We live in eight dimensions and no, they are not hidden

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Abstract

In this paper, we will show how Geometric Algebra expand the three spatial dimensions into entities of 8 degrees of freedom. It is also explained that one of these degrees of freedom (the trivector) can be considered to be the time (so no ad-hoc extra dimension is necessary). The square of the trivector is negative, solving this way the issue of the negative signature of the time (not necessary any ad-hoc metric indicating this, it is a property of time that appear naturally).

Also, we will show that we can try to prove this experimentally looking for the electromagnetic trivector, an entity that should exist according to GA.

Also, some comments regarding the similarities with E8 theory are given. Mainly that E8 theory considers 8 dimensions, exactly the same, emerging naturally in this paper. But not only that, also some similarities regarding how gravity can be understood, and others are presented.

Keywords

Geometric Algebra, Eight Dimensions, Dirac Equation, Gravitation, E8 theory

1. Introduction

In this paper, we will use the mathematical discipline known as Geometric Algebra to show that the three spatial dimensions create a mathematical framework with 8 degrees of freedom. We will check that these degrees are sufficient to explain different disciplines of Physics (Dirac Equation, Gravitation, comparison with E8 theory…).

2. We live in eight dimensions

There is a discipline in mathematics that is called Geometric Algebra [1][3] also known as Clifford Algebras. One curious thing of this Algebra is that if you consider a certain number of spatial dimensional (a certain number of independent vectors), automatically appear other dimensions (or if you want to call them, new degrees of freedom or other entities other than vectors).

In fact, the total number of degrees of freedom in an n-dimensional (understanding n as the number of special dimensions or independent vectors) in Geometric Algebra is:

$$\boldsymbol{Total\ number\ of\ degrees\ of\ freedom} = \mathbf{2}^{\boldsymbol{n}}$$

If we consider that our world has three spatial dimensions (in Geometric Algebra it is called $Cl_{3,0}$), we will have:

$$\boldsymbol{Total\ number\ of\ degrees\ of\ freedom} = \mathbf{2}^{\mathbf{3}} = \mathbf{8}$$

And in fact, we can check that this is true:

In three dimensions, we have three independent vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{z}}$:

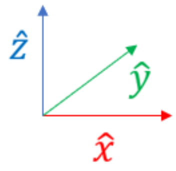

Fig. 1 Basis vectors in three-dimensional space.

In geometric algebra, these three vectors create 5 other entities.

The first other three entities are the bivectors. The bivectors are created multiplying perpendicular vectors. The result of this product is the bivector, an independent entity from the vectors that represent oriented planes. For example, the $\hat{x}\hat{y}$ bivector:

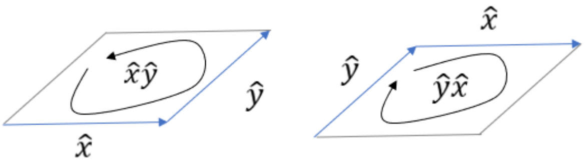

Fig.2 Representation of the bivectors $\hat{x}\hat{y}$ and $\hat{y}\hat{x}$. They represent the same plane with opposite orientation. In fact, $\hat{x}\hat{y} = -\hat{y}\hat{x}$.

There are three independent bivectors: $\hat{\boldsymbol{x}}\hat{\boldsymbol{y}}, \hat{\boldsymbol{y}}\hat{\boldsymbol{z}}$ and $\hat{\boldsymbol{z}}\hat{\boldsymbol{x}}$.

Another appearing entity is the trivector. It is formed by the product of the three independent vectors (and represent an oriented element of volume):

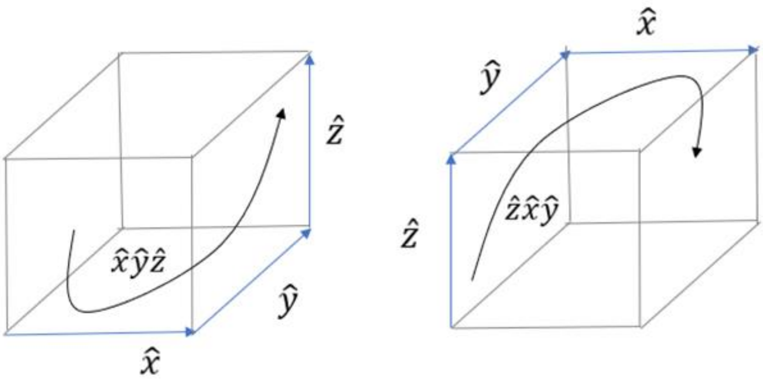

Fig.3 Representation of the two possible orientations of the trivector.
We can check that $\hat{x}\hat{y}\hat{z} = -\hat{y}\hat{x}\hat{z}$.

One important thing of the trivector is that in three dimensions there is only "one trivector". I mean, it can be bigger or smaller or with opposite direction (this means it can be escalated by a real scalar -positive or negative-), but the trivector itself as basis or unit trivector is always the same. You can check Annex A1 to check wheat I am talking about.

Another special property of the bivectors and the trivectors is that the square of a bivector or a trivector is -1. This you can check in all the papers of GA [1][3][5][6][26][27]. And the square of a vector is 1. Always talking in Euclidean metric. If this is not the case, you can check [2][4].

That the square of the bivectors and the trivectors is -1, means that they are a clear candidate for the imaginary unit *i* in certain circumstances. And we will see that this property is key for the trivector in the next chapter.

The last entity exiting in Geometric Algebra are the scalars (the numbers). They exist in their own space (are not linear as vectors, surface as bivectors or volume as trivector).

So, in total you can check that we have 8 entities when we have three spatial dimensions: 3 vectors, three bivectors, one trivector and the scalars.

But why are they "degrees of freedom"?

Ok, I will define another concept, the multivector. A multivector is just a sum of all the commented previous entities. This is, for example:

$$\boldsymbol{A} = \boldsymbol{\alpha_0} + \boldsymbol{\alpha_1}\hat{\boldsymbol{x}} + \boldsymbol{\alpha_2}\hat{\boldsymbol{y}} + \boldsymbol{\alpha_3}\hat{\boldsymbol{z}} + \boldsymbol{\alpha_4}\hat{\boldsymbol{x}}\hat{\boldsymbol{y}} + \boldsymbol{\alpha_5}\hat{\boldsymbol{y}}\hat{\boldsymbol{z}} + \boldsymbol{\alpha_6}\hat{\boldsymbol{z}}\hat{\boldsymbol{x}} + \boldsymbol{\alpha_7}\hat{\boldsymbol{x}}\hat{\boldsymbol{y}}\hat{\boldsymbol{z}}$$

Being $\boldsymbol{\alpha_i}$ scalars. This means the multivector (whatever it represents) it has eight degrees of freedom (from $\boldsymbol{\alpha_0}$ to $\boldsymbol{\alpha_7}$). Its meaning can vary a lot depending on the context or the discipline we are talking about.

For example, let us check the position multivector:

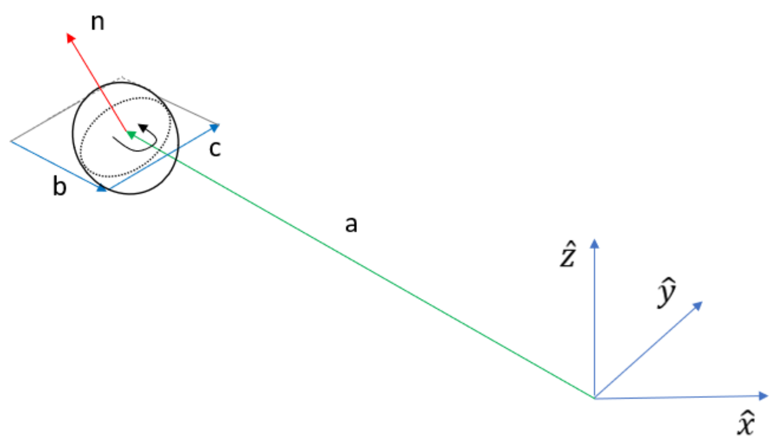

Fig.4 Representation position multivector

This multivector has 8 coordinates (8 degrees of freedom corresponding to the scalar, the three space vectors, the three bivectors and the trivector):

$$R = r_0 + r_x\hat{x} + r_y\hat{y} + r_z\hat{z} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + r_{xyz}\hat{x}\hat{y}\hat{z} \quad (1)$$

We can see that the vector a in the figure corresponds to the linear position of the particle or to the rigid body center of mass:

$$a = r_x\hat{x} + r_y\hat{y} + r_z\hat{z} \quad (2)$$

So, we can simplify the representation of the multivector as:

$$R = r_0 + a + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + r_{xyz}\hat{x}\hat{y}\hat{z} \quad (5)$$

Now let's go to the bivectors. In Fig.4 you can see that there is a bivector b^c that represents the orientation of a preferred plane in the particle/rigid body. This is, if you select a preferred plane solidary to the particle/rigid body, it tells us the orientation of this plane at a certain time. To define this orientation, you need a coefficient per basis bivector (the same as to define a vector you need the sum of three basis vectors, for bivectors works the same). So:

$$b\wedge c = r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} \ (7)$$

Introducing in R:

$$R = r_0 + a + b\wedge c + r_{xyz}\hat{x}\hat{y}\hat{z} \quad (8)$$

You can see that in a unique multivector R we are having the position and the orientation in the same expression. We have the sufficient degrees of freedom in the expression of the multivector R to give all this information just in one entity (the multivector R).

There are two other components r_0 (the scalar) and $\hat{x}\hat{y}\hat{z}$ (the trivector) that I will explain in the next chapter.

For information, this realm of Geometric Algebra that considers three spatial dimensions, and the eight degrees of freedom (or eight type of elements) created by them is called Geometric Algebra $Cl_{3,0}$.

3. Time as the trivector

I am not going to explain a lot here and the reason is because what you are going to hear is very difficult to believe and digest. You can check papers [5][6][26][27] to check all the info that corroborates what I am going to tell now.

In Geometric Algebra, it is not necessary that the time is a fourth dimension of the space-time (the classical 3 space dimensions and one 4th time dimension).

In Geometric Algebra, the time can be the 8th degree of freedom of the 8 degrees of freedom (or dimensions created by the GA itself). The time is emerging as one of the dimensions that appear automatically when the three spatial dimensions exist.

This is, the basis vector of the time is not a separate vector $\hat{\boldsymbol{t}}$ but it is the trivector $\hat{\boldsymbol{x}}\hat{\boldsymbol{y}}\hat{\boldsymbol{z}}$ already commented. The main reasons to consider this are:

- The signature of time is negative in General Relativity [7]. This can only be achieved considering an ad-hoc metric with a -1 signature or considering imaginary numbers. In GA, this is not necessary as the basis vector of time (the trivector) has a negative square as expected.

- I have written three papers [5][6][26] where it is checked that considering this in Dirac Equation, Maxwell equations and Lorentz Force equations match perfectly (see chapters 4, 5 and 6 of this chapter for more information). In fact, that the spinor of the Dirac equation has 8 degrees of freedom, and to consider one of them, the time-trivector, match perfectly with the equations (check chapter 4 and [5]).

So, you will check that from this point on, we will consider always the trivector as the basis vector of time. This does not mean that the trivector could not mean other things depending on the context (sometimes, it could be related to spin [2] or to the electromagnetic trivector see chapter 7). The same than a vector can sometimes represent a position, others a force etc… The trivector is just a tool that has certain properties, and these properties match perfectly with the properties of what we perceive as time.

Anyhow, that the trivector represents at the same time the volume and the time could be a hint that somehow, they are related. And the time could be a kind of measurement of the continuous creation of volume in the universe (you can check different mechanisms of creation of volume by the masses in the universe in [40][41]).

After this shock, we continue with the other pending item of the previous chapter, this is, r_0. The meaning of this element is more obscure. As I have commented, the scalars in the multivectors are a kind of scalation factor that affects all the magnitudes that are multiplied by it.

So, it could be related to a kind of scalation in the metric appearing in non-Euclidean metrics (kind of local Ricci scalar or trace of the metric tensor). See [2] for example.

Another simpler interpretation for r_0, is that the scalars appear when we multiply or divide vectors (or bivectors or the trivector) by themselves. So, sometimes it is necessary a degree of freedom to accommodate these results when they appear. For example, in [6][26] the current density through time, sometimes is accompanied by the trivector and other times is just a scalar depending on the operations that have been performed before.

4. Spinors and the Dirac Equation in Geometric Algebra $Cl_{3,0}$

In the papers [5][31] it was already made a direct relation between a spinor in matrix formalism (a complex 4-vector) with a multivector in Geometric Algebra $Cl_{3,0}$. It was used the Dirac equation, leading for a one-to-one map between these two worlds.

But things could be even more simple. Let, us consider this spinor:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

If we want to project it, in the normally considered space-time 4 dimensions, we can do it multiplying by a raw vector composed by its basis vectors:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} (\hat{x} \quad \hat{y} \quad \hat{z} \quad \hat{t}) = \psi_1 \hat{x} + \psi_2 \hat{y} + \psi_3 \hat{z} + \psi_4 \hat{t}$$

Considering that the time is the trivector, as commented in chapter 3, and using the straightforward convention:

$$\hat{t} = \hat{x}\hat{y}\hat{z}$$

We will get:

$$\psi_1 \hat{x} + \psi_2 \hat{y} + \psi_3 \hat{z} + \psi_4 \hat{x}\hat{y}\hat{z}$$

(For information, in other papers [4][5][6][26][27][31]it is used the opposite convention $\hat{t} = \hat{z}\hat{y}\hat{x}$).

With this move, there is little gain, as we have just associated the four components of the column vector to a dimension. And, if consider that each component ψ_i is a complex number, we are in the darkness again.

The solution is simple. Let's make the same thing but now, knowing that each component is a complex number and taking action on that.

Again, we have:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

Now, let's decompose each component in the corresponding complex number:

$$\psi = \begin{pmatrix} \psi_{1r} + i\psi_{1i} \\ \psi_{2r} + i\psi_{2i} \\ \psi_{3r} + i\psi_{3i} \\ \psi_{4r} + i\psi_{4i} \end{pmatrix}$$

Now, following the rules that the imaginary unit represents a bivector when it is any specific direction/orientation related and it corresponds to the trivector when this is not the case [5][6][26][27]. For a general complex number, we are always in the second case. So:

$$\psi = \begin{pmatrix} \psi_{1r} + \psi_{1i}\hat{x}\hat{y}\hat{z} \\ \psi_{2r} + \psi_{2i}\hat{x}\hat{y}\hat{z} \\ \psi_{3r} + \psi_{3i}\hat{x}\hat{y}\hat{z} \\ \psi_{4r} + \psi_{4i}\hat{x}\hat{y}\hat{z} \end{pmatrix}$$

Now, projecting again to the usually considered space-time dimensions:

$$\begin{pmatrix} \psi_{1r} + \psi_{1i}\hat{x}\hat{y}\hat{z} \\ \psi_{2r} + \psi_{2i}\hat{x}\hat{y}\hat{z} \\ \psi_{3r} + \psi_{3i}\hat{x}\hat{y}\hat{z} \\ \psi_{4r} + \psi_{4i}\hat{x}\hat{y}\hat{z} \end{pmatrix} (\hat{x} \quad \hat{y} \quad \hat{z} \quad \hat{t})$$
$$= \psi_{1r}\hat{x} + \psi_{1i}\hat{x}\hat{y}\hat{z}\hat{x} + \psi_{2r}\hat{y} + \psi_{2i}\hat{x}\hat{y}\hat{z}\hat{y} + \psi_{3r}\hat{z} + \psi_{3i}\hat{x}\hat{y}\hat{z}\hat{z} + \psi_{4r}\hat{t} + \psi_{4i}\hat{x}\hat{y}\hat{z}\hat{t}$$

Substituting again and operating:

$$\hat{t} = \hat{x}\hat{y}\hat{z}$$

$$\psi_{1r}\hat{x} + \psi_{1i}\hat{y}\hat{z} + \psi_{2r}\hat{y} - \psi_{2i}\hat{x}\hat{z} + \psi_{3r}\hat{z} + \psi_{3i}\hat{x}\hat{y} + \psi_{4r}\hat{x}\hat{y}\hat{z} + \psi_{4i}\hat{x}\hat{y}\hat{z}\hat{x}\hat{y}\hat{z} =$$

$$\psi_{1r}\hat{x} + \psi_{1i}\hat{y}\hat{z} + \psi_{2r}\hat{y} + \psi_{2i}\hat{z}\hat{x} + \psi_{3r}\hat{z} + \psi_{3i}\hat{x}\hat{y} + \psi_{4r}\hat{x}\hat{y}\hat{z} - \psi_{4i}$$

You can see that we have arrived to the already commented multivector of 8 degrees of freedom (8 dimensions if you want) but starting from the three spatial dimensions. No special magic or hidden tiny dimension is necessary. Just geometric. The three special dimensions lead to three vectors, three bivectors and one trivector (plus the scalars), giving a total of 8 dimensions.

Even the time is not an ad-hoc dimension. It (the time trivector) emerges naturally from the three spatial dimensions. Somehow, the human being perceives the odd-grade dimensions (vectors of space and the trivector of time) as real dimensions. And the even-grade dimensions (scalars and bivectors) otherwise. Probably as orientations or forces or relations between entities. Just as we perceive the visible light with our eyes and the infrared light as heat, somehow, we perceive differently the dimensions (or degrees of freedom if you want) of the world.

If you want to check a proper formal relation between a spinor in matrix algebra and in GA algebra using the Dirac equation you can check [5][31] to arrive to:

Dirac Equation in GA:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t} - \hat{y}\hat{z}\frac{\partial}{\partial x} - \hat{z}\hat{x}\frac{\partial}{\partial y} - \hat{x}\hat{y}\frac{\partial}{\partial z}\right)\psi - m\psi_{even}\hat{z} + m\psi_{odd}\hat{z} = 0$$

Where:

$$\psi_{even} = \psi_0 + \hat{x}\hat{y}\psi_{xy} + \hat{y}\hat{z}\psi_{yz} + \hat{z}\hat{x}\psi_{zx}$$
$$\psi_{odd} = \hat{x}\psi_x + \hat{y}\psi_y + \hat{z}\psi_z + \hat{x}\hat{y}\hat{z}\psi_{xyz}$$
$$\psi = \psi_{even} + \psi_{odd} = \psi_0 + \hat{x}\psi_x + \hat{y}\psi_y + \hat{z}\psi_z + \hat{x}\hat{y}\psi_{xy} + \hat{y}\hat{z}\psi_{yz} + \hat{z}\hat{x}\psi_{zx} + \hat{x}\hat{y}\hat{z}\psi_{xyz}$$

If the wavefunction solution in Matrix Algebra is defined as:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \psi_{1r} + i\psi_{1i} \\ \psi_{2r} + i\psi_{2i} \\ \psi_{3r} + i\psi_{3i} \\ \psi_{4r} + i\psi_{4i} \end{pmatrix}$$

There is a one-to-one mapping of both representations:

$$\psi_{1r} = -\psi_y$$
$$\psi_{1i} = -\psi_x$$
$$\psi_{2r} = \psi_{xyz}$$
$$\psi_{2i} = \psi_z$$
$$\psi_{3r} = -\psi_{yz}$$
$$\psi_{3i} = \psi_{zx}$$
$$\psi_{4r} = \psi_{xy}$$
$$\psi_{4i} = \psi_0$$

So, we have seen that considering only three spatial dimensions, and the time as the trivector appearing naturally in the Geometric Algebra, is sufficient to accommodate the Dirac Equation. This means, not an added extra-time dimension is needed in this algebra. Time emerges naturally from the three spatial dimensions of the Geometric Algebra $Cl_{3,0}$ and is coherent with the results.

5. The Lorentz Force in Geometric Algebra $Cl_{3,0}$ (in three spatial dimensions, with the time as the trivector)

In [6] you can find a complete mapping of Electromagnetism and Lorentz force between classical tensor notation to Geometric Algebra $Cl_{3,0}$ realm, where specifically the Lorentz Force equation is represented as:

$$\frac{dp}{d\tau} = qFU \quad (21)$$

Where:

$$\frac{dp}{d\tau} = \frac{dp_0}{d\tau} + \frac{dp_{yz}}{d\tau}\hat{x} + \frac{dp_{zx}}{d\tau}\hat{y} + \frac{dp_{xy}}{d\tau}\hat{z} + \frac{dp_x}{d\tau}\hat{y}\hat{z} + \frac{dp_y}{d\tau}\hat{z}\hat{x} + \frac{dp_z}{d\tau}\hat{x}\hat{y} + \frac{dp_{xyz}}{d\tau}\hat{x}\hat{y}\hat{z} \quad (20)$$

$$F = E_x\hat{x} + E_y\hat{y} + E_z\hat{z} + B_x\hat{y}\hat{z} + B_y\hat{z}\hat{x} + B_z\hat{x}\hat{y} \qquad (19)$$

$$U = U_{xyz}\hat{x}\hat{y}\hat{z} + U_x\hat{y}\hat{z} + U_y\hat{z}\hat{x} + U_z\hat{x}\hat{y} \qquad (18)$$

Getting the following equations:

$$\frac{dp_x}{d\tau} = q\left(E_x U_{xyz} - B_y U_z + B_z U_y\right) \qquad (24)$$
$$\frac{dp_y}{d\tau} = q\left(E_y U_{xyz} + B_x U_z - B_z U_x\right) \qquad (25)$$
$$\frac{dp_z}{d\tau} = q\left(E_z U_{xyz} - B_x U_y + B_y U_x\right) \qquad (26)$$
$$\frac{dp_{xyz}}{d\tau} = q\left(E_x U_x + E_y U_y + E_z U_z\right) \qquad (30)$$

That corresponds one to one with the Lorentz Force in the covariant formalism:

$$\frac{dp_4}{d\tau} = q\left(E_x u^1 + E_y u^2 + E_z u^3\right) \qquad (13)$$
$$\frac{dp_1}{d\tau} = q\left(-E_x u^4 - B_z u^2 + B_y u^3\right) \qquad (14)$$
$$\frac{dp_2}{d\tau} = q\left(-E_y u^4 + B_z u^1 - B_x u^3\right) \qquad (15)$$
$$\frac{dp_3}{d\tau} = q\left(-E_z u^4 - B_y u^1 + B_x u^2\right) \qquad (16)$$

With the following equivalences when comparing Geometric Algebra notation to covariant tensor notation [6], so all the considerations taken into account in Geometric Algebra notation (for example, considering time as the trivector of the $Cl_{3,0}$ Geometric Algebra) are substantiated:

$$u^4 = U_{xyz} \qquad u^1 = U_x \qquad u^2 = U_y \qquad u^3 = U_z \qquad (18.1)$$

$$\frac{dp_{xyz}}{d\tau} = \frac{dp_4}{d\tau} \qquad \frac{dp_x}{d\tau} = -\frac{dp_1}{d\tau} \qquad \frac{dp_y}{d\tau} = -\frac{dp_2}{d\tau} \qquad \frac{dp_z}{d\tau} = -\frac{dp_3}{d\tau} \qquad (32)$$

6. The Maxwell Equation in Geometric Algebra $Cl_{3,0}$ (in three spatial dimensions, with the time as the trivector)

Similarly, in [26] we make a complete comparison of Maxwell laws from classical tensor notation to Geometric Algebra $Cl_{3,0}$. Specifically, the Maxwell laws in Geometric Algebra are put together in one just equation:

$$\nabla F = J$$

Where:

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} + \frac{\partial}{\partial t}$$
$$F = E_x\hat{x} + E_y\hat{y} + E_z\hat{z} + B_x\hat{y}\hat{z} + B_y\hat{z}\hat{x} + B_z\hat{x}\hat{y}$$
$$J = J_x\hat{x} + J_y\hat{y} + J_z\hat{z} + J_0$$

This is:

$$\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} + \frac{\partial}{\partial t}\right)\left(E_x\hat{x} + E_y\hat{y} + E_z\hat{z} + B_x\hat{y}\hat{z} + B_y\hat{z}\hat{x} + B_z\hat{x}\hat{y}\right)$$
$$= J_x\hat{x} + J_y\hat{y} + J_z\hat{z} + J_0$$

Operating and obtaining an equation for each element of the Geometric Algebra (scalars, vectors, bivectors and trivector) we get eight equations:

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = J_0$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_z}{\partial t} = 0$$
$$-\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} + \frac{\partial B_y}{\partial t} = 0$$
$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$
$$-\frac{\partial B_y}{\partial x} + \frac{\partial B_x}{\partial y} + \frac{\partial E_z}{\partial t} = J_z$$
$$\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} + \frac{\partial E_y}{\partial t} = J_y$$
$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial t} = 0$$
$$-\frac{\partial B_z}{\partial y} + \frac{\partial B_y}{\partial z} + \frac{\partial E_x}{\partial t} = J_x$$

Which are exactly the Maxwell laws in classical tensor notation if we consider the following relations between both realms (Geometric Algebra and classical tensor notation):

$$J_x = -J^x$$
$$J_y = -J^y$$
$$J_z = -J^z$$
$$J_0 = J^t$$

So, all the considerations taken into account in Geometric Algebra notation (for example, considering time as the trivector of the $Cl_{3,0}$ Geometric Algebra) are substantiated. You can check all the details in [26].

7. The trivector

More about the trivector. We have commented that the trivector is the basis vector of time, but it is involved in more tricky things.

In the papers [6][26] when considering the electromagnetic field, we had the vectors and the bivectors as the electric and the magnetic field. But we saw that also the electromagnetic trivector could affect the particles. Probably just rotating them or creating a zitterbewegung movement. The issue is that the thing could go even further and be (the trivector) the creator of the Electromagnetic Field itself.

Check it this way. Imagine that there is an electromagnetic trivector field everywhere acting on currents (vectors) and on magnetic fields (bivectors).

If we have a current in direction $\hat{x}$, the "omnipresent" trivector acts (it is post-multiplied) on it, creating a bivector:

$$(\hat{x})(\hat{x}\hat{y}\hat{z}) = \hat{x}\hat{x}\hat{y}\hat{z} = \hat{y}\hat{z}$$

(We are in the convention of the trivector $\hat{x}\hat{y}\hat{z}$, in the case of $\hat{z}\hat{y}\hat{x}$ we should premultiply).

This means, the omnipresent trivector converts a current (a "vector-directed" charge) in a bivector (magnetic field).

The opposite can also happen. A magnetic bivector field (under certain circumstances as varying during time) could create vector-directed currents:

$$(\hat{x}\hat{y})(\hat{x}\hat{y}\hat{z}) = \hat{x}\hat{y}\hat{x}\hat{y}\hat{z} = -\hat{x}\hat{x}\hat{y}\hat{y}\hat{z} = -\hat{z}$$

This is nothing but the Maxwell equations or the Lorentz force. But with a difference. If we lived in a world with an omnipresent trivector but in with opposite direction, the Maxwell laws and the Lorentz force would be inverted. The right-hand rule would be transformed to the left-hand rule when calculating the direction of a magnetic field created by a current and vice-versa.

$$(\hat{x})(\hat{z}\hat{y}\hat{x}) = \hat{x}\hat{z}\hat{y}\hat{x} = \hat{z}\hat{y} = -\hat{y}\hat{z}$$

The magnetic field would be the opposite as before.

The same with a current:

$$(\hat{x}\hat{y})(\hat{z}\hat{y}\hat{x}) = \hat{x}\hat{y}\hat{z}\hat{y}\hat{x} = -\hat{x}\hat{x}\hat{y}\hat{z}\hat{y} = -\hat{y}\hat{z}\hat{y} = \hat{z}$$

So, the question is clear, can we create a "world" with a trivector acting in the opposite direction than the one existing and check if this inversion of the handed-rules could happen?

In principle, we should be able. We can create in a laboratory a current with the shape of a trivector:

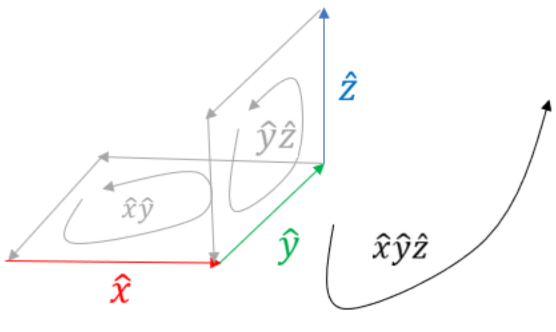

Fig. 5 Conductor (current) with the shape of a trivector

and then an opposite current with exactly the same shape.

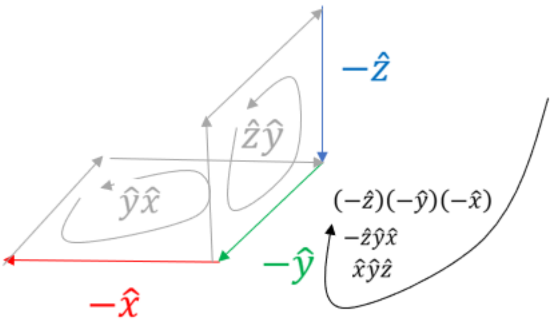

Fig. 6 Conductor (current) with the shape of a trivector (opposite direction of currents as Fig 5.)

We can see that the currents and the created magnetic fields are opposite in Fig. 5 and Fig 6. (current $-\hat{x}$ in opposition of $\hat{x}$, and magnetic field bivector $\hat{y}\hat{z}$ in opposition with magnetic bivector $\hat{z}y$ for example).

But we can check that the trivector created in both cases is the same. It is the trivector $\hat{x}\hat{y}\hat{z}$. You can check on the right in Fig 6. the arithmetic operation to arrive to that result or see it geometrically, as follows.

If we rotate Fig. 6 by $\hat{y}$ axis clockwise direction (90º), we get:

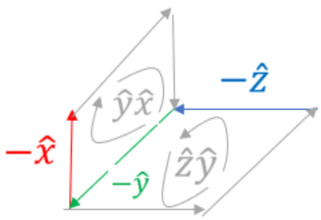

Fig. 7 It is the same as Fig. 6 rotated by $\hat{y}$ axis clockwise.

Now, we rotate by $\hat{x}$ axis 180º:

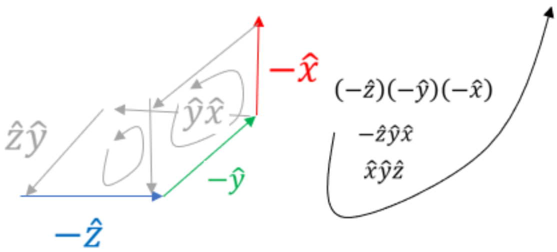

Fig. 8 It is Fig. 7 rotated by $\hat{x}$ axis 180º.

You can check that the trivector created in Fig 8. And in Fig 5 is the same.

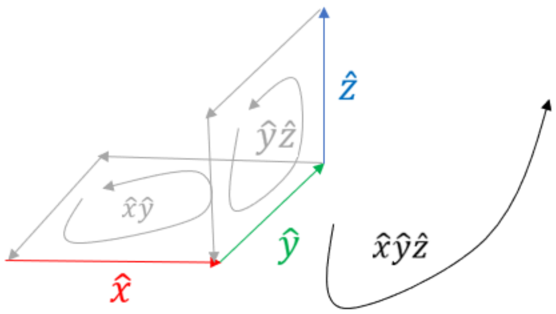

Fig. 5 Conductor (current) with the shape of a trivector

The name of the axes has changed but the physical trivector created in Fig. 5 and Fig. 8 is the same. Meaning the physical effect (if it exists) of this trivector should be the same in both cases. If you do not understand that both trivectors are the same, you can check Annex A1 for a better explanation.

This means, if we superimpose Fig. 5 and Fig 6.:

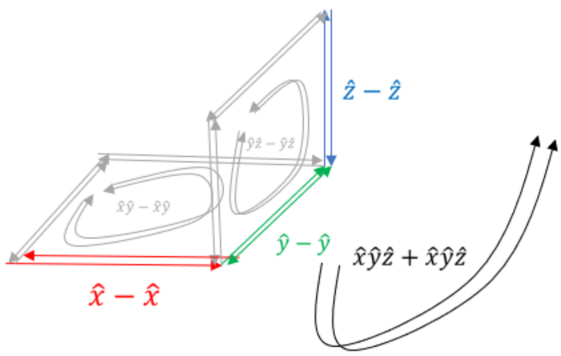

Fig. 9 Two opposite conductors (currents) with the shape of a trivector

We are superimposing two conductors with opposite direction of currents. We see that the vectors (currents) cancel. Also, the bivectors (magnetic fields) cancel.

But the trivector, is not cancelled. In fact, it is doubled. Both conductors create the same trivector and one is added to the other. Remind that the trivector of Fig. 6 is the same as the trivector of Fig. 8 (that is the same as Fig.5).

In classical electromagnetism Fig. 9 is just a loss of power with no effects. You are consuming electricity to create two opposite currents that do not create anything by themselves. All the effects (magnetic fields) cancel.

In Geometric algebra, it is not the case. It is different a situation with these currents than a situation without them.

If we have a conductor $\hat{v}$ with no other currents involved, it will create a magnetic field $M_{\hat{v}}$:

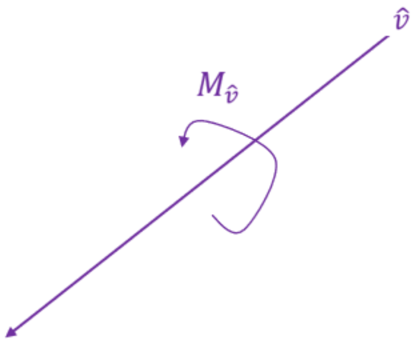

Fig. 10 Magnetic field $M_{\hat{v}}$ created by the current vector $\hat{v}$.

Considering that the magnetic field is created by the current with omnipresent trivector we have:

$$M_{\hat{v}} = \hat{v}\hat{x}\hat{y}\hat{z}$$

But if we have created an artificial trivector in the acting area of $\hat{v}$, the result could be different:

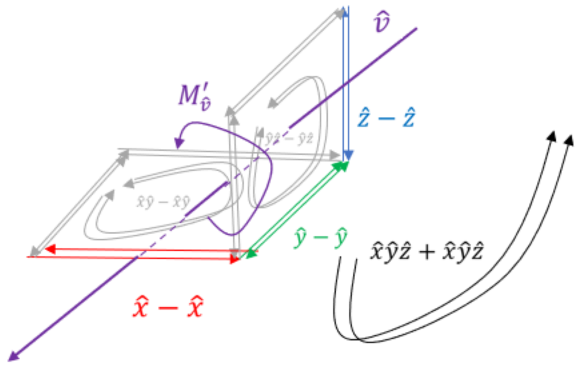

Fig. 11 Magnetic field $M'_{\hat{v}}$ created by the current vector $\hat{v}$.

If we consider that the environmental omnipresent trivector is the same value and in the same direction as the ones we have created, we will have:

$$M'_{\hat{v}} = \hat{v}(\hat{x}\hat{y}\hat{z} + 2\hat{x}\hat{y}\hat{z}) = 3\hat{v}\hat{x}\hat{y}\hat{z}$$

If we consider that the ones, we have created have the same value as the environmental one but in opposite direction, we will have:

$$M'_{\hat{v}} = \hat{v}(\hat{x}\hat{y}\hat{z} - 2\hat{x}\hat{y}\hat{z}) = -\hat{v}\hat{x}\hat{y}\hat{z}$$

This means the Maxwell laws would be inversed, getting an opposite magnetic field than expected.

Of course, this is an extreme case. What we would expect in a real experiment is a little difference between $M_{\hat{v}}$ and $M'_{\hat{v}}$. And we would notice that this difference changes the sign if we inverse the currents in our artificial trivector.

In fact, I have tried this experiment with very low power and with the cables that convey the opposite currents completely twisted. No difference was observed. Only when the cables were not perfectly twisted, some difference in the magnetic field could be noticed but completely explainable with the magnetic field created by parallel conductors [35].

The issue is that a high-power experiment should be necessary to counterbalance whatever the trivector environmental value has. It is like trying to detect gravitational effects between two football balls in the surface of the earth. The gravitational force of the earth will shadow whatever gravitational effects between masses in its surface. Only a very precise, controlled experiment can check minimal effects of a force/field when a much bigger force/field of the same type is present during the experiment.

If the environmental trivector is something ad-hoc of the universe or depends somehow on other parameters (mainly the quantity of matter -in opposition of antimatter- present in the area spreading the “matter laws” as right-handed rule etc..) is something that could be studied if finally, this effect of the trivector is found. And its effects can be compared in different situations, (surface of earth, space etc….).

It is important to notice that the trivector here is just considered as another element of the electromagnetic field, separated from other forces. But as we will see later, a field that is everywhere affecting all the particles has all the chances to be also related to gravity or other interactions. Also, to be noted that the trivector also represent volume, time, spin [2]. Somehow, there is a relation between all these elements mathematically that could imply a real relation in the physical world.

8. Gravity

In the paper [2] gravity is considered as a non-Euclidean metric that can be managed in GA via the scalar products among the basis vectors. This means, we have to define the products:

$$\hat{x}^2 = \|\hat{x}\|^2 = g_{xx}$$
$$\hat{y}^2 = \|\hat{y}\|^2 = g_{yy}$$
$$\hat{z}^2 = \|\hat{z}\|^2 = g_{zz}$$
$$\hat{x}\hat{y} = 2g_{xy} - \hat{y}\hat{x}$$
$$\hat{y}\hat{z} = 2g_{yz} - \hat{z}\hat{y}$$
$$\hat{z}\hat{x} = 2g_{zx} - \hat{x}\hat{z}$$

So, we have only 6 variables to be defined, to define the metric. This is an issue, because in General Relativity (as time is considered a 4^{th} dimension), we need 10 parameters to define the metric tensor[7][12]. The metric tensor has 16 parameters, but it is symmetric leading to 10[7].

But it has to be considered two things more:

- The Ricci relations, reducing from 10 to 8 the free parameters[7][12].
- There is one degree of freedom related to the definition and relation of the elements of the metric (saying it in another way, it can always be “normalized” or defined in another way). This would reduce one degree more leading to 7 parameters.

It should be studied if this extra parameter is really necessary or again there is a degree of freedom in the definition of the metric leaving that 6 is sufficient. Or it could be that in GA also the following product should be defined (but it should have been already defined with the previous relations except something I cannot see):

$$\hat{x}\hat{y}\hat{z} = 2g_{xyz} - \hat{y}\hat{x}\hat{z}$$

9. The E8 theory

A. Garret Lisi created the E8 theory [28][29][30]. The summary of this theory is that all the particles and forces existing in our world can be explained using a semi-regular figure of eight dimensions called the E8 polytope. I am very far to have the knowledge-comprehension to understand its depth. The idea is that transformations of particles into other ones and the different interactions among them could be explained as existing in different edges of the figure or via rotations of it.

For me, the incredible thing of this theory is that it has been created as an ad-hoc theory (not related with GA in a direct manner) but leading to the same conclusions, as we will see now.

In fact, this is not exactly correct, as yes there is a relation between both approaches, at least in an indirect manner. All the Lie groups SU(2), (3) (used in the E8 theory) have its direct correspondence with GA or Clifford Algebras. Anyhow, it is surprising how they lead to very similar conclusions anyhow.

Ones to be remarked:

- It considers a figure of exactly 8 dimensions. This is exactly the number of dimensions corresponding to a GA with 3 spatial dimensions as commented in chapter 2.
- It considers that the gravity is related to spin [29]. As I have commented before, the trivector can represent the spin (chapter 7 and [2]) and also a field that is everywhere (like the electromagnetic trivector) affecting all the particles (another way of calling the Higgs field? or any other relation with gravity?). Also, an omnipresent trivector could explain other issues as the quantum entanglement [36]. See Annex A2.
- In E8 [30] gravity is related with the "principal bundle" of the geometry. In chapter 8 of this paper and [2] it has been already commented that gravity could be related with the definition of the relation of the basis vectors among them (its scalar products) in GA. Very similar of what a principal bundle is by definition [42].
- As commented, all the Lie groups SU(2), (3) have its direct correspondence with a realm in GA. A study of the detailed correspondence could be done to try to relate why these groups lead to 8 dimensions, the same way than GA does (as explained throughout this paper).
- The total degrees of freedom of E8 theory is 248. If we consider that the multivectors (it does not matter if they represent spinors, positions etc…) have 8 degrees of freedom. If we multiply three multivectors among them, we will have 512 parameters. Considering that it could happen that the result should be symmetric somehow, it corresponds to 256 parameters. If 8 of them are dependent on others, we will have this 248 parameters. In the papers [5][31] we could arrive to the most generalized both sided Dirac equation (In fact Klein-Gordon in these regards):

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m+\hat{\boldsymbol{x}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}+\hat{\boldsymbol{y}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}+\hat{\boldsymbol{z}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}\right)$$

$$(\psi_0+\hat{x}\psi_x+\hat{y}\psi_y+\hat{z}\psi_z+\hat{x}\hat{y}\psi_{xy}+\hat{y}\hat{z}\psi_{yz}+\hat{z}\hat{x}\psi_{zx}+\hat{x}\hat{y}\hat{z}\psi_{xyz})\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m+\hat{\boldsymbol{x}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}+\hat{\boldsymbol{y}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}+\hat{\boldsymbol{z}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}\right)=0$$

  This type of equation (with other multivectors) could lead to the 512 (or 248 free parameters). In this case, this equation as such is never used as the last element of the product is eliminated (as was de facto done in original Dirac equation). See chapter 4 and [5][31].

The only point I would try to do another way is the transformation that is done in [30] from the imaginary unit *i* to matrices. I would clearly exchange the imaginary units and the SU matrices to their equivalents in GA (mainly vectors, bivectors or trivectors) to simplify the calculations but more important to simplify the geometric understanding of the model.

10. Conclusions

In Geometric Algebra mathematical framework, the three spatial dimensions, expand naturally to entities with 8 degrees of freedom. It has been shown that one of these degrees of freedom (the trivector) can be considered to be the time (so no ad-hoc extra dimension is necessary). Even the issue of the negative signature of time is solved this way, as the square of the trivector is negative by definition.

It has been shown that all this can be checked experimentally looking for the electromagnetic trivector, an entity that should exist according to GA and could be checked experimentally.

Also, some comments regarding the similarities with E8 theory are given. Mainly that E8 theory considers 8 dimensions, exactly the same as emerging naturally in this paper.

But not only that, also some similarities regarding how gravity can be understood, and others are presented.

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If you consider this helpful, do not hesitate to drop your BTC here:

bc1q0qce9tqykrm6gzzhemn836cnkp6hmel5lmz36f

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A1. Annex A1. Equality of vectors, bivectors and trivectors in the different dimensions

If we lived in only one dimension (if we were unidimensional animals in a unidimensional world), we would know only one vector. And always the same. It could change of course, its magnitude or its direction. This means it can be escalated by a scalar real number (positive or negative) but the "original unit" vector will always be the same.

Fig. A1.1 The same vector escalated (in magnitude or in direction -negatively-).

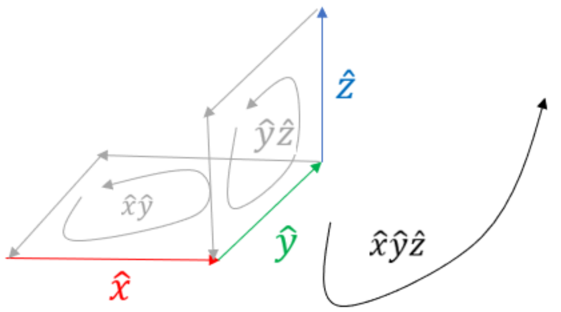
If we lived in two dimensions, we would know only one bivector. The same, it can be escalated or even change its orientation. But the bivector will always represent the same plane. The same as before, that a vector in a unidimensional world represented always the same straight line.

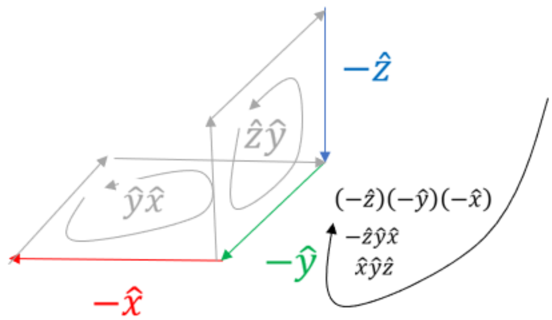
Fig. A1.2 The same bivector all the time. Rotating its vectors, does not change the bivector (it is always the same as long as it represents the same plane -magnitude and orientation left aside-).

In Fig. A1.2 all the colors (the dotted grey line, the purple, lines, the orange lines, the original red and green lines) represent the same bivector. Those rotations do not mean anything for the bivector. It is the same bivector, as long as it is in the same plane. If you want more info regarding this, you can check [1][3][2][4][26]

Now, we come to our world. We are three-dimensional beings in a three-dimensional world.

These two trivectors are the same and even have the same orientation:

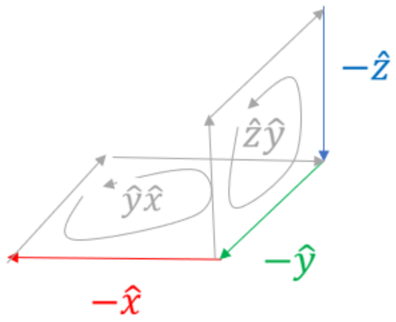




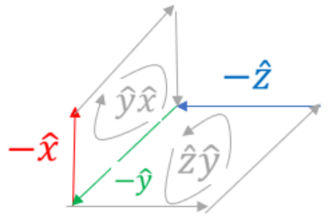
Why? Because we can rotate them in our world to get to the same trivector. The same as we have seen in the two-dimensional world, where all the bivectors were the same because they were in the same plane. All the trivectors in our world are the same (again, magnitude and orientation aside) because they are just rotations of the same trivector in the same 3-D world. A 4-D entity, yes, it could rotate our trivector in a way that we cannot imagine and create a new trivector. But we, poor 3-D mortals will see always the same trivector (magnitude and orientation aside) our whole life.

The transformation of one into another can be done with the rotations commented in chapter 7.

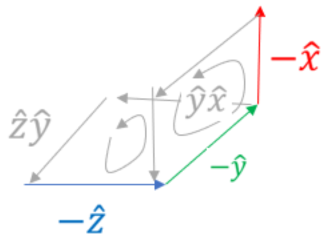
Starting from:



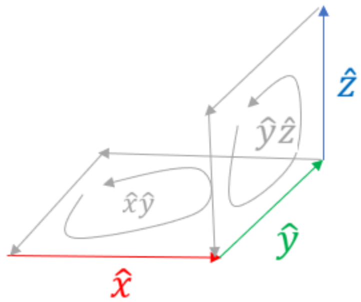
If we rotate above figure by $\hat{y}$ axis clockwise direction (90º), we get:



Now, we rotate above figure by $\hat{x}$ axis 180º:



We have arrived to the same trivector (and in this case, even with the same orientation) as the first figure, that seemed completely opposite in the beginning:



You can check that above both figures represent the same physical entity, even if the nomenclature of the vectors is different.

A2. Annex A2. Quantum Entanglement using an omnipresent trivector or any other quantum field

In [37], I already commented that the quantum entanglement [36] could be explained as caused by a hidden field omnipresent everywhere. The "random" value (or alternative value) of this field at a given moment would affect both particles in the same way at a certain time, giving the impression that both particles are talking to each other. While in reality, it is the field itself which provokes this "coherent" behavior between both particles without the necessity of any faster than light information.

In [38] Joy Christian, already proposes a disproof of Bell's theorem [39] using bivectors and the trivector (what he calls I), as the anti-commutative properties of the geometric algebra are sufficient to make his demonstration.

Making a much less formal way of explaining it (my style), we can go with an omnipresent field (that could be formed by vectors, bivectors or the trivector).

Let's imagine a particle "Amber" which has an internal property (it could be orientation of spin, or whatever even not still known property) that is defined by bivectors:

$$\alpha\hat{y}\hat{z} + \beta\hat{z}\hat{x} + \gamma\hat{x}\hat{y}$$

Another particle "Blue" quantum entangled with "Amber" (for example its twin in a double particle creation-annihilation process), would have this property as:

$$-\alpha\hat{y}\hat{z} - \beta\hat{z}\hat{x} - \gamma\hat{x}\hat{y}$$

This is, the opposite of Amber, as quantum entanglement states.

Imagine that there exists an omnipresent (meaning omnipresent as it has the same value in a very large area of the space) trivector field that has an alternative value given by:

$$\delta(t)\hat{x}\hat{y}\hat{z}$$

Where δ is an alternative scalar that could have a form like this or whatever alternative or random equation depending on time (and only at very large, large scales on space coordinates, so we can consider depending only on time in our experiment):

$$\delta = \sin(t) \quad or \quad \delta = 5\cos(t) - \sin(4t)$$
$$or\ whatver\ other\ equation\ depending\ on\ time$$

If the omnipresent trivector field acts in the commented property of the particles post-multiplying by it, we would get for "Amber":

$$(\alpha\hat{y}\hat{z} + \beta\hat{z}\hat{x} + \gamma\hat{x}\hat{y})(\delta\hat{x}\hat{y}\hat{z}) = \alpha\delta\hat{y}\hat{z}\hat{x}\hat{y}\hat{z} + \beta\delta\hat{z}\hat{x}\hat{x}\hat{y}\hat{z} + \gamma\delta\hat{x}\hat{y}\hat{x}\hat{y}\hat{z}$$
$$= -\alpha\delta\hat{x} - \beta\delta\hat{y} - \gamma\delta\hat{z}$$

If you do not understand how this product is done (why have some vectors disappear?) and where these negative signs come from, you can check [1] to [6].

If we do the same for the particle "Blue":

$$(-\alpha\hat{y}\hat{z} - \beta\hat{z}\hat{x} - \gamma\hat{x}\hat{y})(\delta\hat{x}\hat{y}\hat{z}) = \alpha\delta\hat{x} + \beta\delta\hat{y} + \gamma\delta\hat{z}$$

Now, if we want to measure whatever property observing in the x axis (let's consider that this means, for example, we have to post-multiply by the x unit vector):

In Amber:

$$(-\alpha\delta\hat{x} - \beta\delta\hat{y} - \gamma\delta\hat{z})(\hat{x}) = -\alpha\delta + \beta\delta\hat{x}\hat{y} - \gamma\delta\hat{z}\hat{x}$$

In Blue:

$$(\alpha\delta\hat{x} + \beta\delta\hat{y} + \gamma\delta\hat{z})(\hat{x}) = \alpha\delta - \beta\delta\hat{x}\hat{y} + \gamma\delta\hat{z}\hat{x}$$

Whatever is the result we want to measure (the scalar αδ or the bivectors in xy or zx plane) we see that the result in Amber and Blue are opposite. This is true as far as δ -that depends on time- is "approximately" the same in both cases. This means, the measurements are really done in both particles at "approximately" the same time.

For the distant observers the result seems "magical", as the result of the observable measured in one particle is "magically" related to the other one. But in fact, the only issue is that they have measured when a "hidden" field, that is acting on both particles, had the same value at a certain moment. As the field is "hidden", the result seems "random" and the same for both particles. But it is not random, it is just "unknown" until measured.

Imagine that we measure at a moment when the specific value of δ is $-\delta_0$, we will have a result with different signs as before but, they will always be the opposite in both particles. The "magical" entanglement is always kept, as far the field acting on them keeps being the same. This is, they are measured at "approximately" the same time and in an area where the field does not depend in space coordinates (the field has to be the same or "smooth" in very large areas).

In Amber:

$$(-\alpha(-\delta_0)\hat{x} - \beta(-\delta_0)\hat{y} - \gamma(-\delta_0)\hat{z})(\hat{x}) = \alpha\delta_0 - \beta\delta_0\hat{x}\hat{y} + \gamma\delta_0\hat{z}\hat{x}$$

In Blue:

$$(\alpha(-\delta_0)\hat{x} + \beta(-\delta_0)\hat{y} + \gamma(-\delta_0)\hat{z})(\hat{x}) = -\alpha\delta_0 + \beta\delta_0\hat{x}\hat{y} - \gamma\delta_0\hat{z}\hat{x}$$

As commented in chapters 7 to 9, it could be that the big masses (for example the Earth) force the trivector to be of a certain value near their area of influence (the value of the trivector will depend on time, but negligibly will depend on space coordinates while they are in the area of influence). This would mean that all the experiments near the Earth would result as "entanglement" working well, as the predominant trivector is always the same. Probably at really very large distances where local gravities or other effects are really different, the "entanglement" measurements start to differ. One possible experiment will be to check in the surface of Earth and outside the Earth to check if the entanglement starts failing in the experiments with a higher rate than when they are done for both particles in the surface of Earth.

A3. Annex A3. Trivector effects

We have seen that if we can create an artificial trivector which direction is opposite to the existing trivector in space, Maxwell laws could work in reverse direction in its area of influence. We could invert the "right-handed" rule. Or even it could be that whatever effect that we are accustomed to see, works in the opposite way (reversion of time, reversion of entropy inside its area of influence)? Too much sci-fi.

What is real is that we could try to create this "opposite trivector" in a laboratory with the sufficient power to overwhelm the "omnipresent" one. Even, we could try to reduce the needed power creating a "coil of trivector".

This is, the same that we create a coil of current to sum up its effect to create magnetic fields, we could do something similar to this to help the creation of the trivector:

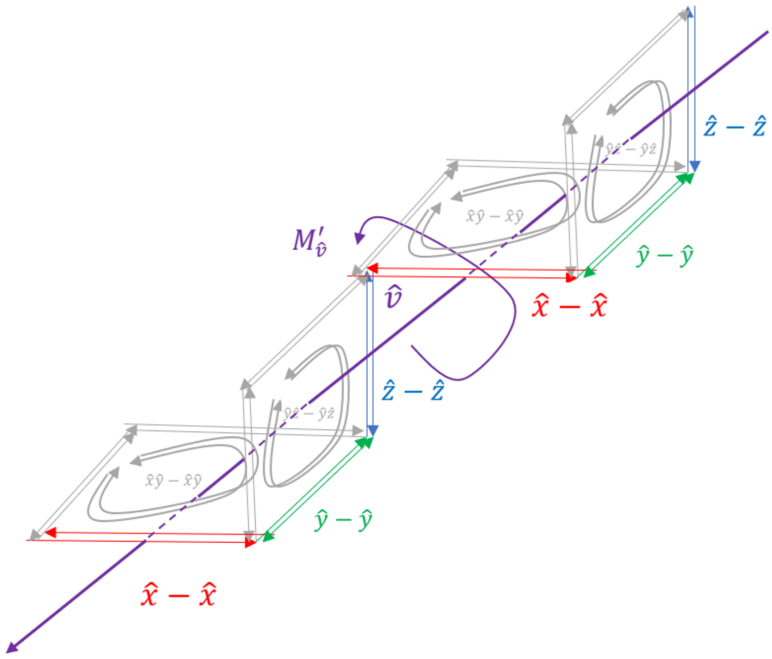


Fig. A3.1 Adding consecutive trivectors

Adding consecutive trivectors, we can try to sum up its effects. This figure A3.1 is very theoretical and not practical. In fact, whatever coil of two twisted cables that is rolled in a cylinder from left to right (or the opposite) creates a trivector. The longer the cylinder, the longer its effect. But it has to be rolled from one side to the other in the same direction all the time, you cannot randomly move from left to right or viceversa. You have to keep the direction of rolling always the same, to sum the effects.

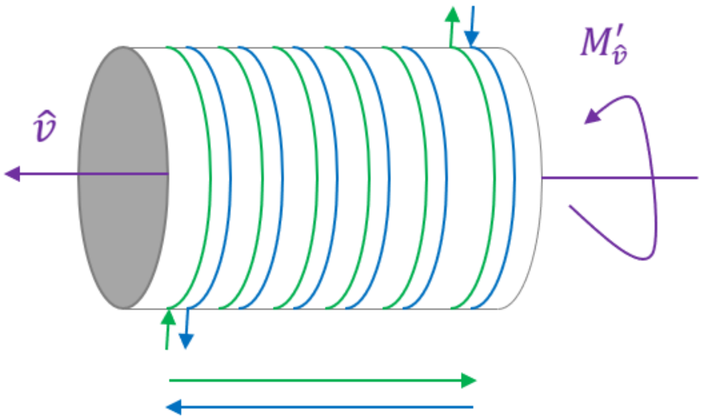


Fig. A3.2 Creating a trivector in a coil. The green and blue cables in reality are twisted (represented parallel -instead of twisted- in the figure for simplicity). And the current will go in opposite direction in green and blue cables.

You can check figure A3.2. The green and blue cables are twisted (but represented parallel in the figure for simplicity). The currents in blue and green cables are opposite.

According to Maxwell laws and classical electromagnetism, this configuration is useless. It is only a loss of power, as green and blue cables effect cancel.

In Geometric Algebra, the vectors (currents) and the bivectors (magnetic fields) cancel also. But the trivector acts in the same direction for both cables. As the opposite currents move also in an opposite spatial direction -the axial direction of the cylinder- so both cables act creating the same trivector, and its effects (trivector-wise) are summed.

This is, the individual coil created by each cable creates an opposite bivector in both cables -because their currents are opposite-. But the direction of the rolling of the cable in the other dimension -the vector representing the axis of the cylinder- is also opposite. So, the product between the bivector and the vector in both cables leads to the same trivector (the product of two positives is equal to the product of two negatives).

This means, inside the cylinder, the Maxwell laws are altered, and the $M'_{\hat{v}}$ created by the purple vector $\hat{v}$ in Fig A3.2 would be different than the magnetic field, $M_{\hat{v}}$ that would be created in empty space (Fig A3.3). In the most extreme case, they could even have different direction. See chapter 7 for more information.

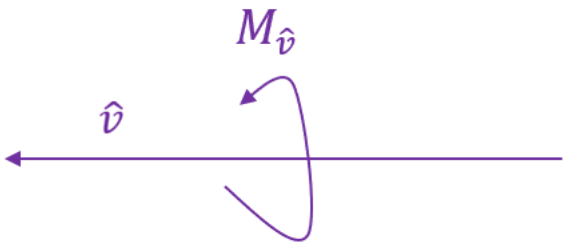


Fig. A3.3 The vector $\hat{v}$ creates a magnetic field $M_{\hat{v}}$ in empty space.

As a general comment, the figure A3.2 in a real experiment very probably would be more practical in vertical position (the axis of the cylinder and the $\hat{v}$ conductor in vertical position, having the coil planes horizontal).

Also, another thing that could be measured inside the cylinder, apart from the magnetic field $M'_{\hat{v}}$, would be the time. This means, we could put an atomic clock inside the cylinder and another one outside the cylinder. Both synchronized in the beginning of the experiment. And to check their values afterwards.

It could be that to keep the general symmetry of all the laws (including gravity, and the arrow of time/entropy, not only electromagnetism) these laws could also change (at least slightly) if an artificial trivector is modifying the omnipresent one (that normally would define how these laws work). So yes, it could be that we see a difference in the speed of time (in the most extreme sci-fi case a reversion of it, as it could happen with the Maxwell law right-hand rule, but this is just sci-fi, as commented). What we could really expect is a slight change in the measurement of the clocks -if the sufficient power or number of turns is applied-.

Also, as commented in chapter 7, it is expected that the Earth field increases the effect of the omnipresent trivector. So, it is expected that the changes in time or in the magnetic field $M'_{\hat{v}}$, provoked by the new created artificial trivector, would be bigger in space, outside the Earth orbit, than on the Earth surface. So making this experiment outside the Earth surface would increase the possibilities of success.

A4. Annex A4. Off-topic

One off-topic thing to burn your head. If the number of degrees of freedom depend on the equation:

$$\boldsymbol{Total\ number\ of\ degrees\ of\ freedom = 2^n}$$

It is clear that three spatial dimensions create 8 degrees of freedom, as commented throughout the paper.
Two spatial dimensions create 4 degrees (the two vectors, one bivector and the scalars).
One spatial dimension create 2 degrees (one vector and the scalars).
But…
0 spatial dimensions (the nothing, the emptiness)… create 1 degree of freedom! In fact, the scalars. They will live somewhere even with no space… Probably they rioted until they got the spatial dimensions to scape? :)
If you like these stupidities, you can also check [34] or [40] if you want.

A5. Annex A5. Off-topic: The anti-spatial dimensions

I have checked what could happen if the 8 degrees of freedom of the multivector created by the three spatial dimensions would not be sufficient. The solution could be to go again to the 4 space dimensions. But the negative signature of the trivector and the Dirac equations [5] work so well that I would keep the definition of time as it is, the trivector.

A solution could be to have the three spatial dimensions x, y, z and three anti-spatial dimensions u,v,w that we cannot perceive. The same as the matter has won the antimatter, there could be another three dimensions with opposite definition (whatever this means), that usually do not interfere with our reality. These three extra dimensions would create multivectors of 64 degrees of freedom (2^6=64), leaving more freedom for the magnitudes. The product between two of them would lead to 4096 free parameters that seem too much for what it is necessary in the theory. I guess (and hope) that the 8 degrees of freedom of the multivector of just three spatial dimensions (x,y,z) should be sufficient to explain the interactions. As commented, a product of three of them leads to a maximum of 512 parameters that fits with the necessary 248 free parameters (according E8 theory) taking into account symmetries and other relations.

The reason of considering the possibility of these extra three anti-space dimensions is more related to symmetry than to necessity. The same that exists matter and anti-matter and left-handed and right-handed particles, to leave the possibility that anti-dimensions exist to keep a complete symmetry. Besides that, of course they give more degrees of freedom in case they would be necessary.

A6. Annex A6. Geometric Algebra $Cl_{3,0}$ vs $Cl_{0,3}$

In all the papers [4], [5], [6], [26], [27], [31], I have considered the Geometric Algebra $Cl_{3,0}$ but there is another possibility, that is $Cl_{0,3}$. The difference is in the square signature of the vectors and the trivector.

Considering always orthonormal bases, in Geometric Algebra $Cl_{3,0}$:

$$1^2 = 1$$
$$\hat{x}^2 = \hat{x}\hat{x} = 1$$
$$\hat{y}^2 = \hat{y}\hat{y} = 1$$
$$\hat{z}^2 = \hat{z}\hat{z} = 1$$
$$(\hat{x}\hat{y})^2 = \hat{x}\hat{y}\hat{x}\hat{y} = -1$$
$$(\hat{y}\hat{z})^2 = \hat{y}\hat{z}\hat{y}\hat{z} = -1$$
$$(\hat{z}\hat{x})^2 = \hat{z}\hat{x}\hat{z}\hat{x} = -1$$
$$(\hat{x}\hat{y}\hat{z})^2 = \hat{x}\hat{y}\hat{z}\hat{x}\hat{y}\hat{z} = -1$$

On the other hand, in Geometric Algebra $Cl_{0,3}$:

$$1^2 = 1$$
$$\hat{x}^2 = \hat{x}\hat{x} = -1$$
$$\hat{y}^2 = \hat{y}\hat{y} = -1$$
$$\hat{z}^2 = \hat{z}\hat{z} = -1$$

$$(\hat{x}\hat{y})^2 = \hat{x}\hat{y}\hat{x}\hat{y} = -1$$
$$(\hat{y}\hat{z})^2 = \hat{y}\hat{z}\hat{y}\hat{z} = -1$$
$$(\hat{z}\hat{x})^2 = \hat{z}\hat{x}\hat{z}\hat{x} = -1$$
$$(\hat{x}\hat{y}\hat{z})^2 = \hat{x}\hat{y}\hat{z}\hat{x}\hat{y}\hat{z} = 1$$

In both Algebras (as commented, in at least orthogonal bases) the anticommutative relations are held:

$$\hat{x}\hat{y} = -\hat{y}\hat{x}$$
$$\hat{y}\hat{z} = -\hat{z}\hat{y}$$
$$\hat{z}\hat{x} = -\hat{x}\hat{z}$$

I have made comparisons between both Algebras regarding the the Maxwell Equations, Lorentz Force (and I will do also for the Dirac equation). The difference is mainly (as expected) in signs. In general, they are equivalent just changing some sign conventions or changing the signs when assigning correspondences between Geometric Algebra and its equivalents in Tensor or matrix Algebras.

Anyhow, none of them match so perfectly right that you can consider that one of them is the correct one against the other.

Although it is not the one I have used, I see some advantages in the $Cl_{0,3}$ version.

A6.1. In $Cl_{0,3}$, time could be considered either the trivector or the scalar element, depending on the situation, not breaking "the opposite signature rule"

In the Geometric Algebra $Cl_{3,0}$ version, the time is the trivector as commented. It has a signature (negative) that is opposite to the vectors (space dimensions) whose signature is positive.

But as the bivectors have also negative signature (and sometimes represent magnitudes as linear momentum or velocity), this means that in some cases some magnitudes (as linear momentum) would have the same signature as time (both negatives) and this is not ok in certain disciplines as General Relativity for example.

Also, we have seen that in certain cases, it could be convenient to consider the time as the scalar (see A6.2). This happens in certain magnitudes that should be time related but somehow, due to some operations with the dimension units, in the end the magnitude finish being a scalar. An example of this happens at the end of Chapter 8 of [26] where the time component of the electrical current multivector J happens to be J_0 (the scalar part) instead of the expected J_{xyz} (the trivector element).

In Geometric Algebra $Cl_{0,3}$, there is a solution to this. The vectors (that could represent space) and the bivectors that could represent velocity or linear momentum (see [5] or [6] for example), both have the same signature, negative in this case (see the $Cl_{0,3}$ relations at the beginning of this chapter A6).

In parallel, both the scalars and the trivector have the same signature (positive in this case) opposite to the vectors and bivector ones. This means, depending on convenience or just a forced situation, if can be used the trivector or the scalars as the time magnitude and they will always have the same signature (positive, making both equivalents) and always opposite to the vectors and bivectors that normally represent space related dimensions.

We can say that we can divide the eight degrees of freedom in two halves (each of them, with its own representation of time):

The even grade will have the three bivectors and the scalars. The bivectors (grade 2) will have negative signature (representing magnitudes normally related to space dimensions) and the scalars (grade 0) will have positive signature (this is, opposite signature, with the possibility of representing time in some cases if necessary).

$$(\hat{x}\hat{y})^2 = \hat{x}\hat{y}\hat{x}\hat{y} = -1$$
$$(\hat{y}\hat{z})^2 = \hat{y}\hat{z}\hat{y}\hat{z} = -1$$
$$(\hat{z}\hat{x})^2 = \hat{z}\hat{x}\hat{z}\hat{x} = -1$$
$$1^2 = 1$$

The odd grade will have the three vectors and the trivector. The vectors (grade 1) will have negative signature (representing something related to space) and the trivector (grade 0) will have positive signature (opposite signature, representing the time).

$$\hat{x}^2 = \hat{x}\hat{x} = -1$$
$$\hat{y}^2 = \hat{y}\hat{y} = -1$$
$$\hat{z}^2 = \hat{z}\hat{z} = -1$$
$$(\hat{x}\hat{y}\hat{z})^2 = \hat{x}\hat{y}\hat{z}\hat{x}\hat{y}\hat{z} = 1$$

On the other hand, in $Cl_{3,0}$ the scalars are positive signature and the trivector negative signature. So, if the time is represented by either one of them depending on the situation, this could have issues. The same for vectors and bivectors that also have opposite signatures. Anyhow, as commented, it is not clear which algebra represents better the reality, but it seems that $Cl_{0,3}$ seems more coherent.

A6.2. The Algebra of the Physical Space (APS) is a subset of $Cl_{0,3}$

The Algebra of the Physical Space (APS) [43] is a discipline that considers the time as the scalar instead of the trivector.

As commented before, sometimes it is not clear which one to use (scalars or trivector) or even if both are valid to represent the time. Anyhow, I will not comment about this here. I did it already in Annex A1 of [26].

What I will comment is its mapping with Algebra $Cl_{0,3}$. We can consider two options.

- One is that APS is a subset of $Cl_{0,3}$ that considers scalars (representing the time with positive signature) and vectors (representing time with negative signature) this way:

$$\hat{x}^2 = \hat{x}\hat{x} = -1$$
$$\hat{y}^2 = \hat{y}\hat{y} = -1$$
$$\hat{z}^2 = \hat{z}\hat{z} = -1$$
$$1^2 = 1$$

  With the component relations among them:

$$\hat{x}\hat{y} = -\hat{y}\hat{x}$$
$$\hat{y}\hat{z} = -\hat{z}\hat{y}$$
$$\hat{z}\hat{x} = -\hat{x}\hat{z}$$

  And their bivectors and trivector square:

$$(\hat{x}\hat{y})^2 = \hat{x}\hat{y}\hat{x}\hat{y} = -1$$
$$(\hat{y}\hat{z})^2 = \hat{y}\hat{z}\hat{y}\hat{z} = -1$$
$$(\hat{z}\hat{x})^2 = \hat{z}\hat{x}\hat{z}\hat{x} = -1$$
$$(\hat{x}\hat{y}\hat{z})^2 = \hat{x}\hat{y}\hat{z}\hat{x}\hat{y}\hat{z} = 1$$

- The other option is that they are a "hidden form" of the even-grade subset of the Algebra $Cl_{0,3}$. This is, the bivectors and the scalars. I say "hidden form" because in this form they are not considered bivectors but quaternion vectors [44], but in fact they are same thing, as I will show:

$$1$$
$$\hat{\imath} = \hat{y}\hat{z}$$
$$\hat{\jmath} = \hat{z}\hat{x}$$
$$\hat{k} = \hat{x}\hat{y}$$

The scalars represent the time and the "vectors" (in fact, bivectors) $\hat{\imath}, \hat{\jmath}, k$ represent the space related magnitudes. They fulfill the quaternion relations as:

$$1^2 = 1$$
$$\hat{\imath}^2 = (\hat{y}\hat{z})^2 = \hat{y}\hat{z}\hat{y}\hat{z} = -1$$
$$\hat{\jmath}^2 = (\hat{z}\hat{x})^2 = \hat{z}\hat{x}\hat{z}\hat{x} = -1$$
$$\hat{k}^2 = (\hat{x}\hat{y})^2 = \hat{x}\hat{y}\hat{x}\hat{y} = -1$$

$$\hat{\imath}\hat{\jmath} = \hat{y}\hat{z}\hat{z}\hat{x} = -\hat{y}\hat{x} = \hat{x}\hat{y} = \hat{k}$$
$$\hat{\jmath}\hat{k} = \hat{z}\hat{x}\hat{x}\hat{y} = -\hat{z}\hat{y} = \hat{y}\hat{z} = \hat{\imath}$$
$$\hat{k}\hat{\imath} = \hat{x}\hat{y}\hat{y}\hat{z} = -\hat{x}\hat{z} = \hat{z}\hat{x} = \hat{\jmath}$$

$$\hat{\jmath}\hat{\imath} = \hat{z}\hat{x}\hat{y}\hat{z} = -\hat{x}\hat{y} = -\hat{k}$$
$$\hat{k}\hat{\jmath} = \hat{x}\hat{y}\hat{z}\hat{x} = -\hat{y}\hat{z} = -\hat{\imath}$$
$$\hat{\imath}\hat{k} = \hat{y}\hat{z}\hat{x}\hat{y} = -\hat{z}\hat{x} = -\hat{\jmath}$$

$$\hat{\imath}\hat{\jmath}\hat{k} = \hat{y}\hat{z}\hat{z}\hat{x}\hat{x}\hat{y} = \hat{y}(-1)(-1)\hat{y} = \hat{y}\hat{y} = -1$$

The only point I want to comment here is that APS is a subset of $Cl_{0,3}$ (and probably a subset of $Cl_{3,0}$ -to be checked-). So, it is not necessary that it is an independent discipline in its own. In fact, it is a subset (or another perspective if you want) of the $Cl_{0,3}$ or $Cl_{3,0}$. So whatever shown in the papers [4], [5], [6], [26], [27], [31] in the field of $Cl_{0,3}$ or $Cl_{3,0}$, is based in the same ground where APS is stablished.

A7. Annex A7. Space Time Algebra (STA)

Until now, I have tried to delay as possible the comment about Space Time Algebra STA [3][45].

Space Time Algebra is the father of all the Geometric Algebra theories in Physics. The gods of Geometric Algebra (David Hestenes, Chris Doran, Anthony Lasenby, Garret Sobczyk, Joy Christian) have used it as the framework for their theories.

They normally consider $Cl_{1,3}$ where the three spatial dimensions which vectors are γ_1, γ_2 and γ_3 have a negative signature and the added dimension of time γ_0 has the positive signature.

$$(\gamma_0)^2 = +1$$
$$(\gamma_1)^2 = -1$$
$$(\gamma_2)^2 = -1$$
$$(\gamma_3)^2 = -1$$

We can see that the time is considered an extra dimension and the issue that has an opposite signature than the rest, has to be added as an ad-hoc property.

This algebra has 16 degrees of freedom (2^4) that are: the scalars, four vectors, six bivectors, 4 trivectors, and one tetravector. This means, the double degrees of freedom than $Cl_{0,3}$ or $Cl_{3,0}$ (that have only eight).

One important thing regarding STA is that when we enter in quantum mechanics, a clear example is the Dirac Equation, instead of using the complete algebra (the 16 degrees of freedom), they use the even subalgebra that only has 8 degrees of freedom. They use the following bivectors [3]:

$$\sigma_1 = \gamma_1\gamma_0$$
$$\sigma_2 = \gamma_2\gamma_0$$
$$\sigma_3 = \gamma_3\gamma_0$$

So, in the end, they end up using the equivalent of a $Cl_{3,0}$ or $Cl_{0,3}$ algebra with the 8 degrees of freedom emerging from these three bivectors.

This means, there are at least two issues:

- Time is an ad-hoc added extra dimension with an opposite signature, breaking the symmetry of the initial hypothesis just because it is considered necessary. And we have shown that it is not necessary to add it. Time emerges naturally and with the correct opposite signature from the more symmetrical $Cl_{3,0}$ or $Cl_{0,3}$ algebra directly. Being the time, the scalar or the trivector of these algebras, not an ad-hoc added dimension with convenient signature, but an element that emerges naturally form the algebra.
- The STA has 16 degrees of freedom but in general, only 8 are necessary. And in fact, for quantum mechanics directly the $Cl_{1,3}$ is de facto converted into a $Cl_{3,0}$ algebra as no more degrees of freedom are needed. The same can be said regarding other disciplines where even normally just the four linear dimensions (the three space dimensions and the time) are used from the 16 possible. The Occam razor, tells us that if you have a simpler solution, use it. The $Cl_{3,0}$ or $Cl_{0,3}$ are sufficient in the fields of physics I have checked. If in the end, 16 degrees of freedom are necessary for whatever discipline, ok we can go back.

As commented, this is not an annex to prove anything, just to say that when two possible options are the solution, going for the simplicity, normally is the best way. In this case, $Cl_{3,0}$ or $Cl_{0,3}$ seem sufficient to explain most of the disciplines of physics, if we accept the issue that the time could be an emergent dimension coming from the three spatial dimensions in the form of scalar or trivector and not an ad-hoc added dimension to the reality.

A8. Annex A8. Gell-Mann matrices (Strong force Interaction)

Another thing to be commented also related to the previous chapter are the Gell-Mann matrices [46]. They are the matrices used in the SU(3) group for the strong force interaction. They are elements that fulfill certain properties regarding their multiplications and commutations. In the standard algebra, the elements used are matrices.

The important thing here is that the number of matrices necessary are again… eight matrices. This means for this interaction again, only 8 elements (a space of 8 degrees of freedom if you want) are necessary.

This means 8 elements resulting from a combination of the 8 components of the $Cl_{3,0}$ or $Cl_{0,3}$ (scalars, three vectors, three bivectors and the trivector):

$$1, \hat{x}, \hat{y}, \hat{z}, \hat{x}\hat{y}, \hat{y}\hat{z}, \hat{z}\hat{x}, \hat{x}\hat{y}\hat{z}$$

Would be sufficient to create the equivalent to the strong force interaction in Geometric Algebra $Cl_{3,0}$ or $Cl_{0,3}$. Again, the message is that only the 8 degrees of freedom coming from $Cl_{3,0}$ or $Cl_{0,3}$ are necessary. If instead, we want to use $Cl_{1,3}$ (with 16 possible degrees of freedom), in the end we will only use 8 of them (probably the even grade subalgebra) making the $Cl_{1,3}$ not necessary in the first place.

At this stage, I am working in finding these 8 combinations of the 8 components of the $Cl_{3,0}$ or $Cl_{0,3}$. For this case, $Cl_{0,3}$ seems more convenient as 6 elements (vectors and bivectors) share the same signature. But the symmetry of $Cl_{3,0}$ (4 elements of each signature) could also have an advantage. To be checked.