The Čerenkov Radiation from Dipole and the Lorentz Contraction

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Abstract

The power spectral formula of the Čerenkov radiation of the system of two opposite charges is derived in the framework of the source theory. The distance between charges is supposed to be relativistically contracted which manifests in the spectral formula. The knowledge of the spectral formula then can be used to verification of the Lorentz contraction of the relativistic length of dipole. A feasible experiment for the verification of the dipole contraction is suggested.

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1 Introduction

The Čerenkov radiation of magnetic and electric dipoles is of interest even though the dipole radiation for individual particles (neutron, electron) is very weak. We present a method of calculation which is somewhat different from that used in literature (Ginzburg et al., 1959).

A physical dipole consists of two equal and opposite point charges. Its field at large distances (i.e., distances large in comparison to the separation of the poles) depends almost entirely on the dipole moment. A point (electric) dipole is the limit obtained by letting the separation tend to 0 while keeping the dipole moment fixed. The field of a point dipole has a particularly simple form, and the order-1 term in the multipole expansion is precisely the point dipole field.

The power spectral formula of the Čerenkov radiation of the system of two opposite charges (dipole) is here derived in the framework of the source theory. The distance between charges is supposed to be relativistically contracted which manifests in the spectral formula. The knowledge of the spectral formula then can be used to verification of the Lorentz contraction of the relativistic length of dipole.

2 The Čerenkov effect

The fast moving charged particle in a medium when its speed is faster than the speed of light in this medium produces electromagnetic radiation which is called the Čerenkov radiation.

The prediction of Čerenkov radiation came long ago. Heaviside (1889) investigated the possibility of a charged object moving in a medium faster than electromagnetic waves in the same medium becomes a source of directed electromagnetic radiation. Kelvin (1901) presented an idea that the emission of particles is possible at a speed greater than that of light. Somewhat later, Sommerfeld (1904) proposed the hypothetical radiation with a sharp angular distribution. However, in fact, from experimental point of view, the electromagnetic Čerenkov radiation was first observed in the early 1900's by experiments developed by Marie and Pierre Curie when studying radioactivity emission. In essence they observed the emission of a bluish-white light from transparent substances in the neighborhood of strong radioactive source. But the first attempt to understand the origin of this was made by Mallet (1926; 1929a; 1929b), who observed that the light emitted by a variety of transparent bodies placed close to a radioactive source always had the same bluish-white quality, and that the spectrum was continuous, with no line or band structure characteristic of fluorescence. Unfortunately, these investigations were forgotten for many years. Čerenkov experiments (Čerenkov, 1934) was performed at the suggestion of Vavilov who opened a door to the true physical nature of this effect (Bolotovskii, 2009).

This radiation was first theoretically interpreted by Tamm and Frank (1937) in the framework of the classical electrodynamics. The source theoretical description of this effect was given by Schwinger et al. (1976) at the zero temperature regime and the classical spectral formula was generalized to the finite temperature situation in the framework of the source theory by author (Pardy, 1989). The similar problems was solved and published by author in some articles (Pardy, 1983; 2000b; 2002a; 2004).

The question arises, what is the relation of the Čerenkov radiation to the relativistic length. The relativistic length can be formed by the system of charges of the linear chain, or, only by the two charges of the rest distance l. The problem of the radiation of the composed systems of charges is not new and it was defined for the first time in the pioneering work of Ginzburg (1940). Later by Frank (1942; 1946), it was given the solution of the problem of the Čerenkov radiation of the electrical and magnetic dipoles oriented parallel and perpendicularly to the direction of motion. While the parallel orientation gives no surprising result the situation with the perpendicular orientation gives special anomaly which has been frequently discussed in the physical journals. In year 1952 was published the article discussing the Čerenkov radiation of the arbitrary electrical and magnetic multipoles (Frank, 1952). The review of the problems of the Čerenkov radiation of the magnetic and electrical multipoles was given by Frank (1984). The extensive work concerning the radiation by uniformly moving sources is elaborated (Ginzburg, 1986). However, the problem of testing the Lorentz contraction by Čerenkov effect is here considered for the first time (Pardy, 1997a; Cavalleri, 2000).

While the original articles on the Čerenkov radiation involve only determination of the spectral formula, it is possible interest the question on the relationship between the spectral formula and the Lorentz contraction of the length of some linear object. The specific situation forms the system of two equal or opposite charges of the rest distance l. Then, we can expect that the spectral formula of the Čerenkov radiation involves the Lorentz contraction which follows immediately from the Lorentz transformation for coordinates between systems S' and S:

$$x' = \gamma(x - vt); \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{1}$$

where x are coordinates in the system S and x' are corresponding coordinates in the system S' which is moving with velocity v relative to the system S. If the left and right points of the moving rod are x_1, x_2 in the system S and x'_1 and x'_2 in the system S', then from equation (1) we have:

$$x'_2 - x'_1 = \gamma(x_2 - x_1), \tag{2}$$

which can be transcribed in the form

$$a = l\sqrt{1 - v^2/c^2}, \tag{3}$$

where l is the rest length of the rod and a is the length of the moving rod.

The formula (3) is well known and there was general belief since the formulation of the special theory of relativity by Einstein that the so called Lorentz contraction (3) should be visible to the eye. Also Lorentz stated in 1922 that the contraction could be photographed. Similar statements appear in other references concerning the special theory of relativity.

However, the special theory of relativity predicts that the contraction can be observed by a suitable experiment with the nuance that there is distinction between observing and seeing. The situation was analyzed for instance by Terrell (1959) and Weisskopf (1960) and others (Dreissler, 2005), who proved that the photograph obtained by an observer depends only on the place and time of taking the picture and is independent of the relative motion of observer and object photographed.

It would be incorrect to state that we see the length contraction, or, that the length "appears" to be contracted by the factor $\sqrt{1 - v^2/c^2}$. As first pointed out by Lampa (1924) and later by Penrose (1959), Terrell (1959) and Weisskopf (1960) what one sees and how an object appears are very different from what is given by the Lorentz contraction. The reason is that various parts of the object are different distances from the observer, and in order for the light rays from the various parts to arrive at the observer at the same time, they must have left the object at different times. It follows from the special theory of relativity that the length contraction is the result of the measurement procedure and the time dilation is also the measurement procedure as was shown by Fok (1961) and author (Pardy, 1969).

If the Fok interpretation of the relativistic measurement is correct, then it involves the Lorentz contaction as the measurement procedure. Such statements are not involved in the Einstein publications (Einstein, 1916, 1919).

In other words, an observation of the shape of a fast moving object involves simultaneous measurement of the position of a number of points on the object. If done by means of light, all the quanta should leave the surface simultaneously, as determined in the observer position at different times. In such observation the data received must be corrected for the finite velocity of light, using measured distances to various points of the moving object. In seeing the object, on the other hand, or photographing it, all the light quanta arrive simultaneously at the eye having departed from the object at various earlier times. In such a way this should make a difference between contracted shape which is in principle observable and the actual visual appearance of a fast-moving object. The photograph of a relativistically moving object with a camera using, instead of photons, particles moving much faster than the velocity of light, eliminates the non-desired optical effects and the film would show the object shortened by a factor of $\sqrt{1 - v^2/c^2}$. in the direction of motion. However, such a camera is not physically possible, and we can ask how to correct for the optical effects so that only the relativistic effects will be observed on a photograph taken by an ordinary camera.

Obviously, the Čerenkov radiation of the charged two-particle system involves the Lorentz contraction of their rest distance. We will consider the system of two equal charges e which have the mutual rest distance l. The Lorentz contraction will be involved in the power spectral formula for this linear system.

In this article we evaluate in source theory the power spectral formula of the Čerenkov radiation of the dipole moving with velocity v in the dielectric medium. Radiative corrections to this dipole Čerenkov radiation are not considered. In conclusion, a feasible experiment is suggested for the verification of the Lorentz contraction.

3 The field formulation of the problem

Source theory (Schwinger, et al. 1976; Schwinger, 1970; Dittrich, 1978) is the theoretical construction which uses quantum-mechanical particle language. Initially it was constructed for description of the particle physics situations occurring in the high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by photon or graviton respectively.

The basic formula in the source theory is the vacuum to vacuum amplitude (Schwinger, et al. 1976):

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W(S)}, \tag{4}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add (Schwinger, 1970; Dittrich, 1978).

The electromagnetic field is described by the amplitude (4) with the action

$$W(J) = \frac{1}{2c^2}\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{5}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities. The symbol $D_{+\mu\nu}(x-x')$, is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger, et al. 1976):

$$| < 0_+|0_- > |^2 = \exp\{-\frac{2}{\hbar}\mathrm{Im}\, W\} \stackrel{d}{=} \exp\{-\int dt d\omega \frac{P(\omega,t)}{\hbar\omega}\}, \tag{6}$$

where we have introduced the so called power spectral function (Schwinger, et al. 1976) $P(\omega,t)$. In order to extract this spectral function from $\mathrm{Im}\, W$, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x-x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\phi,\mathbf{A})$ and it is generated by the four-current $J^\mu(c\varrho,\mathbf{J})$ according to the differential equation (Schwinger, et al. 1976):

$$(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2})A^\mu = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu)J_\nu \tag{7}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_{+}^{\mu\nu} = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^{\mu}\eta^{\nu})D_{+}(x-x'), \tag{8}$$

where $\eta^{\mu} \equiv (1, \mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_{+}(x-x')$ was derived by Schwinger et al. (1976) in the following form:

$$D_{+}(x-x') = \frac{i}{4\pi^2 c}\int_{0}^{\infty} d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{9}$$

Using formulas (5), (6), (8) and (9), we get for the power spectral formula the following expression (Schwinger et al., 1976):

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]\times$$

$$\times\left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\}. \tag{10}$$

Now, we are prepared to apply the last formula to the situations of the two equal charges moving in the dielectric medium.

4 The Čerenkov radiation of the dipole

It is usually supposed that the Čerenkov radiation in electrodynamics is produced by uniformly moving charge with the constant velocity. Here we consider the system of two particles with the oposite charges e with the constant mutual distance $a = |\mathbf{a}|$ moving with velocity $\mathbf{v}$ in dielectric medium. We follow the author articles (Pardy, 1997a; 2007).

In this situation the charge and the current densities for this system are given by the by the following equations:

$$\varrho = -e\delta(\mathbf{x}-\mathbf{v}t) + \delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t) \tag{11}$$

$$\mathbf{J} = -e\mathbf{v}\delta(\mathbf{x}-\mathbf{v}t) + e\mathbf{v}\delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t). \tag{12}$$

where $\mathbf{a}$ is the vector going from the left charge to right charge with the length of $a = |\mathbf{a}|$ in the system S.

Let us suppose that $\mathbf{v} \parallel \mathbf{a} \parallel x$. Then, after insertion of eq. (11) and (12) into eq. (10), putting $\tau = t'-t$, and $\beta = v/c$, where $v = |\mathbf{v}|$, we get instead of the formula (10) the following relation:

$$P(\omega,t) = 2P_1(\omega,t) - P_2(\omega,t) - P_3(\omega,t), \tag{13}$$

where

$$P_1(\omega,t) = \frac{1}{4\pi^2}\frac{e^2\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]\int_{-\infty}^{\infty} d\tau\frac{\sin n\omega\beta\tau}{\tau}\cos\omega\tau \tag{14}$$

$$P_2(\omega,t) = \frac{1}{4\pi^2}\frac{e^2\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]\int_{-\infty}^{\infty} d\tau\frac{\sin n\omega\beta|\frac{a}{v}+\tau|}{|\frac{a}{v}+\tau|}\cos\omega\tau \tag{15}$$

$$P_3(\omega,t) = \frac{1}{4\pi^2}\frac{e^2\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]\int_{-\infty}^{\infty} d\tau \frac{\sin n\omega\beta|\frac{a}{v}-\tau|}{|\frac{a}{v}-\tau|}\cos\omega\tau. \tag{16}$$

The formula (14) contains the known integral:

$$J_1 = \int_{-\infty}^{\infty} d\tau \frac{\sin n\omega\beta\tau}{\tau}\cos\omega\tau = \left\{ \begin{array}{ll} \pi; & n\beta > 1 \\ 0; & n\beta < 1 \end{array} \right. . \tag{17}$$

Formulas (15) and (16) contain the following integrals:

$$J_2 = \int_{-\infty}^{\infty} d\tau \frac{\sin n\omega\beta|\frac{a}{v}+\tau|}{|\frac{a}{v}+\tau|}\cos\omega\tau \tag{18}$$

and

$$J_3 = \int_{-\infty}^{\infty} d\tau \frac{\sin n\omega\beta|\frac{a}{v}-\tau|}{|\frac{a}{v}-\tau|}\cos\omega\tau. \tag{19}$$

Using the integral (17) we finally get the power spectral formula P_1 of the produced photons:

$$P_1(\omega,t) = \frac{e^2}{4\pi}\frac{\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]; \quad n\beta > 1 \tag{20}$$

and

$$P_1(\omega,t) = 0; \quad n\beta < 1. \tag{21}$$

Using transformations

$$\frac{a}{v}+\tau = T, \quad \frac{a}{v}-\tau = T, \tag{22}$$

we get after evaluations of the corresponding integrals J_2, J_3 the corresponding spectral formulas P_2, P_3:

$$P_2(\omega,t) = \frac{e^2}{4\pi}\frac{\mu\omega}{c^2}\cos\left(\frac{\omega a}{v}\right)v\left[1-\frac{1}{n^2\beta^2}\right] = P_3; \quad n\beta > 1 \tag{23}$$

and

$$P_2(\omega,t) = P_3(\omega,t) = 0; \quad n\beta < 1. \tag{24}$$

The sum of the partial spectral formula form the total radiation emitted by the Čerenkov mechanism of the two-charge system:

$$P(\omega,t) = 2(P_1-P_2) = \sin^2\left(\frac{a\omega}{2v}\right)\frac{e^2}{4\pi}\frac{\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]; n\beta > 1 \tag{25}$$

and

$$P(\omega,t) = 0; \quad n\beta < 1. \tag{26}$$

The zero point of function $P(\omega,t)$ are as follows:

$$\omega_0 = 0; \quad \frac{\omega_n a}{2v} = n\pi; \quad n = 1,2,3,\ldots. \tag{27}$$

From the last equation follows

$$a = \frac{2v}{\omega} = l\sqrt{1 - \frac{v^2}{c^2}}, \tag{28}$$

or,

$$l = \frac{2\pi v}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{1}{\omega}. \tag{29}$$

If we know the n-th and m-th zero points with the corresponding ω-s and velocity of the charges we can exactly determine their rest distance. Then, the rest distance determined by the formula (29) can be compared with the rest distance of the charges obtained by direct measurement and in such a way we can verify the Lorentz contraction.

5 A feasible experiment for dipole

While the simultaneous acceleration of the system of the two equal charges can be performed immediately in every laboratory with the circle accelerator, the simultaneous acceleration of the system of two opposite charges can be performed only with the laser accelerator (Pardy, 1997b, 2000b, 2001, 2002b, 2003a). In this equipment the opposite charges are accelerated at the same acceleration as a result of the Compton effect. It is not excluded that the successful acceleration can by performed by the magnetronic laser (Pardy, 2003b), or by the lasers with two beams (Pardy, 2005, 2006).

6 Discussion

We have demonstrated in the past (Pardy, 1997a) that in case of the system of two equal charges - double pole, the Lorentz contraction can be determined from the spectral formula of the Čerenkov radiation. Obviously this effect can be involved into the group of the classical relativistic effects. In case of the system of opposite charges, or, in other words, of the dipole we have instead of $\cos(\omega a/2v)$ function $\sin(\omega a/2v)$ in the final formula. To our knowledge the determination of the Lorentz contraction using the Čerenkov effect was not considered in theory and in experiment. After performing the experiment with the Čerenkov radiation of the system of the two charges it will be definitely confirmed the Lorentz contraction.

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