Lorentz Covariance of the Octonion Covariant Derivative

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Abstract

The Octonion covariant derivative form applied to an inertial frame of reference velocity transformation requires Lorentz covariance if the two components in rotational and irrotational field types are to maintain proper relative sign and equivalent scale post transformation. Similar to the other Octonion fields different from the magnetic and electric fields, the expected hyperbolic rotation of electric field into the magnetic field and magnetic field into electric field are produced by application of the covariant derivative on Octonion potential function algebraic elements. Also presented are 8-current, 8-work-force and Octonion Poynting vector covariant forms for inertial frame of reference velocity transformations.

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I first presented the Lorentz covariance provided by the Ensemble Derivative, the Octonion covariant derivative, back in 2012 in an FQXi essay I titled “The Algebra of Everything” (ref[1]). Due to maximum length requirements, I could only mention the results in passing. In what follows, I will derive and expound on this.

General considerations for a velocity transformation

If our Ensemble Derivative (refs[1], [2]) form is generally covariant, we should expect to see proper Lorentz type rotations of electric field into magnetic field, and magnetic field into electric field when the observation perspective is in motion relative to the field sources. Repeating our covariant derivative form for Octonion Algebra operating on potential functions **A** we have

$$\mathbf{E}(\mathbf{A}(\mathbf{v})) = 1/J\ \partial/\partial v_i\ [\ C_{ij}\ \ T_{kl}\ \ A_k\ ]\ \ e_j * e_l$$

Once again, **T** is the basis transformation between the intrinsic e basis with position $\mathbf{u} = u_i\ e_i$ and the g basis with position $\mathbf{v} = v_i\ g_i$ where $g_a = T_{ab}\ e_b$. C_{ij} is the cofactor of T_{ij} and J is the Jacobian of **T**. In a stationary system, **T** and **C** are the matrix identity and the Jacobian J=1. As shown in reference [3], in the intrinsic e basis we form the left side application Octonion fields with the covariant derivative operating on the potential functions **A**, and drop the scalar portion since it is not needed, giving the following written in algebraic orientation covariant form (reference [3]):

$\mathbf{F}_L =$

$\{+\nabla_0(A_1) + \nabla_1(A_0) + (\nabla_5(A_4) - \nabla_4(A_5))\, s_5 + (\nabla_2(A_3) - \nabla_3(A_2))\, s_9 + (\nabla_7(A_6) - \nabla_6(A_7))\, s_{13}\}\, e_1$

$\{+\nabla_0(A_2) + \nabla_2(A_0) + (\nabla_6(A_4) - \nabla_4(A_6))\, s_3 + (\nabla_3(A_1) - \nabla_1(A_3))\, s_9 + (\nabla_5(A_7) - \nabla_7(A_5))\, s_{11}\}\, e_2$

$\{+\nabla_0(A_3) + \nabla_3(A_0) + (\nabla_7(A_4) - \nabla_4(A_7))\, s_7 + (\nabla_1(A_2) - \nabla_2(A_1))\, s_9 + (\nabla_6(A_5) - \nabla_5(A_6))\, s_{15}\}\, e_3$

$\{+\nabla_0(A_4) + \nabla_4(A_0) + (\nabla_2(A_6) - \nabla_6(A_2))\, s_3 + (\nabla_1(A_5) - \nabla_5(A_1))\, s_5 + (\nabla_3(A_7) - \nabla_7(A_3))\, s_7\}\, e_4$

$\{+\nabla_0(A_5) + \nabla_5(A_0) + (\nabla_4(A_1) - \nabla_1(A_4))\, s_5 + (\nabla_7(A_2) - \nabla_2(A_7))\, s_{11} + (\nabla_3(A_6) - \nabla_6(A_3))\, s_{15}\}\, e_5$

$\{+\nabla_0(A_6) + \nabla_6(A_0) + (\nabla_4(A_2) - \nabla_2(A_4))\, s_3 + (\nabla_1(A_7) - \nabla_7(A_1))\, s_{13} + (\nabla_5(A_3) - \nabla_3(A_5))\, s_{15}\}\, e_6$

$\{+\nabla_0(A_7) + \nabla_7(A_0) + (\nabla_4(A_3) - \nabla_3(A_4))\, s_7 + (\nabla_2(A_5) - \nabla_5(A_2))\, s_{11} + (\nabla_6(A_1) - \nabla_1(A_6))\, s_{13}\}\, e_7$

Below, to simplify things as done reference [3], we can substitute in Octonion orientation **R0** centric rotational field **R** and irrotational field **I** definitions repeated here for clarity:

$\mathbf{F}_L =$

$\{-I_1 + R_{54}\, s_5 + R_{23}\, s_9 + R_{76}\, s_{13}\}\, e_1$

$\{-I_2 + R_{64}\, s_3 + R_{31}\, s_9 + R_{57}\, s_{11}\}\, e_2$

$\{-I_3 + R_{74}\, s_7 + R_{12}\, s_9 + R_{65}\, s_{15}\}\, e_3$

$\{-I_4 + R_{26}\, s_3 + R_{15}\, s_5 + R_{37}\, s_7\}\, e_4$

$\{-I_5 + R_{41}\, s_5 + R_{72}\, s_{11} + R_{36}\, s_{15}\}\, e_5$

$\{-I_6 + R_{42}\, s_3 + R_{17}\, s_{13} + R_{53}\, s_{15}\}\, e_6$

$\{-I_7 + R_{43}\, s_7 + R_{25}\, s_{11} + R_{61}\, s_{13}\}\, e_7$

Anticipating the possible general form, we will try the following for a prototype transformation matrix **T** and resultant cofactor matrix **C**, where a, b, c and d are yet unspecified beyond the requirement that **T** has Jacobian ad – bc = 1.

Prototype **T** =

$$\begin{bmatrix} +a & 0 & 0 & 0 & 0 & 0 & 0 & +b \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ +c & 0 & 0 & 0 & 0 & 0 & 0 & +d \end{bmatrix}$$

Prototype **C** =

$$\begin{bmatrix} +d & 0 & 0 & 0 & 0 & 0 & 0 & -c \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ -b & 0 & 0 & 0 & 0 & 0 & 0 & +a \end{bmatrix}$$

Calculating the physical fields $\mathbf{F_L}$ by left application of the Octonion covariant Ensemble Derivative on potential functions **A** we have

$\{$
$+d\nabla_0(A_1) +a\nabla_1(A_0) +c\nabla_1(A_7) -b\nabla_7(A_1) -\nabla_4(A_5)s_5 +\nabla_5(A_4)s_5 +\nabla_2(A_3)s_9 -\nabla_3(A_2)s_9$
$-c\nabla_0(A_6)s_{13} -b\nabla_6(A_0)s_{13} -d\nabla_6(A_7)s_{13} +a\nabla_7(A_6)s_{13}$
$\}\ e_1$

$\{$
$+d\nabla_0(A_2) +a\nabla_2(A_0) +c\nabla_2(A_7) -b\nabla_7(A_2) -\nabla_4(A_6)s_3 +\nabla_6(A_4)s_3 -\nabla_1(A_3)s_9 +\nabla_3(A_1)s_9$
$+c\nabla_0(A_5)s_{11} +b\nabla_5(A_0)s_{11} +d\nabla_5(A_7)s_{11} -a\nabla_7(A_5)s_{11}$
$\}\ e_2$

$\{$
$+d\nabla_0(A_3) +a\nabla_3(A_0) +c\nabla_3(A_7) -b\nabla_7(A_3) -c\nabla_0(A_4)s_7 -b\nabla_4(A_0)s_7 -d\nabla_4(A_7)s_7 +a\nabla_7(A_4)s_7$
$+\nabla_1(A_2)s_9 -\nabla_2(A_1)s_9 -\nabla_5(A_6)s_{15} +\nabla_6(A_5)s_{15}$
$\}\ e_3$

$\{$
$+d\nabla_0(A_4) +a\nabla_4(A_0) +c\nabla_4(A_7) -b\nabla_7(A_4) +\nabla_2(A_6)s_3 -\nabla_6(A_2)s_3 +\nabla_1(A_5)s_5 -\nabla_5(A_1)s_5$
$+c\nabla_0(A_3)s_7 +b\nabla_3(A_0)s_7 +d\nabla_3(A_7)s_7 -a\nabla_7(A_3)s_7$
$\}\ e_4$

$\{$
$+d\nabla_0(A_5) +a\nabla_5(A_0) +c\nabla_5(A_7) -b\nabla_7(A_5) -\nabla_1(A_4)s_5 +\nabla_4(A_1)s_5 -c\nabla_0(A_2)s_{11} -b\nabla_2(A_0)s_{11}$
$-d\nabla_2(A_7)s_{11} +a\nabla_7(A_2)s_{11} +\nabla_3(A_6)s_{15} -\nabla_6(A_3)s_{15}$
$\}\ e_5$

{
$+d\nabla_0(A_6)$ $+a\nabla_6(A_0)$ $+c\nabla_6(A_7)$ $-b\nabla_7(A_6)$ $-\nabla_2(A_4)s_3$ $+\nabla_4(A_2)s_3$ $+c\nabla_0(A_1)s_{13}$ $+b\nabla_1(A_0)s_{13}$
$+d\nabla_1(A_7)s_{13}$ $-a\nabla_7(A_1)s_{13}$ $-\nabla_3(A_5)s_{15}$ $+\nabla_5(A_3)s_{15}$
} e_6

{
$+bd\nabla_0(A_0)$ $-ac\nabla_0(A_0)$ $+d^2\nabla_0(A_7)$ $-c^2\nabla_0(A_7)$ $-b^2\nabla_7(A_0)$ $+a^2\nabla_7(A_0)$ $-bd\nabla_7(A_7)$ $+ac\nabla_7(A_7)$
$-\nabla_3(A_4)s_7$ $+\nabla_4(A_3)s_7$ $+\nabla_2(A_5)s_{11}$ $-\nabla_5(A_2)s_{11}$ $-\nabla_1(A_6)s_{13}$ $+\nabla_6(A_1)s_{13}$
} e_7

Our transformation must not break the relative signs in any rotational field or irrotational field component pair, and we must scale them both with the same magnitude. This requires a = d and b = c.

Substituting in for d and c in the Jacobian we require to be +1, we have $a^2 - b^2 = 1$. The a and b solutions for this are

$a = \cosh(\zeta)$
$b = \pm\sinh(\zeta)$

This is precisely the hyperbolic rotation specified in 4D space-time by the Lorentz Transformation.

Lorentz covariance for the Octonion electric and magnetic fields

Now change **T** to represent an inertial frame of reference constant velocity transformation in the polar e_7 direction analogous to the 4D Lorentz transformation but now in our 8D Octonion system. We have then in familiar terms given for constant velocity magnitude v and speed of light c:

$\beta = v/c$ $\qquad$ $\gamma = 1 / \sqrt{(1 - \beta^2)}$

T[row][column] =

$$\begin{bmatrix} +\gamma & 0 & 0 & 0 & 0 & 0 & 0 & +\gamma\beta \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ +\gamma\beta & 0 & 0 & 0 & 0 & 0 & 0 & +\gamma \end{bmatrix}$$

The Jacobian for this **T** is +1. The **T** cofactor matrix for use in the covariant derivative, **C** is

C[row][column] =

$$\begin{bmatrix} +\gamma & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma\beta \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ -\gamma\beta & 0 & 0 & 0 & 0 & 0 & 0 & +\gamma \end{bmatrix}$$

We of course want to express the velocity transformed results in algebraic orientation covariant form. Doing the math for the covariant derivative with this **T** and **C** with results in field form we have after dropping the scalar portion:

$\mathbf{F_L}$ =
$\{ -\gamma [I_1 - R_{17} \beta] + R_{54} s_5 + R_{23} s_9 + \gamma [R_{76} + I_6 \beta] s_{13} \} e_1$
$\{ -\gamma [I_2 + R_{72} \beta] + R_{64} s_3 + R_{31} s_9 + \gamma [R_{57} - I_5 \beta] s_{11} \} e_2$
$\{ -\gamma [I_3 - R_{37} \beta] + \gamma [R_{74} + I_4 \beta] s_7 + R_{12} s_9 + R_{65} s_{15} \} e_3$
$\{ -\gamma [I_4 + R_{74} \beta] + R_{26} s_3 + R_{15} s_5 + \gamma [R_{37} - I_3 \beta] s_7 \} e_4$
$\{ -\gamma [I_5 - R_{57} \beta] + R_{41} s_5 + \gamma [R_{72} + I_2 \beta] s_{11} + R_{36} s_{15} \} e_5$
$\{ -\gamma [I_6 + R_{76} \beta] + R_{42} s_3 + \gamma [R_{17} - I_1 \beta] s_{13} + R_{53} s_{15} \} e_6$
$\{ -I_7 + R_{43} s_7 + R_{25} s_{11} + R_{61} s_{13} \} e_7$

As usual, the right-side application of the covariant derivative $\mathbf{F_R}$ is $\mathbf{F_L}$ with algebraic variant terms negated. To achieve covariance, the effective rotational and irrotational field components in the moving system must have the same algebraic orientation status indicators as in the stationary system. One more beautiful example of algebraic orientation covariance benefits is our results are automatically sieved, indicating in the moving system what are now the effective irrotational and rotational field components. We see the mappings are

$I_1 \rightarrow \gamma [I_1 - R_{17} \beta]$ $\quad$ $I_2 \rightarrow \gamma [I_2 + R_{72} \beta]$ $\quad$ $I_3 \rightarrow \gamma [I_3 - R_{37} \beta]$
$I_4 \rightarrow \gamma [I_4 + R_{74} \beta]$ $\quad$ $I_5 \rightarrow \gamma [I_5 - R_{57} \beta]$ $\quad$ $I_6 \rightarrow \gamma [I_6 + R_{76} \beta]$

$R_{76} \rightarrow \gamma [R_{76} + I_6 \beta]$ $\quad$ $R_{57} \rightarrow \gamma [R_{57} - I_5 \beta]$ $\quad$ $R_{74} \rightarrow \gamma [R_{74} + I_4 \beta]$
$R_{37} \rightarrow \gamma [R_{37} - I_3 \beta]$ $\quad$ $R_{72} \rightarrow \gamma [R_{72} + I_2 \beta]$ $\quad$ $R_{17} \rightarrow \gamma [R_{17} - I_1 \beta]$

All other field components, including all in the direction of motion, are unmodified. The stationary reference frame algebraic variant rotational fields are changed to algebraic invariants such that they are additive to the irrotational fields by noticing the β scalings are the outcome of algebraic variant cross products with $\beta\ e_7$. Same for changing the algebraic invariant irrotational fields to algebraic variants so they can be additive to the algebraic variant rotational fields.

Recognizing I_5 and I_6 are the electric field E_x and E_y respectively, and R_{76} and R_{57} are the magnetic field components B_x and B_y respectively, and our velocity is in the z direction, these results are precisely those provided by the 4D Lorentz transformation in the z direction on the combined 4D space-time field tensor. Using algebraically covariant differentiation Octonion velocity transformations properly demonstrates electric and magnetic field Lorentz covariance.

8–current Lorentz covariance

Now for the 8–current under the same constant velocity transformation in the e_7 direction given in reference [3] as:

$\mathbf{j} = \frac{1}{2}\ \{\ \mathbf{E}(\mathbf{F_L}) + (\mathbf{F_R})\mathbf{E}\ \}$

Doing the math on the potential function version of the fields we have

$$
\begin{aligned}
\{\ & \\
&+\gamma\beta\nabla^2{}_0(A_7) +\gamma\beta\nabla_0\nabla_7(A_0) -\gamma\nabla_0\nabla_1(A_1) -\gamma\nabla^2{}_1(A_0) -\gamma\beta\nabla^2{}_1(A_7) +\gamma\beta\nabla_1\nabla_7(A_1) -\gamma\nabla_0\nabla_2(A_2) -\gamma\nabla^2{}_2(A_0) \\
&-\gamma\beta\nabla^2{}_2(A_7) +\gamma\beta\nabla_2\nabla_7(A_2) -\gamma\nabla_0\nabla_3(A_3) -\gamma\nabla^2{}_3(A_0) -\gamma\beta\nabla^2{}_3(A_7) +\gamma\beta\nabla_3\nabla_7(A_3) -\gamma\nabla_0\nabla_4(A_4) -\gamma\nabla^2{}_4(A_0) \\
&-\gamma\beta\nabla^2{}_4(A_7) +\gamma\beta\nabla_4\nabla_7(A_4) -\gamma\nabla_0\nabla_5(A_5) -\gamma\nabla^2{}_5(A_0) -\gamma\beta\nabla^2{}_5(A_7) +\gamma\beta\nabla_5\nabla_7(A_5) -\gamma\nabla_0\nabla_6(A_6) -\gamma\nabla^2{}_6(A_0) \\
&-\gamma\beta\nabla^2{}_6(A_7) +\gamma\beta\nabla_6\nabla_7(A_6) -\gamma\nabla_0\nabla_7(A_7) -\gamma\nabla^2{}_7(A_0) \\
\}\ & e_0
\end{aligned}
$$

$$
\begin{aligned}
\{\ & \\
&+\nabla^2{}_0(A_1) +\nabla_0\nabla_1(A_0) +\nabla_1\nabla_2(A_2) -\nabla^2{}_2(A_1) +\nabla_1\nabla_3(A_3) -\nabla^2{}_3(A_1) +\nabla_1\nabla_4(A_4) -\nabla^2{}_4(A_1) \\
&+\nabla_1\nabla_5(A_5) -\nabla^2{}_5(A_1) +\nabla_1\nabla_6(A_6) -\nabla^2{}_6(A_1) +\nabla_1\nabla_7(A_7) -\nabla^2{}_7(A_1) \\
\}\ & e_1
\end{aligned}
$$

$$
\begin{aligned}
\{\ & \\
&+\nabla^2{}_0(A_2) +\nabla_0\nabla_2(A_0) -\nabla^2{}_1(A_2) +\nabla_1\nabla_2(A_1) +\nabla_2\nabla_3(A_3) -\nabla^2{}_3(A_2) +\nabla_2\nabla_4(A_4) -\nabla^2{}_4(A_2) \\
&+\nabla_2\nabla_5(A_5) -\nabla^2{}_5(A_2) +\nabla_2\nabla_6(A_6) -\nabla^2{}_6(A_2) +\nabla_2\nabla_7(A_7) -\nabla^2{}_7(A_2) \\
\}\ & e_2
\end{aligned}
$$

$$
\begin{aligned}
\{\ & \\
&+\nabla^2{}_0(A_3) +\nabla_0\nabla_3(A_0) -\nabla^2{}_1(A_3) +\nabla_1\nabla_3(A_1) -\nabla^2{}_2(A_3) +\nabla_2\nabla_3(A_2) +\nabla_3\nabla_4(A_4) -\nabla^2{}_4(A_3) \\
&+\nabla_3\nabla_5(A_5) -\nabla^2{}_5(A_3) +\nabla_3\nabla_6(A_6) -\nabla^2{}_6(A_3) +\nabla_3\nabla_7(A_7) -\nabla^2{}_7(A_3) \\
\}\ & e_3
\end{aligned}
$$

$$\{$$
$$+\nabla^2{}_0(A_4) + \nabla_0\nabla_4(A_0) - \nabla^2{}_1(A_4) + \nabla_1\nabla_4(A_1) - \nabla^2{}_2(A_4) + \nabla_2\nabla_4(A_2) - \nabla^2{}_3(A_4) + \nabla_3\nabla_4(A_3)$$
$$+\nabla_4\nabla_5(A_5) - \nabla^2{}_5(A_4) + \nabla_4\nabla_6(A_6) - \nabla^2{}_6(A_4) + \nabla_4\nabla_7(A_7) - \nabla^2{}_7(A_4)$$
$$\}\ e_4$$

$$\{$$
$$+\nabla^2{}_0(A_5) + \nabla_0\nabla_5(A_0) - \nabla^2{}_1(A_5) + \nabla_1\nabla_5(A_1) - \nabla^2{}_2(A_5) + \nabla_2\nabla_5(A_2) - \nabla^2{}_3(A_5) + \nabla_3\nabla_5(A_3)$$
$$-\nabla^2{}_4(A_5) + \nabla_4\nabla_5(A_4) + \nabla_5\nabla_6(A_6) - \nabla^2{}_6(A_5) + \nabla_5\nabla_7(A_7) - \nabla^2{}_7(A_5)$$
$$\}\ e_5$$

$$\{$$
$$+\nabla^2{}_0(A_6) + \nabla_0\nabla_6(A_0) - \nabla^2{}_1(A_6) + \nabla_1\nabla_6(A_1) - \nabla^2{}_2(A_6) + \nabla_2\nabla_6(A_2) - \nabla^2{}_3(A_6) + \nabla_3\nabla_6(A_3)$$
$$-\nabla^2{}_4(A_6) + \nabla_4\nabla_6(A_4) - \nabla^2{}_5(A_6) + \nabla_5\nabla_6(A_5) + \nabla_6\nabla_7(A_7) - \nabla^2{}_7(A_6)$$
$$\}\ e_6$$

$$\{$$
$$+\gamma\nabla^2{}_0(A_7) + \gamma\nabla_0\nabla_7(A_0) - \gamma\beta\nabla_0\nabla_1(A_1) - \gamma\beta\nabla^2{}_1(A_0) - \gamma\nabla^2{}_1(A_7) + \gamma\nabla_1\nabla_7(A_1) - \gamma\beta\nabla_0\nabla_2(A_2) - \gamma\beta\nabla^2{}_2(A_0)$$
$$-\gamma\nabla^2{}_2(A_7) + \gamma\nabla_2\nabla_7(A_2) - \gamma\beta\nabla_0\nabla_3(A_3) - \gamma\beta\nabla^2{}_3(A_0) - \gamma\nabla^2{}_3(A_7) + \gamma\nabla_3\nabla_7(A_3) - \gamma\beta\nabla_0\nabla_4(A_4) - \gamma\beta\nabla^2{}_4(A_0)$$
$$-\gamma\nabla^2{}_4(A_7) + \gamma\nabla_4\nabla_7(A_4) - \gamma\beta\nabla_0\nabla_5(A_5) - \gamma\beta\nabla^2{}_5(A_0) - \gamma\nabla^2{}_5(A_7) + \gamma\nabla_5\nabla_7(A_5) - \gamma\beta\nabla_0\nabla_6(A_6) - \gamma\beta\nabla^2{}_6(A_0)$$
$$-\gamma\nabla^2{}_6(A_7) + \gamma\nabla_6\nabla_7(A_6) - \gamma\beta\nabla_0\nabla_7(A_7) - \gamma\beta\nabla^2{}_7(A_0)$$
$$\}\ e_7$$

The result is seen to be an algebraic orientation invariant as it is in the stationary reference frame. Filling in the stationary frame of reference definition for **j** from reference [3] we have

$$\mathbf{j} = \{j_0\gamma + j_7\gamma\beta\}\ e_0 + j_1\ e_1 + j_2\ e_2 + j_3\ e_3 + j_4\ e_4 + j_5\ e_5 + j_6\ e_6 + \{j_7\gamma + j_0\gamma\beta\}\ e_7$$

The result is seen to be hyperbolic rotations of j_0 into j_7 and j_7 into j_0. Stationary frame 8–charge rotates into 8–current in the moving frame of reference, and stationary frame 8–current rotates into 8–charge in the moving frame of reference. Inserting stationary frame of reference values $\beta = 0$ and $\gamma = 1$ produces the proper stationary frame of reference **j** definition as it must.

Running the divergence of the 8–current, we see it remains equal to zero independent of the definitions for the potential functions. Conservation of 8–charge is a Lorentz invariant.

8–work–force Lorentz covariance

The potential function version for 8–work–force **wf** gets ugly so for clarity I will only show it in field definition form after doing the following **wf** product definition using the potential function forms for the moving reference frame definitions for **j**, $\mathbf{F_L}$ and $\mathbf{F_R}$ from above. As given in reference [3] we have:

$$\mathbf{wf} = -\tfrac{1}{2}\ \{\ \mathbf{j} * \mathbf{F_R} + \mathbf{F_L} * \mathbf{j}\ \}$$

Sorting by algebraic variance the results are

Invariant **wf**

$\{$
$+[-j_1I_1 - j_2I_2 - j_3I_3 - j_4I_4 - j_5I_5 - j_6I_6 - j_7I_7]\ \gamma$
$+[-j_0I_7 + (j_1R_{17} - j_6R_{76}) + (j_5R_{57} - j_2R_{72}) + (j_3R_{37} - j_4R_{74})]\ \gamma\beta$
$\}\ e_0$
$\{ j_0I_1 + (j_2R_{12} - j_3R_{31}) + (j_5R_{15} - j_4R_{41}) + (j_7R_{17} - j_6R_{61}) \}\ e_1$
$\{ j_0I_2 + (j_3R_{23} - j_1R_{12}) + (j_6R_{26} - j_4R_{42}) + (j_5R_{25} - j_7R_{72}) \}\ e_2$
$\{ j_0I_3 + (j_1R_{31} - j_2R_{23}) + (j_7R_{37} - j_4R_{43}) + (j_6R_{36} - j_5R_{53}) \}\ e_3$
$\{ j_0I_4 + (j_1R_{41} - j_5R_{54}) + (j_2R_{42} - j_6R_{64}) + (j_3R_{43} - j_7R_{74}) \}\ e_4$
$\{ j_0I_5 + (j_4R_{54} - j_1R_{15}) + (j_7R_{57} - j_2R_{25}) + (j_3R_{53} - j_6R_{65}) \}\ e_5$
$\{ j_0I_6 + (j_1R_{61} - j_7R_{76}) + (j_4R_{64} - j_2R_{26}) + (j_5R_{65} - j_3R_{36}) \}\ e_6$
$\{$
$+[j_0I_7 + (j_6R_{76} - j_1R_{17}) + (j_2R_{72} - j_5R_{57}) + (+j_4R_{74} - j_3R_{37})]\ \gamma$
$+[+j_1I_1 +j_2I_2 +j_3I_3 +j_4I_4 +j_5I_5 +j_6I_6 + j_7I_7]\ \gamma\beta$
$\}\ e_7$

Looking at the algebraic invariant e_0 term, recognizing β represents the velocity in the e_7 direction, this is a hyperbolic rotation of the non-scalar inner product $\mathbf{j} \cdot \mathbf{I}$ work into the work form given by the inner product of the force and velocity given here in the e_7 direction. For the invariant e_7, we see the hyperbolic rotation of the e_7 force into the inner product $\mathbf{j} \cdot \mathbf{I}$ work scaling the velocity in the e_7 direction. The forces orthogonal to the velocity are unchanged.

The algebraic orientation variants for **wf** are as follows:

wf variance s_2
$\{ [-j_3R_{57} + j_5R_{37} + j_7R_{53}]\ \gamma + [j_0R_{53} + j_3I_5 - j_5I_3]\ \gamma\beta \}\ s_2\ e_1$
$\{ [j_1R_{57} - j_5R_{17} + j_7R_{15}]\ \gamma + [j_0R_{15} + j_5I_1 - j_1I_5]\ \gamma\beta \}\ s_2\ e_3$
$\{ [-j_1R_{37} +j_3R_{17} +j_7R_{31}]\ \gamma + [j_0R_{31} + j_1I_3 - j_3I_1]\ \gamma\beta \}\ s_2\ e_5$
$\{ [-j_1R_{53} - j_3R_{15} - j_5R_{31}] \}\ s_2\ e_7$

wf variance s_4
$\{ [j_3R_{76} + j_6R_{37} - j_7R_{36}]\ \gamma + [-j_0R_{36} + j_3I_6 - j_6I_3]\ \gamma\beta \}\ s_4\ e_2$
$\{ [-j_2R_{76} + j_6R_{72} + j_7R_{26}]\ \gamma + [j_0R_{26} - j_2I_6 + j_6I_2]\ \gamma\beta \}\ s_4\ e_3$
$\{ [-j_2R_{37} - j_3R_{72} - j_7R_{23}]\ \gamma + [-j_0R_{23} + j_2I_3 - j_3I_2]\ \gamma\beta \}\ s_4\ e_6$
$\{ [+j_2R_{36} - j_3R_{26} + j_6R_{23}] \}\ s_4\ e_7$

wf variance s_6
$\{ [j_2R_{65} + j_5R_{26} - j_6R_{25}] \} s_6 e_1$
$\{ [-j_1R_{65} + j_5R_{61} + j_6R_{15}] \} s_6 e_2$
$\{ [-j_1R_{26} - j_2R_{61} - j_6R_{12}] \} s_6 e_5$
$\{ [+j_1R_{25} - j_2R_{15} + j_5R_{12}] \} s_6 e_6$

wf variance s_8
$\{ [-j_5R_{76} - j_6R_{57} - j_7R_{65}] \gamma + [-j_0R_{65} - j_5I_6 + j_6I_5] \gamma\beta \} s_8 e_4$
$\{ [+j_4R_{76} - j_6R_{74} + j_7R_{64}] \gamma + [j_0R_{64} + j_4I_6 - j_6I_4] \gamma\beta \} s_8 e_5$
$\{ [+j_4R_{57} + j_5R_{74} - j_7R_{54}] \gamma + [-j_0R_{54} - j_4I_5 + j_5I_4] \gamma\beta \} s_8 e_6$
$\{ [+j_4R_{65} - j_5R_{64} + j_6R_{54}] \} s_8 e_7$

wf variance s_{10}
$\{ [-j_3R_{64} - j_4R_{36} - j_6R_{43}] \} s_{10} e_1$
$\{ [+j_1R_{64} - j_4R_{61} + j_6R_{41}] \} s_{10} e_3$
$\{ [+j_1R_{36} + j_3R_{61} - j_6R_{31}] \} s_{10} e_4$
$\{ [+j_1R_{43} - j_3R_{41} + j_4R_{31}] \} s_{10} e_6$

wf variance s_{12}
$\{ [+j_3R_{54} - j_4R_{53} + j_5R_{43}] \} s_{12} e_2$
$\{ [-j_2R_{54} - j_4R_{25} - j_5R_{42}] \} s_{12} e_3$
$\{ [+j_2R_{53} + j_3R_{25} - j_5R_{23}] \} s_{12} e_4$
$\{ [-j_2R_{43} + j_3R_{42} + j_4R_{23}] \} s_{12} e_5$

wf variance s_{14}
$\{ [+j_2R_{74} - j_4R_{72} + j_7R_{42}] \gamma + [+j_0R_{42} + j_2I_4 - j_4I_2] \gamma\beta \} s_{14} e_1$
$\{ [-j_1R_{74} - j_4R_{17} - j_7R_{41}] \gamma + [-j_0R_{41} - j_1I_4 + j_4I_1] \gamma\beta\} s_{14} e_2$
$\{ [+j_1R_{72} + j_2R_{17} - j_7R_{12}] \gamma + [-j_0R_{12} + j_1I_2 - j_2I_1] \gamma\beta \} s_{14} e_4$
$\{ [-j_1R_{42} + j_2R_{41} + j_4R_{12}] \} s_{14} e_7$

All γ scaled terms in our algebraic orientation variants for **wf** are of course their stationary frame of reference algebraic orientation variants. If we adopt the idea that we can make the full **wf** an algebraic orientation invariant, we would assign a value of zero to their sums. Doing so first in the stationary reference frame, there is no counterbalance to zero out our moving frame of reference algebraic variance, so we must conclude the terms scaling $\gamma\beta$ also sum to zero. These are new to our presentation in reference [3], a manifestation of our Lorentz covariance.

Octonion Poynting vector Lorentz covariance

Finally for our constant velocity Lorentz transformation, the Octonion Poynting vector constructed as done in reference [3]. We find it to be an algebraic orientation invariant as it is in the stationary frame of reference. In the moving frame of reference, it takes the following form:

$$\{$$
$$+[\ (I_3R_{31} - I_2R_{12}) + (I_4R_{41} - I_5R_{15}) + (I_6R_{61} - I_7R_{17})\ ]\ \gamma$$
$$+[\ I_1I_7\ -R_{12}R_{72} - R_{31}R_{37} + R_{41}R_{74} + R_{15}R_{57} + R_{61}R_{76}\ ]\ \gamma\beta$$
$$\}\ e_1$$

$$\{$$
$$+[\ (I_1R_{12} - I_3R_{23}) + (I_4R_{42} - I_5R_{25}) + (I_7R_{72} - I_6R_{26})\ ]\ \gamma$$
$$+[\ I_2I_7 - R_{12}R_{17} - R_{26}R_{76} + R_{23}R_{37} + R_{42}R_{74} + R_{25}R_{57}\ ]\ \gamma\beta$$
$$\}\ e_2$$

$$\{$$
$$+[\ (I_2R_{23} - I_1R_{31}) + (I_5R_{53} - I_6R_{36}) + (I_4R_{43} - I_7R_{37})\ ]\ \gamma$$
$$+[\ I_3I_7 - R_{53}R_{57} - R_{36}R_{76} + R_{17}R_{31} + R_{23}R_{72} + R_{43}R_{74}\ ]\ \gamma\beta$$
$$\}\ e_3$$

$$\{$$
$$+[\ (I_5R_{54}\gamma - I_1R_{41}) + (I_6R_{64} - I_2R_{42}) + (I_7R_{74} - I_3R_{43})\ ]\ \gamma$$
$$+[\ I_4I_7 - R_{42}R_{72} - R_{54}R_{57} + R_{17}R_{41} + R_{37}R_{43} + R_{64}R_{76}\ ]\ \gamma\beta$$
$$\}\ e_4$$

$$\{$$
$$+[\ (I_1R_{15} - I_4R_{54}) + (I_2R_{25} - I_7R_{57}) + (I_6R_{65} - I_3R_{53})\ ]\ \gamma$$
$$+[\ I_5I_7 - R_{15}R_{17} - R_{54}R_{74} + R_{25}R_{72} + R_{37}R_{53} + R_{65}R_{76}\ ]\ \gamma\beta$$
$$\}\ e_5$$

$$\{$$
$$+[\ (I_7R_{76} - I_1R_{61}) + (I_2R_{26} - I_4R_{64}) + (I_3R_{36} - I_5R_{65})\ ]\ \gamma$$
$$+[\ I_6I_7 - R_{36}R_{37} - R_{64}R_{74} + R_{17}R_{61} + R_{26}R_{72} + R_{57}R_{65}\ ]\ \gamma\beta$$
$$\}\ e_6$$

$$\{$$
$$+[\ -I_1^2 - I_2^2 - I_3^2 - I_4^2 - I_5^2 - I_6^2\ ]\ \gamma^2\beta$$
$$+[\ -R_{17}^2 - R_{72}^2 - R_{37}^2 - R_{74}^2 - R_{57}^2 - R_{76}^2\ ]\ \gamma^2\beta$$
$$+[\ (I_1R_{17} - I_6R_{76}) + (I_5R_{57} - I_2R_{72}) + (I_3R_{37} - I_4R_{74})\ ]\ [2\gamma^2 - 1]$$
$$\}\ e_7$$

The Octonion Poynting vector components orthogonal to our velocity all get modified with a hyperbolic rotation between the γ scaled stationary reference frame value and $\gamma\beta$ scalings we recognize within sign from reference [3] within the Octonion stress–energy–momentum “tensor” component $\mathbf{\Omega}_7$ which also holds the Poynting vector component in the direction of our velocity.

It is starting to look more like the quotes around “tensor” can be removed, although as stated in reference [3], we do not transform them directly here as done with true tensors, we count on the general covariance of the Ensemble Derivative form used to create them.

References

[1] Richard D. Lockyer, 2012, FQXi Essay *The Algebra of "Everything"*, August 31, 2012
https://fqxi.org/data/essay-contest-files/Lockyer_fqxi_essay_RickLock.pdf

[2] Richard D. Lockyer, February 2022, *Division Algebra Covariant Derivative*
https://vixra.org/pdf/2202.0154v1.pdf

[3] Richard D. Lockyer, January 2023, *Octonion Dynamics*
https://vixra.org/pdf/2301.0005v1.pdf

