

Shortest Proof of Dark Numbers

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Definition: Dark numbers are numbers that cannot be chosen as individuals.

Example: All \aleph_0 unit fractions $1/n$ lie between 0 and 1. But not all can be chosen as individuals.

Proof of the existence of dark numbers.

Let $\text{SUF}(x)$ be the Set of Unit Fractions in the interval $(0, x)$ between 0 and $x \in (0, 1]$.

Between two adjacent unit fractions there is a non-empty interval defined by

$$\forall n \in \mathbb{N}: 1/n - 1/(n+1) = 1/(n(n+1)) > 0$$

In order to accumulate a number of \aleph_0 unit fractions, \aleph_0 intervals have to be summed.

This is more than nothing.

Therefore the set theoretical result

$$\forall x \in (0, 1]: |\text{SUF}(x)| = \aleph_0$$

cannot be correct.

Nevertheless no real number x with finite $\text{SUF}(x)$ can be shown. They are dark.