}{N}$$

Since the integration interval in this study is one week, the equation above represents the weekly number of cases, which has plenty of data available.

Tuning and verification of the model with real Covid 19 data

The first step for the model tuning was the β determination. The previous paper was done before one entire year of the epidemic has passed, so part of the β annual profile was estimated. This was recalculated using real data, considering from 2020 week 20 to 2021 week 19. Real data was obtained from Our World in Data [2] and US CDC [3]. So, below are presented the graphs of β for Florida, Germany and India. The blue line is the estimated β and the orange line is the calculated one. Notice that the estimation, roughly from week 8 to 18 is always smaller than the real data, since a conservative approach was adopted to avoid unrealistic high Covid 19 cases forecasts. Real data is especially high for Germany and India, which will cause a major effect on the number of cases, as will be shown in the simulation later.

For contextualization, β is supposed to be seasonal in this work and a function of the weather and the consequent human behavior. It might be roughly the same every year with some variations due to weather and behavior, for example, a colder year or intense propaganda.
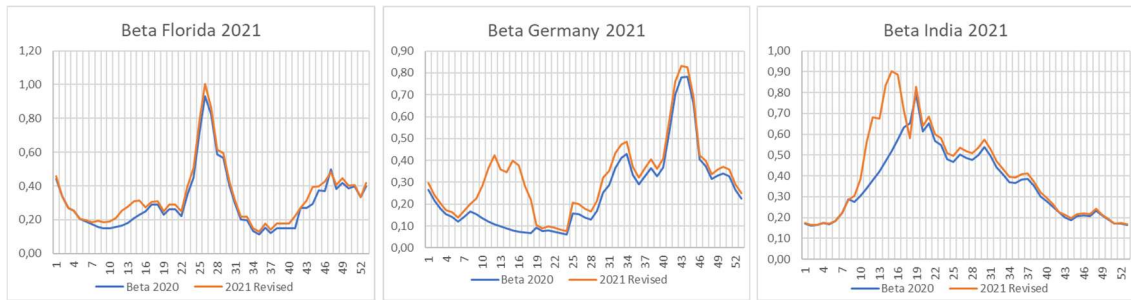
Figure 3

Next, as a conservative estimation, the vaccine efficacy was considered to be in average 90% as a simpler approach for the several types of vaccines and several countries studied.

Finally, α should be estimated and γ should be fine-tuned. This was done manually by graphical analysis and binary search, having in mind that the uncertainty of the simulation was calculated in the previous work as 25% [1]. So, this order of magnitude of the difference between simulation and real data will be considered good enough. The value of γ was changed from the original value of the previous work which was 7/30 = 0,2333 to 7/28 = 0,25. This was done in conjunction to the convergence of the value of α to 0,06. The goal was to make the simulation fit in average in the best manner to the real data. For this simulation was used the vaccination real data for 2021 [2], not the estimated profile presented in the previous work. The results are shown in Figure 4. In the end of 2020 (around week 50) the difference was bigger but a good fit in the quantity of cases was obtained in the middle of 2021 (around week 30). As can be seen, the quantity of cases fitted well, but with a time displacement. This displacement uses to happen because, as have been said, the spread depends on the weather and behavior, which are not constant or absolute. Again, must be said that was used a fixed annual profile of β, which modulates the shape of the cases. This can also be not always true but can be a good approximation.
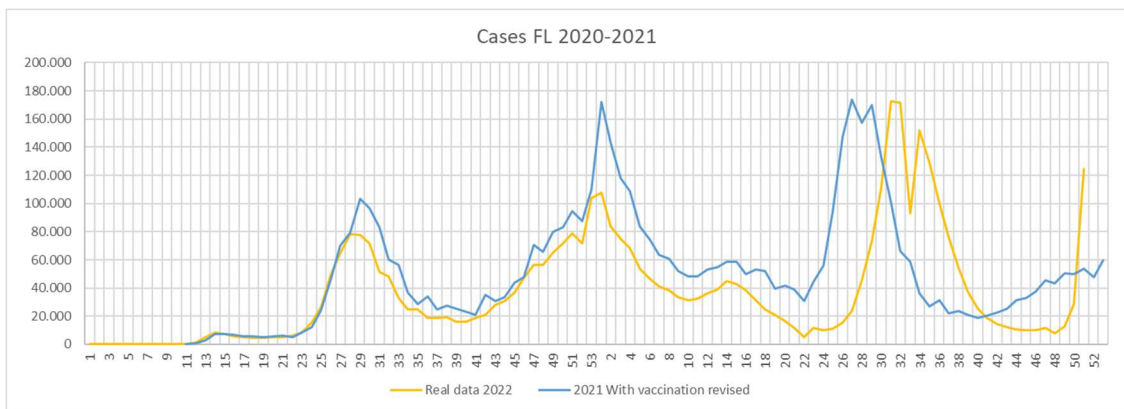

Figure 4

Those parameter values were confirmed checking the fit for Germany and India, shown in
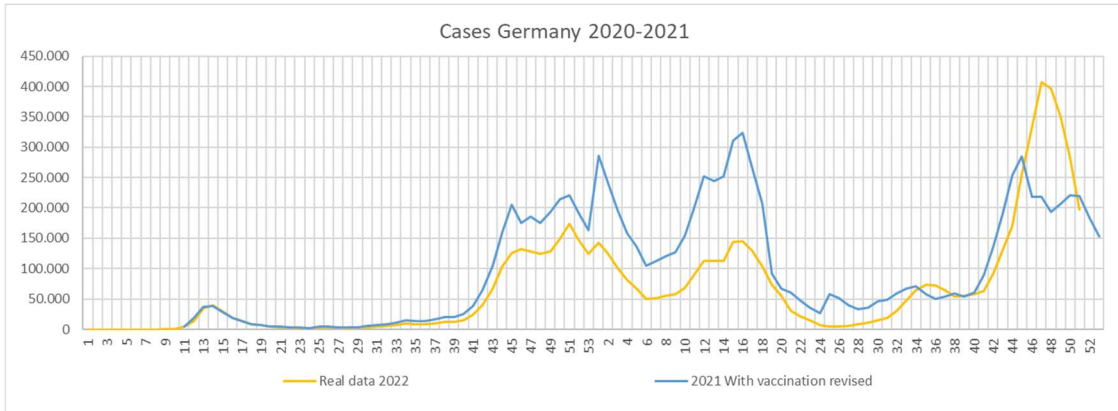

Cases Germany 2020-2021

Figure *5* and Figure 6. For Germany can be seen that the simulation and real data are well synchronized, the amplitude of the simulated waves are roughly the same for 2020 and 2021 while the real data showed that the wave for 2022 is significantly bigger than 2021. This is the limitation of using a single β profile for all the years.
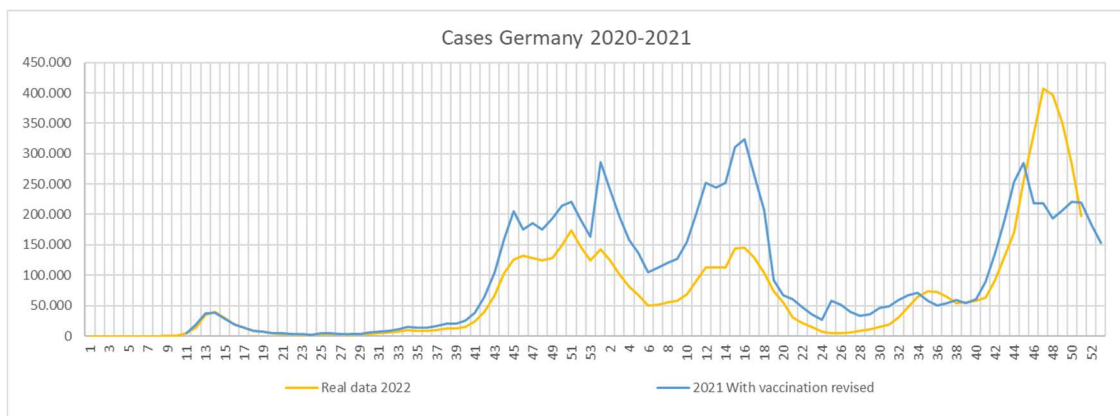

Figure 5

For India the simulation is well synchronized with real data as well, but in 2021 the real data only shows the first part of the wave, with a smaller duration and reaching a smaller peak. Anyway, in all three cases the resemblance of the simulation with the real data is astonishing.
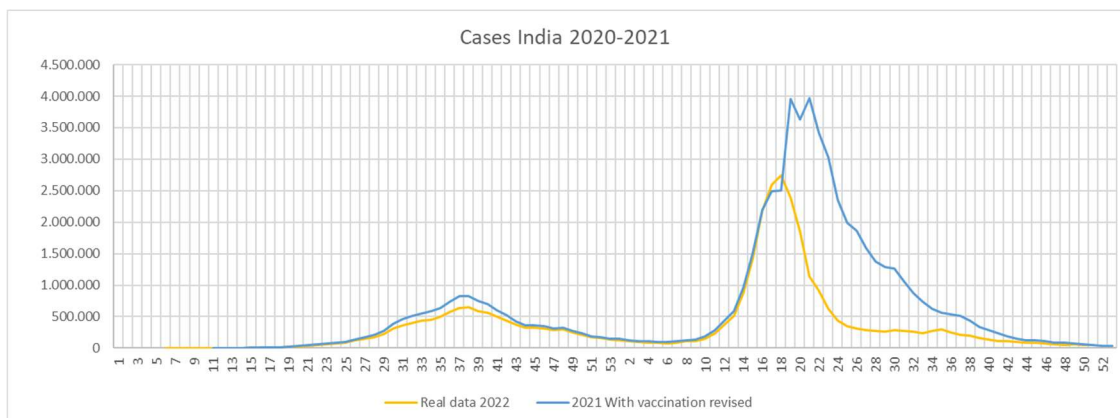

Figure 6

Once tuned the model using 2021 data, the next step is its verification for 2022. For that, we will make the assumption that the vaccination occurred only in 2021 for the sake of simplification. This is reasonable because it was significantly smaller in 2022 as can be seen in the graph for United States of Figure 7 [2]. For further simplification, this will be considered also true for the other countries.
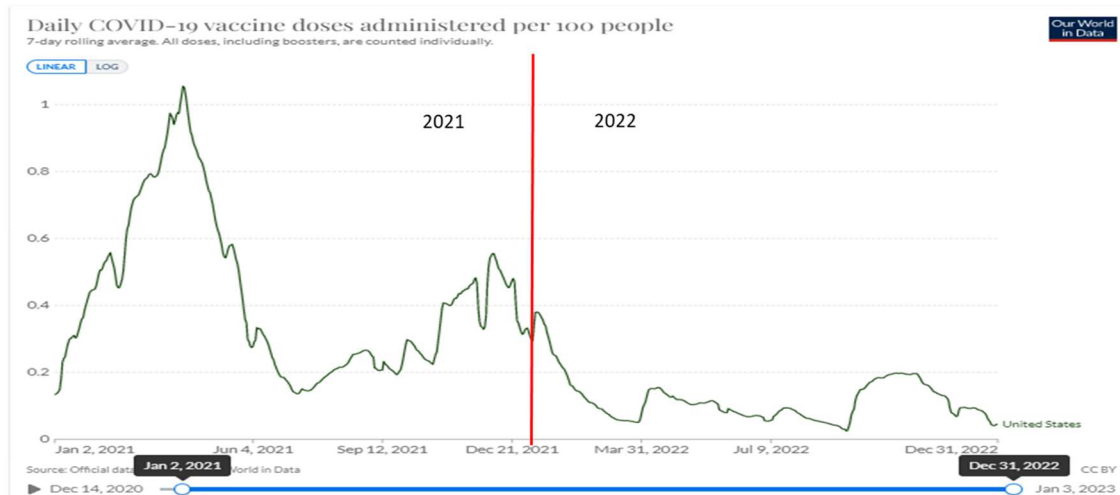

Figure 7

This way, the simulation results for 2022 presented in Figure 8 was obtained for Florida. There, it seems that the waves of week 3 and 30 of 2022 are switched in simulation when compared to the real data. We must highlight that the wave of week 3 of 2022 is the so-called Omicron wave. One hypothesis to explain this is that a big commotion occurred in the beginning of Omicron wave, maybe leading most people to stay indoors in a kind of quarantine, exposing themselves to the virus, according to the conjectures presented in the first paper of this series [4]. So, a large amount of cases arose. And in week 30 there was more people immune and the number of cases was smaller than initially forecasted. In fact, Figure 9 depicts around week 36 of 2022 an increase in immune people in the simulation, represented by the recovered ones (R, blue line), just after the peak of infected (I, orange line) around week 30, showing that should happen the same thing for the real data Omicron wave. Figure 9 is a plot of the three states S, I and R and we can also see the increase in immune people around week 20 of 2021 because of vaccination. If it wasn't that, the peak of infected at week 30 of 2021 would be bigger. But we also can see how fast the quantity of immune people vanishes and at week 27 of 2022 it has fallen around 70% from the peak value. In other words, this difference is a consequence of a fixed β profile, which is not true. It changes depending on the human behavior, modulated by weather, socioeconomic stimulus etc. Notice that in this figure the right vertical axis shows the values of infected people (I).
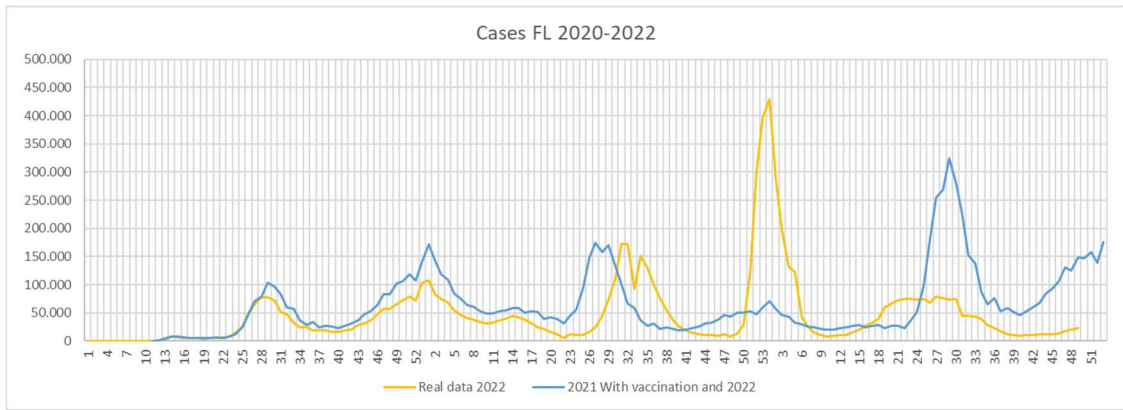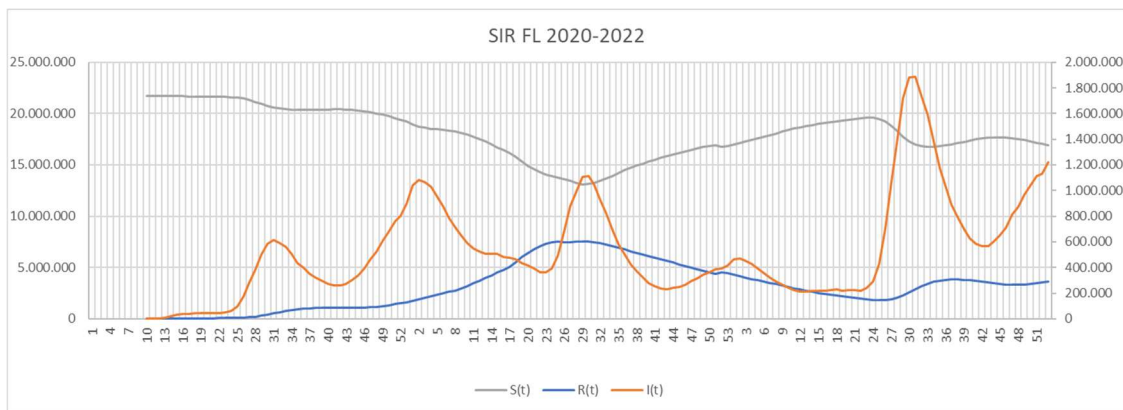
Figure 8

Figure 9

Figure 10 presents the simulation result for Germany in 2022. The same behavior for Omicron wave is observed in relation to the wave around week 40 and the explanation can be the same. Besides that, an unexpected wave around week 25 occurred. A wave in this period was not observed in 2020 and 2021 in Germany. Analyzing Figure 11 one can reach the very same conclusions of the analysis of Figure 9, demonstrating that the behavior of the model is consistent.
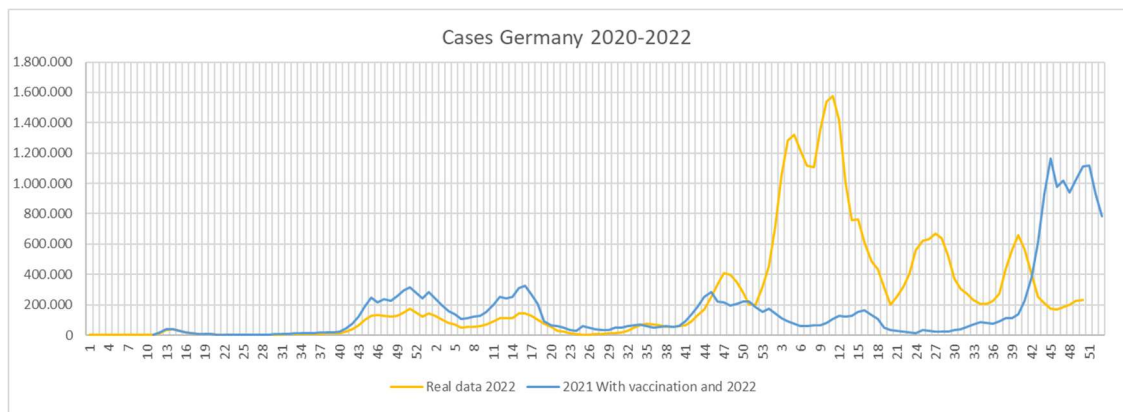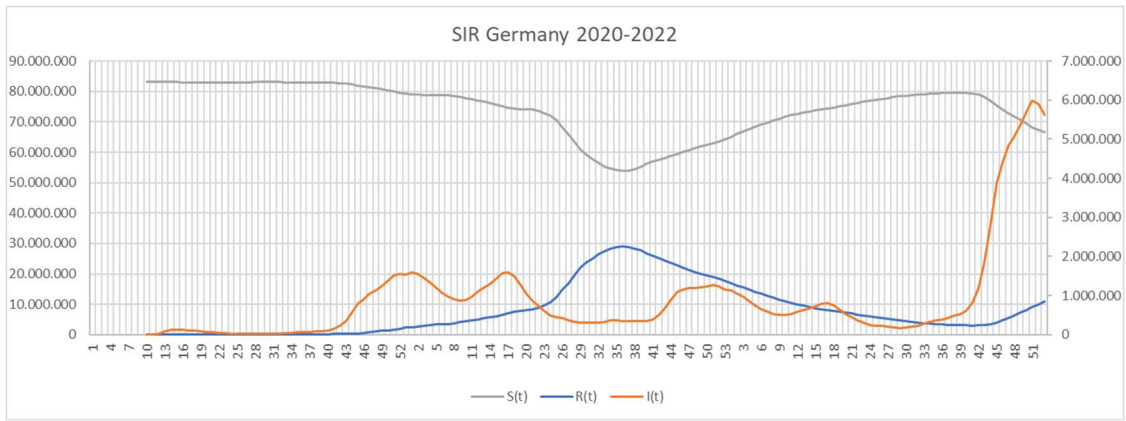

Figure 10

Figure 11

The very same behavior is seen for India (Figure 12). There was the Omicron wave around week 3 of 2022 and the wave expected for around week 30 was much smaller than simulated. Again, the cause might be the amount of people immune after Omicron. Again, analyzing Figure 13 with the variables S, I and R takes to same conclusions of the analysis of Figure 9. In this figure the succession of waves of infected and immune people (blue and orange lines, respectively) is even clearer to see.
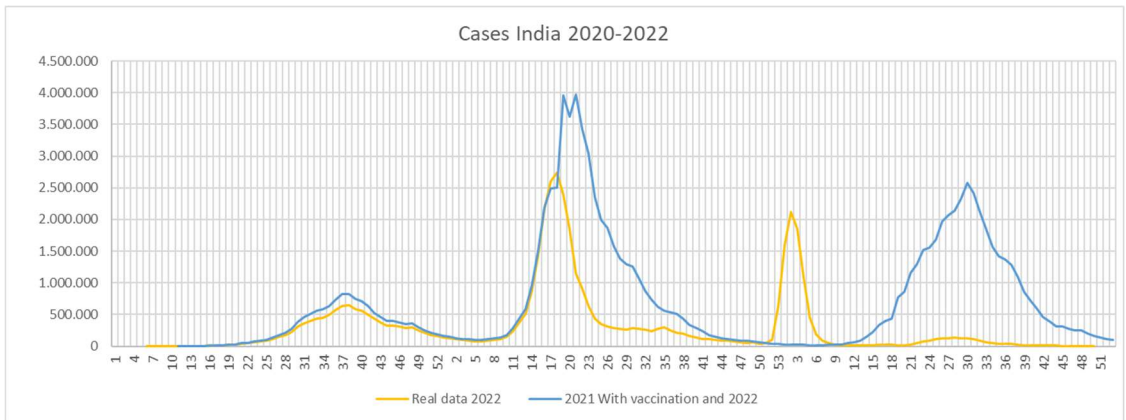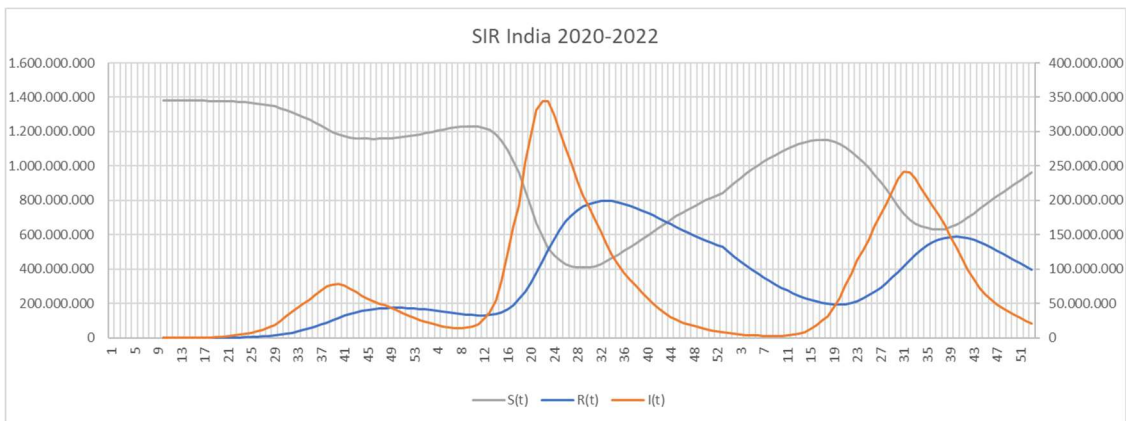

Figure 12

Figure 13

Analysis of the general case

Even considering that the curve fit was good enough with the calculated parameters, one can observe that a value of 0,06 for α is quite large. This means that the immunity loss is considerable, not to say huge. This takes to the need to check if the fit would be better considering two different immunity losses – one for natural immunity, acquired with the disease, and other for the vaccine immunity [5]. This verification will be done along to the simulation of long term epidemy. This simulation considers an ideal seasonal variation of β with sinusoidal shape, with peaks and valleys with values compatible with the observed in Figure 3 along 10 years, with an initial surge to start the epidemy, which is the same methodology adopted in the previous paper [1]. Its period was set in 35 weeks, which was observed in several countries. Figure 14 shows β shape for 10 years. The straight line beyond week 415 will be explained later.
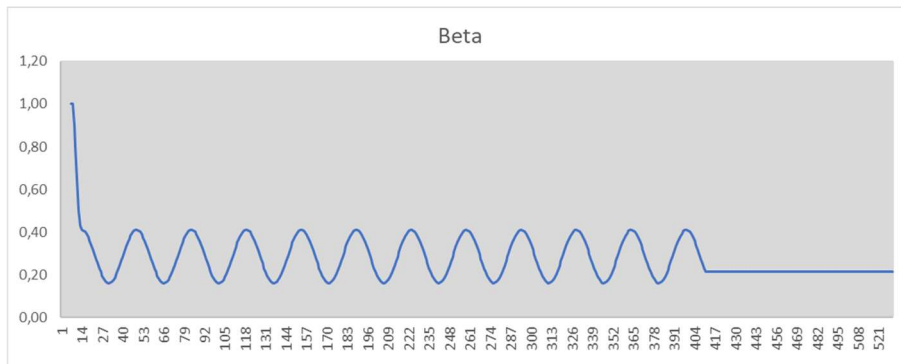

Figure 14

To do that simulation, S equation should be enhanced to:

$$\frac{dS}{dt} = -\frac{\beta SI}{N} - \frac{d\left(\varepsilon\frac{dVac}{dt} - \alpha_{vac}Vac\right)}{dt} + \alpha R$$

This would consider the decrease in the total vaccinated people (Vac) caused by the vaccine immunity loss (α_{vac}). Then, is calculated the derivative of the accumulated vaccinated people, which is the value needed for dS/dt. And Figure 2 should be redrawn as:
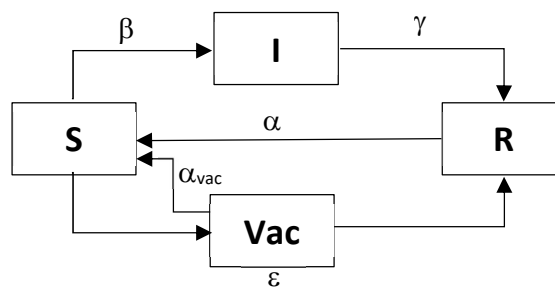
Figure 15

Besides that, this simulation results seemed to be unrealistic with the values tuned for the parameters in the previous session. So, γ was set back to 7/3 = 0,2333. Literature shows that vaccine immunity loss, α_{vac}, ranges from 0,02 to 0,08 [6, 7] and that natural immunity loss is at least three times smaller than this [8]. This way was done a simulation using a more realistic value for α for natural immunity of 0,005 and a value of 0,04 for vaccine immunity, α_{vac}, which

considers that the vaccine immunity vanishes eight times faster than the natural immunity. Was obtained the result of Figure 16.

Can be observed in the first graph that the vaccination can decrease the number of cases in a first moment, but soon the fast immunity loss observed in the blue line of the third graph makes the epidemic to rise again. But this is a self-contained process, since the natural immunity acquired, that lasts longer, makes the number of cases fall. The second graph is the same of the first one, but with logarithmic scale to allow to see more details of the small numbers. Even being a self-contained process, it would take a while to vanish completely, being possible to rise a new epidemic due to some environmental or behavior change. This vanishing time could be significantly long, taking years, as can be seen in the second graph, considering the rate at which the sinusoidal number of cases is falling. This way, after week 415 was tested a value of β that would make the epidemic vanishes. This value is the straight line presented in Figure 14. The big question is how to limit β to this level. Observe that the number of cases falls exponentially to zero (or linearly, in a logarithmic scale), faster as expected.
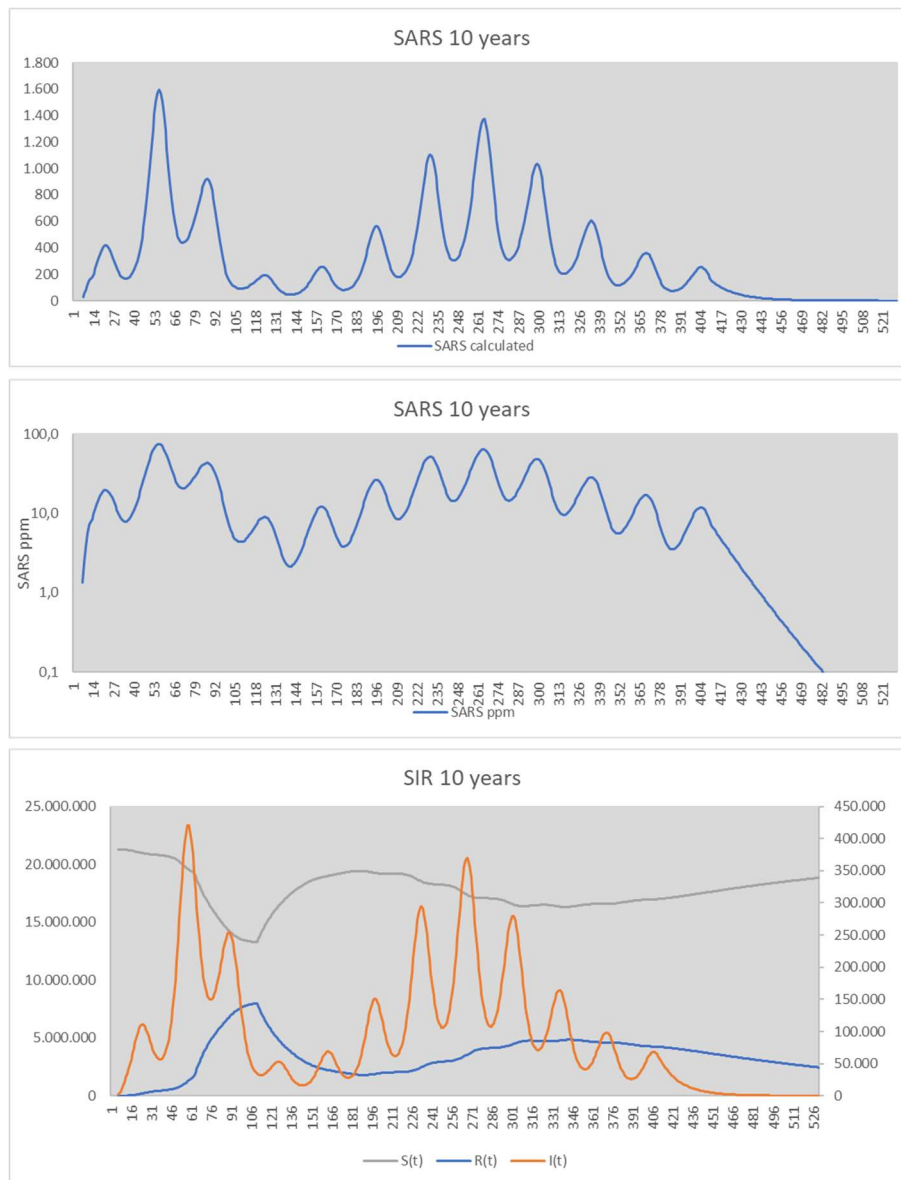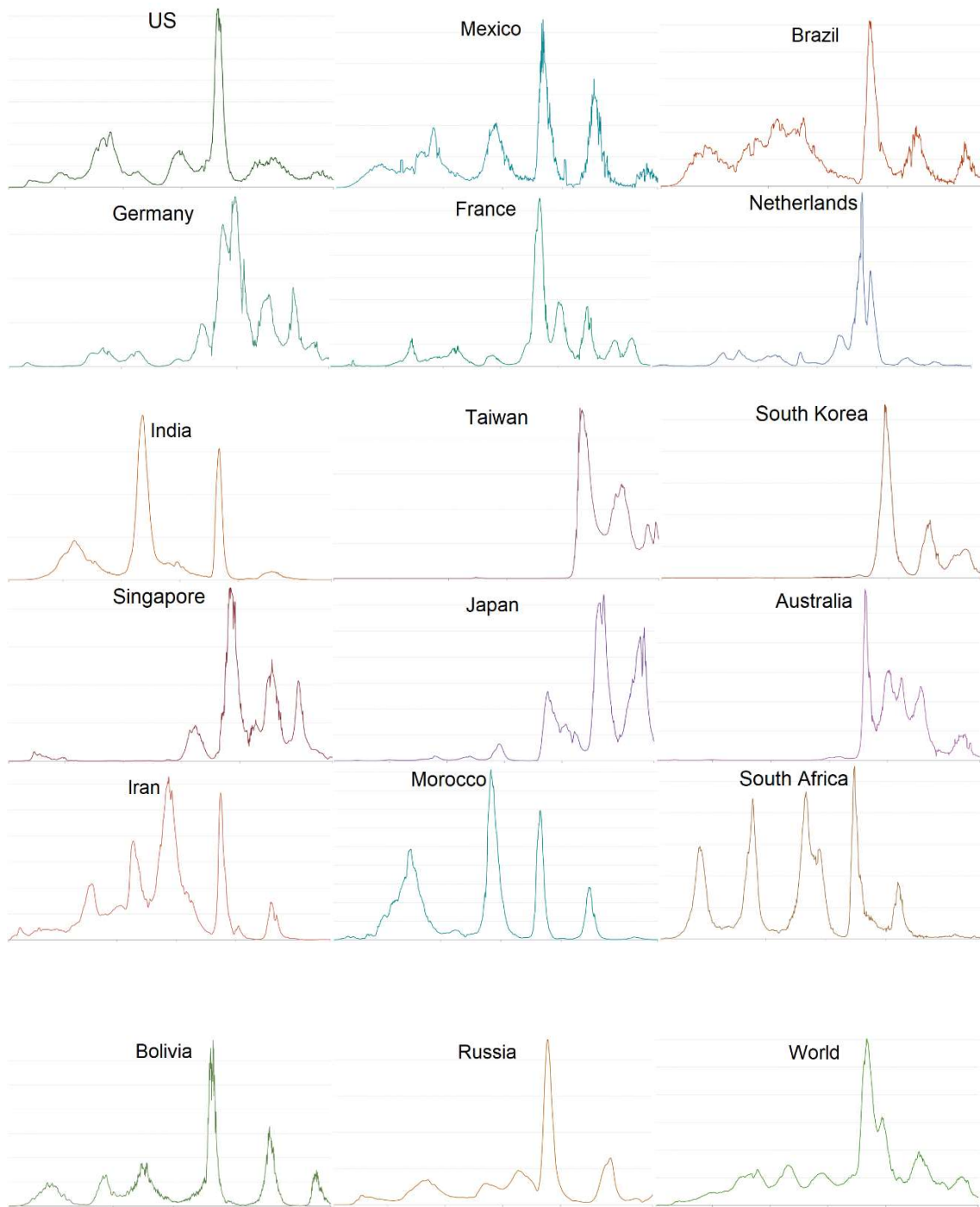

Figure 16

But the profile of the number of cases shown in Figure 16 doesn't match well to the profile observed in the real data presented in Figure 17, gathered from Our World in Data [2]. In the figure are presented profiles of number of cases per million in selected countries from February 2020 to February 2023, performing 3 years. Is possible to see in this figure that the rush of the epidemic occurred in different times according to the country. In some of them it was in the very beginning and in others it was delayed. Anyway, is possible to see the typical dumped cycle in all of them. It is also possible to realize that it seems to have a typical behavior depending on the region of the world in which the country is. Checking their log version is also insightful.

Weekly cases per million from February 2020 to February 2023
Figure 17

So, since the number of cases shown in Figure 16 doesn't match well, another hypothesis must be verified. It must be said that this is a theoretical study of a limit case. This way, Figure 18 presents the results using the very same parameters of the previous one, changing only the transmission by vaccinated people. In other words, the simulation of Figure 18 considers that vaccination would not avoid transmission. As can be seen, the profile in the first years has a good match to the observed Covid-19 data in several countries of Figure 17. Similar results for the first years were already presented in the previous paper [1]. In later years, due to the immunity loss that can be seen in the blue line of the third graph, the number of cases would rise again, maybe keeping an eternal cycle. The logarithmic graph is useful to apprehend this. It is shown again that keeping β below some limit would end the process.
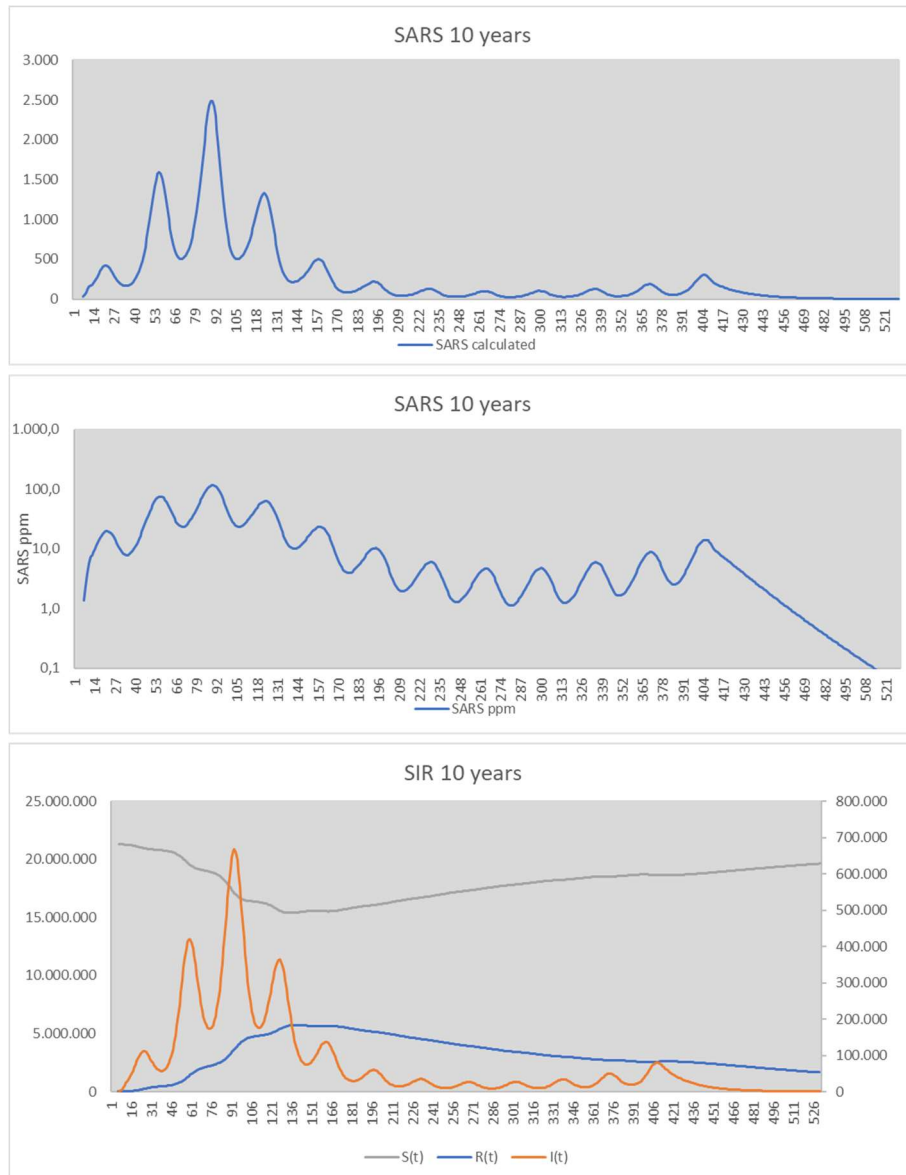

Figure 18

Maybe a more realistic case would be to consider the efficacy of the vaccine for the transmission of 20%. This way would be obtained the results of Figure 19, which also represents well the profile observed in the data for several countries. Again, must be observed that this is a theoretical exercise and was not checked against real data. This aims only to show several scenarios.
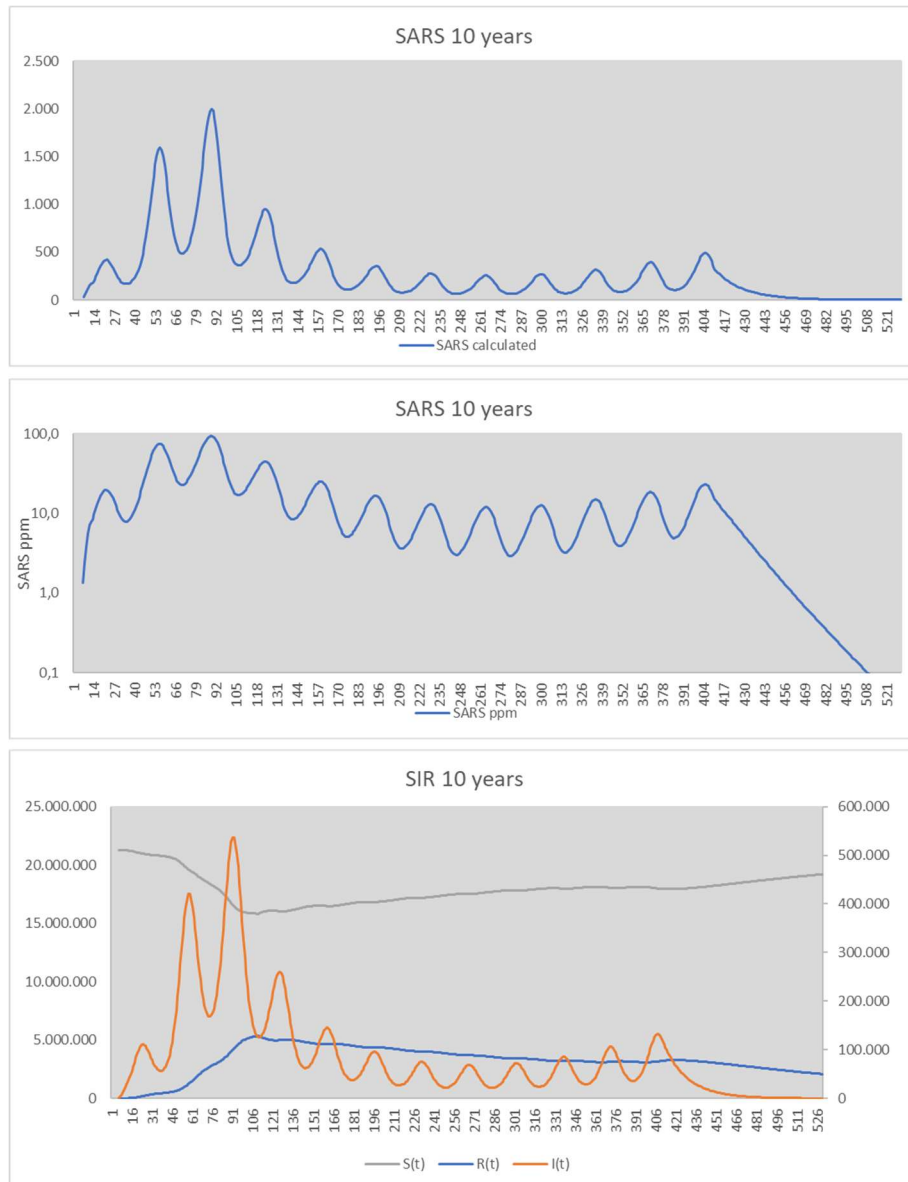
Figure 19

Additional option to try to control the epidemic

What is already known and must be chased is that if β is below some limit the epidemic will vanish. The large-scale experience made in the last few years (2020-2022) showed that lockdowns and face masks are not enough to achieve this target. A good example of that is the case of China. There, was used a rigorous contact tracking system, quarantine, lockdown, face masks and mass vaccination and even so the epidemic arose, as can be seen in Figure 20. It only took more time, but the process was inexorable, as can be seen in the logarithmic scale graph.

Figure 20

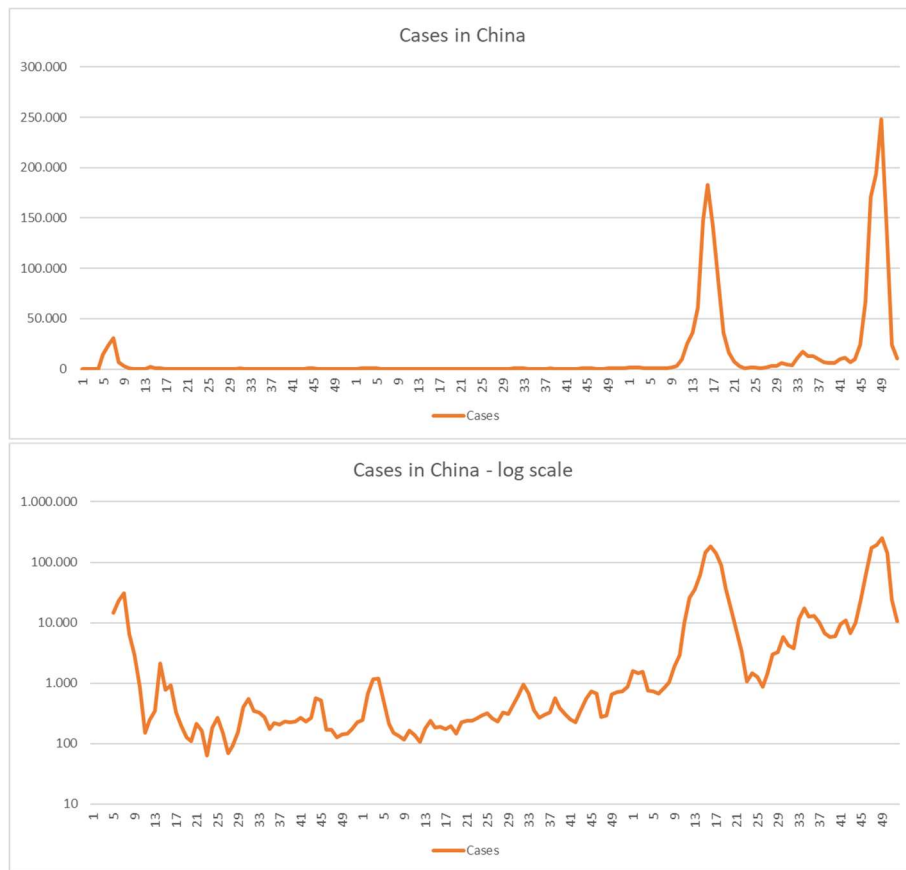It was an error to state just like as a dogma that those would be the only and the best solutions. If there was the liberty for each city, state or country to experiment different additional approaches maybe some of them would present a promising result. This diversity is beneficial for this kind of development, just like the genetic diversity is for evolution. The complementary effect of some of those measures might have a better performance than observed so far.

In the first paper on this subject [4] was proposed the hypothesis that ventilation would be a good option face to the nature of the transmission mechanism. But it would be limited in places with harsh environment that demands air conditioning or heating. In the second paper [1], there was the hypothesis that it would be also a function of GDP. For example, in countries in development people in general can't afford for air conditioning. One can notice, for example, that in Africa Covid-19 rates are low. Those papers where published in January and March of 2021, respectively. Also in March, Nardell published a paper [9] about the advantages of germicidal UV (GUV) over other techniques in indoors environments, where natural ventilation is not possible due to the weather. In April Marr published a paper [10] about the benefits of a better ventilation on Covid-19 control. In summary, two more options could be used in the tentative of the control of a respiratory virus epidemic, besides, for example, vaccination: better ventilation, natural or forced, and germicidal UV.

There is no need to wait until the next epidemic to test the validity of those two hypothetical approaches. The so-called flu is endemic and has an annual cycle. Could be used a controlled environment, for example a hospital. This would be convenient in three aspects – hospitals are an enclosed and controlled environment, already have an issue with nosocomial infections and already have statistical control over it. Could be separated wings of the hospital according to the

kind of patients and activities. Where possible, could be improved and controlled the natural ventilation. Where not, could be installed a GUV system. In two or three years monitoring nosocomial data a conclusion could be reached. Independent of Covid-19 outcomes, this could be a breakthrough in the control of all the respiratory transmitted diseases in a hospital that challenges their infectologists so far. For the records, GUV was used for tuberculosis prevention in the era before antibiotics, which showed to be a more effective approach. For viruses the experience of the latest years showed that there is not such artifact available yet. This way, GUV and ventilation seems to be good promises.

Conclusions

Considering that the worldwide mass vaccination of 2021 didn't finished Covid-19 epidemic as showed by the SIR model simulation developed in the previous work, was developed an enhancement in the model including immunity loss. The review of historic data in the tuning phase of the model showed that the previous β profile was very conservative. In other words, the real data showed infection peaks bigger than in the simulation done in the beginning of 2021. The tuning allowed a good conformance of the model to the real data for the year of 2021 for Florida, Germany and India, roughly around the calculated uncertainty of 25%.

Once tuned with 2021 data, the results for 2022 presented peaks equivalents to omicron wave, but displaced in time, synchronized with the seasonal β profile adopted. In fact, it seems that the peaks of January and July were exchanged, for Florida and India, and January and November, for Germany. One hypothesis for this is the modulation of the behavior due to the commotion in the beginning of omicron wave, increasing abnormally β profile, besides it being also more virulent. Then, with a bigger number of infected and so immune people, the wave in the expected period was smaller.

The value estimated for α, 0,06, is quite big. This way was verified what would happen if there were different values for natural and vaccinated immunity losses. The simulation using a theoretical ideal scenario showed an evolution comparable to the real data. But the similarity was bigger when considered that the vaccination didn't avoid transmission or avoid only up to 20% of the transmission. This was a mathematical exercise of limit situations for study purposes only.

Was showed that, as already known, if β is below some limit the epidemic would vanish. Is also known by the historic that the measures adopted in Covid-19 pandemic weren't enough to reach this limit, being them non-pharmaceutical or vaccination. Data shows that it seems that this pandemic reached its natural end again, like the other ones in history. From now on it will be endemic, also like the other ones. So, additional measures not used could be efficient for epidemic or endemic phases, like ventilation improvement and air sterilization using GUV, for example.

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