MNRAS **000**, 1–5 (2023) Preprint 13 May 2023 Compiled using MNRAS LATEX style file v3.0

The GW of the Vela Pulsar and the Receiving Pattern of the Livingston Interferometer

Herbert Weidner[1]★
[1]*Am Stutz 3, D-63864 Glattbach*

13 May 2023

ABSTRACT

After compensation for phase modulation and frequency drift, the pulsar's GW can be detected in the records of the interferometer Livingston. The signatures agree with the known values measured with electromagnetic waves. The measured amplitude modulation of the GW in the daily rhythm shows the directivity of the antenna.

Key words: Gravitational waves

1 INTRODUCTION

The Vela Pulsar probably generates continuous gravitational waves (GW), which could not be detected despite repeated searches with different methods. This is amazing because the LIGO interferometers are extremely sensitive and have recorded large amounts of data. The present study has for the first time succeeded in detecting the GW using standard methods of communications engineering.

This is also due to the fact that the frequency is precisely known from electromagnetic wave observations. Astronomers have been observing the pulsar, which spins about 11 times per second. In communications, very weak signals are always received using the same principle: Interfering noise is removed with narrowband filters. The bandwidth of the filter can only be selected to be particularly small if the signal frequency is constant. Unprepared, no GW meets this requirement, as some effects increase the minimum bandwidth:

- The Vela pulsar radiates energy and in 2020, the frequency $f_{GW} \approx 22.37$ Hz of the radiation decreases with $\dot{f}_{pulsar} = -3.11 \times 10^{-11}\ s^{-2}$. One must double the values given in Espinoza (2021) because for theoretical reasons $f_{GW} = 2 f_{spin}$ holds.
- As the interferometer orbits the axis of the Earth in 24 hours, the Doppler effect produces a small periodic frequency shift of $\Delta f \approx \pm 21\ \mu$Hz (equation (1)).
- The high orbital velocity of the Earth around the Sun (about 14 km s^{-1}) produces periodic frequency changes of about ± 1 mHz (Doppler effect). This frequency uncertainty can be reduced by choosing special observation periods.
- Occasionally and at irregular intervals, the pulsar changes its rotation frequency by several microhertz. Such events are unlikely to interfere if the investigation is limited to a few days.

The prospects of detecting the GW of the Vela pulsar improve if all known modulations are identified and eliminated to reduce the interfering noise by minimizing the signal processing bandwidth. The aim of this investigation is to prove the GW of the Vela pulsar and to measure the amplitude modulation in the diurnal rhythm.

★ E-mail: herbertweidner@gmx.de

2 THE OBSERVATION PERIOD

During a short period of a few days around February 23, the frequency change caused by the Earth's orbit is very small and changes proportionally to the time (NRAO). Since no sun interferes between the pulsar and the Earth during this period, there is no need to discuss whether the sun influences the propagation of GW.

A measurement period of 240 hours ensures a high frequency resolution for the following investigation. During this time, the Earth rotates several times around its axis and an interferometer at the zero meridian of the Earth ($\lambda = 0^o, \varphi = 0^o$) receives twice a day – at midnight and noon – the "true" frequency of the GW generated by the Vela pulsar. At these points in time, the frequency shift caused by the Doppler effect due to the Earth's rotation disappears and the signal amplitude reaches a maximum. If the antenna had an isotropic sensitivity, it would measure the maximum redshift at 6:00:00 UTC and the maximum blueshift at 18:00:00 UTC. According to the geographic position of the Livingston interferometer, the times of the extremes of daily redshift and blueshift are delayed by six hours (Figure 1) and the maximum signal amplitudes are different because the interferometer is not at the equator.

An interferometer is insensitive when the source of the GW is close to the horizon (node of the receiving pattern). But this is the best time period to measure the maximum daily frequency shift. Correspondingly, the declination of the GW source can be determined inaccurately. When receiving a GW, the position of the Sun does not matter. Therefore, the sidereal daylength of 23.93447192 hours applies in all calculations.

3 SIGNAL PROCESSING

The Vela pulsar's GW is searched for in a ten-day dataset, starting at 2020-02-20 at 01:14:22 UTC. This time span should be sufficient to measure the periodic frequency shift as a result of the rotation of the Earth. The small relative frequency change $\Delta f / f_{GW} \approx 10^{-6}$ (see equation (1)) is easier to measure after the signal frequency is greatly reduced. Therefore, one mixes f_{GW} with a locally gener-

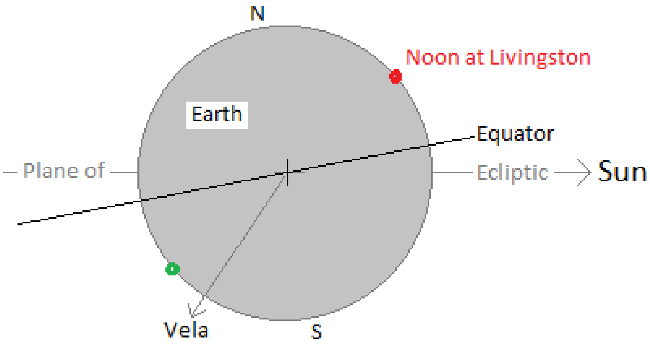

Figure 1. View of the Pacific on February 23 at 18:00 UTC. The red dot marks the position of the Livingston interferometer. The upper-right *zenith main lobe* only receives signals from undesired GW sources. The opposite *nadir main lobe* (towards the green dot) points close to the Vela pulsar and receives a particularly strong signal during a period of time because the earth is transparent to GW.

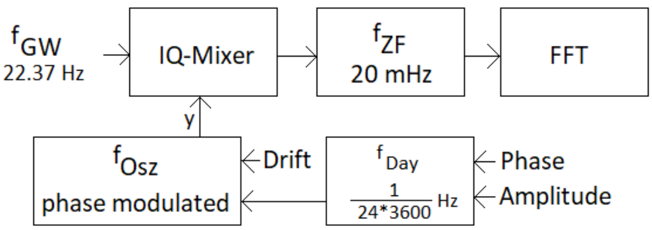

Figure 2. Principle of the MSH method: The parameters for frequency drift and phase modulation of the auxiliary oscillator are iterated until the amplitude of the spectral line at f_{ZF} reaches a maximum. Only a narrow frequency range around f_{ZF} is evaluated (see figure 4). f_{ZF} has no sideband frequencies because the MSH process has removed the modulations from f_{GW}.

ated frequency f_{Osz} (Figure 2) to produce a lower frequency f_{ZF} (heterodyning). For the frequencies, $f_{ZF} = |f_{GW} - f_{Osz}|$ is valid.

Usually, the value f_{Osz} is constant in order not to modify the modulation content of the signal. In the search for GW, the opposite is true: one must remove the known but unwanted phase modulation (PM) in order to reduce the bandwidth. This leads to the development of the "Modified SuperHet" (MSH): One modulates the frequency f_{Osz} with the goal of obtaining a *constant* diffence frequency f_{ZF} (figure 3). When the modulation of the received signal and the oscillator coincide, the "picket fence-like" spectrum of a PM signal turns into a single high-amplitude spectral line. Illustratively speaking: The many spectral lines inside the bandwidth are rearranged so that they add up to a large total length. The MSH method does not require spectral decomposition of the GW signal as an intermediate step; it combines the sidebands of *one* signal to produce a single, strong spectral line. Neighboring signals are distorted. A detailed description of the MSH method and the operating instructions can be found in Weidner (2023).

An analysis shows the advantages of reducing the signal bandwidth by the MSH method: the daily rotation of the interferometers around the earth axis with $f_{day} = 11.6\ \mu$Hz produces a phase modulation with the peak frequency-deviation

$$\Delta f = f_{GW}\left(\sqrt{\frac{c + v_{equator}}{c - v_{equator}}} - 1\right)\cos(\varphi)\cos(\delta) = 21.1\ \mu\text{Hz} \qquad (1)$$

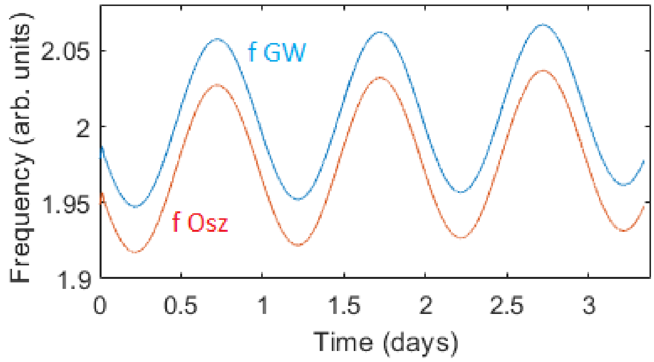

Figure 3. The idea behind the MSH method: The frequency of the GW oscillates around an average value that slowly increases. If one succeeds to generate an auxiliary frequency f_{Osz} with identical modulation, the differential frequency $f_{GW} - f_{Osz}$ (= vertical distance between the two curves) is constant.

The variables mean:
c is the propagation velocity of the GW
$v_{equator} = 464$ m s^{-1}
$\varphi = 30.6^o$ is the geographic latitude of the antenna
$\delta = -45^o$ is the declination of the GW source Vela pulsar

The aim of this study is to confirm the GW of the Vela pulsar with all properties. This requires that the phase modulated signal is processed with the Carson bandwidth *BW* to avoid distortions.

$$BW \geq 2(\Delta f + f_{day}) = 66\ \mu\text{Hz} \qquad (2)$$

The intermediate frequency f_{ZF} must be sufficiently high to allow this bandwidth. The wide range *BW* contains the seven major spectral lines that represent the spectrum of the Vela pulsar, noise and additional spectral lines produced by other, previously undetected pulsars. The MSH procedure compensates for the phase modulation and the frequency drift of *one* signal frequency. The seven spectral lines recombine to a single one and the bandwidth may be reduced from 66 μHz to about 1 μHz. The consequences:

Assuming that the signal processing bandwidth contains only device noise plus a single GW, the amplitude of the noise decreases by a factor $a_1 = \sqrt{66/1} \approx 8$.

The MSH process ensures that the energy content of the GW, which was previously distributed over seven spectral lines, is now concentrated in the *unmodulated* carrier frequency f_{ZF}. Therefore, the amplitude increases by a factor of $a_2 \approx 3$.

It is not necessary to identify, measure and recombine of the phases and amplitudes of about seven spectral lines in the noise.

Overall, the amplitude of the single spectral line at f_{ZF} may be $a_1 \cdot a_2 = 24$ times higher than the amplitude of the surrounding noise. Thus, MSH enables the detection and analysis of signals below the noise floor. To my knowledge, no comparable method has ever been used to remove phase modulation from a signal.

4 THE SENSITIVITY OF THE LIGO INTERFEROMETERS

The S/N determines the quality of the signal reception. The average noise amplitude A_{noise} depends on the inherent noise of the receiver, described by the PSD value, and the bandwidth BW of the receive channel.

$$A_{noise} = \sqrt{PSD \cdot BW} \quad (3)$$

The LIGO interferometers have PSD values of around 10^{-46} s Biscoveanu (2020). One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because then the necessary time T_{min} that the filter needs to settle down increases. This relationship was first formulated by Küpfmüller and is reminiscent of the Heisenberg uncertainty principle.

$$T_{min} \cdot BW \geq 0.5 \quad (4)$$

Each record of LIGO (LIGOdata) lasts 4096 seconds and limits the spectral resolution to 122 μHz. A comparison of this value with the result of the equation (1) shows that one-hour recordings are too short to detect phase modulation in the diurnal rhythm. A minimum recording duration 240 hours improves the frequency resolution to 0.6 μHz. Filtering the received data with this bandwidth, the noise floor is as low as 0.8×10^{-26} according to equation (3). Since rotating neutron stars are expected to have strains around 10^{-26} Riles (2017), the GW of strong sources should be visible in the records of the LIGO interferometers. The low frequency resolution does not allow to accurately measure the daily frequency modulation of ± 21 μHz. The frequent interruptions of the data recording additionally reduce the quality.

5 MATHEMATICAL MODELING

One goal of the investigation is to identify a signal in the records of the antenna which has the properties of the GW of the Vela pulsar. For this purpose, one generates an auxiliary frequency f_{Osz} and iterates drift and phase modulation until it agrees with the suspected GW (lower part of the figure 2). Then, f_{ZF} is constant and has especially large amplitude because the total energy of the GW is concentrated in a narrow frequency range. The auxiliary oscillation y is generated by the following Ansatz:

$$y = \sin(2\pi t(f_{GW} + f_{ZF} + \dot{f}t + A_{day} \cdot \sin(2\pi t f_{day} + \phi_{day}))) \quad (5)$$

This equation contains a second oscillator of frequency f_{day} to generate the PM in daily rhythm. The parameters have the following meaning:

5.1 Frequency of the GW (f_{GW})

The frequency of the Vela pulsar was measured several times at intervals of years. Assuming $f_{GW} = 2f_{spin}$, the approximation $f_{GW} = (22.5260377 - 2.65259655 \times 10^{-6} \cdot MJD)$ Hz holds. The initial value f_{GW} is calculated for the MJD of the data file and corrected in steps of 2 μHz until we find a high amplitude signal at f_{ZF} that has the expected properties of the GW of the pulsar.

5.2 Intermediate frequency of the MSH method (f_{ZF})

This value is arbitrary, but should be as small as possible so that the daily frequency changes $\Delta f / f_{ZF}$ can be easily seen. Minimum value is the necessary Carson bandwidth BW (equation (2)).

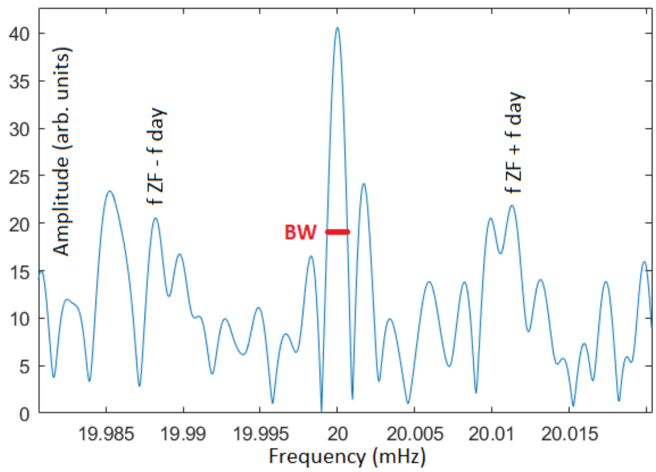

Figure 4. Spectrum of the environment of the GW of the Vela pulsar after removal of the phase modulation and drift and reduction of the frequency to 20 mHz. Only the narrow range BW is evaluated, corresponding to a final band-pass filter. The cause of the sidebands at $f_{ZF} - f_{day}$ and $f_{ZF} + f_{day}$ is discussed in section 6.

5.3 Frequency drift of GW ($\dot{f}$)

In addition to the inherent frequency drift of the Vela pulsar of $\dot{f}_{pulsar} = -3.11 \times 10^{-11}$ s^{-2}, the Doppler effect produces a time-proportional frequency shift of $\dot{f}_{orbit} = 22.44 \times 10^{-11}$ s^{-2} NRAO because of the Earth's orbit. The two components cannot be separated. The nominal value is $\dot{f}_{pulsar} + \dot{f}_{orbit} = +19.33 \times 10^{-11}$ s^{-2}.

5.4 Modulation index (A_{day})

Because of the Earth's rotation, the frequency of the pulsar changes daily by a maximum of ± 21 μHz (equation (1)). One models this variation by a sinusoidal PM. The initial value for the modulation index of the Vela pulsar is $A_{day} = \Delta f / f_{day} \approx 1.8$. Any deviation from the expected value indicates that the GW is *not* coming from the direction of the Vela pulsar.

5.5 Phase (ϕ_{day})

The GW of the Vela pulsar can only be detected with a targeted search because the narrow frequency range around f_{GW} apparently contains several GWs. From the known R.A. of the pulsar follows when the maximum frequency shift of the GW occurs. The point in time has to be kept during the iteration in order not to lose sight of the target pulsar. The initial value for ϕ_{day} is calculated from the start time of the analyzed chain of records.

6 MEASUREMENTS

The data source is created by concatenating 209 consecutive strain datasets from the Livingston interferometer, starting with $L1266196480$. Conspicuous glitches and data gaps are replaced by zeros. The additional information in the *hdf5* files is not required. To reduce computing time, the sampling frequency is reduced from 4096 Hz to 0.128 Hz.

In the first experiments, no signal with the characteristic properties of the Vela pulsar could be detected in a wide search range

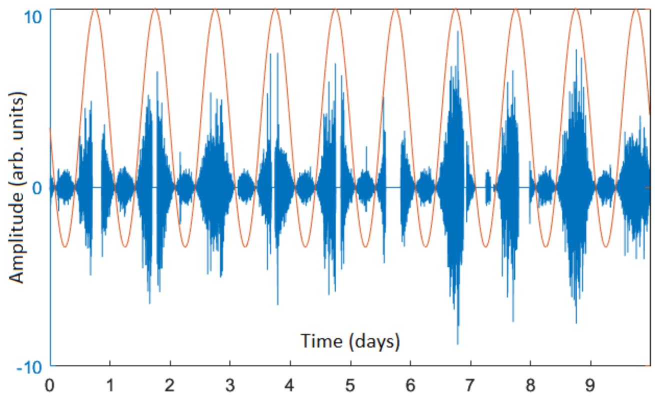

Figure 5. The red sine wave shows the receiver's programmed sensitivity as a function of time. The blue curve is the (mathematical) product of the antenna signal and the sensitivity function. The gaps show how often the interferometer is out of order.

$f_{GW} \pm 30$ μHz. One reason might be the high instrument noise of the interferometer in the range around 22 Hz. In addition, only the *nadir main lobe* of this antenna sometimes points in the direction of Vela and receives interference from other directions during the rest of the time. The *zenith main lobe* always points into wrong directions and increases the noise level. In order to suppress these disturbances as far as possible and to improve the S/N, a periodic time window is constructed: Whenever a main lobe points close to the pulsar (figure 1), you activate reception for several hours. Since a weaker signal with the opposite phase follows twelve hours later, reception is activated there too and the signal phase is inverted. The rest of the time you reduce the sensitivity to suppress unnecessary noise. Extensive tests have shown that the exact form of the sensitivity function (red curve in figure 5) hardly affects the result. With sufficiently wide time windows, the receiving pattern of the antenna is not distorted. Technically, the process is an amplitude modulation that creates two sidebands at $f_{ZF} - f_{day}$ and $f_{ZF} + f_{day}$. These are clearly recognizable in the spectrum, but do not interfere because the MSH method only evaluates the much narrower range $BW = f_{ZF} \pm 0.6$ μHz.

In the prepared signal (blue signal mixture in figure 5) one looks for the spectral line of the Vela pulsar using the MSH method. Figure 4 shows the resulting spectrum in which only the narrow red area around 20 mHz is of interest. This represents the modulation-free GW and corresponds to a filter with a bandwidth of 0.6 μHz. The surrounding area is a mixture of noise and distorted spectra of other GWs, which the interferometer cannot reject because of its low directivity. Some sidebands are probably caused by the frequent interruptions in the interferometer measurements, which act like digital modulation.

If you display the amplitude of f_{ZF} as a function of time, you get a little meaningful wavy line. Much more interesting is the representation as a function of the earth's rotation, which shows the receiving pattern of the antenna (figure 6). The GW of the Vela pulsar can obviously only be received by the *nadir main lobe*. It is noteworthy that the phase of the received GW signal changes by less than $\pm 3^o$ in the entire angular range $10^o < \varphi < 170^o$. Since the maximum of the amplitude is very wide ($\Delta\varphi \approx \pm 30^o$), the R.A. of the GW source can only be determined imprecisely.

Calculation of the strain. In order to measure the strain of the GW, one has to calibrate the vertical scale of figure 4. The *pwelch* function is used to calculate the area under a spectrum peak that exceeds the

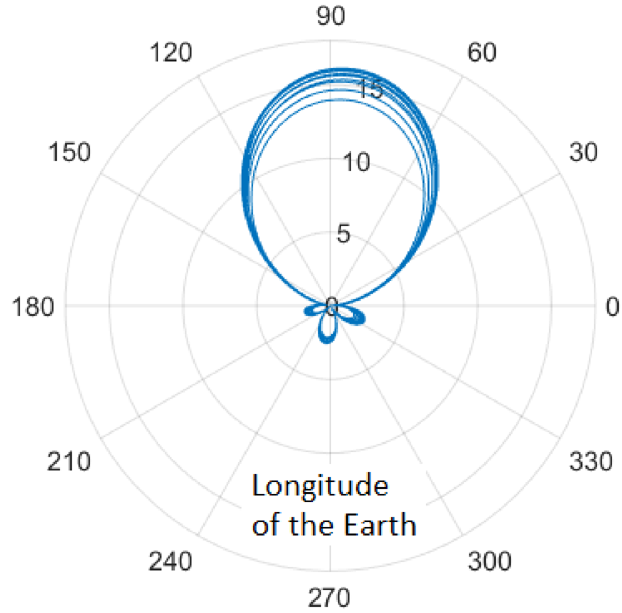

Figure 6. The *nadir main lobe* of the Livingston interferometer at 12 h (UTC). View of the earth from the north, the sun is far to the right. Six hours later, the *nadir main lobe* points to the Vela pulsar, which is far to the left. The *zenith main lobe* ($\varphi \approx 268^o$) is hardly usable. The globe rotates counterclockwise.

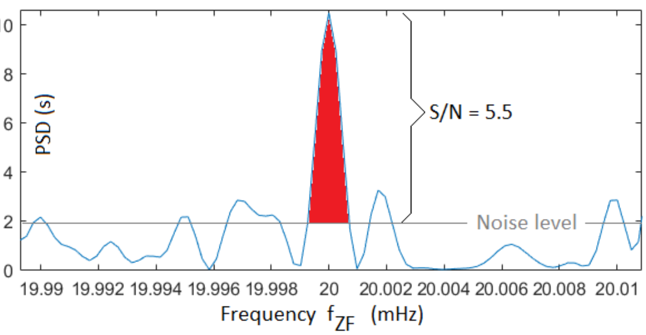

Figure 7. Welch transform of figure 4. The total energy of the GW is distributed over several bins due to the short total duration of the records. The PSD is therefore calculated from the area marked in red. The horizontal gray line marks the estimated noise level.

mean noise level (compare figure 2 in LIGO; Biscoveanu (2020)). The strain h_{GW} is calculated step by step:

The base of the red area in figure 7 is 1.465 μHz wide and corresponds to 6 bins. Each bin is $BW = 0.244$ μHz wide.

The noise level reaches the value of 1.9 s, corresponding to $PSD = 10^{-46}$ s of the interferometer.

The summed height of the red area is 29.3 s (below is just noise). With a significantly longer recording duration, the bandwidth would decrease. Then the GW would fit into a single bin and would have the height $PSD_{GW} = 15.4 \times 10^{-46}$ s.

With equation (3) we get for the strain of the GW:

$$h_{GW}(minimum) = \sqrt{PSD_{GW} \cdot BW} = 1.9 \times 10^{-26} \quad (6)$$

This result agrees well with previous estimates Riles (2017) and is a lower bound.

7 SUMMARY

Result of this investigation:

- The spectral line of the GW of the Vela pulsar is clearly identifiable if one hides the periods in which the *main lobes* of the antenna point into unfavorable directions.
- The *zenith main lobe* and the *nadir main lobe* receive the signal from the GW with opposite phases. Without synchronous phase reversal, the mean amplitude of the GW decreases.
- On 2020-02-20 (MJD=58899) $f_{GW} = 22.369782(4)$ Hz. This value deviates from the expected value by 0.9 ppm (section 5.1).
- The frequency drift is $\dot{f}_{pulsar} + \dot{f}_{orbit} = 1.858(1) \times 10^{-10} s^{-2}$. After subtracting $\dot{f}_{orbit}$, we get $\dot{f}_{pulsar} = -3.85(9) \times 10^{-11} s^{-2}$.
- The modulation index $A_{day} = 1.60(2)$ is 12% smaller than the expected value 1.826. The cause is insufficient antenna sensitivity near the horizon.
- From $\phi_{day} = 0.769$ and the start time of the data chain, one calculates that the redshift of f_{GW} reaches its maximum value at 10.1 h (UTC). Due to the geographical position of the antenna, the maximum is not expected until two hours later. The reason for the discrepancy are the nulls of the interferometers when the pulsar is close to the local horizon.
- Although f_{GW} is at the edge of the usable range of the interferometer, the GW of the Vela pulsar can still be measured with an acceptable $S/N \approx 5.5$ (figure 7).

When working with the MSH method, one gets the impression that the interferometer records are not unstructured noise, but consist of many GWs with closely spaced frequencies and numerous sidebands. For those GWs, the parameters drift ($\dot{f}_O + \dot{f}_P$) and ecliptic longitude can be determined with surprising accuracy, but not (yet) assigned to any known astronomical object. As soon as the interferometers can measure uninterrupted data series lasting several days, the ecliptic latitude of the GW source can also be determined more precisely.

DATA AVAILABILITY

The data underlying this article are available in the Gravitational Wave Open Science Center LIGOdata.

REFERENCES

Espinoza, C. et al., Small glitches and other rotational irregularities of the Vela pulsar, A&A 647, A25 (2021)

NRAO, Calculate radial velocities, www.gb.nrao.edu/GBT/setups/radvelcalc.html

Biscoveanu S. et al., 2020, Quantifying the Effect of Power Spectral Density Uncertainty on Gravitational-Wave Parameter Estimation, https://arxiv.org/pdf/2004.05149.pdf

Davis, D. et. al, LIGO Detector Characterization in the Second and Third Observing Runs, https://arxiv.org/pdf/2101.11673.pdf

Gravitational Wave Open Science Center, O3b Data Release, https://www.gw-openscience.org/data/

Riles K., 2017, Recent searches for continuous Gravitational Waves, Mod. Phys. Lett. A 2017, 32, 730035, https://arXiv:712.05897v1

Weidner, H., Detection of the GW of the Crab Pulsar in the LIGO and Virgo O3b series, https://vixra.org/pdf/2305.0096v1.pdf

This paper has been typeset from a TEX/LATEX file prepared by the author.