The foundations of mathematical physics and its relationship with physical reality

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Abstract

The Hilbert Book Model project produced this publication. The search for a reliable foundation of physical reality has had many setbacks and is slow. As a result, mainstream physics got at a sidetrack. Quantum Field Theory, Quantum Electro Dynamics, and Quantum Chromodynamics use the minimal action principle as their base. The Hilbert Book Model shows that continuums belong to the third phase of a special set and cannot work as a foundation of mathematical physics. This document shows how the three phases of the special set lead to a vector space and number systems, which apply to a system of Hilbert spaces in which the local universe and a parallel multiverse can pose. Also, the document shows that science must not consider the Higgs particle or the Higgs field as part of the Standard Model. Instead, the Standard Model of experimental particle physicists should restrict to elementary fermions.

Most physicists interpret photons as excitations of the electric field. In contrast, the HBM interprets photons as chains of dark energy objects, and the dark energy objects are shock fronts that excite the field, representing the local universe. Hop landings of the state vectors of the fermions produce spherical shock fronts that move with light speed away from the location of this landing. This conflicts with the ideas of conservation laws that play in mainstream physics. According to the HBM, a big bang never occurred. The model considers two episodes, and at the beginning of the second episode, time starts running together with an ongoing creation of fermions.

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1 Justification

During my study, I used my scientific books to quietly read the formulas and make notes between the text. In that way, I got a better understanding of the content of the book. This book reflects most of the content of a multipart PowerPoint presentation of the Hilbert Book Model. PowerPoint presentations are not suitable for adding notes.

https://www.youtube.com/playlist?list=PLRn2RuujW3IJsZPxh7iNvlajFWY12G2Af

The HBM is a private research project I started more than a decade ago when I was 70 years old to investigate the foundation of theoretical physics and the relation between mathematical physics and physical reality. As a long-retired physicist, I had ample time to rethink the physics I used most of my career. In the university, I discovered that the lectured physics was incorrect and held flaws and omissions. However, I am convinced I found and repaired most defects and significantly reduced the remaining mysteries.

Building on the discussion results between Hilbert, von Neumann, Cantor, and Zermelo that stopped in the fourth decade of the twentieth century, I accidentally discovered a special set that features phases and phase transitions. Phases of the special set cannot pass the phase transitions step-by-step. This set becomes the foundation of the Hilbert Book Model (HBM).

My targets are students and young scientists that, like me, are curious about the universe in which they live.

I keep the price of this book as low as possible. The price of paper and the cost of pressing and managing the book decide this price. I use Pumbo.nl for managing the book. Pumbo offers the book as a print-on-demand service and in massive quantities as offset. A freely accessible paper will reflect the content of the book and will contain the active versions of the URLs to which the book refers.

2 Introduction

With some arrogance, I dare to say that the Hilbert Book Model now exposes the essential part of the foundations of physical reality. Some mysteries stay, but the model describes these clearly. For me, these mysteries exist because my knowledge of mathematics does not allow me to explain the origin of these mysteries. It is also possible that this mathematics does not yet exist. The multipart PowerPoint presentation offers suggestions for solving these mysteries via the modular structure of the universe's content. A single sentence can shrink the essence of the structure and behavior of the observable universe. ***"The universe that manifests itself to researchers is one continuous film of the possible coverages of space with versions of number systems belonging to the associative division rings."***

The HBM shows that each Hilbert space applies this version in the archival of the members of the division ring that the Hilbert space uses.

3 Explanation

The observation that ***humans cannot think and communicate about things without providing these things with identification in the form of a name or pointer and a short compact description*** establishes a brief explanation. Indirectly, the Hilbert spaces provide the identifiers and descriptions humans require. The curious thing is that ***physical reality can function***

without these limitations. Yet physical reality also appears to adhere to strict rules and existing structures. Many researchers have come to know these rules and structures substantially and formulate them in what they call ***mathematics and physics***.

Several researchers doubt whether people can discover the calculation rules that physical reality uses. However, the author of this publication does not belong to that group.

My arrogance results from my conviction that those with education at the level of a bachelor in the exact sciences of mathematics or physics should easily be able to follow the argument given here and check it as desired. With less prior knowledge, much of the debate is easy to follow. I, as an author, have done my best to make as many as possible of the by me retrieved details freely accessible via the included URLs. The text points in enumerated brackets to the URLs that make the subject accessible online. The book publishes the URLs in the references chapter. On the internet, the free accessible pdf paper offers the URLs actively. The book displays the text primarily in grayscale because colors increase the costs of printing books. The corresponding pdf file shows the full-text colors. The author publishes both the book and the related pdf for this reason.

Because formulas scare off several readers, they house in separate places. This applies to the calculation rules, the bra-ket procedure of Paul Dirac, and essential equations. The formulas locate in an individual chapter. Previous papers already published the formulas. Many of these formulas retrieve from publicly accessible resources such as Wikipedia. Compared to the publication “Setbacks of Theoretical Physics,” this treatise adds the formulas for lattice theory and puts more emphasis on the special set. This publication introduces the name “special_set” for the mentioned special set. This new name produces fewer problems with spelling and grammar correction tools.

4 Clarification

When people focus their research on space, they quickly realize that ***an empty space stands for the ultimate nothingness***. There is nothing in this space to which one could orient oneself. There is no center, and there are no boundaries. It is not hard to imagine that the space could hold many ***anonymous locations***. However, for humans, tracking the behavior of these locations without giving them identification and a precise description is impossible.

Locations are point-shaped objects that can occupy a position in space. That position differs per location. ***Applying number systems provides the required identification***. The ***values*** of the number system elements show the locations' positions.

Without the locations, the container is empty. What results is a ***simple space*** that can function as a container. It is possible to interpret this simple space as ***the ultimate nothingness***. As a container in which locations reside, the simple space functions as a ***vector space***. Two locations and their connecting direction line form a vector.

The vector space owns a simple arithmetic. That arithmetic enables the speciation of the more complicated arithmetic of number systems. Hilbert spaces apply these number systems. They select a private version of a number system.

This paper introduces a structure that harbors a ***system of Hilbert spaces that all share the same underlying vector space***. Moreover, that system puts number systems in a well-defined interrelationship.

5 Vector space

There is still no possibility to point to the position. ***The pointer can consist of a base location and a pointing location connected by a direction line***. Scientists call this pointer a vector and a space in which vectors occur a ***vector space***. A simple scalar number characterizes the length of the vector. The HBM applies the name "vector space" and manages the vector space accordingly.

Physicists give reality the extra adjective "physical" to indicate that this notion concerns the structure and behavior of what experimenters can observe from what they experience about the university. The HBM copies that habit by using the name physical reality for this notion. This publication will apply "physical_reality" to ease the spelling and grammar-checking tools.

The HBM will use the name “direction_line” for the notion of a direction line. This renaming happens for the same reason that the HBM uses the name special_set. Direction_lines obey simple arithmetic.

The direction_line and the length fully characterize the vector. The integrity of the vector does not change when it shifts parallel. The parallel shift can occur on the direction_line but may also occur in another direction. ***Direction_lines can therefore move parallel in the vector space.*** They have no beginning and no end. This situation at once provides the operation with which two vectors can add. If the base point shifts from one vector to the pointer of the other vector, then the non-overlapping points form a new vector called the ***sum vector***. If the direction_lines differ, the sum vector uses a new direction_line.

The two possibilities form a parallelogram in which the sum vectors are parallel and have equal lengths.

By multiplying the vector by a scalar, the length multiplies by that scalar. This action creates a new vector. When the scalar is negative, the base and pointer point change function and the vector gets the opposite direction. At the same time, its length may change. These simple calculation rules allow vectors to pinpoint all locations in the vector space. The section Vector arithmetic in the chapter Formulas contains the formulas.

5.1 Independent directions

Vector arithmetic enables ***a scalar product of two vectors***. The scalar product can show the independence of the direction_lines of vectors. ***The scalar product of independent vectors is equal to zero.*** This way, several mutually independent basic direction_lines exist in the vector space. Since direction_lines can shift in parallel, ***a raster of direction_lines*** can cover the vector space. The raster can form a primitive ***coordinate system***.

6 Number systems

The HBM applies the arithmetic of its vector space to derive the more complicated arithmetic of the number systems that it uses.

6.1 Real numbers

With their calculation rules, vectors can help to construct number systems. For example, an ongoing addition of a starting vector and vectors equal to the starting vector and located on the same direction_line yields an ordered series of designated locations collectively representing the ***natural numbers***. Using the natural numbers as a label, we can ***count*** collections of locations. The ***subtraction*** procedure appears by removing locations from the collection and introduces the ***countdown*** procedure. Finally, we meet the ***number zero*** on the base point of the original starting vector and subsequently follow the ***negative integers***. The method for multiplying numbers appears by adding groups of vectors frequently. That does not supply new integers. The name of the reversal of multiplication is

division and delivers ***fractions***. Fractions can be new numbers. The ***integer numbers,*** together with the fractions, form the ***rational numbers***.

6.2 Phase transitions

Scholars have shown that there are as many rational numbers as natural ones. David Hilbert and his followers knew this. This size equality means all rational numbers can label with a natural number. However, this procedure only works if both number sets hold ***infinite elements***. The transition from finitely many elements to infinitely many elements implies a ***change in state*** for the special_set. In the new ***phase***, the collection shows different behavior. For this particular set, achieving this phase transition step-by-step is impossible. Also, the way back does not go in a step-by-step manner. Scientists do not often use the terms phase and phase transition when concerning number systems. This paper uses these terms to show the ***change in the status of the number system*** that derives from the special_set.

David Hilbert used the parable of the Hilbert hotel to show that countable infinity introduces another behavior of the considered set. He did not use “phase transition” to classify the set's behavior change. He and his followers did not consider the different behavior as a different phase of the set. The HBM assumes the different behavior as a ***distinct phase*** and the behavior change as a ***phase transition***. Accepting the particularities of the special_set has this consequence.

Adding or removing elements does not change the state of the infinite special_set. The infinite set of well-ordered rational numbers fills a large part of the same direction_line. A rational number can ***arbitrarily close approach*** any location on this line. Nevertheless, there are still many locations on this line that rational numbers cannot appoint. We call the numbers that these places show ***irrational numbers***. Irrational numbers include transcendental numbers, and rational numbers include prime numbers. The third phase of the set consists of both. The third phase is infinite and not countable.

Thus, the set of rational and irrational numbers again form a set that can show as another phase of the special_set. The phase transition happens again ***in one go*** and cannot occur step-by-step. Counting the elements of the special_set in its third phase is no longer possible. In this phase, all series of converging members end in a limit that is a set member. The phase transition adds several new calculation rules that manage the change of cohesive parts of the collection. We obtained a special_set that features particular behavior by adding the irrational numbers. Mathematicians call the extra calculation rules ***differential calculus***. The author applies this name for the additional arithmetic rules of the third phase of the special_set. Differential calculus is closely related to the calculation rules of rational numbers. The calculation rules can even mix. ***Without disturbing actuators, nothing will change in the new phase*** of the special_set. If something disrupts, this collection phase tends to remove the disturbance as quickly as possible by sending away the consequences of the disruption in all directions until the effects eventually disappear into infinity. We know this because differential calculus shows this. As mentioned, the disturbance never reaches disappearance step-by-step. The result is that the ***number-covered area expands***. ***The differential calculus tells precisely how that happens***. On the so-far-considered direction_line, the response acts ***in a single dimension***.

When multiplied by themselves, the rational numbers treated so far yield a positive number on the direction_line of the natural numbers. We call the numbers that behave in this way ***real numbers***. We use this name for all numbers on this direction_line and, therefore, for all phases of the numbers on this direction_line. ***Squaring*** is the name for multiplying by oneself. The section Arithmetic of the real numbers holds the formulas.

The phase transitions cause the underlying set to be particular. In this way, this set differs from standard sets. The set exists because simple space holds it and only consists of point-like locations. The author discovered this set by accident. He never found a set with these features published beforehand.

6.3 Spatial numbers

There also appear to be systems of numbers that yield a negative number that shares the direction_line of the real numbers when multiplied by themselves. We call these ***spatial***

numbers. Often the name used for these numbers is ***imaginary numbers***. The HBM does not use that name because the qualification imaginary also has different meanings. The spatial numbers no longer fit on the direction_line of the real numbers. They ***occupy one or three dimensions***. Suppose spatial numbers fall outside the first spatial dimension. In that case, the calculation rules of the spatial numbers ensure that a third spatial dimension covers with spatial numbers in addition to the second spatial dimension. ***The result of the product of two spatial numbers consists of an internal product that supplies a real number and an external product that is zero or produces a result in a direction that is independent of the direction_lines of both factors***. The internal product is the reason for the negative square. Therefore, the spatial numbers' calculation rules differ from the calculation rules of the real numbers. The reaction to a disturbance of the third phase of spatial numbers is more spectacular in the three-dimensional spatial number system than in the one-dimensional spatial number system. The section Arithmetic of spatial numbers holds the formulas.

6.4 Division rings

Nevertheless, real numbers can add with spatial numbers, and spatial numbers can multiply with real numbers. This addition creates new number systems. The real and one-dimensional spatial numbers form the two-dimensional set of what the model calls ***complex numbers***. The HBM shares this name with common mathematics. The real and three-dimensional spatial numbers form the four-dimensional set of what the model calls

quaternions. Again, the HBM shares this name with common mathematics. This name sharing shows that the HBM applies existing names for its number systems where no conflicts arise and sufficient similarity exists. The Mixed Arithmetic section of the chapter Formulas holds the corresponding formulas.

6.5 Confusing calculation rules

Two vectors can together deliver a scalar product. That scalar product is zero or positive, and for two equal vectors, the scalar product supplies the square of the length of the vector. This length is the ***norm*** of the vector. The almost identical effect of the inner product of spatial numbers has led to confusion among many mathematicians and physicists, so these scientists sometimes confuse spatial numbers with vectors. This confusion happened, among other things, with the discoverer of the quaternions. This confusion led to a public scandal that caused the ***quaternions to fall into oblivion*** after the sixties of the last century. As we will see, this had significant consequences for mathematics and physics. [2]

7 History

Before Christ, Egyptians discovered simple fractions. Cantor found the second and third phases of real numbers around 1870. Cantor did not use the designations phase and phase transition. Instead, he and others turned their attention to various kinds of infinities of sets. Cantor called them ***transfinite numbers***. Together with natural numbers, they form the ***cardinal numbers***. The Hilbert Book Model deals with only two forms of infinity. These are the ***countable infinity*** of the second phase of numbers and the ***uncountable infinity*** of the third phase of numbers.

Gerolamo Cardano discovered the complex numbers as early as 1545. In 1854 Sir William Rowan Hamilton discovered the quaternions. He formulated his discovery using the ***four base numbers***. The base numbers are one real base number and three spatial base numbers. The external product appears in the outcome of the product of the first two spatial base numbers. Hamilton discovered this formula while walking with his wife over a sandstone bridge in Dublin. Out of joy, he scratched the formula into the bridge's wall. The rain quickly erased the inscription. Hamilton's students immortalized the formula on the bridge through a bronze commemorative plaque. [3]

8 Set theory.

8.1 Collections in space

Around the turn of the nineteenth to the twentieth century, a group of mathematicians and mathematical physicists led by David Hilbert had an intense discussion about set theory. [4] [5]

David Hilbert intended to establish an axiomatic theory of both mathematics and physics. Unfortunately, he retired before he could finish that target.

The discussion focused on the various forms of infinity and countability. The discussion partners also paid significant attention to the phases and phase transitions of the collection.

For example, they paid attention to the ***continuum hypothesis***. [6] However, they never used the words phases and phase transitions. The HBM applies these names for the special_set to distinguish this set from other sets.

The mentioned discussion ignored the container of the set and paid no attention to the type of objects that formed the set. These choices are significant in physical_reality and the Hilbert Book Model. By choosing space as a container and locations as elements of the set, the number systems the HBM uses to discover the locations obtain added properties that human researchers and physical_reality must consider. These added properties are the symmetries that stand for the freedom of choice that the calculation rules of the number systems do not define. As a result, in the HBM, the number systems exist in several versions that their symmetry distinguishes. For example, the location of the geometric center of the number system can, in principle, be anywhere in the vector space. Also, the arrangement of the numbers can occur along the direction_lines in one or the opposite direction. Physical_reality must adhere to the calculation rules and will use as many symmetry choices as possible. A different choice of symmetry yields a different version of the number system. The word symmetry has various meanings. These distinct meanings also occur in this publication. In the HBM, geometric symmetries play a prominent role. ***Differences between geometric symmetries*** are essential.

9 Coordinates

Three associative division rings exist. [7]

These are the real numbers, the complex numbers, and the quaternions. Each of these number systems exists in several versions that ***differ in their symmetry***. Recording the symmetry is possible with ***coordinate markers***. These markers use the location that shows the value of the number. In the HBM, a ***Cartesian coordinate system*** records all the selection freedoms of a version of a number system. The record removes the selection freedom and helps establish the version of the number system.[8]

In this way, the HBM connects the selected version to the geometric symmetry of the number system and the symmetry of everything that exclusively applies that version.

The limitations imposed by the vector space create geometric symmetry. Therefore, if a model designs number systems without these limitations, then that model does not meet geometric symmetries.

9.1 Hops and symmetries.

A hop can split in partial hops that occur only along the cartesian coordinate lines. The first part jumps along a selected coordinate line. The second part jumps along a perpendicular coordinate line, and the third part occurs along a coordinate perpendicular to both the first and second. This procedure takes a choice at each of these jumps. These selections concern the up or down direction along the coordinate line. These selections correspond to the symmetries that we discussed before. The partial jumps lead to the Frenet–Serret formulas. These formulas form the base of differential geometry.

Willard Gibbs promoted differential geometry, and Oliver Heaviside advanced vector calculus. Both used complex numbers rather than obliviated quaternions. Mainstream physicists quickly embraced the suggested approaches, and many of these scientists rejected quaternionic field theory. The mainstream physicists spent little attention to the symmetries of versions of number systems. Instead, symmetry groups and Lie groups draw their attention. Universities wanted to coordinate their lectures on theoretical physics and wanted to avoid confusion. That is why most universities follow what they now consider ***mainstream physics***. Also, the part of the press that treats science tends to follow mainstream physics and ignores new developments in theoretical physics. This history explains why theoretical physics appears to have entered a dead end. Investigate:
https://www.researchgate.net/publication/363541991_The_setbacks_of_theoretical_physics

Still, Gibbs and Heaviside stimulated the development of multidimensional differentiation technology and indirectly promoted mathematical quaternionic differential analysis development. The introduction of time as a progression indicator produced the quaternionic differential analysis that the Hilbert Book Model advocates. This development preceded and took place independent of the discussion of Hilbert, von Neumann, Cantor, and Zermelo on set theory. The HBM combines and exploits the results of differential calculus and set theory.

10 Mainstream Science

10.1 Warning

Mainstream science still plays a crucial role in promoting a standard reference for teaching and comparing science. This role limits confusion for students and scientific institutions. However, being promoted by mainstream science is not synonym with granting the truth.

This warning especially holds for mathematics, theoretical physics, and mathematical physics.

11 Hilbert spaces

David Hilbert discovered an extension of the concept of vector space. His assistant John von Neumann provided the name “Hilbert space” to this widened vector space. The Hilbert spaces have the surprising property that they can archive elements of the version of the number system used by the Hilbert space. After the archival in an ***abstract*** structure, the stored quaternions retrieve in an orderly manner. A dedicated ***operator*** manages the archival and the retrieval.

Scientists often describe the Hilbert space as a vector space that owns an internal product. However, as previously argued, each vector space has a scalar product, not an internal product. Moreover, it is difficult to imagine that a vector that depicts itself via the scalar product yields a complex number or quaternion as an eigenvalue.

Instead, Paul Dirac discovered a trustworthy procedure for converting a vector space into a Hilbert space. This procedure combines ***covariant ket vectors*** and ***contravariant bra vectors***. These are not vectors but are closely related to them. One problem is that Dirac only showed this for real and complex numbers. In that period, scientists showed little interest in quaternionic Hilbert spaces. However, a small effort can adapt the procedure to apply for quaternions. Hilbert spaces can thus work with any of the associative division rings.

The HBM restricts the archival to the second phase of the special_set. This choice limits the defined archival capability to the separable Hilbert spaces.

Each separable Hilbert space chooses a private version of one of these number systems. As mentioned, the separable Hilbert space can archive collections of elements of this version and retrieve them in an orderly manner. This capability also applies to the entire chosen version of this number system. There is a devoted operator who manages this collection. The HBM calls this operator the ***reference operator***. This assignment means that ***each Hilbert space has a private parameter space***. The HBM gives that parameter space the name ***natural parameter space*** of the Hilbert space. The natural parameter space of a separable Hilbert space is countable. It also means that the symmetry of the version of the selected number system characterizes the Hilbert space. The first version of the bra-ket process works with countable number systems and yields Hilbert spaces that use a countable number of independent base vectors. Therefore, the HBM calls them ***separable***. Section Dirac's bra-ket procedure treats the formulas.

11.1 Function space

The private parameter space turns every Hilbert space into a ***function space***. Through the functions, Dirac's bra-ket procedure defines ***new operators*** who manage the target space of the sampled function as eigenspace.

11.2 Quantum logic

To the surprise of many mathematicians, the set of the closed subspaces of Hilbert space appears to have a ***lattice structure*** that is slightly different from the lattice structure of classical logic. Some scientists suggested that this deviation could be the cause of the quantum structure of the energy exchange seen in small particles and atoms. Therefore, they assigned the name ***quantum logic*** to this new lattice. [9] A closed subspace of a Hilbert space is again a Hilbert space. Differential calculus offers a more obvious explanation. ***Differential calculus only comes into effect in the third phase of number systems***. Function theory and differential calculus describe the third phase of number systems. The Arithmetic of changes section describes the formulas that govern the third phase of number systems. The formula chapter treats ***lattice theory*** in a separate section.

The countable parameter space of the separable Hilbert space concerns the first two phases of the number systems, or it is uncountable and concerns the undisturbed third phase. In that case, the Hilbert space is no longer separable. The ***non-separable Hilbert space*** provides operators with uncountable eigenspaces or can manage multiple phases of the chosen number system. The non-separable Hilbert space uses a modified version of Paul Dirac's bra-ket procedure that uses integrals of functions instead of sums of series. This changed version supplies insight into the workings of uncertainties and the expectation value of a stochastically spread series of numbers.

The extension to non-separable Hilbert spaces uses Dirac distributions rather than standard functions.

Not all features of standard functions hold for Dirac distributions which are generalized functions. This distinction is why non-separable Hilbert spaces do not behave like separable Hilbert spaces. This distinction becomes actual in the system of non-separable Hilbert spaces.

11.3 Other features of Hilbert spaces

Several unique features reveal by playing with subspaces of the Hilbert space. First, subdividing into subspaces does not prohibit the content of the subspace from functionally relating to the content of other subspaces.

11.3.1 Subdividing into Hilbert spaces

Every closed subspace of a Hilbert space is a Hilbert space. The set of closed subspaces of a Hilbert space is lattice isomorphic with quantum logic.

The version of the number system that defines the private parameter space subdivides into other number systems with a lower number of dimensions. For example, the quaternionic number system holds a complex number system for every direction_line in the spatial part of a quaternionic number system that crosses the number 0. The complex number system contains a real number system. Thus, the quaternionic Hilbert space holds complex-number-based Hilbert spaces as subspaces. These complex-number-based Hilbert spaces have real-number-based Hilbert spaces as a subspace. These Hilbert spaces support their own function space.

11.3.2 Subdividing into parameter space and target space

When visualizing functions, humans intuitively put the parameter and target spaces into separated independent space parts. The HBM shares that habit.

The parameters relate to the target values. In non-separable Hilbert spaces, functions usually act in the third phase of the number system. However, the model applies sampled functions in separable and non-separable Hilbert spaces.

The subdivisions require extra dimensions. The vector space owns ample space to harbor these extra dimensions. We call the subspace space that holds the target spaces of all functions the common target space. In a separable Hilbert space, an orthonormal set of base vectors representing a target value of one or more functions can span the common target space.

11.3.2.1 How and why the HBM creates time

The Hilbert Book Model applies the real part of the parameter space to implement the indicator for the progression of change. It uses the common target space to harbor a collection of target spaces of static functions that each belong to the values of the corresponding progression indicator. We will call the value of the progression indicator a ***timestamp***. **This replacement of the real parts of the quaternions by a progression indicator introduces the notion of time into the model**. This subdivision acts as the functionality of a book in which each page stands for an instant of the history of the usual target space. Thus, ***time is an artificial parameter***. ***The hop landings never coincide.*** Therefore, time can intercalate, and the model can sequence the real parts of quaternions in the archived hopping paths.

The model applies this opportunity by exchanging the real parts of the hop landings against the artificial progression steps that the HBM introduces as instances of time.

Humans created differential calculus as part of mathematics. The creation of the artificial time concept allows humans to apply differential calculus.

11.3.2.2 Keeping the relation between parameter value and target value

The original arrangement of locations in the parameter space can be demolished in the target space. This demolishment would occur when oscillations or rotations are involved. The demolishment endangers the relation between parameter value and target value. In the model, embedding other Hilbert spaces or clusters of Hilbert spaces into the target space resolves this. The embedding plots the image of the Hilbert space or the cluster of Hilbert spaces into the target space. The embedded Hilbert spaces or Hilbert space clusters will implement the oscillations and rotations. Section A system of Hilbert spaces treats this. Embedding floating Hilbert spaces, or clusters of Hilbert spaces, disrupts the relation with the background parameter space.

Consequently, these objects own a different time sequence than the elementary floating Hilbert spaces. That time sequence depends on the local gravitational potential in the embedding field. See the presentations of Carlo Rovelli about the notion of time and gravitational time dilatation. The following section explains how the HBM introduces time.

11.3.2.3 The Hilbert Book model

11.3.2.4 Separating the target space into a mirror-symmetric and an anti-mirror-symmetric part

Along direction_lines on each page of the usual target space, superpositions of cosine functions can stand for the ***mirror-***

symmetric functions. Likewise, the superpositions of sine functions can stand for the ***anti-mirror-symmetric functions***.

At the geometrical center of the parameter space, the cosine functions have a maximum. At the geometrical center of the parameter space, the sine functions switch from negative to positive. The anti-mirror-symmetric target spaces realize in a separate subspace. In the formulas, the imaginary factor i shows this. In Hilbert space, this ***imaginary factor*** stands for a split into another subspace.

A cosine function can combine with a sine function with the same frequency into a complex number-valued exponential function. This combination is allowed because the imaginary factor i belongs to the direction of that same direction_line. The resulting ***complex exponential function*** has the remarkable property that it relates to the partial differential change operator that belongs to the selected direction_line. The section Fourier transform in the formula chapter presents the details.

The sine and cosine functions use spatial frequencies as their parameters. This application introduces a frequency parameter space parallel to the spatial position parameter space. The frequency parameter space covers three spatial dimensions in the quaternionic Hilbert space. The frequency parameter space serves spectral functions that populate the common target space. We also call this representation the ***change space***.

The HBM does not restrict frequencies to a single direction_line. It enables spatial frequencies up to three dimensions and quaternionic frequencies that cover four dimensions.

11.3.2.5 Separating the target space into scalar function targets and spatial function targets

The split into mirror symmetric target space and anti-mirror symmetric target space can occur separately for the scalar and spatial function targets.

11.3.3 Adding change with time

If the change with time also includes the split into mirror-symmetric and anti-mirror-symmetric dependency, then the ***frequency parameter space will cover four dimensions***. Fourier series show that the base vectors that span the location parameter space are superpositions of the base vectors of the frequency parameter space with all coefficients having the same amplitude. This statement also holds vice-versa.

12 Potentials and forces

In physics, potential energy is energy held by an object because of its position relative to other objects.

The potential at a location is equal to the work (energy transferred) per unit of actuator influence that physics requires to move an object to that location from a reference location where the value of the potential equals zero.

The Hilbert Book Model considers the potential to be zero at infinity. Suppose the model selects infinity as the reference location. In that case, the potential at a regarded location is equal to the work (energy transferred) per unit of actuator influence that involves moving an object from infinity to that location. In that case, the potential at a location stands for the reverse action of the combined actuator influences that act at that location.

12.1 Center of Influence of Actuators

The influence of similar actuators can superimpose. Thus, a geometrical center of these influences defines the location of the ***virtual location*** of a representant of the considered group of actuators. In physical_reality, ***virtual locations do not exist***. It is a theoretical concept.

This virtual representant has a potential that has the same potential that a point-like actuator of the same influence type would possess. In the Hilbert Book Model, static point-like actuators other than charges do not exist because the embedding field tends to remove them as quickly as possible. However, a model can define static virtual point-like actuators.

12.2 Forces

The first-order change holds five terms, two scalar terms, and three spatial terms. In each of these subgroups, the terms can compensate for each other. For example, in the group of spatial terms, the gradient of the scalar part of the quaternionic field can compensate for the time variation of the spatial component of the quaternionic field. If we neglect the curl of the part of the quaternionic field, then the gradient of a local potential can cause a time variation of a spatial field that describes the movement of influenced objects. If these are uniformly moving massive objects, then these objects will ***accelerate***. So, the spatial field will stand for a ***force field***.

12.3 Actuators

We list the actuators of spherical responses discussed in this paper in the table below.

Actuator	Description	Influenced objects	Symbol Θ	Symbol θ
Actual electric charge	Electric charges are the sources or sinks of electrical fields and cause potentials in the electrical field. The influenced objects are other electric charges. In the HBM, these charges exist at the geometrical centers of floating Hilbert spaces.	Other electric charges	Q	q
Virtual electric Charge	*Virtual charges* stand for a collection of electric charges	Other electric charges	Q	q
Isotropic pulse	Isotropic pulses are embeddings of hop landings of the state vector of floating Hilbert spaces into the dynamic universe field. These pulse responses are spherical in the form of spherical shock fronts.	Other massive objects	M	m
Floating Hilbert space	*Virtual mass* represents a collection of isotropic pulses that a floating Hilbert space generates.	Other massive objects	M	m
Virtual mass	Virtual masses stand for a collection of masses of floating Hilbert spaces.	Other massive objects	M	m

The Hilbert Book Model also explains the notions of attracting and repelling by introducing progression as time.

Electric fields and gravitational fields differ fundamentally in their start and boundary conditions.

Electric charges can attract or repel each other.

Masses will always attract each other.

Spherical pulse responses in the form of spherical shock fronts are dark matter objects. However, the qualification “dark” not justifies when vast numbers of these objects cooperate such that they become perceivable.

13 Stochastic processes

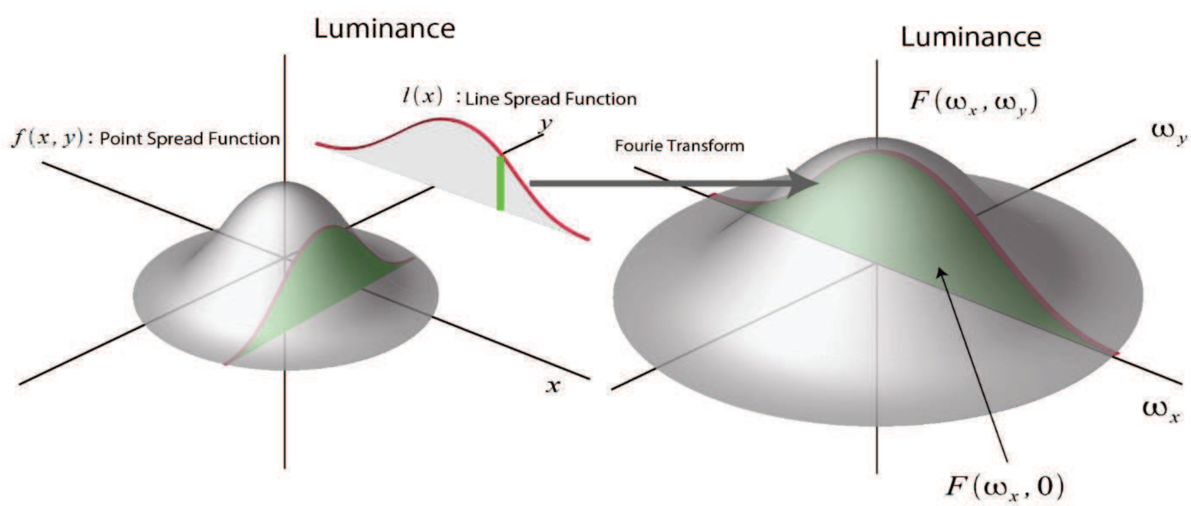
Replacing the real parts of archived quaternions with progression indicators introduces a stochastic process. The HBM ***suggests*** that this stochastic process is a combination of a Poisson Process and a binomial process. If we consider this process as a combination of a Poisson process and a binomial process, and if a location density distribution that owns a Fourier transform in the form of a frequency spectrum that describes the effect of the binomial process, then the stochastic process ***holds a characteristic function***. In the HBM, the frequency spectrum can cover up to four dimensions.

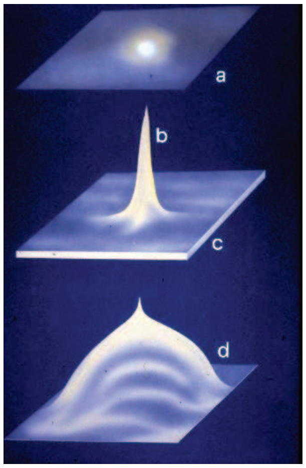
The characteristic function of a stochastic process in the change space can control the spread of the location density distribution of the produced location swarm in position space.

A dedicated footprint operator archives the production of the stochastic process in its eigenspace. After reordering the timestamps, the footprint operator stores its eigenvalues in the quaternionic storage bins. The storage bin contains a real number valued timestamp and a three-dimensional spatial number value for the archived hop landing location. After sequencing the timestamps in equidistant steps, the hop landing locations stand for a hopping path of a point-like object. The hopping path regularly regenerates a coherent hop landing location swarm. The location density distribution describes this swarm.

If this location density distribution is a Gaussian distribution, then its Fourier transform decides exactly the location density distribution of the swarm. The Fourier transform is again a Gaussian distribution but has distinctive characteristics.

The author dares to suggest that the stochastic process combines a Poisson and binomial process because he measured the spatial frequency characteristics of many imaging spots and line images in images produced by lenses and image intensifier devices.





The optical transfer function is the Fourier transfer of the point spread function (PSF), shown in the second picture (b,c).

The modulation transfer function (MTF) is the modulus of the optical transfer function. Each cut through the center of the MTF is ***symmetric.*** Therefore, it suffices to specify half of that curve.

Often, a peak appears at the center of the MTF. Optical experts call the cause of the peak ***veiling glare***. Picture (d) shows this peak.

Analyzing the Fourier transfer of the line spread function (LSF) is more manageable because it covers more contributing imaging objects and corresponds with a cut through the central axis of the MTF.

The central axis of the MTF shows the distribution of the imaging objects in the image. The HBM states that photons are one-dimensional chains of shock fronts. Thus, if the imaging objects are photons, then according to the HBM, the central axis of the MTF shows the distribution of the energy that the shock fronts carry. In the peak, the shock fronts are less spatially related than in the broader part of the MTF. In analyzing the image of a galaxy, the veiling glare might stand for the halo that cosmologists see around these galaxies.

The notion of the MTF does not restrict to photons. The imaging objects can form a mixture of photons, elementary fermions, and conglomerates of elementary fermions. In that case, the MTF is a function of these contributors' angular, chromatic, and phase distribution. The author participated in developing world standards for specifying and measuring the OTF and the MTF. It started with a STANAG standard, the ISO and IEC standard, and included country-wide standards such as the German DIN standard accepted these worldwide standards. At low dose rates, the relative contribution of noise will increase. The Detective Quantum Efficiency (DQE) objectively measures this influence. The author also participated in standardizing the DQE for IEC and DIN.

The described stochastic process can deliver the actuators that generate the pulse responses that may deform the dynamic universe field. In some way, an ongoing embedding process must map the eigenspace of the footprint operator onto the embedding field. As previously argued, the footprint operator's eigenspace corresponds to a dynamic footprint vector that defines a location density function and a probability amplitude. The footprint vector exists in the underlying vector space and has a representation in Hilbert space via the footprint operator. The footprint vector acts as the state vector of the separable

Hilbert space, and the probability amplitude corresponds to what physicists call the wave function of the represented moving particle.

13.1 Optical Transfer Function and Modulation Transfer function

Some stochastic processes own a characteristic function. This characteristic function is the Fourier transform of a location density distribution. Experimenters commonly use such stochastic processes to qualify imaging excellence via the Optical Transfer Function of an imaging process or imaging equipment. The Optical Transfer Function is the Fourier transform of the Point Spread Function. For spatial locations, the PSF acts as a location density distribution. The Modulation Transfer Function is the modulus of the Optical Transfer Function and is a symmetric function. The vertical axis of the MTF shows the energy distribution of the spatial spectrum. In the case of light, it is the chromatic distribution of the PSF. A central peak in the form of a rapid decrease of the MTF at low spatial frequencies shows the existence of a veiling glare or halo. It is energy that is less correlated to location.

The Line Spread Function (LSF) equals the integral over the Point Spread Function in the direction of the line. The Fourier transform of the Line Spread Function equals the cut through the center of the Optical Transfer Function. The cut runs perpendicular to the direction of the line. The LSF can be a function of the direction of the line. In that case, the PSF has a non-isotropic angular distribution. The Fourier transform of the convolution of two functions equals the product of the Fourier transforms of the functions. The result of the Fourier transform conforms to the convolution of the OTF with the Fourier transform of the blade sharp pulse that corresponds to the Fourier transform of the integral along the line.

A phase distribution will also occur if an ongoing dynamic process generates the PSF. The Optical Transfer Function combines the Modulation Transfer Function and the Phase Transfer Function. In complex number-based descriptions, the Phase Transfer Function is the argument of the Optical Transfer Function.

A system of Hilbert spaces that share the same underlying vector space can perform the job of the imaging platform. In this system, the embedding process is the alternative name for the imaging process. However, this explanation still says nothing about the essence of the underlying stochastic selection process. That ***stays a mystery***.

The concept of the Optical Transfer Function also makes sense for dependence on time. For time dependence, the name of the tool is also Fourier analysis. Together the two tools perform ***a four-dimensional spectral analysis***.

13.2 Photons

Photons are not electromagnetic waves. Instead, photons consist of chains of equidistant one-dimensional shock fronts that travel along a geodesic. The one-dimensional shock fronts are shock fronts that often get the name ***dark energy objects***. However, when cooperating in huge quantities, the objects become observable, and then the name “dark object” becomes confusing; see the section on differentiation.

13.3 Light

Light is a distribution of photons. A beam of light can have an angular distribution, a chromatic distribution, and a phase distribution. A homogeneous light beam holds a single frequency and usually a narrow angular distribution.

13.4 Refraction

Refraction occurs at the borders of transparent media in which information transfer occurs with constant speed. The information transfer can take place through chains of absorption and reemission cycles. In free space, nothing exists that absorbs or emits photons, but photons can travel through free space along geodesics [10].

Refraction enables the construction of lenses, fiber plates, optical fibers, prisms, and mirrors.

A separate part of optics covers refraction. [11]

13.5 Holographic imaging

Transparent optical lenses and tiny apertures can function as Fourier transformers. They map distributions of photons in position space into distributions in frequency space. The name of these distributions is a hologram. [12]

Photographs can capture holograms. Also, metal mirrors imprinted with phase patterns can generate holograms when the imprinted mirror reflects a coherent beam of light.

13.6 Electron optics

Electron optics concerns imaging charged particles by artificially constructed electric or magnetic fields or electromagnetic fields [13][14]. Construction elements are metallic electrodes at a given voltage or coils that carry electric currents.

Radio transmission is a special kind of electron optics.

14 Social influences

The rise of National Socialism in Hitler's Nazi Germany disrupted the promising discussion about set theory and number systems. Nazism threatened key discussion participants, or they had to flee to safer places. Many fled to the United States of America, where the government morally obliged them to cooperate in the fight against Nazism by taking part in the development of new weapon systems, such as the atomic bomb. Sets and number systems no longer attracted their attention. The success of the complex functional analysis, which can treat singularities, worsened this effect. [15]

Joshua Willard Gibbs and Oliver Heaviside led the physicists toward geometric differential theory and vector analysis. [16][17]

In this way, many scientists thought the spatial functions would be sufficient to explain physical phenomena. However, this choice is at the expense of the relationship with the real functions, which quaternionic function theory regulates more clearly. Many physicists no longer understood the reason Hilbert spaces attracted their attention. The complex Hilbert spaces became a toy of the mathematicians who developed all kinds of fancy complex Hilbert spaces.

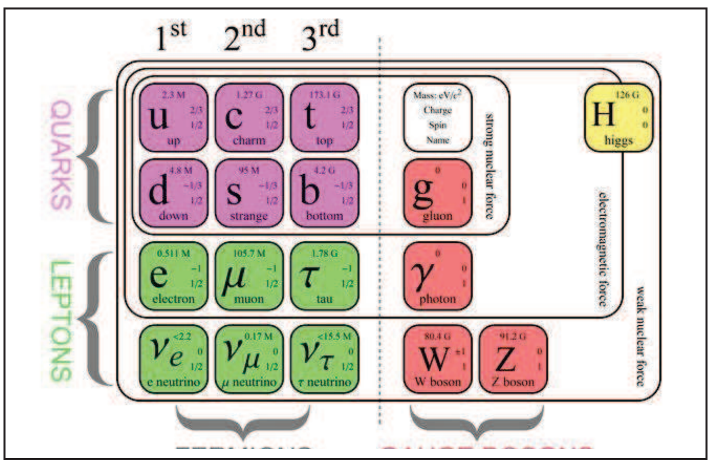
15 Ongoing investigation

At CERN in Geneva, sufficiently far from the Nazi sphere of influence, a small group continued with quantum logic and Hilbert spaces. The book "Foundations of quantum mechanics" by Josef M. Jauch guided my attention to quaternionic Hilbert spaces. [18]

Due to too few results, this research languished and died out in the sixties.

16 New insight

Now we are taking a giant step. This step concerns a significant difference in understanding between me and mainstream theoretical physics. The curious shortlist of properties of the electric charges of the first generation of elementary fermions prompted this difference. This list covers charges with values -1, -2/3, -1/3, 0, +1/3, +2/3, and +1. This list is part of the Standard Model of the experimental particle physicists who have summarized their main observations in that Standard Model. [19]



Multiplying with 3 turns the list into a list of integers -3, -2, -1, 0, +1, +2, and +3. This series is the list of differences between a reference symmetry and other symmetries of versions of quaternionic number systems when the coordinate axes restrict to be parallel.

We limit our use of the Standard Model to a subset and exclude the bosons and the gluons. We exclude theoretical theories like Quantum Field Theory, Quantum Electro Dynamics, and Quantum Chromo Dynamics. Opportunistic theoretical physicists introduced QFT, QED, and QCD that spoiled the experimental results with these not-so-well-founded theoretical ideas by inserting them into the Standard Model. The minimal action principle from which a Lagrangian derives forms the foundation of these theories. These concepts play in the third phase of number systems. The calculation rules and restrictions of the third phase exist in the first and second phases. Therefore, these theories cannot explain the existence of electric charges and diverse types of fermions. Furthermore, these theories have no reasonable explanation for the presence of the wave function, and their rationale for the existence of conglomerates is questionable.

The similarity with the symmetries of versions of number systems stimulated me. However, it is not the similarity with the symmetries themselves that provides the reason. Instead, one of the Hilbert spaces plays the role of a background system. All other system members float with their geometric center over the parameter space of this background system. Especially the difference between the symmetries of the versions of the number systems that float with their separable Hilbert space and the symmetry background platform control the situation. This opportunity occurs in a system of separable Hilbert spaces that all apply the same underlying vector space.

17 A System of Hilbert spaces

The author calls the system of Hilbert spaces the ***Hilbert repository*** because it stores all data of a multiverse. Two types of systems of Hilbert spaces exist.

The first type is a system of ***separable*** Hilbert spaces.

The second type is a system of ***non-separable*** Hilbert spaces.

Both systems hold a member that acts as a ***background platform***.

17.1 A System of separable Hilbert spaces

The background platform owns a companion non-separable Hilbert space that embeds its separable companion. This companion archives a dynamic universe field. The floating separable members can harbor an electric charge at their geometric center. A ***dark hole*** holds the countable parameter space of the separable Hilbert space that functions as the background platform. The HBM employs the name "dark hole" because continuous objects cannot penetrate this countable subset and cannot leave the encapsulated region. It is a second phase contained in a third phase surround.

We limit ourselves to Hilbert spaces derived from the same vector space. Furthermore, we choose four mutually independent directions in the underlying vector space. The axes of the Cartesian coordinate system of the number system shall be parallel to one of the chosen direction_lines. This choice, therefore, leaves only a few different symmetry types. The exact reason, which enforces this restriction, is not apparent.

However, the limitation makes comparing symmetries and computing symmetry differences easier. To understand the consequences of the limitation, we put the symmetries of the remaining versions of the quaternionic number system in a table whose lines we arrange with binary written hexadecimal rank numbers. We choose one of the sixteen remaining versions as a frame of reference platform and place this version at the front of the queue. The table mentions the fitting fermions by name.

You will notice that the anti-attribute raises a conflict between symmetries and the electric charges of the Standard Model. The reason might be that the anti-attribute is not measurable.

No		R	G	B	real	Difference	charge	type	Rgb
0		0	0	0	0	0	0	background	
1		1	0	0	0	1	-1/3	down	R
2		0	1	0	0	1	-1/3	down	G
3		1	1	0	0	2	-2/3	anti-up	B
4		0	0	1	0	1	-1/3	down	B
5		1	0	1	0	2	-2/3	anti-up	G
6		0	1	1	0	2	-2/3	anti-up	R
7		1	1	1	0	3	-3/3	electron	
8		0	0	0	1	0	0	neutrino	
9		1	0	0	1	-1	1/3	anti-down	B
A		0	1	0	1	-1	1/3	anti-down	G
B		1	1	0	1	-2	2/3	up	R
C		0	0	1	1	-1	1/3	anti-down	R
D		1	0	1	1	-2	2/3	up	G
E		0	1	1	1	-2	2/3	up	B
F		1	1	1	1	-3	3/3	positron	
		B	G	R					

All these Hilbert spaces are separable and use number systems that belong to the first or second phase.

The remaining system of Hilbert spaces holds a Hilbert space that can serve as a background platform. Therefore, the HBM assumes that the reference version functions as a background platform.

The background platform must have an infinite number of subspaces. An infinite number of subspaces means that the version of the number system chosen by this Hilbert space has an infinite number of elements.

17.2 A modeling platform

A system of Hilbert spaces that all share the same underlying vector space can function as a modeling platform that not only supports dynamic fields that obey quaternionic differential equations. The model can, in principle, capture all phenomena in a dynamic universe.

The system of separable Hilbert spaces applies the structured storage ability of the Hilbert spaces that are members of the system. The requirement that all member Hilbert spaces must share the same underlying vector space restricts the types of Hilbert spaces that can be a member of the system of separable Hilbert spaces. In the change chapter, we already restricted the definition of partial change along the directions of the Cartesian coordinate system. It appears that the coordinate systems that decide the symmetry type of the members of the system of separable Hilbert spaces must have the Cartesian coordinate axes in parallel. The Cartesian coordinate system is due to the existence of the primitive coordinate system in the underlying parameter space. The restriction enables the determination of differences in symmetry. The Model selects the sequence along the axis only up or down. It also means that partial change has a systemwide significance. Thus, the model tolerates only a small set of symmetry types. One of the Hilbert spaces will function as the background platform, and its symmetry will serve as background symmetry. Its ***natural parameter space*** will act as the background parameter space of the system. All other system members will float with the geometric center of their parameter space over the background parameter space. These features already generate a dynamic system. The symmetry differences cause symmetry-related sources or sinks that will exist at the geometric center of the natural parameter space of the corresponding floating Hilbert space. The sources and sinks correspond to symmetry-related charges that generate symmetry-related fields. In physics, these symmetry-related charges are electric charges.

Not the symmetries of the floating Hilbert spaces are essential. Instead, the differences between the symmetry of the floating member and the background symmetry are crucial for showing the type of the member Hilbert space. The counts of the differences in symmetry restrict to the shortlist -3, -2, -1, 0, +1, +2, +3.

It is possible to understand the existence of symmetries and symmetry differences. However, the presence of corresponding symmetry-related charges is counterintuitive. The Model does not yet explain the realization of these charges as sources or sinks of symmetry-related fields.

All floating Hilbert spaces are separable. The background Hilbert space is an infinite-dimensional separable Hilbert space. It owns a non-separable companion Hilbert space that embeds its separable partner.

The system of separable Hilbert spaces supports the containers of footprints that can map into the quaternionic fields. The vectors that stand for the footprint vectors originate in the underlying spatial field. They function as state vectors for the Hilbert spaces that serve as containers for the footprints. The state vector stands for the vector from the underlying vector space that aims at the geometric center of the floating Hilbert space. This picture enables the maps of these state vectors and the corresponding footprint in the dynamic universe field. The state vector stands for a vector from the underlying vector space that tries to find the position of the floating platform's geometric center in the background platform's parameter space. State vectors are particular footprint vectors. Together this entwined locator installs an ongoing embedding process that acts as an imaging process that maps the geometric center of the floating platform onto the background parameter space.

Finally, the eigenspace of a dedicated footprint operator maps this image into the dynamic field that stands for the universe.

In this way, the image maps a vast number of ongoing hopping paths onto the embedding field. Physicists call this dynamic field the universe. On the floating platforms, the hopping paths close. The movement of the floating platforms breaks the closure of the images of the hopping paths.

17.2.1 Conglomerates

Elementary fermions behave as elementary modules. The conglomerates of these elementary modules populate the dynamic field that we call our universe. All massive objects, except black holes, are conglomerates of elementary fermions. Therefore, all conglomerates of elementary fermions own mass. This mass ownership of modules means that massive modular systems cover the universe.

A private stochastic process decides each elementary fermion's complete local life story. The fermion controls that stochastic process in the change space of its private Hilbert space. The private stochastic process produces an ongoing hopping path. This hopping path corresponds to a footprint vector that consists of a dynamically changing superposition of the reference operator's eigenvectors. The section of the formula chapter that treats the arithmetic of change explains this. Each floating platform of the system of separable Hilbert spaces owns a single private footprint vector. The footprint vector acts as the state vector of the elementary fermion, and the probability amplitude corresponds to what physicists call the particle's wave function.

This picture invites the idea that stochastic processes whose characteristic functions define in the change space of the background platform represent conglomerates of elementary fermions. In this change space, the characteristic function of a stochastic process that specifies a conglomerate is a superposition of the characteristic functions of the components of the conglomerate. The dynamic superposition coefficients function as displacement generators. This functionality means that these displacement generators define the internal oscillations of the components within the conglomerates. It might not hold for higher-order conglomerates, but in the HBM, it fits for lower-order conglomerates.

Since the HBM does not define the position in change space, the fact that a component belongs to a conglomerate does not restrict the distance between the components. This way of determining the membership of a conglomerate introduces entanglement. Independent of their mutual distance, components of a conglomerate must still obey the Pauli exclusion principle.

17.2.2 Interaction with black holes

Field excitations cannot enter or leave black holes, but the Hilbert spaces that stand for elementary fermions may hover over the enclosed region of the black hole. So, part of the footprint of the elementary particle may map into the territory of the black hole. The mass of the black hole attracts nearby elementary fermions. Together with the effect of hovering, this may enable the growth of black holes and the merge of approaching black holes. It may also explain the merge of a black hole and a dense star.

17.2.3 Hadrons

Hadrons can be mesons or baryons. They are conglomerates of quarks. Quarks can only bind via oscillations and via the attraction that their electric charges induce. Since the symmetry of quarks does not differ from the background symmetry in an isotropic way, the footprint of quarks does not deform the embedding field. So, mass does not help to bind the quarks until they reach an isotropic symmetry difference. Scientists call this phenomenon ***color confinement***. Hadrons feature mass. Thus, these conglomerates are sufficiently isotropic to deform the embedding field. Once configured, the mutual binding of baryons is strong. Baryons make up the nuclei of atoms.

17.2.4 Atoms

Compound modules are composite modules for which the images of the geometric centers of the platforms of the components coincide in the background platform. The charges of the elementary module platforms show the corresponding platforms' primary binding. Physicists and chemists call these compound modules atoms or atomic ions.

In free compound modules, the geometric symmetry-related charges do not participate in internal oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This oscillation means that the hopping path of the elementary module folds around the oscillation path, and the hop landing location swarm smears along the oscillation path. The oscillation path is a solution to the Helmholtz equation. Each fermion must use a different oscillation mode. A change in the oscillation mode goes together with the emission or absorption of a photon. As suggested earlier, the emission or absorption of a photon involves a switch from the quaternionic Hilbert space to a subspace, which a complex-number-based Hilbert space stands for. The duration of the switch lasts an entire particle regeneration cycle. During that cycle, the stochastic mechanism does not produce a swarm of hop landing locations that produce pulses that generate spherical shock fronts. Instead, it creates a one-dimensional string of equidistant pulse responses that cause one-dimensional shock fronts. The center of emission coincides with the geometrical center of the compound module. This location ensures that the emitted photon does not lose its integrity. All photons will share the same emission duration, which will coincide with the regeneration cycle of the hop landing location swarm. This coincidence is the reason that photons obey the Planck-Einstein relation $E = h\nu$. Absorption cannot interpret so easily. It only becomes understandable as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon. ***The number of one-dimensional***

pulses in the string corresponds to the step in the energy of the Helmholtz oscillation.

The stochastic process that controls the binding of components appears to manage the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy spends on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy distributes over the available oscillation modes, and the model spends the rest on the kinetic energy of the shared platform, or it the energy escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably, the current value of the dynamic superposition coefficient archives in the eigenspace of a particular superposition operator.

17.2.5 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center. However, the compound modules share electron oscillations. This sharing binds the compound modules with geometric symmetry-related charges into the molecule.

17.2.6 Earth

On Earth, conglomerates of molecules can form living species. Living species archive essential properties in RNA and DNA molecules.

17.2.7 Particles and fields

The model interprets the floating elements of the system as particles. In contrast, the model does not interpret the background platform as a particle. Still, all elements of the system of Hilbert spaces are platforms that show similar capabilities and properties. For example, all floating platforms act like symmetry-related fields, and these fields correspond to symmetry-related charges. However, the background platform does not offer a symmetry-related field or a symmetry-related charge. Instead, it acts as a universe-wide embedding field, which the presence of floating members deforms. Mainstream physics considers the Higgs particle responsible for the capabilities that this paper assigns to the background platform. In this paper, the background platform, including its non-separable companion, implements the origin of the gravitational potential via the action of spherical shock fronts that actuators that cause isotropic pulses generate.

17.2.8 Modular system communities

The modular construction of objects in the universe invites the consideration of communities of modular systems that belong to the same type or species. Examples are ants, bees, herd animals, and humans. These are living species. Type communities offer the advantage that the community members can cooperate to perfect the community's resilience by exploiting the members' diversity. Usually, the community survives orders of magnitude longer than the individual members. The community can promote, support, and guard the culture and intercommunication of the community. A danger of this effect is that the community grows to such an extent that it endangers running out of resources. Another risk is that the community oppresses its members to support the interests of the community against the interests of the individual members. After the arrival of intelligent species, modular design, and construction can change from stochastic design and construction into intelligent design and construction. This capability accelerates the generation of modular systems and produces incredibly sophisticated modular systems in a brief period. This trend can even create new intelligent species. Robots occupy this next generation. It is astonishing to see how badly human communities can manage the outrun of resources, can prevent wars, can stop terrorism, and can prevent economic crises.

On the other hand, humanity has difficulty managing pandemic outbreaks. Moreover, humans consider democracy the most effective solution and nationalism a lousy solution. Unfortunately, none of these views appear to be correct.

17.3 A System of non-separable Hilbert spaces

The second type is a system of ***non-separable*** Hilbert spaces. The background platform is a non-separable Hilbert space, which archives a dynamic multiverse field. The parameter space of the background platform is a continuum; therefore, ***a black hole does not encapsulate it***. The floating members of the system are the background platforms of systems of separable Hilbert spaces that own a companion non-separable Hilbert space that embeds its separable partner. A black hole holds the parameter space of the separable part of the background platform.

This system shows similarities with the holographic principle that some theoretical physicists promote [20]. However, the model reaches this resemblance without the tools of string theory or quantum gravity because, in this paper, the black hole is supposed to hold a countable parameter space related to a continuous surrounding common target space. The system does not show the recycling universe of Sir Roger Penrose.

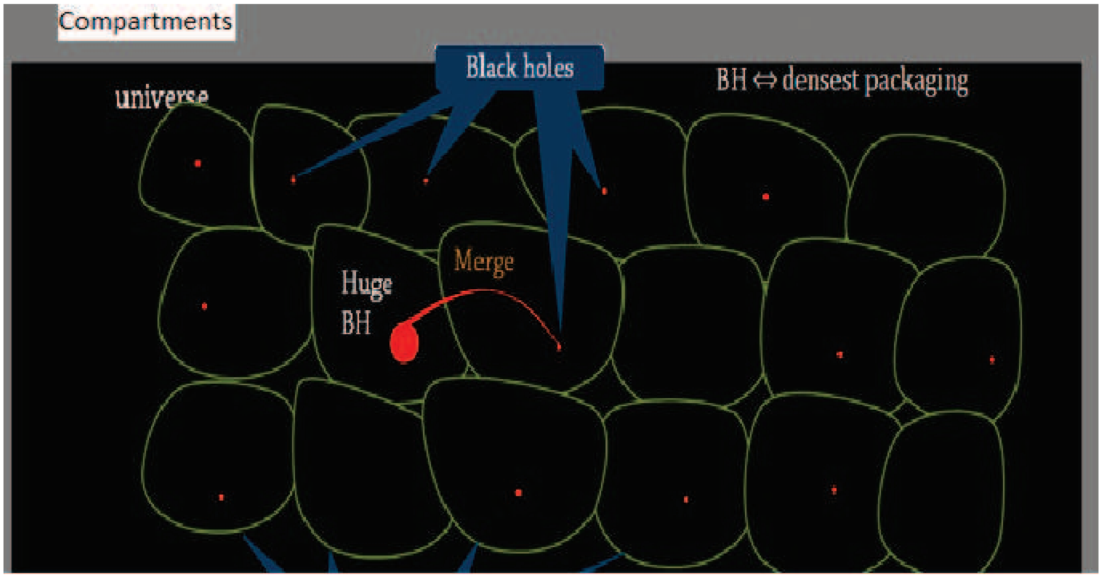
The floating members of the system are universes that connect relationally to a private black hole. The corresponding space compartment stands for this black hole's influence range. The countable parameter space contained in the black hole relates to the content of the compartment. The borders of the compartments do not function as barriers for photons, fermions, atoms, planets, or stars. The background member of the system holds the continuum parameter space of the whole multiverse. It relates to all the contained universes.

Astronomers see that black holes can merge and that neutron stars can collapse into new black holes. These events redistribute the compartments. These events cause graphical shock fronts that constitute an enormous number of superimposed spherical shock fronts that arise in a small region and a short period. A gravitational wave is a misnomer for these phenomena.

The parameter space of the multiverse adapts to the changes in the covered compartments.

17.3.1.1 Compartments

This configuration stands for a dynamic multiverse that divides part of the underlying vector space into a set of compartments. Each compartment supports a dynamic universe.



This picture shows an artist's impression of a simple space covered by compartments.

Objects can pass the borders of the regions.

These objects range from particles to galaxies.

18 Conclusions

The Hilbert Book Model applies the system of Hilbert spaces that all share the same underlying vector space. The author calls this system the ***Hilbert repository***. This approach differs on various essential points from the approach that mainstream physics follows. Still, an astonishing agreement exists between the Standard Model of the elementary fermions that ***the Standard Model of the experimental particle physicists*** holds and the system of separable Hilbert spaces.

The HBM considers ***physical_reality*** to represent what experimental physicists want to observe. Humans can only control physical_reality indirectly by altering the conditions on which physical_reality (PR) reacts. Physical_reality reacts according to its laws and does not consider collateral damage, and does not negotiate. PR is a harsh and consistent controller. It is the essential opponent for humanly installed governments and institutions. For example, on Earth, PR reacts to climate deterioration with disasters that touch all humans and all living species.

In the system of separable Hilbert spaces, spatial coordinate axes play a vital role. These axes must be systemwide in parallel. The reason for this restriction is not apparent. The HBM seeks the logic in that the simple coordinate system of the vector space relates to the coordinate system that exists in the version of the number system that a Hilbert space selects.

In spatial continuums, first-order change usually occurs along the spatial coordinate axes. In locally spherical symmetric conditions, change covers all directions. The freedom of choice left by spatial arithmetic always appears along the Cartesian coordinate axes. This fact is due to the adaptation to the primitive coordinate system in the underlying parameter space.

In the Hilbert Book Model (HBM), the footprints of all massive objects recurrently regenerate with a high repetition rate that corresponds with the duration of the emission of photons.

Mainstream physics still has not found a suitable explanation for dark matter objects and dark energy objects. The HBM explains these objects as field excitations that behave as shock fronts. Solutions of second-order quaternionic partial differential equations describe the shock fronts in detail. The spherical shock fronts are the only field excitations that deform the field that embeds them. According to the HBM, photons are strings of equidistant one-dimensional shock fronts. Black holes are slowly varying objects that hold a countable content. Black holes deform their continuous surround. From enough distance, the black hole can appear as a point-like object. In this situation, the description of the black hole becomes simple. However, for the HBM, nearby black holes are complicated objects.

Elementary fermions are complicated objects in which a private quaternionic separable Hilbert space manages the fermion's properties. These Hilbert spaces own a private parameter space and a private symmetry. The separable Hilbert spaces float with the geometric center of their parameter space over a background parameter space, which a background separable Hilbert space manages. This background Hilbert space owns a non-separable Hilbert space. The non-separable Hilbert space embeds its separable companion. The non-separable Hilbert space manages several continuums in the eigenspace of a corresponding dedicated operator. One of the continuums is a dynamic field, which physicists call the universe. The universe field embeds the images of the geometric centers of the floating separable Hilbert spaces.

Stochastic disturbances of the locator vector in the underlying vector space and points to the geometric center of the floating Hilbert space blur this map. Depending on the difference in symmetry, the embedding of the image may cause a spherical shock front response that will temporarily deform the universe field. The corresponding shock front moves away in all directions until it vanishes at infinity. This conflicts with the conservation laws that mainstream physics supports. An ongoing creation of fermions that compensates for the vanishing of spherical shock fronts but does not compensate for the expansion of the covered part of the simple space compensates for this. The HBM turns the simple space into a vector space. The simple space and, thus, the vector space has no boundaries. The content of the shock front expands the covered volume of the field that corresponds to the background Hilbert space. The generation of the spherical shock front needs an isotropic symmetry difference with the background platform. Only a few fermion types feature an isotropic symmetry difference. Isolated quarks do not own the required isotropic symmetry difference and will not produce a deformation of the universe. However, combined in a hadron such that the combination features an isotropic symmetry difference, the hadron can cause deformation. This phenomenon is known as ***color confinement***.

The non-separable Hilbert space embeds its separable partner. Consequently, the parameter space of the non-separable Hilbert space is the parameter space of the separable companion Hilbert space where the irrational numbers add to the rational numbers. The result is a ***continuum***. Deforming actuators do not affect the parameter spaces. However, the continuum eigenspaces of other operators than the reference operator of the non-separable Hilbert space can vibrate, deform, and expand.

Symmetry-related charges exist at the geometric centers of the floating Hilbert spaces. The charges depend on the difference in symmetry between the floating platform and the background platform. The charges function as sources or sinks of corresponding symmetry-related fields. These fields differ fundamentally from the universe field. However, both types of fields obey the same quaternionic field equations. They differ in their start and boundary conditions. Mainstream physics supports general relativity (GR). However, GR does not consider the electromagnetic and gravity fields fundamentally different. GR applies tensor calculus, whereas the HBM applies quaternionic field theory and disruption of the gravitation field by spherical actuators. In the HBM, the gravitation field is the local representant of the universe field. At quantum scales, the gravitation field does not meet the condition that the distance to the observed scene is considerable, such that the formulas can simplify. Therefore, the HBM does not apply the gravitation field at quantum scales. Still, shock fronts are valid solutions to quaternionic differential equations at quantum scales.

In the HBM, the archival of the footprint in the floating separable Hilbert space enables the independent retrieval of that footprint at a later instance. Thus, the footprint can generate in an episode before the beginning of the flow of time. The retrieval can occur as a function of the flow of time and uses the archived timestamps for synchronizing the retrieval. This conclusion means that the model retrieved no archived footprint data before the instant of time zero. Without deforming actuators, the embedding field stays flat. Thus, the embedding field was in its maiden state at the beginning of the flow of time. The function that described the universe field was equal to its parameter space. Immediately after that instant, the locator landings started, distributed randomly over that parameter space, to mark the locations of the geometrical centers of the floating Hilbert spaces. Depending on the symmetry of the floating Hilbert space, this resulted in a corresponding spherical shock front. This description certainly does not look like the Big Bang that mainstream physics promotes. Instead, already at its start, the ongoing embedding was a quiet imaging process.

The background non-separable Hilbert space defines the conglomerates of elementary fermions as superpositions in change space. For that reason, it applies the characteristic functions of the stochastic mechanisms that generate the footprints of the elementary fermions. However, the model does not define the position in the change space. This inability is the reason for the existence of entanglement. The Pauli exclusion principle works independently of the distance between the elements of the conglomerate.

Elementary fermions act like elementary modules. Together they make up all massive objects that occur in the universe. Black holes form the notorious exception. For the rest, the universe's content is one extensive modular system that produces an enormous number and diversity of modular subsystems. Atoms, molecules, rocks, planets, stars, galaxies, and living species are all examples of modular systems. Every human is a modular system. Humanity is a modular system community. On planet Earth, before the arrival of humans, modularization happened in a stochastic way. Since the arrival of humans, modularization can happen intelligently. Computers and robots are excellent examples of this development.

Once the elementary fermions formed, the rest of the universe's content followed automatically. Modular systems that care for their community and the modular systems on which they depend have the highest chance of survival. See “A law of nature” in [21].

Mainstream physics usually bases on the steady action principle. Currently, mainstream physicists prefer the name minimal action principle. The minimal action principle does not request a recurrent regeneration of the objects that occur in the universe. It does not require that conglomerates generate in a modular way. It also does not oppress the strange reaction of continuums to disruptions by actuators. Also, mainstream physics does not explain the origin of electric charges and the diversity of elementary particles.

Forces require a point of engagement. Fields do not own a point of engagement. For quaternionic functions, the first-order change already connects the gradient of a scalar field to the time variation of the corresponding spatial part of the field. It suffices that the universe field shows a gradient in its scalar part and that the spatial part of the field moves uniformly. Thus, a gravitational potential raises an acceleration of the moving spatial field. Intuition cannot tell you this. But mathematics does.

Finally, the paper introduces the system of non-separable Hilbert spaces. This system concerns a multiverse consisting of universes that all apply a black hole to archive the private parameter space of the background platform of the system of separate Hilbert spaces that stands for the considered dynamic universe. The system of non-separable Hilbert spaces corresponds to a coverage of space by compartments holding a dynamic universe and a private black hole.

Astronomers see that black holes can merge and that neutron stars can collapse into new black holes. These events redistribute the compartments.

The HBM does not provide a clear explanation for multiverses' coverage of simple space. The artist's impression of the coverage by compartments is its best guess.

19 Formulas

19.1 Relativity and curvature

Most formulas feature Euclidean format. In this chapter, the HBM, in first approximation, ignores the relative speed difference between the source of information and the observer of the information. In the embedding field, the information follows geodesics. In this chapter, the formulas do not treat this extra complication. General relativity claims to consider both influences but does not recon the coupling of the embedding field and the electric field via electric charges [22].

19.2 Physical units

This chapter applies mathematical formulas that do not hold physical units. Physical units are the adaptation of the considered subject to units that experimental physicists use to measure that subject.

Lightspeed c is such a physical unit because it stands for a physical unit measured in meters per second. Physicists use permittivity $\varepsilon = \varepsilon_0 \varepsilon_1$ for the electrical field. In free space $\varepsilon_1 = 1$. Physicists use permeability $\mu = \mu_0 \mu_1$ for the magnetic field. In free space $\mu_1 = 1$.

The two physical units are related via light speed c [23] [24].

$$c^2 = \frac{1}{\varepsilon_0 \mu_0} \tag{19.2.1}$$

19.3 Vector arithmetic

In this section, vectors in a vector space will be in boldface font, and scalars will be in italic font.

The ***addition*** of vectors is commutative. Addition occurs by shifting one of the vectors in parallel until it coincides with the alternative point of the other vector. Now the two resulting points stand for the vector ***sum***. The arithmetic of scalars resembles the arithmetic of rational members of the real number systems. Vector addition is ***commutative***. The addition creates new vectors;

$$\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v} \tag{19.3.1}$$

Vector addition is also ***associative***;

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}) \tag{19.3.2}$$

Multiplication with a scalar is commutative. This multiplication may change the length and, thus, the integrity of the vector. In addition, it may create a new vector;

$$\mathbf{w} = a\mathbf{v} = \mathbf{v}a \tag{19.3.3}$$

Multiplication with scalars is ***distributive*** for scalars and vectors;

$$\begin{aligned} (a+b)\mathbf{v} &= a\mathbf{v} + b\mathbf{v} \\ a(\mathbf{v} + \mathbf{w}) &= a\mathbf{v} + a\mathbf{w} \end{aligned} \tag{19.3.4}$$

Multiplication with negative scalars ***reverses the direction*** of the vector. In particular;

$$(-1)\mathbf{v} = -\mathbf{v} \tag{19.3.5}$$

Vectors obey a ***scalar product***. However, they do not obey an outer product. Otherwise, their arithmetic would equal the arithmetic of the spatial numbers, and the vector space's dimension would be restricted to three.

19.3.1 Base vectors

A selected ***base*** $\{\mathbf{u}_i\}$ is a subset of the vectors that the model uses to define another vector as a superposition of the members of the base;

$$\mathbf{v} = \sum_{i=0}^{i=N} v_i \mathbf{u}_i \tag{19.3.6}$$

A ***scalar product*** $\langle \mathbf{v}, \mathbf{w} \rangle$ of two vectors $\mathbf{v}$ and $\mathbf{w}$ would be defined in terms of the orthonormal base $\{\mathbf{u}_i\}$ as;

$$\langle \mathbf{v}, \mathbf{w} \rangle = \sum_{i=0}^{i=N} v_i w_j \langle \mathbf{u}_i, \mathbf{u}_j \rangle \ , \tag{19.3.7}$$

while;

$$\langle \mathbf{u}_i, \mathbf{u}_j \rangle = \delta_{ij} \tag{19.3.8}$$

If the orthonormal base spans the entire vector space, then the vector space holds N dimensions. N can be infinite.

The scalar product that covers all dimensions generates a ***metric***. That metric can show the ***length*** $\ell_{\mathbf{a}}$ of the vector $\mathbf{a}$ as a scalar. The scalar product can show the length of a vector;

$$\begin{aligned} \ell_{\mathbf{a}} &= \|\mathbf{a}\| \\ \langle \mathbf{a}, \mathbf{a} \rangle &= \|\mathbf{a}\|^2 \end{aligned} \tag{19.3.9}$$

If the scalar product equals zero, either one of the vectors has zero length, or the two vectors live in different dimensions. In that case, the vectors are ***independent***. In a N dimensional vector space, precise N vectors can be mutually independent.

The model applies the scalar product to construct a set of ***coordinate markers*** forming a ***coordinate system***.

19.3.2 Grid of coordinate markers

Coordinate markers can form a grid. The human brain can imagine a spatial grid of maximally three dimensions. Humans picture this grid as equidistant Dirac pulses that constitute a comb function. Mathematicians extend this to much higher dimensions and towards infinite dimensional spaces. The superposition of functions adapts to this extension. Moreover, the adaptation enables infinite-dimensional function spaces. In this way, the Fourier series that the special_set supports in its first two phases turn into support of Fourier integrals in the third phase of this set.

19.4 Arithmetic of real numbers

We will show the real numbers with the suffix r.

For real numbers, ***addition and multiplication are commutative, associative, and distributive***;

$$\begin{aligned} b_r + a_r &= a_r + b_r \\ (a_r + b_r) + c_r &= a_r + (b_r + c_r) \end{aligned} \qquad (19.4.1)$$

$$\begin{aligned} b_r a_r &= a_r b_r \\ (a_r b_r) c_r &= a_r (b_r c_r) \end{aligned} \qquad (19.4.2)$$

$$a_r (b_r + c_r) = a_r b_r + a_r c_r \qquad (19.4.3)$$

For real numbers, the square is zero, or it is positive;

$$a_r a_r \geq 0 \qquad (19.4.4)$$

For ***spatial numbers***, addition is ***commutative and associative***;

$$\vec{b}+\vec{a}=\vec{a}+\vec{b}$$
$$\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right) \qquad (19.5.1)$$

The product d of two spatial numbers $\vec{a}$ and $\vec{b}$ results in a real scalar part d_r and a new spatial part $\vec{d}$;

$$d=d_r+\vec{d}=\vec{a}\vec{b} \qquad (19.5.2)$$

$d_r=-\left\langle\vec{a},\vec{b}\right\rangle$ is the ***inner product*** of $\vec{a}$ and $\vec{b}$

For the inner product and the ***norm*** $\|\vec{a}\|$ holds $\langle\vec{a},\vec{a}\rangle=\|\vec{a}\|^2$.

$$\left\langle\vec{a},\vec{b}\right\rangle=\|\vec{a}\|\left\|\vec{b}\right\|\cos(\alpha) \qquad (19.5.3)$$

The angle α between the spatial numbers $\vec{a}$ and $\vec{b}$ is measured in ***radians***.

The ***square*** of a spatial number equals zero, or it is a negative real number;

$$\vec{a}\vec{a}=-\left\langle\vec{a},\vec{a}\right\rangle\le 0 \qquad (19.5.4)$$

$\vec{d}=\vec{a}\times\vec{b}$ is the ***outer product*** of $\vec{a}$ and $\vec{b}$.

The spatial part $\vec{d}$ is independent of $\vec{a}$ and independent of $\vec{b}$. This independence implies that $\left\langle\vec{a},\vec{d}\right\rangle=0$ and $\left\langle\vec{b},\vec{d}\right\rangle=0$.

$$\left\|\vec{a}\times\vec{b}\right\|=\|\vec{a}\|\left\|\vec{b}\right\||\sin(\alpha)|$$
$$\vec{a}\times\vec{b}=-\vec{b}\times\vec{a} \qquad (19.5.5)$$

It is possible to write spatial numbers as superpositions of base numbers. For the three-dimensional spatial numbers, this means;

$$\begin{aligned}&\vec{a} = a_i\vec{i} + a_j\vec{j} + a_k\vec{k}\\&\vec{i}\ \vec{j} = \pm\vec{k}\end{aligned} \tag{19.5.6}$$

The $\pm$ sign shows the ***chiral choice*** of the ***handedness*** of the outer product.

19.6 Mixed arithmetic

The addition and multiplication of real numbers with spatial numbers are commutative;

$$\begin{aligned}&a_r + \vec{b} = \vec{b} + a_r\\&a_r\vec{b} = \vec{b}a_r\end{aligned} \tag{19.6.1}$$

Mixed numbers show without suffixes and caps. For example, inside the following formula c acts as a mixed number.

$$c = c_r + \vec{c} \tag{19.6.2}$$

Quaternionic multiplication obeys the equation;

$$\begin{aligned}c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right)\\&= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r\vec{b} + \vec{a}b_r \pm \vec{a} \times \vec{b}\end{aligned} \tag{19.6.3}$$

The $\pm$ sign shows the ***freedom of choice*** of the handedness of the product rule when selecting a version of the quaternionic number system. In this way***, the handedness of the product rule is a special kind of geometric symmetry***.

The application must select the version of the number system before it can use the product in calculations.

Two quaternions that are each other's inverse can ***rotate*** the spatial part of another quaternion;

$$c = ab / a \qquad (19.6.4)$$

The construct rotates the spatial part of b that is perpendicular to $\vec{a}$ ***over an angle that is twice the angular phase*** θ of $a = \|a\| e^{\vec{i}\theta}$ where $\vec{i} = \vec{a} / \|\vec{a}\|$.

Cartesian quaternionic functions apply a quaternionic parameter space. A Cartesian coordinate system sequences this parameter space. The users of quaternionic functions tend to interpret the real part of the quaternions in the parameter space as instances of (proper) time. The spatial parts often appear as spatial locations. With these interpretations, the real parts of quaternionic functions stand for dynamic scalar fields. The spatial parts of quaternionic functions stand for dynamic spatial fields. Often, those users call these fields vector fields. "Vector field" is a misleading name. Vectors obey different arithmetic.

19.7 Arithmetic of change

In continuums, all convergent series of numbers end in a limit that is a member of that continuum. This fact enables the differentiation of the continuum. Differential calculus shows that a continuum can change. In the Hilbert Book Model, the continuum shows astonishing behavior. It has the habit of removing deformations. Without disturbing actuators, the continuum stays flat.

19.7.1 Differentiation

Along a direction_line, a partial differential describes the change. If in a region of the space coverage inside this direction_line, all converging series of coordinate markers result in a ***limit*** that is a coordinate marker, then the partial change of the space coverage along the direction of r is defined as the limit

$$\frac{\partial \psi}{\partial r} = \lim_{\delta r \to 0} \frac{\psi(r + \delta r) - \psi(r)}{\delta r} \qquad (19.7.1)$$

If all its irrational numbers cover the region, then this limit exists. The region does not ensure the existence of the limit. If the limit does not exist, then the location stands for a ***singular point***. It is also possible that an enclosed discrete set of point-like objects covers the surrounding region. ***In that situation, this set is not a continuum.***

If the spatial part of the neighborhood is isotropic and the limit also exists in the real number space, then the ***total differential change*** df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (19.7.2)$$

In this equation, the ***partial differentials*** $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ ***behave like quaternionic differential operators***.

The quaternionic nabla ∇ assumes the ***particular condition*** that partial differentials direct along the axes of the Cartesian coordinate system in a natural parameter space of a non-separable Hilbert space. Thus,

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \qquad (19.7.3)$$

The following section will apply this by splitting the quaternionic nabla and the function into scalar and spatial parts.

The first-order partial differential equations divide the first-order change of a quaternionic field into five distinct parts that each represent a new field. We will replace the quaternionic field change operator with a quaternionic nabla operator. ***This operator behaves like a quaternionic multiplier.***

The first-order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{19.7.4}$$

The spatial nabla $\vec{\nabla}$ is well-known as the ***del operator***. Wikipedia treats the del operator in detail. The partial derivatives in the change operator only use parameters that they take from ***the natural parameter space***.

$$\begin{aligned} \phi = \nabla \psi &= \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right) (\psi_r + \vec{\psi}) \\ &= \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \end{aligned} \tag{19.7.5}$$

Only the corresponding version of the quaternionic nabla is active in a selected version of the quaternionic number system. ***In a selected Hilbert space, this version is always the same everywhere***.

19.7.1.1 Five terms

The differential $\nabla \psi$ describes the change of field ψ. The five separate terms in the first-order partial differential ***have separate physical meanings***. All basic fields feature this decomposition. The terms may stand for ***new fields***;

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \tag{19.7.6}$$

ϕ_r is a scalar field.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{19.7.7}$$

$\vec{\phi}$ is a spatial field.

$\vec{\nabla} f$ is the gradient of f .

$\langle \vec{\nabla}, \vec{f} \rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

Important ***properties of the del operator*** are;

$$\left(\vec{\nabla}, \vec{\nabla}\right)\psi = \Delta\psi = \nabla^2\psi \tag{19.7.8}$$

$$\left(\vec{\nabla}, \vec{\nabla} \times \vec{\psi}\right) = 0 \tag{19.7.9}$$

$$\vec{\nabla} \times \left(\vec{\nabla}\psi_r\right) = 0 \tag{19.7.10}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi}\right) = \vec{\nabla}\left(\vec{\nabla}, \vec{\psi}\right) - \left(\vec{\nabla}, \vec{\nabla}\right)\vec{\psi} \tag{19.7.11}$$

Sometimes parts of the change get ***new symbols***;

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla}\psi_r \tag{19.7.12}$$

$$\vec{B} = \vec{\nabla} \times \vec{\psi} \tag{19.7.13}$$

The formula (19.7.5) is complete and does not leave room for gauges. However, ***Maxwell's equations*** treat the equation (19.7.6) as a ***gauge***.

The parts with new symbols obey;

$$\left(\vec{\nabla}, \vec{B}\right) = 0 \tag{19.7.14}$$

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{\nabla} \times \vec{\psi} - \vec{\nabla} \times \vec{\nabla}\psi_r = -\nabla_r \vec{B} \tag{19.7.15}$$

$$\left(\vec{\nabla}, \vec{E}\right) = -\nabla_r \left(\vec{\nabla}, \vec{\psi}\right) - \left(\vec{\nabla}, \vec{\nabla}\right)\psi_r \tag{19.7.16}$$

19.7.1.2 The conjugate equation

The conjugate of the quaternionic nabla operator defines another type of first-order field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{19.7.17}$$

$$\begin{aligned}\zeta = \nabla^*\phi &= \left(\frac{\partial}{\partial \tau} - \vec{\nabla}\right)\left(\phi_r + \vec{\phi}\right) \\ &= \nabla_r\phi_r + \left\langle \vec{\nabla}, \vec{\phi} \right\rangle + \nabla_r\vec{\phi} - \vec{\nabla}\phi_r \mp \vec{\nabla} \times \vec{\phi}\end{aligned} \tag{19.7.18}$$

All dynamic quaternionic fields obey the same first-order partial differential equations (19.7.5) and (19.7.18).

$$\nabla^\dagger = \nabla^* = \nabla_r - \vec{\nabla} = \nabla_r + \vec{\nabla}^\dagger = \nabla_r + \vec{\nabla}^* \tag{19.7.19}$$

19.7.1.3 Other normal differential operators

In the Hilbert space, the quaternionic nabla is a normal operator. The operators

$$\nabla^\dagger\nabla = \nabla\nabla^\dagger = \nabla^*\nabla = \nabla\nabla^* = \nabla_r\nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \tag{19.7.20}$$

are ***normal operators who are also Hermitian operators***.

The separate operators $\nabla_r\nabla_r$ and $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ are also ***Hermitian operators***.

$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ is known as the ***Laplace operator***.

The two operators can also combine as $\square = \nabla_r\nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$. This construction is the ***d'Alembert operator***.

The solutions to $\nabla_r\nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle = 0$ and $\nabla_r\nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle = 0$ differ.

These two equations offer different solutions, so they deliver different dynamic behavior of the field. The equations control the behavior of the embedding field that physicists call their universe. This dynamic field exists everywhere within the reach of the parameter space of the function. Both equations also control the behavior of the symmetry-related fields. The homogeneous d'Alembert equation, known as the wave equation, ***offers waves and wave packages*** as its solutions. Both equations provide shock fronts as solutions, but only the operators in the equation (19.7.18) deliver shock fronts with a ***spin or polarization vector***. Integration over the time domain turns both equations in the ***Poisson equation*** and removes the spin or polarization vector.

Shock fronts require a corresponding actuator and occur only in odd numbers of participating dimensions.

19.7.1.4 Continuity equations

Continuity equations are partial quaternionic differential equations.

Users interpret the dynamic changes in the field as field excitations, field deformations, or field expansions.

The here-discussed field excitations are solutions to the mentioned second-order partial differential equations. Without a corresponding actuator, the field will not react. Spherical pulses are the only actuators that deform the field. The field responds to these pulses by quickly removing the deformation. The removal sends the deformation away in all directions in the form of shock fronts until these fronts vanish at infinity. This behavior follows from the solutions presented in (19.7.29) and (19.7.31).

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\zeta = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla}\right)\left(\nabla_r - \vec{\nabla}\right)\left(\psi_r + \vec{\psi}\right)$$
$$= \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi$$

(19.7.21)

All other terms vanish. $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ is the ***Laplace operator***.

The integration over the time domain results in the ***Poisson equation***;

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \qquad (19.7.22)$$

Under isotropic conditions, a particular solution of the Poisson equation is the ***green's function*** $\frac{1}{4\pi\left|\vec{q} - \vec{q'}\right|}$ of the affected field. This solution is the spatial ***Dirac*** $\delta(\vec{q})$ ***pulse response*** of the field under strict isotropic conditions.

$$\nabla \frac{1}{\left|\vec{q} - \vec{q'}\right|} = -\frac{\left(\vec{q} - \vec{q'}\right)}{\left|\vec{q} - \vec{q'}\right|^3} \qquad (19.7.23)$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \frac{1}{\left|\vec{q} - \vec{q'}\right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left|\vec{q} - \vec{q'}\right|} \right\rangle$$
$$= -\left\langle \vec{\nabla}, \frac{\left(\vec{q} - \vec{q'}\right)}{\left|\vec{q} - \vec{q'}\right|^3} \right\rangle = 4\pi\delta\left(\vec{q} - \vec{q'}\right) \qquad (19.7.24)$$

This solution corresponds with an ongoing source or sink in the field.

A point-like stationary spatial pulse cannot start a shock front.

Therefore, the stationary spatial point-like object must be a sink or a source.

In mathematical physics and for physical_reality, stationary point-like masses do not persist. Instead, the embedding field sends them away.

Change can occur either in one spatial dimension or in two or three spatial dimensions.

19.7.1.5 Dynamic Pulse Response

Under the proper conditions, the field's dynamic pulse response is a solution to a particular form of the equation (19.7.21).

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q'}\right)\theta\left(\tau \pm \tau'\right) \qquad (19.7.25)$$

Here $\theta(\tau)$ is a temporal step function and $\delta(\vec{q})$ a spatial Dirac pulse response. ***For the spherical pulse response, the pulse must be isotropic***.

After the instant τ', the equation turns into a ***homogeneous equation***.

The ***shock front in one dimension*** along the line $\vec{q} - \vec{q'}$ is a straightforward solution;

$$\psi = f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\vec{n}\right) \qquad (19.7.26)$$

Here $\vec{n}$ is ***a normed spatial quaternion***. This spatial quaternion has an arbitrary direction that does not vary in time. In this equation, the normalized spatial number $\vec{n}$ also interprets as the ***polarization*** of the solution. We intentionally placed the normalized spatial number $\vec{n}$ close to speed c. The function f can be a primitive shock front or a ***superposition*** of primitive shock fronts. The single primitive shock front solution stands for a ***dark energy object***. It stands for a quantum of energy.

The adjective "dark" is confusing because shock fronts become observable when they cooperate in huge quantities.

19.7.1.6 Trick

In isotropic conditions, we better switch to spherical coordinates. Then the equation gets the form;

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial^2}{\partial r^2}+2\frac{\partial}{r\partial r}\right)\psi = \left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial^2}{\partial r^2}\right)(\psi r)=0 \qquad (19.7.27)$$

The second line describes the second-order change of ψr in one dimension along the radius r. The above text describes the solution. A solution to this equation is;

$$\psi r = f\left(r \pm c\tau\vec{n}\right) \qquad (19.7.28)$$

The solution of (19.7.27) is described by;

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right| \pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\vec{q'}\right|} \qquad (19.7.29)$$

The normalized spatial number $\vec{n}$ can show as the ***spin*** of the solution. It might be related to the direction the model selects when the quaternion-based Hilbert space temporarily reduces to a subspace that holds a complex-number-based Hilbert space. The spherical pulse response acts as an expanding or contracting spherical shock front. Over time this pulse response integrates into the green's function. This integration means that the isotropic pulse injects the volume of the green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the green's function and sucks it out of the field. The $\pm$ sign in the equation (19.7.25) selects between injection and suction. The shock front moves away from the pulse that caused the front. Finally, it vanishes at infinity. The inserted volume expands the field.

Spherical shock fronts are ***dark matter objects***.

When they cooperate in huge quantities, they may become perceivable. Then they are no longer dark.

Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond. The HBM does not go into details of that situation.

Equations (19.7.21) and (19.7.22) show that the operators $\frac{\partial^2}{\partial\tau^2}$ and $\langle\vec{\nabla},\vec{\nabla}\rangle$ are valid second-order partial differential operators.

These operators combine in the quaternionic equivalent of the wave equation;

$$\varphi = \left(\frac{\partial^2}{\partial\tau^2} - \langle\vec{\nabla},\vec{\nabla}\rangle\right)\psi = \Box\psi \qquad (19.7.30)$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions;

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q}'\right|} \qquad (19.7.31)$$

$$\psi = f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\right) \qquad (19.7.32)$$

These pulse responses do not hold the normed spatial number $\vec{n}$.

Apart from pulse responses, ***the wave equation offers waves*** as its solutions. That is why scientists named it the wave equation.

19.7.1.7 Split

If, locally, the field can split into a time-dependent part $T(\tau)$ and a location-dependent part $A(\vec{q})$, the homogeneous version of the wave equation can be transformed into the Helmholtz equation.

$$\frac{\partial^2 \psi}{\partial \tau^2} = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2 \psi \qquad (19.7.33)$$

$$\psi(\vec{q}, \tau) = A(\vec{q}) T(\tau) \qquad (19.7.34)$$

$$\frac{1}{T} \frac{\partial^2 T}{\partial \tau^2} = \frac{1}{A} \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \qquad (19.7.35)$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A + \omega^2 A = 0 \qquad (19.7.36)$$

$$\frac{\partial^2 T}{\partial \tau^2} + \omega^2 T = 0 \qquad (19.7.37)$$

ω acts as quantum coupling between(19.7.36) and (19.7.37).

The time-dependent part $T(\tau)$ depends on initial conditions, or it shows the switch of the oscillation mode.

During the switch, the quaternionic Hilbert space temporarily switches to a complex-number-based Hilbert space that is a subspace of the Hilbert space. The switch takes a corresponding interval; during that interval, the subspace emits or absorbs a sequence of equidistant one-dimensional shock fronts. Together, these shock fronts form a photon.

The text above discusses the one-dimensional shock fronts. The switch of the oscillation mode means that the oscillation stops temporarily, and instead, the system emits or absorbs energy that compensates for the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep moving objects within a bounded region.

For three-dimensional isotropic spherical conditions, the solutions have the form.

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \qquad (19.7.38)$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the more straightforward wave solutions to the equation (19.7.33);

$$\psi(\vec{q},\tau)=\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\} \qquad (19.7.39)$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \qquad (19.7.40)$$

A more general solution is a superposition of these basic types.

19.7.1.8 Homogenous Equations

Two relatively similar homogeneous second-order partial differential equations exist. They are the homogeneous versions of equations (19.7.25) and (19.7.30). The equation (19.7.25) has spherical shock front solutions with a spin vector that behaves like the spin of elementary particles. The field only reacts dynamically when corresponding actuators trigger it. For example, pulses may cause shock fronts that, after the trigger, keep traveling. Oscillations of type (19.7.39) and (19.7.40) must be initiated by periodic actuators.

The inhomogeneous pulse-activated equations are;

$$\left(\nabla_r \nabla_r \pm \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q}\,'\right)\theta(\tau \pm \tau') \qquad (19.7.41)$$

Without the interaction with actuators, all vibrations and deformations of the field keep busy vanishing until the affected field resembles a flat field. Only an ongoing stream of actuators can generate a more persistently deformed field.

An ongoing embedding of the actuators into the eigenspaces of operators that archive the dynamic fields provides this.

19.7.1.9 Isotropic conditions

The two shock front solutions show an interesting property of the Laplace operator. In isotropic conditions, the Poisson equation rewrites as;

$$\varphi = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} \right)\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) \qquad (19.7.42)$$

The product $\phi = (r\psi)$ is a solution of a one-dimensional equation in which r plays the variable.

Section Trick treats this situation.

The same thing holds for all differential equations that contain the Laplace operator $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$

So, spherical solutions of the second-order differential equations ξ / r follow from the solutions ξ of one-dimensional second-order differential equations by dividing ξ by the distance r to the center.

In isotropic conditions, the quaternionic differential calculus can scale down to complex-number-based differential calculus. This downscaling already works at local scales. If, on larger scales, the isotropic condition violates, the model must adapt the coordinates of the complex-number-based abstraction to the deformed Cartesian coordinates of the quaternionic platform. This adaptation makes sense in moderate deformations of the quaternionic field. After adaptation, the map of each complex-number-based coordinate line becomes a geodesic.

These tricks are possible because it is possible to consider complex-number-based Hilbert spaces as subspaces of quaternionic Hilbert spaces.

19.7.1.10 Bosons

The trick becomes relevant when rotations obstruct the relation with the geometric center of the natural parameter space. For example, neutrinos that encircle the electrons explain the muon and tau generations of the electron.

These combinations form bosons. They are composites and not elementary particles.

19.7.1.10.1 Spin

Suppose the dimension of the quaternionic Hilbert space reduces to the dimension of a subspace that applies a complex-number-based Hilbert space. In that case, whether the selected direction involves a polar or azimuth angle might become significant. In mathematics, the polar angle range is twice the azimuth angle range. In physics, the two ranges belong to different particle types. Fermions apply odd counts of the azimuth angle range. Bosons apply an even count of the azimuth angle range or an integer count of the polar angle. The choice can take a role as spin value.

In physics, fermions feature a half-integer spin value, and bosons feature an integer spin value.

19.7.2 Enclosure balance equations

Enclosure balance equations are quaternionic integral equations that describe the balance between the inside, the border, and the outside of an enclosure.

These integral balance equations replace the del operator $\vec{\nabla}$ with a normed vector $\vec{n}$. The vector $\vec{n}$ orients outward and perpendicular to a local part of the closed boundary of the enclosed region;

$$\vec{\nabla}\psi \Leftrightarrow \vec{n}\psi \tag{19.7.43}$$

This approach turns part of the differential continuity equation into a corresponding integral balance equation;

$$\iiint \vec{\nabla}\psi \, dV = \oiint \vec{n}\psi \, dS \tag{19.7.44}$$

$\vec{n}\ dS$ plays the role of a differential surface. $\vec{n}$ is perpendicular to that surface.

This result separates into three parts;

$$\begin{aligned}\vec{\nabla}\psi &= -\langle \vec{\nabla}, \vec{\psi} \rangle + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\langle \vec{n}, \vec{\psi} \rangle + \vec{n}\psi_r \pm \vec{n} \times \vec{\psi}\end{aligned} \tag{19.7.45}$$

The first part concerns the gradient of the scalar part of the field;

$$\vec{\nabla}\psi_r \Leftrightarrow \vec{n}\psi_r \tag{19.7.46}$$

$$\iiint \vec{\nabla}\psi_r \, dV = \oiint \vec{n}\psi_r \, dS \tag{19.7.47}$$

An integral balance equation, known as the Gauss theorem, treats the divergence. It is also known as the divergence theorem [25];

$$\left\langle \vec{\nabla},\vec{\psi} \right\rangle \Leftrightarrow \left\langle \vec{n},\vec{\psi} \right\rangle \tag{19.7.48}$$

$$\iiint \left\langle \vec{\nabla},\vec{\psi} \right\rangle dV = \oiint \left\langle \vec{n},\vec{\psi} \right\rangle dS \tag{19.7.49}$$

A corresponding integrated balance equation treats the curl;

$$\vec{\nabla}\,\vec{\psi} \Leftrightarrow \vec{n}\,\vec{\psi} \tag{19.7.50}$$

$$\iiint \vec{\nabla}\times\vec{\psi}\,dV = \oiint \vec{n}\times\vec{\psi}\,dS \tag{19.7.51}$$

Equation (19.7.49) and equation (19.7.51) can combine in the extended theorem;

$$\iiint \vec{\nabla}\,\vec{\psi}\,dV = \oiint \vec{n}\ \vec{\psi}\,dS \tag{19.7.52}$$

The method also applies to other partial differential equations. For example;

$$\begin{aligned}\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right) = \vec{\nabla}\left\langle \vec{\nabla},\vec{\psi} \right\rangle - \left\langle \vec{\nabla},\vec{\nabla} \right\rangle \vec{\psi} \Leftrightarrow \vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right) \\ = \vec{n}\left\langle \vec{n},\vec{\psi} \right\rangle - \left\langle \vec{n},\vec{n} \right\rangle \vec{\psi}\end{aligned} \tag{19.7.53}$$

$$\iiint\limits_V \left\{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)\right\} dV = \oiint\limits_S \left\{\vec{\nabla}\left\langle \vec{\nabla},\vec{\psi} \right\rangle\right\} dS - \oiint\limits_S \left\{\left\langle \vec{\nabla},\vec{\nabla} \right\rangle \vec{\psi}\right\} dS$$

(19.7.54)

One dimension less, a similar relation exists;

$$\iint\limits_S \left(\left\langle \vec{\nabla}\times\vec{a},\vec{n} \right\rangle\right) dS = \oint\limits_C \left\langle \vec{a},d\vec{l} \right\rangle \tag{19.7.55}$$

This equation is known as the Stokes theorem [26]

The curl can show as a line integral;

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \left\langle \vec{\psi}, d\vec{r} \right\rangle \right) \tag{19.7.56}$$

19.7.2.1 Derivation of physical laws

The quaternionic equivalents of Ampère's law are [27];

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \tag{19.7.57}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \tag{19.7.58}$$

The quaternionic equivalents of Faraday's law are [28];

$$\nabla_r \vec{B} = \vec{\nabla} \times \left(\nabla_r \vec{\psi} \right) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times \left(\nabla_r \vec{\psi} \right) = -\vec{\nabla} \times \vec{E}$$

(19.7.59)

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \tag{19.7.60}$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E} \right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v} \rho \tag{19.7.61}$$

$$\iint_S \left\langle \overline{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \left\langle \vec{v} \rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \tag{19.7.62}$$

The equations (19.7.60) and (19.7.62) enable the derivation of the ***Lorentz force*** [29];

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \tag{19.7.63}$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds$$

(19.7.64)

The Leibniz integral equation states [30];

$$\frac{d}{dt}\iint_{S(\tau)}\left\langle \vec{X}(\tau_0),\vec{n}\right\rangle dS$$

$$= \iint_{S(\tau_0)}\left\langle \dot{\vec{X}}(\tau_0)+\left\langle \vec{\nabla},\vec{X}(\tau_0)\right\rangle \vec{v}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{X}(\tau_0),d\vec{l}\right\rangle$$

(19.7.65)

With $\vec{X}=\vec{B}$ and $\left\langle \vec{\nabla},\vec{B}\right\rangle = 0$ follows;

$$\frac{d\Phi_B}{d\tau} =$$

$$\frac{d}{d\tau}\iint_{S(\tau)}\left\langle \dot{\vec{B}}(\tau),\vec{n}\right\rangle dS = \iint_{S(\tau_0)}\left\langle \vec{B}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$

$$= -\oint_{C(\tau_0)}\left\langle E(\tau_0),d\vec{l}\right\rangle - \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$

(19.7.66)

The electromotive force (EMF) ε equals [31];

$$\varepsilon = \oint_{C(\tau_0)}\left\langle \frac{\vec{F}(\tau_0)}{q},d\vec{l}\right\rangle = -\left.\frac{d\Phi_B}{d\tau}\right|_{\tau=\tau_0}$$
$$= \oint_{C(\tau_0)}\left\langle \vec{E}(\tau_0),d\vec{l}\right\rangle + \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle \quad (19.7.67)$$

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} \quad (19.7.68)$$

19.8 Dirac's' bra-ket procedure

Paul Dirac introduced a handy notation for the relationship between the ingredients of a Hilbert space. The bra-ket combination allows using complex numbers and quaternions as superposition coefficients. The bra-ket combination restricts the applied numbers to members of an associative division ring. This restriction reduces the choice to real numbers, complex numbers, and quaternions. The bra-ket combination selects a private version of that associative division ring. First, we focus on separable Hilbert spaces. Inside separable Hilbert spaces, the applied sets of numbers are countable. With that restriction, the bra-ket combination turns the underlying vector space into a separable Hilbert space.

19.8.1 Countable number systems

In this section, we focus on separable Hilbert spaces. Inside separable Hilbert spaces, the applied sets of numbers are countable. With that restriction, the bra-ket combination turns the underlying vector space into a separable Hilbert space. Selecting a version of the number system, fixes the symmetry of the number system. This section treats the case that the Hilbert space applies quaternions to specify the values of bra-ket combinations. The format of the formulas that show also holds for complex numbers and real numbers. The values of bra-ket combinations will apply in linear combinations of vectors and as eigenvalues of operators.

The bra-ket method establishes this by distinguishing the vectors from the underlying vector space into two types of vectors with different arithmetic. The two types stand for separate views of the underlying simple vector space. The ket $\langle \mathbf{f} |$ is a ***covariant vector***, and the bra $| \mathbf{g} \rangle$ is a ***contravariant vector***. The vectors $\mathbf{f}$ and $\mathbf{g}$ exist in the underlying vector space. The arithmetic of the ket vectors differs from the arithmetic of the bra vectors. The bra-ket combination $\langle \mathbf{f} | \mathbf{g} \rangle$ has a quaternionic value. If the underlying vectors $\mathbf{f}$ and $\mathbf{g}$ are equal, then the bra-ket combination can act as a metric. Since the product of quaternions is not commutative, the user must take care with the format of the formulas when quaternions apply.

19.8.1.1 Ket vectors

The addition of ket vectors is commutative and associative;

$$|\mathbf{f}\rangle + |\mathbf{g}\rangle = |\mathbf{g}\rangle + |\mathbf{f}\rangle = |\mathbf{f}+\mathbf{g}\rangle \tag{19.8.1}$$

$$\left(|\mathbf{f}+\mathbf{g}\rangle\right) + |\mathbf{h}\rangle = |\mathbf{f}\rangle + \left(|\mathbf{g}+\mathbf{h}\rangle\right) = |\mathbf{f}+\mathbf{g}+\mathbf{h}\rangle \tag{19.8.2}$$

Together with quaternions, a set of ket vectors forms a ***ket vector space***. Ket vectors are covariant vectors.

A quaternion α can help to construct a ***covariant linear combination*** with the ket vector $|\mathbf{f}\rangle$;

$$|\alpha\mathbf{f}\rangle = |\mathbf{f}\rangle\alpha \tag{19.8.3}$$

19.8.1.2 Bra vectors

For bra vectors hold;

$$\langle\mathbf{f}| + \langle\mathbf{g}| = \langle\mathbf{g}| + \langle\mathbf{f}| = \langle\mathbf{f}+\mathbf{g}| \tag{19.8.4}$$

$$\left(\langle\mathbf{f}+\mathbf{g}|\right) + \langle\mathbf{h}| = \langle\mathbf{f}| + \left(\langle\mathbf{g}+\mathbf{h}|\right) = \langle\mathbf{f}+\mathbf{g}+\mathbf{h}| \tag{19.8.5}$$

Bra vectors are contravariant vectors.

$$\langle\alpha\mathbf{f}| = \alpha^*\langle\mathbf{f}| \tag{19.8.6}$$

Quaternions can form linear combinations with bra vectors. Together with quaternions, a set of bra vectors creates a ***bra vector space***.

A set of bra vectors form the vector space that is ***adjunct*** to the vector space of ket vectors that are the origins of these maps. If the map images the adjunct space onto the original vector space, then the bra vectors may map onto the corresponding ket vector.

19.8.1.3 Bra-ket combination

For the bra-ket combination holds;

$$\langle \mathbf{f} \mid \mathbf{g} \rangle = \langle \mathbf{g} \mid \mathbf{f} \rangle^* \tag{19.8.7}$$

For quaternionic numbers α and β hold;

$$\langle \alpha\mathbf{f} \mid \mathbf{g} \rangle = \langle \mathbf{g} \mid \alpha\mathbf{f} \rangle^* = \left(\langle \mathbf{g} \mid \mathbf{f} \rangle \alpha\right)^* = \alpha^* \langle \mathbf{f} \mid \mathbf{g} \rangle \tag{19.8.8}$$

$$\langle \mathbf{f} \mid \beta\mathbf{g} \rangle = \langle \mathbf{f} \mid \mathbf{g} \rangle \beta \tag{19.8.9}$$

$$\begin{aligned} \langle (\alpha+\beta)\mathbf{f} \mid \mathbf{g} \rangle &= \alpha^* \langle \mathbf{f} \mid \mathbf{g} \rangle + \beta^* \langle \mathbf{f} \mid \mathbf{g} \rangle \\ &= (\alpha+\beta)^* \langle \mathbf{f} \mid \mathbf{g} \rangle \end{aligned} \tag{19.8.10}$$

These formulas correspond with (19.8.3) and (19.8.6);

$$\langle \alpha\mathbf{f} | = \alpha^* \langle \mathbf{f} | \tag{19.8.11}$$

$$| \alpha\mathbf{g} \rangle = | \mathbf{g} \rangle \alpha \tag{19.8.12}$$

We made a choice. Another possibility would be $\langle \alpha\mathbf{f} | = \alpha \langle \mathbf{f} |$ and $| \alpha\mathbf{g} \rangle = | \mathbf{g} \rangle \alpha^*$

19.8.1.4 Operator construction

$|\mathbf{f}\rangle\langle\mathbf{g}|$ is a constructed operator;

$$|\mathbf{g}\rangle\langle\mathbf{f}| = \left(|\mathbf{f}\rangle\langle\mathbf{g}|\right)^\dagger \tag{19.8.13}$$

The superfix † shows the adjoint version of the operator.

For the orthonormal base $\{|q_i\rangle\}$ consisting of eigenvectors of the reference operator holds;

$$\langle q_n \mid q_m \rangle = \delta_{nm} \tag{19.8.14}$$

Eigenvectors belong to the underlying vector space. Eigenvalues belong to the natural parameter space. The natural parameter space is a selected version of the applied number system. The ***bra-ket method*** enables the definition of new operators that quaternionic functions define;

$$\langle \mathbf{g} \,|\, F \,|\, \mathbf{h} \rangle = \sum_{i=1}^{N} \left\{ \langle \mathbf{g} | q_i \rangle F(q_i) \langle q_i | \mathbf{h} \rangle \right\} \tag{19.8.15}$$

The symbol F is used both for the operator F and the sampled quaternionic function $F(q)$. This format enables the shorthand;

$$F \equiv |q_i\rangle F(q_i) \langle q_i| \tag{19.8.16}$$

for operator F. For the adjoint operator;

$$F^{\dagger} \equiv |q_i\rangle F^{*}(q_i) \langle q_i| \tag{19.8.17}$$

For ***reference operator*** $\mathfrak{R}$ holds;

$$\mathfrak{R} = |q_i\rangle q_i \langle q_i| \tag{19.8.18}$$

If $\{q_i\}$ consists of all rational values of the version of the quaternionic number system that Hilbert space $\mathfrak{H}$ applies, then the eigenspace of $\mathfrak{R}$ represents the natural parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the natural operator F in the formula (19.8.16). This formula turns the separable Hilbert space into a sampled function space.

19.8.1.5 Expected value.

Any bra vector $\langle \mathbf{g}|$ can write as a linear combination of the bra base vectors $\{\langle q_i|\}$;

$$\langle \mathbf{g}| = \sum_{i=1}^{N} \left\{ \langle \mathbf{g} | q_i \rangle \langle q_i| \right\} \tag{19.8.19}$$

Any ket vector $|\mathbf{g}\rangle$ can write as a linear combination of the ket base vectors $\{|q_i\rangle\}$;

$$|\mathbf{g}\rangle = \sum_{i=1}^{N} \{|q_i\rangle\langle q_i|\mathbf{g}\rangle\} \tag{19.8.20}$$

The eigenvalues archive as a combination of a real value and a spatial value. These parts take independent dimensions. If the real parts sequence, then the sequence of eigenvalues stands for an ongoing hopping path. Suppose this ongoing hopping path recurrently regenerates the same hop landing location swarm. In that case, the hop landing locations can sum over the regeneration period in the cells of a dense spatial grid. The total sum results in a spatial center location. The sums in the cells describe a location density distribution. The center location acts as the expected spatial value of the hop landing locations. A hop landing location distribution will describe the hop landing location swarm. If the swarm covers a more significant number of locations, then the location density distribution description will be more correct. If the results for the grid cells sample over a more substantial part of the real numbers, then the describing location density distribution approaches a continuous function.

This means that $|\langle \mathbf{g}|\vec{q}_i\rangle|^2 = \langle \mathbf{g}|\vec{q}_i\rangle\langle \vec{q}_i|\mathbf{g}\rangle$ can take the role of a hop landing location distribution.

Here, we only used the spatial parts of the eigenvalues.

The expected spatial value for the operator $\mathfrak{R}$ and vector $\mathbf{g}$ is;

$$\langle \mathfrak{R}\rangle_{\mathbf{g}} = \langle \mathbf{g}|\mathfrak{R}|\mathbf{g}\rangle = \sum_{i=1}^{N} \{\langle \mathbf{g}|\vec{q}_i\rangle \vec{q}_i \langle \vec{q}_i|\mathbf{g}\rangle\} \tag{19.8.21}$$

The expected value plays its role in a series of ordered observations or events. After sequencing the timestamps of the samples, the string of samples stands for an ongoing hopping path. If the vector $\mathbf{g}$ aims at a particular location inside the parameter space of the Hilbert space, then the mechanism that generates the ongoing hopping path recurrently regenerates a hop landing location swarm that a stable location density distribution describes. For large values of N, the location density distribution approaches a continuous function $\langle \mathbf{g}|\vec{q}\rangle\langle\vec{q}|\mathbf{g}\rangle$, and the distribution $\langle \mathbf{g}|\vec{q}\rangle$ interprets as a probability amplitude. The square of the modulus of this probability amplitude is a probability density distribution. What these continuous functions describe are discrete sets. The approach fits better if the number of elements in the set is more significant and there exists a requirement for a considerable coherence of the set.

Suppose at instant zero, the vector equals the eigenvector that belongs to eigenvalue zero, and the expectation value also equals zero. In that case, the hop landing locations $\{q_i\}$ will tend to stay awhile about the geometrical center of the Hilbert space. If the tendency lasts, the vector $\mathbf{g}$ will function as a ***unique state vector*** of the Hilbert space.

An active stochastic selection process will give the location density distribution a statistical sense. A footprint vector $|\mathbf{g}\rangle$ that varies over time stands for that selection process. The selection process's characteristic function checks how $|\mathbf{g}\rangle$ varies over time. A vector $\mathbf{g}$ in the underlying vector space stands for the footprint vector. The Hilbert space can archive the life history of the footprint vector in the form of a cord of quaternionic eigenvalues from a devoted footprint operator.

The state vector of the Hilbert space is a unique footprint vector of the Hilbert space. At every instant of time, the footprint vector has the expectation value of zero. At instant zero, the state vector equals the eigenvector that belongs to location zero. This fact still does not say everything about the essence of the required underlying stochastic selection mechanism. The decision of the HBM to replace the real parts of the geometric eigenvalues against timestamps hides what happened to the hopping path. For example, this description does not explain the value and stability of the recurrence rate of the hop landing location swarm. It is not clear why the characteristic function of the stochastic mechanism is stable. Nevertheless, the decision made the application of differential calculus possible.

19.8.1.6 Operator types

I stands for the identity operator.

For normal operator N holds; $NN^{\dagger} = NN^{\dagger}$.

The normed eigenvectors of a normal operator form an orthonormal base of the Hilbert space.

For unitary operator U holds; $UU^{\dagger} = U^{\dagger}U = I$

For Hermitian operator H holds; $H = H^{\dagger}$

N has a Hermitian part; $\dfrac{N+N^{\dagger}}{2}$ and an anti-Hermitian part; $\dfrac{N-N^{\dagger}}{2}$

For anti-Hermitian operator A holds; $A = -A^{\dagger}$

A Hermitian operator has real eigenvalues. An anti-Hermitian operator has spatial eigenvalues.

The reference operator $\mathfrak{R}$ is a normal operator.

19.8.2 Uncountable number systems

Every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that embeds its separable partner. The non-separable Hilbert space allows operators that maintain eigenspaces that, in every dimension and every spatial direction, hold closed sets of rational and irrational eigenvalues. These eigenspaces are uncountable and behave as dynamic sticky continuums. These continuums can vibrate, deform, and expand.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, the bra-ket method turns from a summation into an integration for operators with continuum eigenspaces;

$$\langle \mathbf{g} | F | \mathbf{h} \rangle \equiv \int \iiint \{ \langle \mathbf{g} | q \rangle F(q) \langle q | \mathbf{h} \rangle \} dV d\tau \qquad (19.8.22)$$

Here we omitted the enumerating subscripts that we used in the countable base of the separable Hilbert space. Instead, the integration applies the infinitesimal $dVd\tau$ originating from the continuum in the private parameter space.

The shorthand for the operator F is now;

$$F \equiv |q\rangle F(q) \langle q| \qquad (19.8.23)$$

For eigenvectors $|q\rangle$, the function $F(q)$ defines as;

$$F(q) = \langle q | Fq \rangle = \int \iiint \{ \langle q | q' \rangle F(q') \langle q' | q \rangle \} dV' d\tau' \qquad (19.8.24)$$

The function $F(q)$ no longer samples.

The reference operator $\mathcal{R}$ that supplies the continuum's natural parameter space as its eigenspace follows from;

$$\langle \mathbf{g} | \mathcal{R} \mathbf{h} \rangle \equiv \int \iiint \{ \langle \mathbf{g} | q \rangle q \langle q | \mathbf{h} \rangle \} dV d\tau \qquad (19.8.25)$$

The corresponding shorthand is;

$$\mathcal{R} \equiv |q\rangle q \langle q| \qquad (19.8.26)$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, the claim becomes clear that every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that embeds its separable companion.

The reverse bra-ket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology. The replacement of the real parts of geometric eigenvalues made this possible. Humans, not physical_reality made this choice!

19.8.2.1 Expected spatial value.

Like the separable Hilbert space situation, the model applies a grid overlay of the spatial part of the parameter space to integrate over the grid cells. The expected spatial value averages over a part of the real part of the parameter space.

In the non-separable Hilbert space, the model defines the expected spatial value as an average over the spatial part of the parameter space;

$$\langle \mathfrak{R} \rangle_{\mathbf{g}} = \langle \mathbf{g} | \mathfrak{R} | \mathbf{g} \rangle = \iiint_0 \{ \langle \mathbf{g} | q \rangle \vec{q} \langle q | \mathbf{g} \rangle \} dV \quad (19.8.27)$$

Usually, the model keeps the real part of the parameter space fixed, and the integration occurs over the spatial part of the parameter space.

The location density distribution is a continuous function with values corresponding to locations in the spatial part of the parameter space;

$$|\langle \mathbf{g} | q \rangle|^2 = \langle \mathbf{g} | q \rangle \langle q | \mathbf{g} \rangle \quad (19.8.28)$$

Thus, the variable $\vec{q}$ can be any value in the spatial part of the parameter space.

19.9 Lattice theory

19.9.1 Relational structures

The interface types, relation types, and module types feature a ***lattice structure***.

Also, the closed subspaces of separable Hilbert spaces feature a similar lattice structure.

Garret Birkhoff and John von Neumann stated that lattice structure in 1936; decades later, Maria Pia Solèr proved this mathematically in Solèr's theorem.

John von Neumann was the assistant of David Hilbert.

Many scientists suspected the lattice structure of the separable Hilbert spaces as a ***foundation of physics***. However, this conclusion does not justify. Instead, the HBM relies on the special_set as a better candidate for the foundation of theoretical physics.

Later, the author suggests that the lattice structure of the closed subspaces of a separable Hilbert space is a requirement for the ***modularization*** of the fermions.

19.9.2 How quantum logic got its name

In 1936 Garret Birkhoff and John von Neumann reported a skeleton relational structure that can function as a foundation of a model of physics.

They called it quantum logic.

~25 axioms define classical logic, which differs in only two axioms from quantum logic.

In ***quantum logic***, the ***distributive law fails,*** and the ***modular law weakens.***

Due to the remarkable resemblance with classical logic, Birkhoff and von Neumann classified their skeleton relational structure as logic and called it quantum logic.

19.9.3 Lattice structure

A lattice is a set of elements $a,\ b,\ c,\ .,.,.$ that closes for the connection's conjunction ∩ and disjunction **∪**.

Symbol ⊂ stands for implication.

These connections obey;

The set orders partially.

With each pair of elements $a,\ b$ belongs an element c, such that $a \subset c$ and $b \subset c$.

The set is a ∩ half lattice if, with each pair of elements $a,\ b$ an element c exists, such that $c = a \cap b$.

The set is a **∪** half lattice if, with each pair of elements $a,\ b$ an element c exists, such that $c = a \cup b$.

The set is a lattice if it is both a ∩ half lattice and a **∪** half lattice.

19.9.3.1 Partially ordered set

The following relations hold in a lattice;

$$a \cap b = b \cap a$$

$$(a \cap b) \cap c = a \cap (b \cap c)$$

$$a \cap (a \cup b) = a$$

$$a \cup b = b \cup a$$

$$(a \cup b) \cup c = a \cup (b \cup c)$$

$$a \cup (a \cap b) = a$$

A lattice has a partial order inclusion **⊂**;

$a \subset b \Leftrightarrow a \subset b = a$

A ***complementary lattice*** contains two elements n and e, with each element a, a complementary element a';

$$a \cap a' = n \quad a \cap n = n$$

$$a \cap e = a \quad a \cup a' = e$$

$$a \cup e = e \quad a \cup n = a$$

19.9.4 Orthocomplemented lattice

This type of lattice contains with each element a an element a'' such that;

$$a \cup a'' = e$$

$$a \cap a'' = n$$

$$(a'')'' = a$$

$$a \subset b \Leftrightarrow b'' \subset a''$$

19.9.4.1 Distributive lattice

In a distributive lattice;

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$$

$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$$

19.9.4.2 Modular lattice

In a modular lattice;

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c))$$

Classical logic is an orthocomplemented modular lattice.

19.9.5 Weak modular lattice

There exists an element d such that;

$$a \subset c \Leftrightarrow (a \cup b) \cap c$$

$$= a \cup (b \cap c) \cup (d \cap c)$$

where d obeys:

$$(a \cup b) \cap d = d$$

$$a \cap d = n$$

$$b \cap d = n$$

$$[(a \subset g) \text{ and } (b \subset g)] \Leftrightarrow d \subset g$$

Quantum logic obeys the weak modular law.

19.9.6 Atomic lattice

In an atomic lattice is;

$$\exists_{p \in L} \forall_{x \in L} \{x \subset p \Rightarrow x = n\}$$

$$\forall_{a \in L} \forall_{x \in L} \{(a < x < a \cap p)$$

$$\Rightarrow (x = a \, or \, x = a \cap p)\}$$

p **is an atom.**

19.9.7 Logics

Classical logic has the structure of an ***orthocomplemented distributive modular*** and ***atomic*** lattice.

Quantum logic has the structure of an ***orthocomplented weakly modular*** and ***atomic*** lattice.

Scientists also call quantum logic an ***"orthomodular lattice***."

19.9.8 Rules and relational structures

The part of mathematics that treats relational structures is ***lattice theory***.

Logic systems are applications of lattice theory.

Classical logic has a simple relational structure.

However, since the paper of Birkhoff and von Neumann in 1936, we know that ***physical_reality cheats*** classical logic.

Since then, many scientists have thought that nature obeys ***quantum logic***.

Quantum logic has a ***much more complicated relational structure*** than classical logic.

Quantum logic enables modularization in abstract structures.

Physical_reality applies these abstract structures.

19.9.8.1 The axioms

The axioms concern ***relations*** between elements

The axioms do not concern the ***content*** of the elements.

The axioms describe ***countable*** discrete sets of elements.

The axioms do not necessarily concern logic systems.

19.10 Fourier transforms.

A cosine function combines with a sine function that owns the same frequency into a complex number-valued exponential function. The imaginary factor i belongs to the direction of that same direction_line.

$$\varphi(2\pi xp) = \cos(2\pi xp) + i\bullet\sin\left(2\pi xp\right) = \exp(i2\pi xp) \qquad (19.10.1)$$

This sum has the remarkable property that p resembles the partial differential change operator for the direction i of x;

$$i\frac{\delta}{\delta x}\varphi = -2\pi p\varphi \qquad (19.10.2)$$

$$i\frac{\delta}{\delta p}\varphi = -2\pi x\varphi \qquad (19.10.3)$$

x and p are related via a Fourier transform [32].

This section does not show the spatial direction number with a vector cap in the exponentials. Instead, we use the convention that complex number versions of the exponential function apply.

The relation between $\psi(x)$ and $\tilde{\psi}(p_{xn})$ in the sum;

$$\psi(x) = \sum_{n=-\infty}^{\infty} \left\{ \tilde{\psi}\left(p_{x,n}\right) e^{2\pi i x p_{x,n}} \left(p_{x,n+1} - p_{x,n}\right) \right\} \tag{19.10.4}$$

gives the Fourier transform in a separable complex-number-based Hilbert space,

In the limit where $\Delta p_x = \left(p_{x,n+1} - p_{x,n}\right) \rightarrow 0$ the sum becomes an integral;

$$\psi(x) = \int_{-\infty}^{\infty} \left\{ \tilde{\psi}\left(p_x\right) e^{2\pi i x p_x} \right\} dp_x \tag{19.10.5}$$

The reverse Fourier transform runs as;

$$\tilde{\psi}(p_x) = \int_{-\infty}^{\infty} \left\{ \psi\left(x\right) e^{-2\pi i x p_x} \right\} dx \tag{19.10.6}$$

In these formulas, the symbol i is a normalized spatial number part of a complex number. i corresponds to the spatial direction that constructing the complex-number-based Hilbert space selected.

The function $e^{2\pi i x p_x}$ is an eigenfunction of the operator $\vec{p}_x = \vec{i} \dfrac{\partial}{\partial x}$, which is recognizable as part of the change operator (19.7.3);

$$\vec{i} \frac{\partial}{\partial x} e^{2\pi i x p_x} = 2\pi \vec{p}_x e^{2\pi i x p_x} \tag{19.10.7}$$

The eigenvalue p_x stands for the eigenfunction and the eigenvector $\vec{p}_x$ in the change space. In the same sense, the function $e^{-2\pi i x p_x}$ is an eigenfunction of the position operator $-\vec{i}\frac{\partial}{\partial p_x}$ and corresponds with the eigenvalue x of that operator;

$$-\vec{i}\frac{\partial}{\partial p_x}e^{-2\pi i x p_x} = 2\pi x e^{-2\pi i x p_x} \tag{19.10.8}$$

The eigenvalue x stands for the eigenfunction and the eigenvector x in the position space.

The Fourier transform of a Dirac delta function is;

$$\tilde{\delta}(p_x) = \int_{-\infty}^{\infty}\left\{\delta(x)e^{-2\pi i x p_x}\right\}dx = 1 \tag{19.10.9}$$

The inverse transform tells;

$$\delta(x) = \int_{-\infty}^{\infty}\left\{1\cdot e^{2\pi i x p_x}\right\}dp_x \tag{19.10.10}$$

In the integral, factor 1 expresses that all superposition coefficients have norm 1.

$$\delta(x-a) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(x-a)p_x}dp_x \tag{19.10.11}$$

$$e^{2\pi i p_x a} = \int_{-\infty}^{\infty}\delta(x-a)e^{2\pi i x p_x}dx \tag{19.10.12}$$

The operator $\vec{p}_x = \vec{i}\frac{\partial}{\partial x}$ is often called the momentum operator for the spatial direction $\vec{i}$ of the coordinate x. $\vec{p}$ differs from classical momentum, which is the product of velocity $\vec{v}$ and mass m. It is vital to notice that every orthonormal base vector of the position space is a superposition of ALL orthonormal base vectors of the change space.

Further, the norms of the superposition coefficients are all equal. Similarly, every orthonormal base vector of the change space is a superposition of ALL orthonormal base vectors of the position space. Again, the norms of the superposition coefficients are all equal. Thus, jumping between different bases randomizes the landing base vector.

Fourier transforms convert convolutions of functions into products of the Fourier transforms of the functions.

19.11 Uncertainty principle

The uncertainty principle states;

$$\left(\int_{-\infty}^{\infty}(x-x_0)^2\left|\psi(x)\right|^2 dx\right)\left(\int_{-\infty}^{\infty}(p_x-p_{x,0})^2\left|\tilde{\psi}(p_x)\right|^2 dp_x\right)\geq\frac{1}{16\pi^2}$$

(19.11.1)

For a Gaussian distribution, the equality sign holds. The Fourier transform of a Gaussian distribution is again a Gaussian distribution with a different standard deviation.

If $\psi(x)$ spreads, then $\tilde{\psi}(p_x)$ shrinks, and vice versa.

19.12 Center of Influence of Actuators

The potential $V(r)$ describes the effect of a local response to an actual or virtual isotropic point-like actuator. It reflects an agent's work to bring a unit amount of the actuator influence from infinity to the considered location;

$$V(r)=\theta_p\varepsilon/r \qquad (19.12.1)$$

Here θ_p stands for the actuator influence. ε takes care of adaptation to physical units. r is the distance to the location of the point-like actuator.

A swarm of point-like actual or virtual actuators that superimpose their potentials in the potential of a single actuator or virtual actuator produces a potential that, viewed from a sufficient distance r, has a shape;

$$V(r)=\Theta\varepsilon / r \tag{19.12.2}$$

Here Θ stands for the actuator influence of the resulting actual or virtual actuator. r is the distance to the center of the actuator influence. This formula is valid at sufficiently large values of r such that a swarm of actuators functions as a point-like object.

In a coherent swarm of actuating objects $\theta_i, i=1,2,3,\ldots n$, each with static influence θ_i at locations r_i, the center of actuation $\vec{R}$ follows from;

$$\sum_{i=1}^{n}\theta_i\left(\vec{r}_i-\vec{R}\right)=\vec{0} \tag{19.12.3}$$

Thus;

$$\vec{R}=\frac{1}{\Theta}\sum_{i=1}^{n}\theta_i\vec{r}_i \tag{19.12.4}$$

Where;

$$\Theta=\sum_{i=1}^{n}\theta_i \tag{19.12.5}$$

In the following, we will consider an ensemble of actuating objects with a center of actuation $\vec{R}$ and a fixed combined actuation influence Θ as a single virtual actuation object that locates at $\vec{R}$. The separate actuators θ_i may differ because, at the instant of summation, the corresponding influence might have partly faded away.

$\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit.

19.13 Forces

The first-order change of the quaternionic field divides into five separate partial changes. Some of these parts can compensate for each other.

Mathematically, the statement that nothing in the field changes in the first approximation indicates that the first-order partial differential will be equal to zero locally.

$$\zeta = \nabla \xi = \nabla_r \xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle + \vec{\nabla} \xi_r + \nabla_r \vec{\xi} \pm \vec{\nabla} \times \vec{\xi} = 0 \qquad (19.13.1)$$

Thus

$$\zeta_r = \nabla_r \xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle = 0 \qquad (19.13.2)$$

$$\vec{\zeta} = \vec{\nabla} \xi_r + \nabla_r \vec{\xi} \pm \vec{\nabla} \times \vec{\xi} = 0 \qquad (19.13.3)$$

It is possible to interpret these formulas independently. For example, according to the equation (19.13.2), the variation in time of ξ_r can compensate the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, we expect the curl $\vec{\nabla} \times \vec{\xi}$ of the spatial field $\vec{\xi}$ to be zero. The resulting terms of the equation (19.13.3) are

$$\nabla_r \vec{\xi} + \vec{\nabla} \xi_r = 0 \qquad (19.13.4)$$

The following text plays the role of the spatial field and ξ_r plays the role of the scalar potential of the considered object. The spatial part $\vec{\xi}$ conforms to the uniform speed of movement of the floating group of influenced objects. The main characteristic of this field is that it tries to keep its overall change at zero. The author calls ζ the ***conservation field***.

At a considerable distance r, we approximate this potential by using the formula;

$$\zeta_r(r) \approx \frac{\Theta\varepsilon}{r} \tag{19.13.5}$$

The new artificial field $\xi = \left\{ \frac{\Theta\varepsilon}{r}, \vec{v} \right\}$ considers a single uniformly moving influenced object or a set of influenced objects that move uniformly as a normal situation. It is a combination of scalar potential $\frac{\Theta\varepsilon}{r}$ and speed $\vec{v}$. This movement speed is the relative speed between the floating and background platforms. At equilibrium, this speed is uniform.

If the gradient of $\frac{\Theta\varepsilon}{r}$ differs from zero, then the artificial field $\left\{ \frac{\Theta\varepsilon}{r}, \vec{v} \right\}$ tries to counteract this by changing field $\vec{v}$ into a field of accelerated objects $\vec{a}$;

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{\Theta\varepsilon}{r} \right) = \frac{\Theta\varepsilon\,\vec{r}}{|\vec{r}|^3} \tag{19.13.6}$$

In reverse, the accelerated spatial field $\vec{a}$ acts on actuator influences $\frac{\Theta\varepsilon}{r}$ that appear in its realm by afflicting a gradient to this potential.

Thus, if two uniformly moving actuator influences Θ_1 and Θ_2 exist in each other's neighborhood, then any disturbance of the equilibrium will cause the force $\vec{F}$;

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = \Theta_1 \vec{a} = \frac{\varepsilon\Theta_1\Theta_2\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} = \frac{\varepsilon\Theta_1\Theta\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{19.13.7}$$

The influenced objects own mass and can possess electric charge. Static electric charges only influence electric charges. Massive actuators only influence massive objects.

19.14 Deformation potentials

The model consider the deformation potential to be zero at infinity. The deformation potential at a considered location equals the work (energy transferred) per unit mass the model would need to move an object from infinity to that location. Isotropic pulses that deform the embedding field introduce an extra complication because the pulse response is a shock front that quickly fades away. Therefore, we reinvestigate this kind of potential.

19.14.1 Center of deformation

Suppose the actuator is a response to an isotropic pulse. In that case, the deformation potential $V(r)$ describes the effect of a local response to an isotropic point-like actuator. It reflects the work that must be done by an agent to bring a unit amount of the injected stuff from infinity back to the considered location;

$$V(r) = m_p G / r \tag{19.14.1}$$

Here m_p stands for the mass that corresponds to the complete pulse response. G takes care of adaptation to physical units. r is the distance to the location of the pulse. The pulse response is a spherical shock front.

A stream of these deforming actuators recurrently regenerates a coherent swarm of embedding locations in the dynamic universe field. Viewed from a sufficient distance r, that swarm generates a potential;

$$V(r) = MG / r \tag{19.14.2}$$

Here M is the mass that corresponds to the considered swarm of pulse responses. r is the distance to the center of the deformation. This formula is valid at sufficiently large values of r such that the whole swarm can function as a point-like object.

In a coherent swarm of massive objects $p_i, i = 1,2,3,...n$, each with static mass m_i at locations r_i, the center of mass $\vec{R}$ follows from;

$$\sum_{i=1}^{n} m_i \left(\vec{r}_i - \vec{R}\right) = \vec{0} \qquad (19.14.3)$$

Thus;

$$\vec{R} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i \qquad (19.14.4)$$

Where;

$$M = \sum_{i=1}^{n} m_i \qquad (19.14.5)$$

In the following, we will consider an ensemble of massive objects with a center of mass $\vec{R}$ and a fixed combined mass M as a single massive object that locates at $\vec{R}$. The separate masses m_i may differ because, at the instant of summation, the corresponding deformation might have partly faded away.

$\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit. The problem with the treatise in this paragraph is that in physical_reality, point-like objects that own a static mass do not exist. Only pulse responses that temporarily deform the field exist. Except for black holes, these pulse responses form all massive objects in the universe.

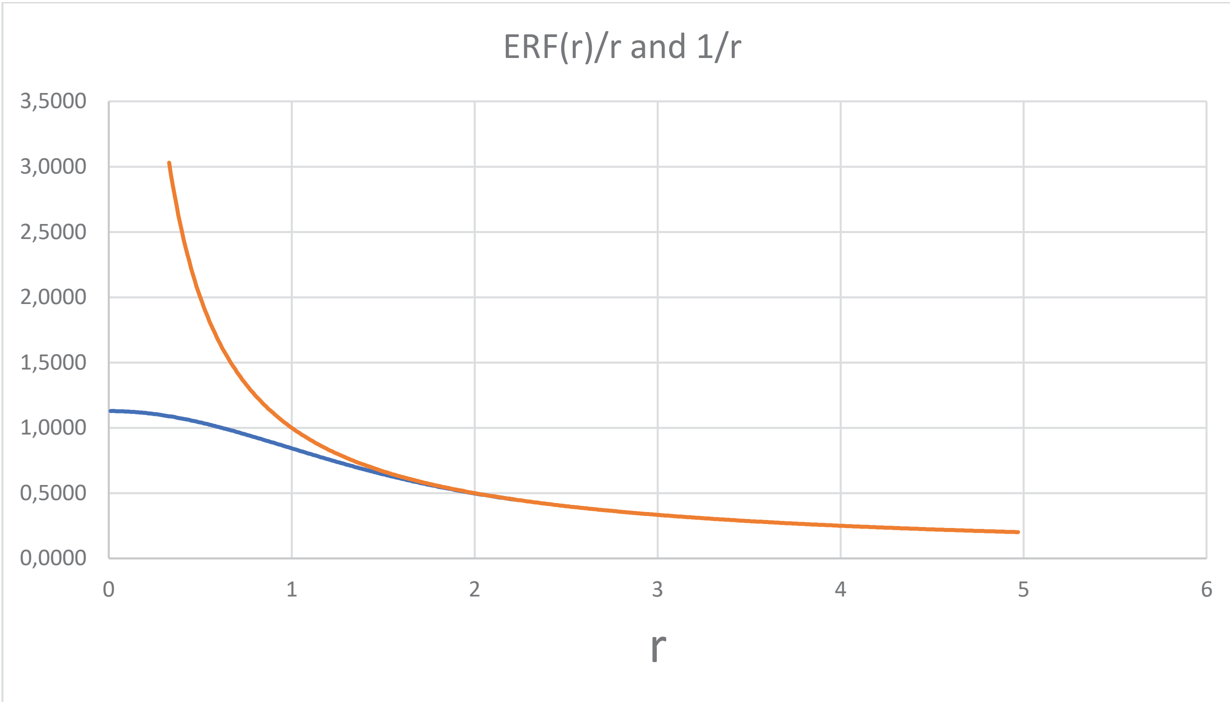
19.15 Pulse location density distribution

It is false to treat a pulse location density distribution as a set of point-like masses, as the formulas (19.14.3) and (19.14.4) show. Instead, the deformation potential follows from the convolution of the location density distribution and the green's function. This calculation is still incorrect because the exact result depends on the fact that the deformation due to a pulse response quickly fades away, and the result also depends on the density of the distribution. If the application can ignore these effects, then the resulting deformation potential of a Gaussian density distribution according to;

$$g(r) \approx GM \frac{ERF(r)}{r} \qquad (19.15.1)$$

Where $ERF(r)$ is the well-known error function. Here the deformation potential is a perfectly smooth function that, at some distance from the center, equals the approximated deformation potential described above in the equation (19.14.2). However, as shown above, the convolution only offers an approximation because this computation does not account for the influence of the swarm's density, and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.



This influence might explain why some elementary module types differ in their mass.

The explanation of the generations of elementary fermions in the section Bosons, differs from the explanation of the different masses of elementary fermions.

This influence might also explain why different first-generation elementary particle types show different masses. Due to the convolution, and the coherence of the location density distribution, the blue curve does not show any sign of the singularity that the red curve, which shows the green's function, holds.

In physical_reality, no point-like static mass object exists. The most important lesson of this investigation is that far from the deformation center of the distribution, the here-shown simplified form of the deformation potential characterizes the deformation of the field as;

$$\phi(r) \approx \frac{GM}{r} \quad (19.15.2)$$

Warning: This simplified form shares its shape with the green's function of the deformed field. This sharing does not mean that the green's function owns a mass that equals $M_G = \frac{1}{G}$. The functions only share the form of their tail.

19.16 Rest mass

The weakness in the definition of the deformation potential is the definition of the unit of mass and the fact that shock fronts move with a fixed finite speed. Thus, the definition of the deformation potential only works properly if the geometric center location of the swarm of injected spherical pulses is at rest in the affected embedding field. The consequence is that the mass that follows from the definition of the deformation potential is the ***rest mass*** of the considered swarm. We will call the mass that corrects for the observer's motion relative to the observed scene the ***inertial mass***.

19.17 Observer

The inspected location is the location of a hypothetical test object that owns an amount of mass. It can stand for an elementary particle or a conglomerate of such particles. This location is the target location in the embedding field. The embedding field is supposed to deform by the embedded objects.

Observers can access information the model retrieved from storage locations with a historic timestamp. That information transfers to them via the dynamic universe field. This dynamic field embeds both the observer and the observed event. The dynamic geometric data of point-like objects archive in Euclidean format as a combination of a timestamp and a three-dimensional spatial location. The embedding field affects the format of the transferred information. The observers perceive in spacetime format. A hyperbolic Lorentz transform converts the Euclidean coordinates of the background parameter space into the spacetime coordinates that the observer perceives. The embedding field may deform. This deformation also affects the transferred information.

Photons follow geodesics in the embedding field [33].

19.17.1 Lorentz transforms.

In dynamic fields, shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \tag{19.17.1}$$

In flat dynamic fields, swarms of triggers of spherical pulse responses move with lower speed v.

For the geometric centers of these swarms still holds;

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \tag{19.17.2}$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v, then;

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \tag{19.17.3}$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \tag{19.17.4}$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{19.17.5}$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{19.17.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{19.17.7}$$

These equations describe a hyperbolic transformation that relates two coordinate systems. The transformation is known as a Lorentz boost.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed.

However, it can also concern the storage location P coordinate time t' and location $\{x',y',z'\}$.

This hyperbolic transform relates two platforms that move with uniform relative speed. One of them may be a floating Hilbert space on which the observer exists. Or it may be a cluster of platforms that cling together and move as one unit. The other may be the background platform on which the embedding process produces the footprint image.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x,y,z\}$ and proper timestamps τ into the perceived coordinate system consisting of the spacetime coordinates $\{x',y',z',ct'\}$, in which t' plays the role of coordinate time. The uniform velocity v causes time dilation; $\Delta t' = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ and length contraction;

$$\Delta L' = \Delta L\sqrt{1-\frac{v^2}{c^2}}$$

19.17.2 Minkowski metric

The Minkowski metric rules spacetime.

In flat field conditions, proper time τ defines as;

$$\tau = \pm\frac{\sqrt{c^2t^2 - x^2 - y^2 - z^2}}{c} \quad (19.17.8)$$

And in deformed fields, still;

$$ds^2 = c^2d\tau^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \quad (19.17.9)$$

Here ds is the spacetime interval and $d\tau$ is the proper time interval. dt is the coordinate time interval?

19.17.3 Schwarzschild metric

Polar coordinates convert the Minkowski metric to the Schwarzschild metric. The proper time interval $d\tau$ obeys;

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) \quad (19.17.10)$$

Under pure isotropic conditions, the last term on the right side vanishes.

According to mainstream physics, the symbol r_s stands for the Schwarzschild radius in the environment of a black hole;

$$r_s = \frac{2GM}{c^2} \quad (19.17.11)$$

The variable r equals the distance to the center of mass of the massive object with mass M .

The Hilbert Book model finds a different value for the boundary of a spherical black hole. That radius is a factor of two smaller.

19.17.4 Event horizon

The deformation potential energy $U(r)$;

$$U(r) = \frac{mMG}{r} \quad (19.17.12)$$

at the event horizon $r = r_{eh}$ of a black hole, is supposed to be equal to the mass-energy equivalent of an object that has unit mass $m = 1$ and is brought by an agent from infinity to that event horizon. Dark energy objects are energy packages in the form of one-dimensional shock fronts that are a candidate for this role. Photons are strings of equidistant samples of these energy packages. The energy equivalent of the unit mass objects is;

$$E = mc^2 = \frac{mMG}{r_{eh}} \qquad (19.17.13)$$

Or;

$$r_{eh} = \frac{MG}{c^2} \qquad (19.17.14)$$

At the event horizon, the system consumes all energy of the dark energy object to compensate for the deformation potential energy at that location. No field excitation, and particularly no shock front, can pass the event horizon. In the equation (19.17.13), the mass m of the test object could have been replaced by the mass m_s of the spherical shock front that represents the dark matter object or by the mass m_e of an electron.

The equivalent energy of this mass is the energy of a dark energy object that a one-dimensional shock front represents.

In the case of m_e the equivalent energy is the energy of the annihilation or creation photon of the electron.

The annihilation and creation photons have the same duration and hold the same number of shock fronts. That number is the same as the number of spherical shock fronts in the footprint of the electron.
The event horizon blocks all field excitations. This blockage means that the image of the floating separable Hilbert space that stands for the electron cannot pass the event horizon. The model postulates that this holds for all floating Hilbert spaces in the system.

19.17.5 Time dilatation and length contraction

Time dilatation defines as;

$$\Delta t = \gamma \, \Delta \tau = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19.17.15}$$

A tick Δt of a coordinate time clock is smaller than the tick $\Delta \tau$ of a proper time clock. In a gravitation potential, the relation is;

$$\Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{MG}{rc^2}}} = \frac{\Delta \tau}{\sqrt{1 - \frac{v_e^2}{rc^2}}} \tag{19.17.16}$$

Here v_e is the escape speed [34].

Length contraction defines as;

$$\Delta L = \frac{\Delta L_o}{\gamma} = \Delta L_o \sqrt{1 - \frac{v^2}{c^2}} \tag{19.17.17}$$

In a gravitation potential, the relation is;

$$\Delta L = \Delta L_0 \sqrt{1 - \frac{MG}{rc^2}} = \Delta L_0 \sqrt{1 - \frac{v_e^2}{rc^2}} \tag{19.17.18}$$

Here L_0 is the length in free space.

19.18 Inertial mass

The Lorentz transform also gives the rest mass to the mass applicable when the embedding field moves relatively with uniform speed $\vec{v}$ to the floating platform of the observed object.

In that case, the inertial mass M relates to the test mass M_0 as

$$M = \gamma M_0 = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19.18.1}$$

This relation indicates that the formula (19.14.2) for the deformation potential at distance r, must be changed to

$$V(r)=\frac{M_0 G}{r\sqrt{1-\frac{v^2}{c^2}}} \tag{19.18.2}$$

19.19 Inertia

The relation between inertia and mass is complicated. We apply an artificial field that resists its change. The condition that for each type of massive object, the deformation potential is a static function and that in free space, the massive object moves uniformly shows that inertia rules the situation's dynamics. These conditions define an artificial quaternionic field that resists change. The scalar part of the artificial field that the deformation potential presents and the massive object's uniform speed represents the field's spatial part.

The first-order change of the quaternionic field divides into five separate partial changes. Some of these parts can compensate for each other.

Mathematically, the statement that in the first approximation, nothing in the field ξ changes indicates that, locally, the first-order partial differential $\nabla\xi$ will equal zero;

$$\zeta=\nabla\xi=\nabla_r\xi_r-\left\langle\vec{\nabla},\vec{\xi}\right\rangle+\vec{\nabla}\xi_r+\nabla_r\vec{\xi}\pm\vec{\nabla}\times\vec{\xi}=0 \tag{19.19.1}$$

Thus;

$$\zeta_r=\nabla_r\xi_r-\left\langle\vec{\nabla},\vec{\xi}\right\rangle=0 \tag{19.19.2}$$

$$\vec{\zeta}=\vec{\nabla}\xi_r+\nabla_r\vec{\xi}\pm\vec{\nabla}\times\vec{\xi}=0 \tag{19.19.3}$$

These formulas interpret independently. For example, according to the equation (19.19.2), the variation in time of ξ_r can compensate the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, we expect the curl $\vec{\nabla}\times\vec{\xi}$ of the spatial field $\vec{\xi}$ to be zero. The resulting terms of the equation (19.19.3) are

$$\nabla_r\vec{\xi}+\vec{\nabla}\xi_r=0 \tag{19.19.4}$$

In the following text plays $\vec{\xi}$ the role of the spatial field and ξ_r plays the role of the scalar deformation potential of the considered object. For elementary modules, this field concerns the effect of the hop landing location swarm on the floating platform on its image in the embedding field. It reflects the activity of the stochastic process and the uniform movement of the geometric center of the floating platform over the embedding field in the background platform. A mass value and the uniform velocity of the floating platform relative to the background platform characterize it. The real (scalar) part conforms to the deformation that the stochastic process causes. The spatial part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change at zero. The author calls ζ the ***conservation field***.

At a considerable distance r, we approximate this potential by using the formula;

$$\zeta_r(r)\approx\frac{GM}{r} \tag{19.19.5}$$

Here M is the inertial mass of the object that causes the deformation. The new artificial field $\xi=\left\{\frac{GM}{r},\vec{v}\right\}$ considers a uniformly moving mass as a normal situation. It is a combination of scalar potential $\frac{GM}{r}$ and speed $\vec{v}$. This movement speed is the relative speed between the floating and background platforms. At rest, this speed is uniform.

If this object accelerates, the new field $\left\{\frac{GM}{r},\vec{v}\right\}$ tries to counteract the change of the spatial field $\vec{v}$ by compensating this with an equivalent change of the scalar part $\frac{GM}{r}$ of the new field ξ . According to the equation (19.19.4), this equal change is the gradient of the real part of the field;

$$\vec{a}=\dot{\vec{v}}=-\vec{\nabla}\left(\frac{GM}{r}\right)=\frac{GM\,\vec{r}}{|\vec{r}|^3} \qquad (19.19.6)$$

The shown generated spatial field acts on masses that appear in its realm.

Thus, if two uniformly moving masses m and M exist in each other's neighborhood, then any disturbance of the situation will cause the deformation force;

$$\vec{F}\left(\vec{r}_1-\vec{r}_2\right)=m_0\vec{a}=\frac{Gm_0M\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^3}=\gamma\frac{Gm_0M_0\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^3} \qquad (19.19.7)$$

Here $M=\gamma M_0$ is the inertial mass of the object that causes the deformation. m_0 is the rest mass of the observer.

The inertial mass M relates to its rest mass M_0 as;

$$M=\gamma M_0=\frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (19.19.8)$$

This formula holds for all elementary particles except for quarks.

The problem with quarks is that these particles do not supply an isotropic symmetry difference. They must first combine into hadrons to generate an isotropic symmetry difference. This phenomenon is known as ***color confinement***.

19.20 Momentum

In the formula (19.19.7), the factor γ that corrects for the relative speed relates mass to force, can be attached to m_0 or to M_0;

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = \gamma \frac{G m_0 M_0 \left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{19.20.1}$$

The force relates to the temporal change of the momentum vector $\vec{P}$ of the observer;

$$\vec{F} = \dot{\vec{P}} = \frac{d\vec{P}}{dt} \tag{19.20.2}$$

The momentum vector $\vec{P}$ is part of a quaternionic momentum P. The momentum depends on the relative speed of the moving object that causes the deformation, which defines the mass. The speed measures relative to the field that embeds the investigated object. The object deforms the field. For free elementary particles, the speed equals the floating speed of the platform on which the particle exists.

$$P = P_r + \vec{P} \tag{19.20.3}$$

$$\|P\|^2 = P_r^2 + \left\|\vec{P}\right\|^2 \tag{19.20.4}$$

$$\vec{P} = \gamma m_0 \vec{v} \tag{19.20.5}$$

$$\left\|\vec{P}\right\|^2 = \gamma^2 m_0^2 \|\vec{v}\|^2 \tag{19.20.6}$$

$$\|P\|^2 = \gamma^2 m_0^2 c^2 = P_r^2 + \gamma^2 m_0^2 \|\vec{v}\|^2 \tag{19.20.7}$$

$$\|P\| = \gamma m_0 c = E / c \tag{19.20.8}$$

$$E = \gamma m_0 c^2 \tag{19.20.9}$$

$$\begin{aligned} P_r^2 &= \gamma^2 m_0^2 c^2 - \gamma^2 m_0^2 \|\vec{v}\|^2 \\ &= \gamma^2 m_0^2 \left(c^2 - \|\vec{v}\|^2\right) = \gamma^2 m_0^2 c^2 \left(1 - \left\|\frac{\vec{v}}{c}\right\|^2\right) = m_0^2 c^2 \end{aligned} \tag{19.20.10}$$

$$P_r = m_0 c = \frac{E}{\gamma c} \tag{19.20.11}$$

$$\|\vec{P}\| = \gamma m_0 \|\vec{v}\| \tag{19.20.12}$$

$$P = P_r + \vec{P} = m_0 c + \gamma m_0 \vec{v} = \frac{E}{\gamma c} + \gamma m_0 \vec{v} \tag{19.20.13}$$

If $\vec{v} = \vec{0}$ then $\vec{P} = \vec{0}$ and $\|P\| = P = P_r = m_0 c$

Here Einstein’s famous mass-energy equivalence is involved.

$$E = \gamma m_0 c^2 = mc^2 \tag{19.20.14}$$

The disturbance by the ongoing expansion of the embedding field suffices to put the deformation force into action. So, the description also holds when the field ξ describes a conglomerate of platforms and M stands for the mass of the conglomerate.

The artificial field ξ represents the underlying model’s habit that ensures the constancy of the deformation potential and the uniform floating of the considered massive objects in free space.

Inertia ensures the minimization of the field's deformation of the third-order differential (the third-order change). It does that by varying the speed of the platforms on which the massive objects exist.

Inertia arises from the definition of mass that applies to the region outside the sphere where the deformation potential behaves like the green's function of the field.

There, the formula $\xi_r = \frac{GM}{r}$ applies.

Further, it shows the modules' intention to keep the deformation potential inside the sphere constant. At least, that holds when this potential averages over the regeneration period. In that case, the overall change $\nabla\xi$ in the conservation field ξ equals zero. Next, the definition of the conservation field supposes that the swarm which causes the deformation moves as one unit. Further, the model uses that the solutions of the homogeneous second-order partial differential equation can superimpose solutions of that same equation.

The popular sketch in which smooth dips show the deformation of our living space is false. This paper's story shows the deformations as local extensions of the field, which stands for the universe. In both sketches, the deformations elongate the information path, but none explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the deformation potential, even when that averaged potential moves uniformly. Without the described practice of stochastic processes, inertia would not exist.

The applied artificial field also explains the deformation attraction by black holes.

The artificial field that implements mass inertia also plays a role in other fields. For example, similar tricks can explain the electrical force from the fact that sources and sinks that the green's function describes producing the electrical field.

19.20.1 Forces

In the system of separable Hilbert spaces, all symmetry-related charges exist at the geometric center of an elementary particle, and all these particles own a footprint that can deform the embedding field for isotropic symmetry differences. In that case, the particle features mass and forces might couple to acceleration via;

$$F = m\vec{a} \tag{19.20.15}$$

Or to momentum via;

$$F = \dot{\vec{P}} \tag{19.20.16}$$

20 Postscript

20.1 The initiator of the project

The Hilbert Book Model Project is ongoing. Hans van Leunen is the initiator of this project. The initiator was born in the Netherlands in 1941. He will not live forever. This project will hold his scientific inheritance.

A Wikiversity project introduced the Hilbert Book Model [35]. In the opinion of the initiator, a Wikiversity project is a perfect way of teaching new science. It primarily serves the needs of independent or retired scientific authors.

The initiator supports a ResearchGate project that considers the Hilbert Book Model Project [36]. In addition, the ResearchGate site supports a flexible way of discussing scientific subjects [37].

The initiator has generated documents that hold highlights as excerpts of the project, and he stored these papers in his personal e-print archive [21]

As long as I can keep it online, the private website http://www.e-physics.eu holds most documents in pdf and docx format. None of these documents claims the copyright. Therefore, everybody is free to use the content of these papers.

20.2 Trustworthiness

Introducing new science always introduces controversial and unorthodox text. The Hilbert Book Model Project is an ongoing enterprise. The author regularly revises its dynamic content.

The author does not make available a peer review of the content of this project. It is the author's task to ensure the correctness of what he writes. In the author's vision, the readers must check the validity of what they read. The peer review process cannot cope with the dynamics of revisions and extensions that become possible via publishing in freely accessible e-print archives. Compared to openly accessible publications on the internet, the peer review process is slow. In addition, it inhibits the usage of revision services, such as those offered by vixra.org and arxiv.org/

Reviewers are biased, and they are never omniscient. Moreover, the peer review process is expensive and often poses barriers to the renewal of science.

One way to check the validity of the text is to expose parts of the text to open scientific discussion sites such as ResearchGate.net.

The initiator challenges everybody to disprove the statements made in this report. He promises a fine XO cognac bottle to anyone who finds a significant flaw in the presented theories.

This challenge stands already for years. Up so far, nobody claimed the bottle [38].

20.3 The author

Hans was born in Helmond in 1941 and visited the Eindhoven HTS in chemistry from 1957-1960.

After his military service from 1960-1963, Hans started at the Technical Highschool Eindhoven (THE) to study applied physics. The university's name changed to the Technical University Eindhoven (TUE).

Hans finished this study in 1970 and then joined Philips Elcoma EOD in developing image intensifier tubes. Later this became a department of Philips Medical Systems division. Hans established the standard for measuring and specifying the Optical Transfer Function for STANAG, ISO, IEC, and DIN as part of his job. He also contributed to the standard for measuring and defining the Detective Quantum Efficiency for IEC and DIN.

In 1987 Hans switched to an internal software house. In 1995 Hans joined the Semiconductor division of Philips. During this period, Hans designed a system for modular software generation.

In 2001 Hans retired.

From 1983 until 2006, Hans owned a software company "Technische en Wetenschappelijke Programmatuur" (TWP).

A private website treats my current activities [39].

I store my papers in a freely accessible e-print archive [21].

To investigate the foundations and the lower levels of physical_reality, Hans started 2009 a personal research project that in 2011 got its current name, "The Hilbert Book Model Project."

The Hilbert Book Model is a purely mathematical unorthodox, and controversial model of the foundations and the lower levels of the structure of physical_reality.
Hans’s motto: If you think, then think twice.

Hans’s conviction: We live in a universe that recurrently renews its content at a high regeneration rate.

20.4 Early encounters

I am born with a deep curiosity about my living environment. When I became aware of this, I was astonished why this environment appeared so complicated, and at the same time, it behaved coherently. In my childhood, I had no idea. Later unique experiences offered me indications. After my retirement, I started 2009 a personal research project to discover and formulate the clues.

My interest in the structure and phenomena of physical_reality started in the third year of my physics study when the configuration of quantum mechanics confronted me for the first time with its extraordinary approach. The fact that its method differed fundamentally from the way that physicists did classical mechanics astonished me. So, I asked my wise lecturer, Professor Broer, what origin this difference relies upon. He answered that the superposition principle caused this difference. I was not happy with this answer because the superposition principle was indeed part of the method of quantum mechanics. Still, in those days, I did not understand how that could present the leading cause of the difference between the two methodologies. So, I decided to dive into literature, and after a search, I met the booklet of Peter Mittelsteadt, “Philosophische Probleme der modernen Physik” (1963). This booklet held a chapter about quantum logic that appeared to maintain a more proper answer. Later, this seemed a far too quick conclusion. In 1936 Garrett Birkhoff and John von Neumann published a paper describing their discovery of “quantum logic.”.

Since then, mathematical terminology has known the discovered quantum logic as an orthomodular lattice [9]. This lattice's relational structure is quite like the relational structure of classical logic. That is why the duo called the orthomodular lattice “quantum logic.” This name was an unlucky choice because no good reason exists to consider the orthomodular lattice as a system of logical propositions. In the same paper, the duo showed that the set of closed subspaces of a separable Hilbert space has exactly the relational structure of an orthomodular lattice. John von Neumann long doubted between Hilbert spaces and projective geometries. Ultimately, he selected Hilbert spaces as the best platform for developing quantum physical theories. That appears to be one of the main reasons quantum physicists prefer Hilbert spaces as a realm in which they model quantum physical systems. Another habit of quantum physicists also intrigued me. My lecturer taught me that all observable quantum physical quantities are eigenvalues of Hermitian operators. Hermitian operators feature real eigenvalues. When I looked around, I saw a world with a structure configured from a three-dimensional spatial domain and a one-dimensional and, thus, scalar time domain. In the quantum physics of that time, no operator stands for the time domain, and no operator delivers the three-dimensional spatial domain compactly. After some trials, I discovered a four-dimensional number system that could provide a suitable normal operator with an eigenspace representing my living environment's full four-dimensional representation. At that moment, I had not yet heard from quaternions. Still, an assistant professor Boudewijn Verhaar quickly told me about the discovery of Rowan Hamilton that happened more than a century earlier. Quaternions are the number system of choice for offering the structure of physical_reality its powerful capabilities.

The introductory paper of Birkhoff and von Neumann already mentioned quaternions. Much later, Maria Pia Solèr offered hard proof that Hilbert spaces can only cope with members of an associative division ring. Quaternions form the most versatile associative division ring. To my astonishment, I quickly discovered that physicists preferred a spacetime structure with a Minkowski signature instead of the Euclidean signature of the quaternions. The devised Hilbert Book Model shows that in physical_reality, the Euclidean and spacetime structures appear parallel. Observers only see the spacetime structure. Physics is a science that focuses on observable information. My university, the TUE, targeted applied physics, and there was not much time nor support for diving deep into the fundamentals of quantum physics. After my study, I started a career in the high-tech industry, where I joined the development of image intensifier devices. There followed my confrontation with optics and with the actual behavior of elementary particles.

In the second part of my career, I devoted my time to establishing a better way of generating software. I saw how the industry was booming in the modular construction of hardware. However, the software still developed as a monolithic system. My experiences in this trial appear in the paper "Story of a War Against Software Complexity;" and the report "Managing the Software Generation Process" [21]. It taught me the power of modular design and modular construction.

Only after my retirement, I got enough time to dive deep into the foundations of physical_reality. In 2009 after my recovery from severe disease, I started my research project that in 2011 got its current name, "The Hilbert Book Model." For the rest of his life, the author takes the freedom to upgrade the related papers at a steady rate.

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