How fictitious forces instead of gravity create tides

Sjaak Uitterdijk

Abstract – It is interesting to realize that a cubic meter of water at the surface of the oceans is subject to a gravity of Earth itself of 9800 N and to 0.03 N of Moon's gravity. So any influence of the Moon on the level of such water must be excluded. Notwithstanding this fact the most weird theories haven been developed to prove the opposite, especially to show which phenomenon creates the mysterious double frequency of the tides. This chapter shows an all-encompassing solution.

1 Introduction

The chapter is divided into two main parts. Section 2 briefly describes why the so-called Solar tide as well as the Lunar tide in the generally accepted tidal theory has to be rejected. Section 3 proves mathematically that fictitious forces, generated by Earth's spinning, cause the tides and how these forces also cause the double frequency of the tides.

2 The generally accepted tidal theory

2.1 Alleged Solar tide

The Earth is orbiting the Sun eternally for the following reasons:

1. The Sun attracts the Earth due to their mutual gravity, also named centripetal force.
2. The Earth is prevented from merging with the Sun due to the centrifugal force generated by Earth's orbital velocity.
3. Centripetal and centrifugal forces are *at any moment* exactly equal.
4. The orbiting takes place in vacuum.

The fundamental consequence is that, apart from the influence just mentioned, the Sun has no gravitational influence at all on Earth, with the result that there is no solar tide at all.

The fact that the Moon revolves around the Earth also means that the Earth doesn't have any other gravitational influence on the Moon, but the Moon has on the Earth, causing *in principle* a Lunar tide.

2.2 Alleged Lunar tide

To try to explain the two low and two high tides every 24 hours, at any applicable place on Earth, the weirdest theory has been introduced. An example of such a theory is found in reference [1]. The approach is the following, with the remark that the applied accelerations in [1] have been changed back to the related forces, based on $F = m \cdot a$.

The well-known expression for the gravitational force between two objects M and m is $F_G = GMm/R^2$, with:

G	gravitation constant: $6.7 \cdot 10^{-11}$	Nm^2kg^{-2}
M	mass of the one object	kg
m	mass of the other object	kg
R	distance between the *gravity centres* of M and m	m

The expression "*gravity centres*" plays a fundamental role in the next consideration.

The explanation continues with changing R into $R \pm \Delta r$, with the following background:

> "If the body of mass m is itself a sphere of radius Δr, then the new particle considered may be located on its surface, at a distance $(R \pm \Delta r)$ from the centre of the sphere of mass M, and Δr may be taken as positive where the particle's distance from M is greater than R."

N.B. The expression "gravity centre" has been avoided by taking the words "centre of the sphere"!

A fundamental fallacy is being made here. Changing R to R ± Δr in the formula for F_G means nothing less, and especially nothing more, than changing the distance between the two ***gravity centres*** of M and m from R into R ± Δr.
The term "gravity centre" expressly indicates that choosing a point in m at a distance Δr from its gravity centre is fundamentally unacceptable from the point of view of the validity of the expression for F_G. After all R is, given its definition, not changed! Only a location in m is chosen, outside its centre of gravity. However there is no physical law describing the force in such a point as a result of the gravity field of M. Besides that, doing so effectively a gravity field has been created on the other side of m (than the side of M) with exactly the same gravity as on the M side. As if two exactly the same Moons are resolving Earth. It should be clear that such a theory must be regarded as far from correct.

Appendix 1 explains that reality also teaches that the influence of the Moon by its gravity must be rejected

3 Mathematical background of fictitious forces

The most basic expression in the field of forces is F = m·dv/dt = ma, with F the force necessary to accelerate mass m with a. The difference between a normal and a fictitious force arises when v is taken as a (normal) linear velocity, respectively when v is the result of an angular velocity, with v = ωr. In case ω and r are both constant the resulting force is called the well-known centrifugal force. Applying m·d(ωr)/dt, with constant ω and r, however leads to F = 0. Apparently another approach has to be taken.

The most simple example of a fictitious force is a mass m orbiting another mass with constant angular velocity at distance r. In vacuum this situation is maintained forever without any external forces, like the Earth orbiting the Sun. The reason for this eternal movement is presented in section 2.1. The mentioned centrifugal force there can be expressed by F = ma, however only in the vector notation $\boldsymbol{F} = m\cdot\boldsymbol{a}$. It is rather well known that $|\boldsymbol{F}| = mv^2/r = m\omega^2 r$. This outcome will be derived below.

Reference [2] presents such a derivation. It defines a coordinate frame A with respect to which a frame B translates and rotates, shown in figure 1. The introductory text reads:

> "Many problems require use of non-inertial reference frames, for example, those involving satellites and particle accelerators. The figure shows a particle with mass m and position vector $\mathbf{x}_A(t)$ in a particular inertial frame A. Consider a non-inertial frame B whose origin relative to the inertial one is given by $\mathbf{X}_{AB}(t)$. Let the position of the particle in frame B be $\mathbf{x}_B(t)$. What is the force on the particle as expressed in the coordinate system of frame B?"

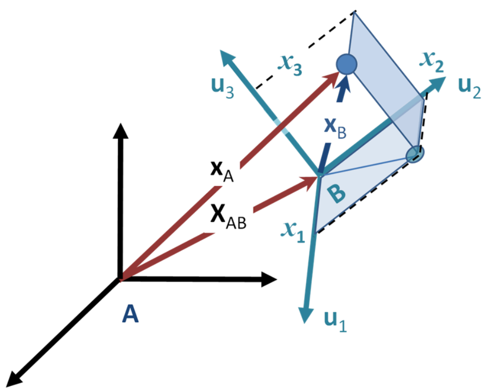

Figure 1

In conformity with figure 1 and leaving out "(t)" from now on, $\mathbf{x}_B = \Sigma\, x_j \mathbf{u}_j$, with Σ representing $\sum_{j=1}^{3}$
The following crucial statement is made in [2]:

> "The interpretation of this equation is that $\mathbf{x}_B$ is the vector displacement of the particle as expressed in terms of the coordinates in frame B at the time (t). From frame A the particle is located at: $\mathbf{x}_A = \mathbf{X}_{AB} + \Sigma\, x_j \mathbf{u}_j$"

As a result: $\mathbf{x}_A = \mathbf{X}_{AB} + \mathbf{x}_B$, suggesting that $\mathbf{x}_B$ is defined in A. But $\mathbf{x}_B$ is clearly defined in B. This crucial mistake is visible in figure 1. But indeed, the figure is confusing.

In order to repair the mentioned mistake the vector $\mathbf{x}_B$ has to be transformed from frame B to frame A.

If R would be the matrix that represents the rotation of B relative to A about its x-, y`- resp. z``-axis, with the angles α, β and γ about these axes respectively, then the matrix-vector multiplication $R * \mathbf{x}_B$ is the projection of $\mathbf{x}_B$ in A. The y`-axis is the axis after the rotation of the x-axis. The z``-axis is the axis after rotation of the x and y`-axis. The matrix R is obtained by the successive multiplications, starting at the right hand side, of the separate matrices shown below and copied from reference [3]:

$$\begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

Reference [2] continues with:

> 'Taking a time derivative, the velocity of the particle is:
>
> $d\mathbf{x}_A/dt = d\mathbf{X}_{AB}/dt + d\mathbf{x}_B/dt = d\mathbf{X}_{AB}/dt + \Sigma\, dx_j/dt \cdot \mathbf{u}_j + \Sigma\, x_j\, d\mathbf{u}_j/dt$
>
> The second term summation is the velocity of the particle, say $\mathbf{v}_B$ as measured in frame B. That is:
>
> $d\mathbf{x}_A/dt = \mathbf{v}_{AB} + \mathbf{v}_B + \Sigma\, x_j\, d\mathbf{u}_j/dt$ "

The mentioned mistake is necessarily repeated until the end, so for $\mathbf{v}_B$ and for $\mathbf{a}_B$ too, both explicitly defined in B. Therefore the explanation in [2] is abandoned and replaced by the proper formulas below.

$$d\boldsymbol{x}_A/dt = d\boldsymbol{X}_{AB}/dt + d(R * \boldsymbol{x}_B)/dt = \boldsymbol{v}_{AB} + R * d\boldsymbol{x}_B/dt + dR/dt * \boldsymbol{x}_B = \boldsymbol{v}_{AB} + R * \boldsymbol{v}_B + dR/dt * \boldsymbol{x}_B$$

$$d^2\boldsymbol{x}_A/dt^2 = d\boldsymbol{v}_{AB}/dt + d(R * \boldsymbol{v}_B)/dt + d(dR/dt * \boldsymbol{x}_B)/dt = \boldsymbol{a}_{AB} + d^2R/dt^2 * \boldsymbol{x}_B + 2(dR/dt) * \boldsymbol{v}_B + R * \boldsymbol{a}_B \qquad (1)$$

In case of no rotation $R = I$, so $dR/dt = 0$ and thus $d\boldsymbol{x}_A/dt = \boldsymbol{v}_{AB} + \boldsymbol{v}_B$, resp. $d^2\boldsymbol{x}_A/dt^2 = \boldsymbol{a}_{AB} + \boldsymbol{a}_B$.

It is interesting to compare the 3 types of fictitious forces, shown in [4], to the expression in (1):

"

Euler force:	$- m \cdot d\boldsymbol{\omega}/dt \times \boldsymbol{r}'$
Coriolis force:	$- 2m \cdot (\boldsymbol{\omega} \times \boldsymbol{v}')$
Centrifugal force	$- m \cdot \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}')$

with:

$\boldsymbol{\omega}$	the angular velocity, of the rotating reference frame relative to the inertial frame
$\boldsymbol{r}'$	the position vector of the object relative to the rotating reference frame
$\boldsymbol{v}'$	the velocity of the object relative to the rotating reference frame

"

Comparison with the term $2(dR/dt) * \boldsymbol{v}_B$ in (1) most likely explains the factor 2 in the Coriolis force.

The pure centrifugal force manifests itself only in a perfect circular motion, represented by $v = \omega r$, with ω and r constant. If these constraints are not met, we have to write: $a = dv/dt = d(\omega r)/dt = d\omega/dt \cdot r + \omega \cdot dr/dt$.

The term $d\omega/dt \cdot r$ thus represents the Euler acceleration and $\omega \cdot dr/dt$ the pure Coriolis one, because this one is related to the pure *linear* velocity dr/dt, as will be shown in the next section too.

An elliptical orbit, for example like a planet orbiting the Sun, fundamentally only consists of Euler and Coriolis forces, because neither ω nor r is constant. But the one of the two forces holding the planet in its orbit is generally only qualified as *centrifugal* force! The opposite one is the gravity / centripetal force, between Sun and planet.

Given the fact that a cross product of vectors obeys the law $\boldsymbol{\omega} \times \boldsymbol{v} = - \boldsymbol{v} \times \boldsymbol{\omega}$ it is remarkable that the above mentioned forces are not expressed as: $m \cdot (\boldsymbol{r}' \times d\boldsymbol{\omega}/dt)$, $2m \cdot (\boldsymbol{v}' \times \boldsymbol{\omega})$, respectively $m \cdot (\boldsymbol{\omega} \times \boldsymbol{r}') \times \boldsymbol{\omega}$!

3.1 Fictitious forces creating air flows above Earth's surface

Reference [4] shows the following example:

> *"The acceleration affecting the motion of air "sliding" over the Earth's surface is the horizontal component of the Coriolis term -2$\boldsymbol{\Omega} \times \boldsymbol{v}$. This component is orthogonal to the velocity over the Earth surface and is given by the expression $\omega v 2 \sin\varphi$, where ω is the spin rate of the Earth and φ the latitude."*

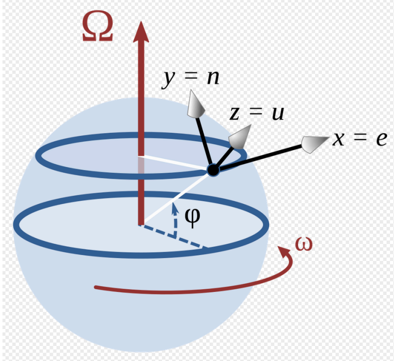

Figure 2 Coordinate system at latitude φ with x-axis east, y-axis north and z-axis upward

The particle with mass m in figure 1 is in figure 2 replaced by a certain part of the air in the atmosphere. "Air "sliding" over the Earth's surface" means more precisely: air moving relative to Earth's surface, so with the velocity $\boldsymbol{v}$ corresponding to $\boldsymbol{v_B}$ in figure 1, with $\boldsymbol{v_B} = (v_e , v_n , v_u)$. Let us have a critical look at the derivation of $\omega v 2 \sin\varphi$ as presented in [4], having $-2\boldsymbol{\Omega} \times \boldsymbol{v}$ replaced by $2\boldsymbol{v} \times \boldsymbol{\Omega}$.

Definition of cross product

In a right-hand frame, with the x-axis oriented at 3 and the y-axis at 12 o'clock, as shown in figure 2 in the black frame, called B, the z-axis is perpendicular to the x-y plane and oriented positive in the same direction as a corkscrew will move, if it is rotated from the x-axis towards the y-axis.
In case $\boldsymbol{\Omega}$ is chosen positive along this z-axis and $\boldsymbol{v}$ positive along this y-axis, rotating $\boldsymbol{v}$ towards $\boldsymbol{\Omega}$ creates a positive acceleration along the positive x-axis. The cross product of these two vectors will be written as $\boldsymbol{v}$ x $\boldsymbol{\Omega}$. The vectors $\boldsymbol{v}$ and $\boldsymbol{\Omega}$ must of course be defined in the same frame. The mathematical definition of this cross product, with $\boldsymbol{v}=(v_x, v_y, v_z)$ and $\boldsymbol{\Omega}=(\omega_x, \omega_y, \omega_z)$, is

$$\boldsymbol{v} \times \boldsymbol{\Omega} = (v_y\omega_z - v_z\omega_y \quad , \quad v_z\omega_x - v_x\omega_z \quad , \quad v_x\omega_y - v_y\omega_x).$$

The black frame B is fixed in frame A at latitude φ and rotated about the x-axis of A by + or -(90°- φ), because for φ = 90° (at the North pole) A and B are oriented equally. A vector in A projected in B will mathematically be described by multiplying the rotation matrix $\mathbf{R}_{AB}$ with the vector in A. In figure 2 the z-axis of A becomes the positive y-axis of B for φ = 0°, so $\mathbf{R}_{AB}$ must be:

$$\mathbf{R}_{AB} = \begin{matrix} 1 & 0 & 0 \\ 0 & \sin\varphi & \cos\varphi \\ 0 & -\cos\varphi & \sin\varphi \end{matrix} \quad \text{with its inverse } \mathbf{R}_{BA} \text{ as} \quad \begin{matrix} 1 & 0 & 0 \\ 0 & \sin\varphi & -\cos\varphi \\ 0 & \cos\varphi & \sin\varphi \end{matrix}$$

Applying $\mathbf{R}_{AB}$ to the vector $\boldsymbol{\Omega_A} = \omega(0 , 0 , 1)$, defined in A, results in $\boldsymbol{\Omega_B} = \omega(0 , \cos\varphi , \sin\varphi)$, defined in B. Thus for $\boldsymbol{v_B} = (v_e , v_n , v_u)$, $2\boldsymbol{v_B} \times \boldsymbol{\Omega_B} = 2\omega(v_n\sin\varphi - v_u\cos\varphi , -v_e\sin\varphi , v_e\cos\varphi) = \boldsymbol{a_{v\Omega}}$, defined in frame B.

Thereafter it is assumed in [4] that $v_u = 0$. Instead of writing $\boldsymbol{a_{v\Omega}} = 2\omega(v_n\sin\varphi , -v_e\sin\varphi , v_e\cos\varphi)$, it is written as f $(v_n , -v_e)$, adding: "*where f = 2ωsinφ is called the Coriolis parameter*."
This mathematical carelessness could have been avoided easily by stating that the horizontal component of $\boldsymbol{a_{v\Omega}}$ is the vector-sum of its x- and y-components, so 2ωvsinφ, with $v = \sqrt{(v_n^2 + v_e^2)}$.

But still the factor 2 falls out of the sky. As mentioned above, this is caused by the assumption that the term $2(d\mathbf{R}/dt)\cdot\boldsymbol{v_B}$ in (1) is a Coriolis acceleration. But the matrix $\mathbf{R}$ in [4] is constant, so $d\mathbf{R}/dt = 0$!

However, much more important is the lack of an order of magnitude larger acceleration, shown hereafter.

3.2 Complete derivation of fictitious forces creating air flows above Earth's surface

In the previous section the acceleration has been calculated which is developed when winds, with speed $\boldsymbol{v}$, are rotated by Earth's spinning, with angular speed $\boldsymbol{\Omega}$. The outcome is simply the cross product $\boldsymbol{v} \times \boldsymbol{\Omega}$.

In this section it will be shown that no winds are needed to develop such accelerations. Just the rotation of the Earth is enough to do so. The resulting accelerations fall under the category "centrifugal accelerations", while those in the preceding section are called "Coriolis accelerations". Both phenomena apply to air as well as to water in the vicinity of Earth's surface. Hereafter it will be shown how these centrifugal accelerations can be calculated.

Finally the strength of both types of accelerations will be compared to each other.

The vector $\boldsymbol{r_A}$ is defined as the vector representing the position of the origin of B in A, at latitude φ. This vector is, together with frame B, rotated about the z-axis of A with the angle ωt, as a result of Earth's spinning. Its coordinates are therefore: $\boldsymbol{r_A} = r \cdot (\cos\varphi\cos\omega t\ ,\ \cos\varphi\sin\omega t\ ,\ \sin\varphi)$, with r Earth's radius. The acceleration in A is $d^2\boldsymbol{r_A}/dt^2 = \omega v_\varphi(-\cos\omega t\ ,\ -\sin\omega t\ ,\ 0) = \boldsymbol{a_{CA}}$, with $v_\varphi = \omega r\cos\varphi$, *valid for any longitude*. The index C is meant to express a centrifugal acceleration.

The projection of $\boldsymbol{a_{CA}}$ as $\boldsymbol{a_{CB}}$ in frame B can be created by the rotation matrix R_{AB}, shown in section 3.1.

$$R_{AB} * \boldsymbol{a_{CA}} = \omega v_\varphi \cdot \begin{matrix} 1 & 0 & 0 \\ 0 & \sin\varphi & \cos\varphi \\ 0 & -\cos\varphi & \sin\varphi \end{matrix} * \begin{matrix} -\cos\omega t \\ -\sin\omega t \\ 0 \end{matrix} = \omega v_\varphi \cdot (-\cos\omega t\ ,\ -\sin\varphi\sin\omega t\ ,\ \cos\varphi\sin\omega t) = \boldsymbol{a_{CB}}$$

The acceleration $\boldsymbol{a_{CB}}$ can now be added directly to $\boldsymbol{a_{v\Omega}} = \omega(v_n\sin\varphi - v_u\cos\varphi\ ,\ -v_e\sin\varphi\ ,\ v_e\cos\varphi)$ (corrected by the factor 2) resulting in the total acceleration in frame B:

$$\boldsymbol{a_{TB}} = \boldsymbol{a_{CB}} + \boldsymbol{a_{v\Omega}} = \omega(-v_\varphi\cos\omega t + v_n\sin\varphi - v_u\cos\varphi\ ,\ \sin\varphi(-v_\varphi\sin\omega t - v_e)\ ,\ \cos\varphi(v_\varphi\sin\omega t + v_e))$$

The absolute value of $\boldsymbol{a_{CB}}$, *inclusive its z-component*, is $\omega v_\varphi\sqrt{\{(-\cos\omega t)^2+(-\sin\varphi\sin\omega t)^2+(\cos\varphi\sin\omega t)^2\}} = \omega v_\varphi$. But the ultimately interesting variable is its horizontal component, as in $\boldsymbol{a_{v\Omega}}$. This variable is, as the expression for $\boldsymbol{a_{CB}}$ shows, a function of time, due to Earth's spinning.

A first impression of the difference in strength of the two types of acceleration is found by comparing the *maximum* value ωv_φ of $\boldsymbol{a_{CB}}$ to the value ωv of the horizontal component of $\boldsymbol{a_{v\Omega}}$.

ωv_φ is, wide on both sides of the equator, much larger than ωv, because v_φ is there ~ ωr, so ~ 460 m/s. That is an order of magnitude larger than a very high wind speed v of 100 km/h ~ 30 m/s.

In reference [4] under *Meteorology and oceanography* the following statement is found:

> *"Perhaps the most important impact of the Coriolis effect is in the large-scale dynamics of the oceans and the atmosphere."*

As said above already, both phenomena indeed apply to air as well as to water near Earth's surface.

It has been shown now that the much larger centrifugal "effect" is most responsible for these "large-scale dynamics" in the atmosphere.

The same applies to the oceans, because the speed of, for example, the Gulf Stream is even an order of magnitude lower than the high wind speed in the atmosphere, just mentioned.

In the next section it will be shown that these centrifugal accelerations, at different latitudes, must cause the tides in the oceans, due to their function of time.

3.3 Fictitious forces creating tides in the oceans

The attraction force of the Moon to *one kg water* at Earth's surface is Gm_M/D^2, with G the gravity constant, m_M the mass of the Moon and D the distance between the *gravity centres* of Earth and Moon. This force equals 0.000033 N. Earth's (own) gravity to this 1 kg water is 9,8 N, so 300000 times larger! Any influence of the Moon on the level of such water by means of its gravity must thus be excluded.

A force per kg is actually an acceleration, as shown by dividing in the equation $\boldsymbol{F} = m \cdot \boldsymbol{a}$ both sides by m. Therefore Earth's mass m_E has been removed in the expression Gm_Em_M/D^2 in order to get the acceleration $\boldsymbol{a_G}$. From now on the term 'force' thus may be read as 'acceleration' and the other way round.

Because D is much larger than Earth's radius r, the approximation of a uniformly distributed $\boldsymbol{a_G}$ over Earth's hemisphere, seen from Moon's position, is sufficient accurate in the situation under consideration. This in contradiction to the centrifugal acceleration at the surface of Earth, caused by its spinning around the axis through the poles. This centrifugal acceleration, calculated in the previous section as $\boldsymbol{a_{CA}}$, is located in frame A at latitude φ, shown in figure 2. Its *maximum* absolute value is $\omega v_\varphi = \omega^2 r\cos\varphi$ and, given its just mentioned location, directly comparable to Moon's gravity acceleration $\boldsymbol{a_G}$. Such a comparison leads to a most surprising result, from the point of view of the generally accepted tidal theory.

The mass of the Moon is $7.35 \cdot 10^{22}$ kg, D = $3.84 \cdot 10^8$ m and G = $6.67 \cdot 10^{-11}$ Nm^2kg^{-2}, so $a_G = 3.3 \cdot 10^{-5}$ m/s^2. The angular velocity of Earth's spinning is $2\pi/(24 \cdot 3600)$ rad/s. The related a_{CA} at latitude 45°, and with Earth's radius r as $6.37 \cdot 10^6$ m, is $2.4 \cdot 10^{-2}$ m/s^2. So roughly a factor of 1000 larger than a_G!

The conclusion at this moment thus is that Moon's gravity has no significant influence at all on the tides.

Hereafter it will be investigated what the importance of the centrifugal forces could be with regard to the, so far not understood, double frequency (2ω) of the tides. As calculated in 3.2, the acceleration $\boldsymbol{a_{CB}}$ is:

$$\boldsymbol{a_{CB}} = \omega v_\varphi(-\cos\omega t \;,\; -\sin\varphi \sin\omega t \;,\; \cos\varphi \sin\omega t)$$

Since its z-coordinate is not relevant for tidal effects, only the horizontal component will be considered. Figure 3 shows the sum of the *squared* x- and y-coordinates. The sought-after double frequency (2ω) is clearly visible! This in contrast to the mathematical expression for $\boldsymbol{a_{CB}}$, where it is not at all visible.

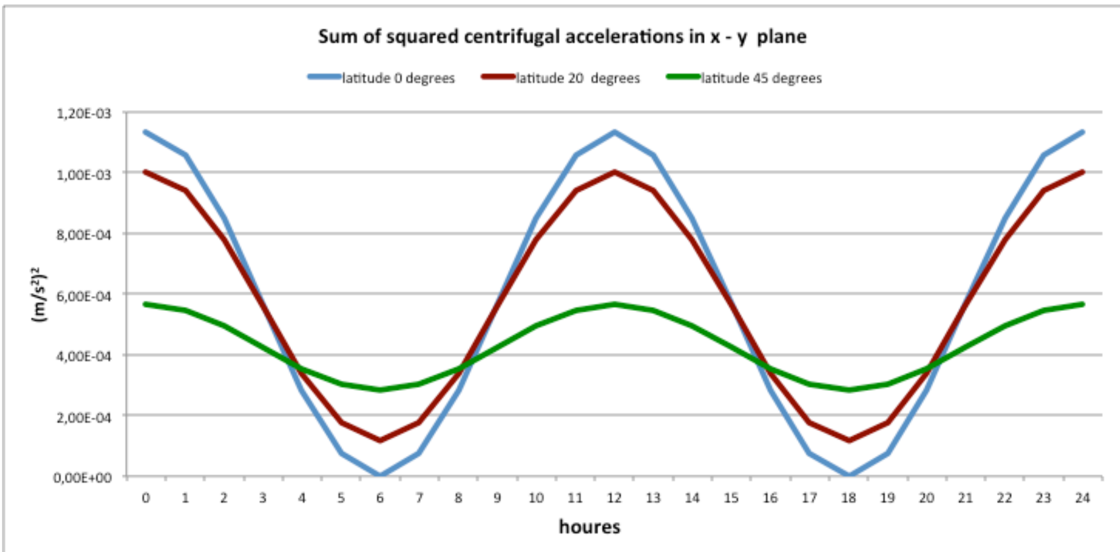

Figure 3

Figure 4 shows the square root of the values in figure 3, proving that in the neighbourhood of the equator the double frequency is maintained, notwithstanding the fact that right at the equator, where $\sin\varphi = 0$, $\boldsymbol{a_{CB}} = \omega^2 r(-\cos\omega t\ ,\ 0\ ,\ \sin\omega t)$, showing that the horizontal component simply equals $\omega^2 r\cos\omega t$.

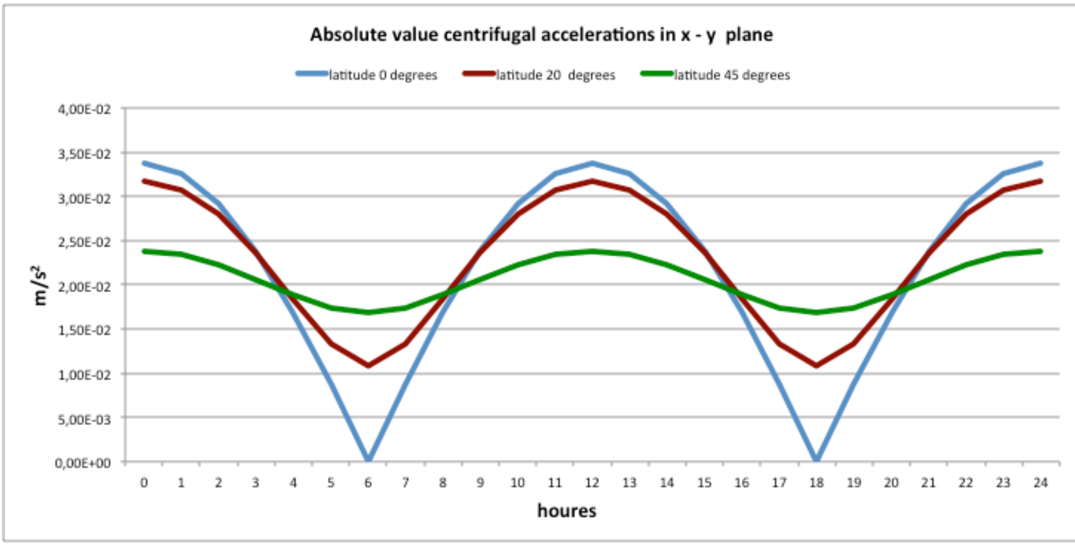

Figure 4

To show a direct comparison with $\boldsymbol{a_{CB}}$ the projection of Moon's gravitational acceleration $\boldsymbol{a_G}$ in frame B has to be calculated. The rotation of the Earth will inevitably affect the behaviour of $\boldsymbol{a_G}$ in the upper part of the oceans, but such projections will not at all produce values close to $|\boldsymbol{a_{CB}}|$. Except in a small area around the poles, where ωv_φ is small due to the low value of $\cos\varphi$. Together with the fact that there is no water in these areas, the calculation of such projections has been omitted.

The conclusion up to now that the Moon doesn't have any influence on the tides seems to be in contradiction with the "observation" that the "double frequency" pattern of the tides is *shifted* each 24 hours by 50 minutes, caused by the a-synchronic orbit of the Moon around the Earth, relative to Earth's spinning.

Moon's *sidereal* period of 27.3 days corresponds to an angular velocity of $2\pi/27.3 = 0.23$ rad/day. Expressed in minutes/day this is $24\cdot60/27.3 = 52.7$.

In Appendix 1 it is shown that this value of 52.7 minutes/day is by far not found in any of the 12 investigated places. Their mean value is 32 minutes/day! The explanation of this significant lower value than the claimed 50 minutes may be found in the following consideration.

The number of sidereal moons per year is $365/27.3 = 13.37$. The fraction 0.37 of this number represents $0.37\cdot27.3 = 10.1$ days/year, so ~40 minutes/24 hours. Such a result is significantly closer to the measured shifts. This could lead to the conclusion that the measured shifts in the normal tide are most likely only caused by the shift in spring and neap tides, which will be explored in more detail in the next section.

3.4 Cause of spring and neap tides

Reference [5] shows the following cause of spring and neap tides:

> **"**The semi-diurnal range (the difference in height between high and low waters over about half a day) varies in a two-week cycle. Approximately twice a month, around new moon and full moon when the Sun, Moon, and Earth form a line, the tidal force due to the Sun reinforces that due to the Moon. The tides's range is then at its maximum; this is called the spring tide. .... There is about a seven-day interval between springs and neaps."

The explanation is clarified with the figure below, copied from [5] too, supplemented with the text:

> "When the Moon is at first quarter or third quarter, the Sun and Moon are separated by 90° when viewed from the Earth, and the solar tidal force partially cancels the Moon's tidal force. At these points in the lunar cycle, the tides's range is at its minimum; this is called the neap tide, or neaps."

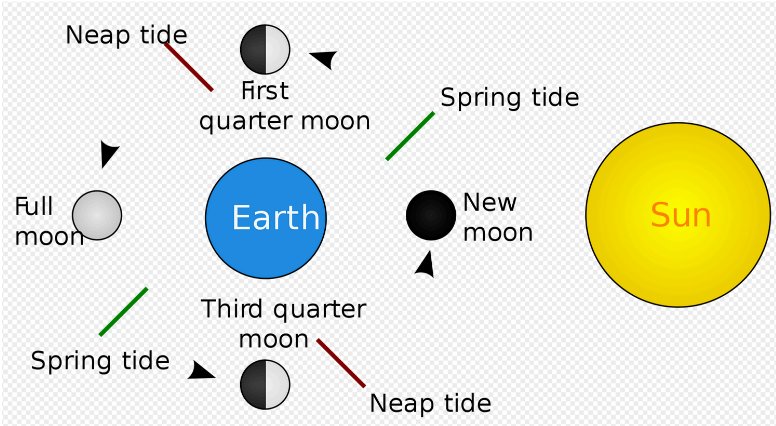

Figure 6

As shown in section 3.3, Moon's gravity is negligible compared to the centrifugal forces at Earth's surface. So any influence of the Moon *along this way* in the form of spring and neap tides must be considered impossible too. In addition, the influence of the Sun's gravity on the tides is completely absent, as argued in section 2.1. Except one remarkable phenomenon!
Given the *observations* in [5], it is most unlikely that the influence of the Moon is *totally* absent. Searching for influences along the way of fictitious forces, compels us to consider motions of Earth superimposed on its orbit around the Sun. These movements, henceforth referred to as wobbles, appear to occur as a result of Moon's orbit around the Earth.

Appendix 2 shows the basics of the digital simulation model with which the amplitude of this wobble has been calculated. The result of the model is: wobble = $4.65 \cdot 10^6 \cdot \sin(\omega_M - \omega_E)t$ meter, with ω_M Moon's *sidereal* angular velocity *about the Earth* of $2\pi/(27.3 \cdot 24 \cdot 3600) = 2.66 \cdot 10^{-6}$ rad/s and ω_E the angular velocity of Earth's *about the Sun* of $2\pi/(365 \cdot 24 \cdot 3600) = 2 \cdot 10^{-7}$ rad/s, resulting in $\omega_M - \omega_E = 2.46 \cdot 10^{-6}$ rad/s.

N.B. The radial frequency $\omega_M - \omega_E$ represents a period of $1/2.46 \cdot 10^{-6} = 4.06 \cdot 10^5$ s/rad $\cdot$ $2\pi/(3600 \cdot 24)$ is 29.5 days, being Moon's *synodic* period.

The wobble is maximally negative at new moon in figure 6. At such a moment, the absolute value of the Earth's acceleration, as a result of the wobble, is also at its maximum, just like at full moon. The agreement with the observations in [5] is that spring and neap tides occur at these same moments. In the approach presented here, the maximum accelerations, experienced by Earth, occur when the Earth comes to its turning points in the wobble.
It is like walking with a shallow bath in your hands, filled with water up to its edge, in the same patron as the wobble does, but then walking from forward to backward, respectively the other way around, at the turning points. The water will for sure flow over the edge at the turning points.

The physical background is the well-known expression p = mv, with in this case m the mass of water in the oceans and v the "wobble velocity" of Earth.

In summary: The only forces left to move the water at Earth's surface are the fictitious horizontal accelerations, mathematically presented above by $\boldsymbol{a_{CB}}$. These accelerations lead to currents, which in turn are blocked by coasts. This translates into a rise in the water level there, which in turn leads to currents in the other direction. All things considered, tides are ultimately the result of water being moved by horizontal centrifugal forces and restrained by coasts. Spring and neap tides are caused by the fluctuation, called wobble, in Earth's orbit around the Sun, which in turn is caused by Moon's orbit around Earth.

Conclusions

1 A cubic meter of water at the surface of the oceans is subject to a gravitational force of 9800 N of Earth itself and to 0.03 N of Moon's gravity. So any influence of the Moon on the level of such water must be excluded.
2 The generally accepted Coriolis forces, influencing winds at Earth's surface, has by far much less effects on the behaviour of the atmosphere than centrifugal forces have.
3 The gravity of the Moon cannot explain the behaviour of the tides, because its force is about 1000 times smaller than the centrifugal forces at Earth's surface around 45^0 latitude.
4 The generally accepted explanation for the double frequency of the tides effectively assumes that there is an equivalent Moon just opposite to the real one on the other side of the Earth, causing the same attraction forces to the oceans as on the Moon's side. Such a theory has to be rejected for more than one reason.
5 The double frequency of the tides is fully explained by the prevailing centrifugal forces at any latitude, except at the poles, as a result of Earth's spinning.
6 The alleged and generally accepted theory regarding the cause of spring and neap tides, claiming the influence of the combined gravity of Moon and Sun, has to be rejected.
7 The cause of spring and neap tides has been found in the acceleration of Earth itself at the turning points of its very small wobbling movements in its orbit around the Sun.
8 The alleged shift of the tides by 50 min./24 hours turns out to be 32 minutes, based on 12 investigated tidal records. This shift is most likely caused by the shift of spring and neap tides.
9 The so-called synodic period of the Moon (29.5 days) turns out to be the period of Earth's wobble, superimposed on its mean orbit around the Sun. Half it's value is the period of spring and neap tides too.
10 Despite the arguments above, presented to NASA, it continues to promote the weird theory in [8]
11 Galileo Galilei was already convinced around the year 1600 that only the movements of the Earth itself are the cause of the tides. Since his theory continues to be rejected, it is time to rehabilitate him in this regard.

References

[1] https://en.wikipedia.org/wiki/Tidal_force
[2] https://en.wikipedia.org/wiki/Fictitious_force
[3] https://en.wikipedia.org/wiki/Rotation_matrix
[4] https://en.wikipedia.org/wiki/Coriolis_force
[5] https://en.wikipedia.org/wiki/Tide
[6] https://www.tide-forecast.com/countries
[7] https://en.wikipedia.org/wiki/Amphidromic_point
[8] https://science.nasa.gov/moon/tides/

Appendix 1 of Encore **The real situation**

Reference [6] apparently, given the accurate looking predictions, calculates the tides for a countless number of places around the world. See figure A1.1 for the place Bournemouth in England.

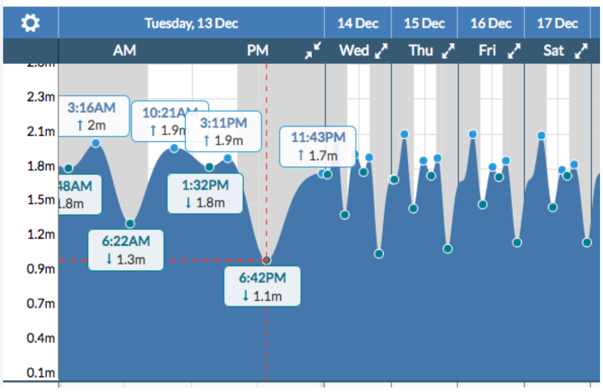

Figure A1.1 Tides in Bournemouth, with the red line showing the requested "present" moment (GMT 18:39)

This example gives already the strong impression that a simple explanation, like shown in the present tidal theory, for the world wide behaviour of the tides most likely is impossible. This impression is confirmed by the content of reference [7]. The dark blue areas in figure A1.2, copied from [7], are the amphidromic points. See 'Formation of amphidromic points' in [7] for what is *meant to be* an explanation of figure A1.2 !

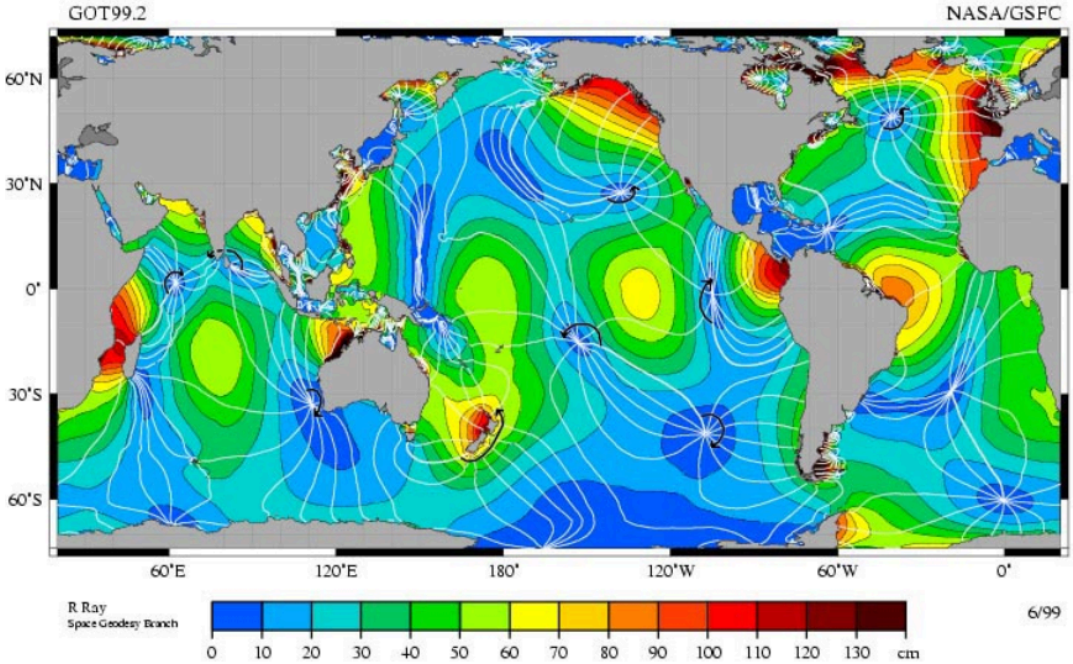

Figure A1.2. The M2 tidal constituent, the amplitude indicated by colour.

The behaviour of tides is presented in reference [6] for a "countless" number of places on Earth. Most of the places with large tide changes, black coloured in figure A1.2, have been investigated regarding their shift. The outcome of this investigation is presented in Table A1.

Netherlands	Ameland	30	France	Bay of Biscay	31	Africa	Senegal	30
Greenland West coast		36	Brasil	Fortaleza	32	Iceland	Reykjavik	29
British Colombia	Vancouver Island North				40	Chile	Antofagasta	33
South Africa west	Blouburgstrand				27	New Zealand	Auckland	38
Panama	Alto del Espino				33	Mozambique	East coast	27

Table A1 Shift's of tides are defined in minutes per 24 hours

The surprising result is that the calculated shift of 49 minutes has not been found in any of these places. The mean value of the presented shifts is 32 minutes each 24 hours! That shows another argument against the alleged *direct* influence of the Moon through its gravity on the tides. And last but not least: the random character of the tides pattern, shown in figure A1.2, does not argue for a uniform influence of the Moon on the level of the water in the oceans. Currents caused by centrifugal forces in the oceans correspond more closely to such a random phenomenon.

Considerations similar to those presented in this appendix led Galileo Galilei, around the year 1600, to reject the influence of the Moon on the tides via gravity. He believed that the movements of the Earth itself cause the water on this planet to move back and forth, amplified by the collisions of the water with the coasts. He has described his vision at length in the last part, called "Day Four," of his book Dialogue.

Appendix 2 of Encore

Mathematical expression for the wobble of 'Earth plus Moon' in their orbit around the Sun

The following parameters and variables play a role in the mathematical consideration.

G	gravity constant	ω_M	angular velocity Moon around Earth
m_S	mass Sun	ω_E	angular velocity Earth around Sun
m_E	mass Earth	φ_M	$\omega_M \cdot t$
m_M	mass Moon	φ_E	$\omega_E \cdot t$
r_{SE}	radius orbit Sun-Earth	C	Center of gravity of Earth plus Moon
r_{EM}	radius orbit Earth-Moon	F_{IJ}	gravity between I and J

Global consideration of F_{SC}

$F_{SE} = Gm_Sm_E/r_{SE}^2$ $\quad$ $F_{SM} = Gm_Sm_M/(r_{SE} + r_{EM})^2$ $\quad$ Each r_{IJ} and F_{IJ} to be considered as a vector

$F_{SC} = F_{SE} + F_{SM} = Gm_Sm_E/r^2_{SE} + Gm_Sm_M/(r_{SE} + r_{EM})^2$

$r_{SE} >> r_{EM}$, so $(r_{SE} + r_{EM})^2 \sim r^2_{SE} + 2r_{SE}r_{EM}$, resulting in: $F_{SC} \sim (Gm_S/r^2_{SE})\cdot\{m_E + m_M(1 - 2r_{EM}/r_{SE})\}$

In case Sun, Earth and Moon are in one line F_{SC} is either:

$$Gm_S/r^2_{SE}\{m_E + m_M - 2m_M\, r_{EM}/r_{SE}\} \quad \text{or} \quad Gm_S/r^2_{SE}\{m_E + m_M + 2m_Mr_{EM}/r_{SE}\}$$

In case the line Earth-Moon is perpendicular to the line Sun-Earth, r_{EM} is not zero, but representing it by a sinusoidal function is not an inaccurate approximation. This function will be derived hereafter.

Mathematical derivation of F_{SC}

The centre of gravity of 'Earth and Moon', indexed by C, lies at distance $r_{EC} = r_{EM}m_M/(m_E + m_M)$ from the centre of Earth, has mass $m_C = (m_E + m_M)$ and orbits around Earth with angular velocity ω_M.

The force F_{SC} is the centripetal force of the system Sun and 'Earth plus Moon' in which 'Earth Plus Moon' orbit the Sun as one object, represented by its centre of gravity C.

$F_{SC} = Gm_Sm_C/r_{SC}^2 = Gm_Sm_C/(r_{SE} + r_{EC})^2$

Given the orbiting of C around Earth r_{EC} is a function of time, expressed in x- and y- values by:

$r_{ECx} = r_{EC}\cos\varphi_M$ $\quad$ resp. $\quad$ $r_{ECy} = r_{EC}\sin\varphi_M$ $\quad$ with $r_{EC} = \sqrt{(r^2_{ECx} + r^2_{ECy})}$

Besides that: $\quad$ $\cos\varphi_E = r_{SEx}/r_{SE}$ and $\sin\varphi_E = r_{SEy}/r_{SE}$

So F_{SC} can be written too as:

$F_{SC} = Gm_Sm_C/[\{r_{SEx} + r_{EC}\cos\varphi_M\}^2 + \{r_{SEy} + r_{EC}\sin\varphi_M\}^2]$

$F_{SC} = Gm_Sm_C/[r^2_{SE} + r^2_{EC} + 2r_{SEx}r_{EC}\cos\varphi_M + 2r_{SEy}r_{EC}\sin\varphi_M\ \}]$ $\quad$ $r^2_{SE} >> r^2_{EC}$, so:

$F_{SC} \sim (Gm_Sm_C/r^2_{SE})\ /\{1 + 2(r_{EC}/r_{SE})\cos\varphi_E\cos\varphi_M + 2(r_{EC}/r_{SE})\sin\varphi_E\sin\varphi_M\ \}$

$F_{SC} \sim (Gm_Sm_C/r^2_{SE})\ /\{1 + 2(r_{EC}/r_{SE})\cos(\varphi_M - \varphi_E)\}$ $\quad$ $2(r_{EC}/r_{SE}) << 1$, so:

$F_{SC} \sim (Gm_Sm_C/r^2_{SE})\ /\{1 - 2(r_{EC}/r_{SE})\cos(\omega_M - \omega_E)t\}$

F_{SC} is a centripetal force. The related centrifugal force is at any moment equal but opposite to F_{SC} and determinative for the acceleration of C, after having divided it by the mass m_C of C. However the wobble is also "hidden" in the vectors r_{EC} and r_{SE}, which makes it impossible to integrate this acceleration mathematically. Therefore a digital simulation model was created (in Excel) to calculate these coordinates.

The principle of the model is based on the numerical calculation of the centripetal force F_{SC}. This force is converted into the centrifugal force experienced by C, by taking the same magnitude *but with opposite sign*. The resulting accelerations of C in the x- and y-coordinates, found by means of $F_{SC}/(m_E + m_M)$, are digitally integrated twice to the new coordinates of r_{SC}. Finally, at each calculation cycle the coordinates of Earth are simply found by: $r_{SEx} = r_{SCx} + r_{ECx}$ and $r_{SEy} = r_{SCy} + r_{ECy}$, with r_{ECx} and r_{ECy} as shown above.
The absolute values of these coordinates: $\sqrt{(r^2_{SEx} + r^2_{SEy})}$ are subtracted from the radius of the perfect circle Earth is assumed to fly without a Moon. These differences are the amplitude of the wobble as function of time.

The described feedback loop appears to be very sensitive to computational inaccuracies, in particular caused by the digital integrators. For that reason the so-called sample-time has to be chosen extremely small: 16 seconds, resulting in an Excel sheet with 200000 rows!
The red graph in figure A2 shows that even for such a small sampling time, the tendency to instability clearly looms.

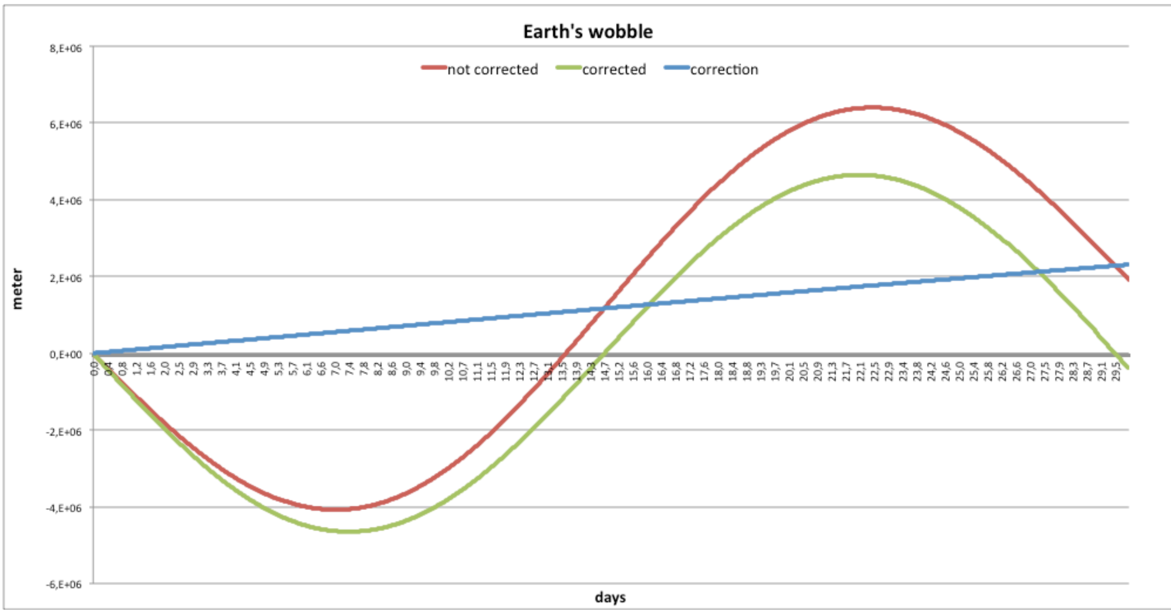

Figure A2

This instability has been calculated by adding a variable with the theoretical function $A_{mpl} \cdot \sin(\omega_M - \omega_E)t$. The amplitude A_{mpl} has been adjusted until the difference with the originally calculated curve is a perfectly linearly increasing function. A "hard copy" of this function (blue graph) has been subtracted from the red graph, resulting in the green one.
The green graph indeed shows a symmetrical function with a period exactly equal to the synodic month of the Moon (29.5 days). The amplitude of the wobble turns out to be $4.65 \cdot 10^6$ meter, or 4650 km, being very small, given the fact that Earth's radius is 6370 km!

The most remarkable part of this outcome thus is:

The so-called *sidereal* period of the Moon (27.3 days), being the orbit period relative to Earth, causes a wobble in the trajectory of Earth about the Sun with the so-called *synodic* period of 29.5 days, arising from the angular velocity $(\omega_M - \omega_E)$.