The Einstein Model of a Solid as a Model of the Mental Apparatus from the Economic Perspective of Psychoanalytic Theory.

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1. Abstract

As Freud has set as early as in the ‘Project for a Scientific Psychology’ (1895), ‘The intention is to furnish a psychology that shall be a natural science’. The aim of this theoretical, interdisciplinary article is at bridging classical psychoanalysis and thermodynamics. The Einstein model of a Solid (ES) is a thermodynamic model with applications in solid state physics. I intend to show that the ES, in the grand canonical ensemble, is a suitable model of the mental apparatus from an economic point of view. The approach I follow is to establish a strict one-to-one correspondence between the constituents of the ES, quantum harmonic oscillators, and energy quanta, and those of the psychic apparatus, ideas, and quota of affect respectively. I also prove that the laws of thermodynamics as applied to the ES, are correlated with the principles of the economic model, Pleasure, Reality, Nirvana, and Constancy. The significance of this research is in introducing science into psychoanalytic theory.

Keywords: Einstein Solid, Psychoanalysis, Thermodynamics, Economic model, Equilibrium Fluctuations.

2. Introduction

As Freud has set as early as in the ‘Project for a Scientific Psychology’ (1895):

> ‘The intention is to furnish a psychology that shall be a natural science: that is, to represent psychical processes as quantitatively determinate states of specifiable material particles, thus making those processes perspicuous and free from contradiction. Two principal ideas are involved: [1] What distinguishes activity from rest is to be regarded as Q, subject to the general laws of motion. (2) The neurons are to be taken as the material particles. N and $Q\eta$ – Similar experiments are now frequent.’

This theoretical, interdisciplinary article aims to bridge classical psychoanalysis and thermodynamics.

Candidate thermodynamic models of the mental apparatus can only be those whose extensive parameters are solely energy and the number of particles. This arises from the requirements described in the ‘Project’ and the entire theory of psychoanalysis. I assert the one-dimensional Einstein model of a solid (ES) as a candidate model of the psychic apparatus from an economic point of view. I particularly assert the case of the ES in the high-temperature limit and in the grand-canonical ensemble, i.e., coupled to a finite reservoir of quantum harmonic oscillators (QHOs) and energy. The ES coupled to the reservoir as defined above will be referred to, henceforward, as the System. In this article, I limit the research to the study of equilibrium fluctuations in the number of QHOs and in the number of energy quanta of the ES. ‘The postulational basis of classical thermodynamics has been expanded to incorporate equilibrium fluctuations’ Mishin (2015g).

The reasoning that I use to equate the mental apparatus to the System is borrowed from group theory and it is based, in principle, on the concept of isomorphism. I, therefore, attempt to establish a strict one-to-one correspondence (mapping) between the mental apparatus and the System. A mapping of the fundamental constituents of the mental apparatus and the principles that these constituents obey to the respective constituents of the System and the physical laws that govern the System. In case this mapping is successful the mental apparatus will inherit the thermodynamic properties of and need not be distinguished from the System.

The fundamental constituents of the mental apparatus are psychical energy (the term libido will be used henceforward to distinguish from the physical energy) and ideas (or presentations, or representations). The respective ones of the System are physical energy and QHOs. The principles of the economic point of view are Pleasure, Reality, Nirvana, and Constancy and I study their relation to the laws of thermodynamics. Additionally, I discuss fundamental entities of the economic point of view: Libido, drives (death and life instinct), cathexis, and how they relate to the System.

I refer to the topographical point of view and I argue for a mapping of the ego to the ES and of the *Ucs* to the reservoir respectively.

The significance of this research is in introducing science into psychoanalysis. I help both the practitioner and the theorist of psychoanalysis in general, as any elucidated theory would.

3. Analysis

The Quantum Harmonic Oscillator (QHO) as an energy container.

The QHO schematically.

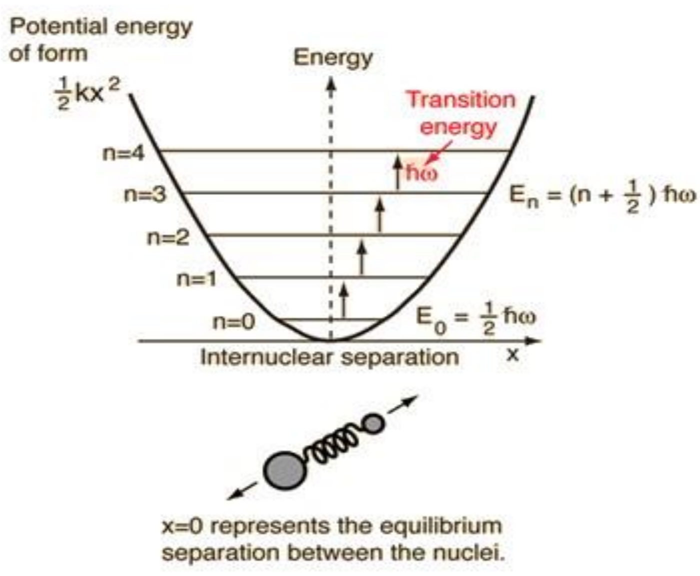

Fig. 1: The QHO schematically, Nave (2023)

The QHO is one of the quantum-mechanical systems for which an exact, analytical solution is known. A quantum of energy is of magnitude hf, where f is the frequency of oscillation and h is Planck's constant. Each QHO can contain a finite but arbitrarily large number of energy quanta.

3.1 Properties of the Einstein model of a Solid (ES).

The ES is a model of a crystalline solid that contains a large number (N) of independent QHOs of the same frequency (f), on the lattice. The ground state energy of the QHOs in this article is omitted and the ES is assigned a total number (q) of energy quanta. The high-temperature limit is defined by assuming $q \gg N$.

The properties of the ES as an ordered set of QHOs can be summarized as follows:

a. Each QHO can contain from zero to q units of energy.

b. The QHOs are disjoint and distinguishable from one another.

c. Non-interacting among them.

d. All the energy levels of all QHOs are equally spaced and equally probable to contain a quantum of energy (by the Fundamental Postulate of Statistical Mechanics), Mishin (2015a).

e. N is large.

3.2 Mapping of the ES to the ego.

3.2.1 Energy to quota of affect (libido).

a. The System is isolated. The total number of energy quanta is conserved.

In the "Project", Freud (1895), there is a reference about energy being transferred from the external to internal of the brain and vice-versa. In this article, I only model the mental apparatus which I consider independent from its somatic equivalent. I, therefore, assume that the mental apparatus is isolated and as a result, no interaction with the physical surroundings is considered. Equivalently, on no account could the model of the living vesicle be within the scope of this article. Since the mental apparatus is isolated, it is natural to assume that the energy (libido) it contains is conserved.

b. Energy is quantized. The quanta of energy of the QHOs of the ES are positive integers. The magnitude of each quantum is hf.

   The quantization of libido is not referred explicitly in the Freudian texts. However, “the quota of affect is one of a number (quantity) attached to an idea”, Laplance and Pontalis (1988e). The quota of affect is proportionate to the quantity of an affect, Laplance and Pontalis (1988e). From the above references and since there is no conflict in the Freudian theory if we assume that libido is quantized, I assert that libido is quantized.

c. Energy is attached to QHOs. In the case that I suggest (high-temperature limit) the energy contained in the ES is always bounded to a QHO

   Libido is always bound to an idea in the ego.

d. The energy quanta are indistinguishable among them.

   “The quota of affect is one of a number (quantity) attached to an idea”, Laplance and Pontalis (1988e). Quantity is measured into units and (quantity) is indistinguishable in its nature. Coarse-grained energy quanta look macroscopically the same.

3.2.2 Quantum Harmonic Oscillators to ideas (or presentations or representations).

a. QHOs are energy carriers, they may contain from zero to q energy quanta.

   Ideas are libido carriers. I have pointed out that there is a direct correspondence between libido and physical energy. QHOs empty of energy may belong to the ES however this occurrence is highly improbable in the high-temperature limit.

b. QHOs are disjoint, and distinguishable.

   Ideas are distinguishable in the sense that they all refer to different objects or entities. Ideas are disjoint in the sense that there is no unification or disunion process taking place among them in the theory of psychoanalysis.

c. QHOs are non-interacting among them. I.e., there are no attraction or repelling forces between QHOs.

Libido can be freely subtracted from one idea and added to another or transferred from the *Ucs* to the ego and vice versa. However, there are no bonds, repelling or attraction forces between the ideas that would cause or mediate in the transfer of energy among them.

d. All the energy levels of all QHOs are equally spaced and equiprobable, as constituents of the ES, Mishin (2015a), to contain a quantum of energy.

   To the best of my knowledge, there is no direct reference in the Freudian texts on these specific issues. As will be shown, there are advantages according to (and for) the theory of psychoanalysis to adopt these specifications for the ideas. For instance, and to differentiate from C. G. Jung, Hull et al. (1972), there can be no definition of constellating power for ideas to attract or cathect additional energy quanta depending on how much libido they are already occupied with. This is because of the equal probability of all energy levels to contain a quantum of energy. Additionally, ‘the quota of affect is proportionate to the quantity of an affect, Laplance and Pontalis (1988e). Consequently, each quantum of energy should produce the same energy shift on an idea. I.e., all the energy levels are equally spaced.

Because of the mapping of the energy quanta to the quota of affect and of ordered QHOs to ideas, I have proved that the constituents of the ES correspond to those of the ego. Additionally:

e. The number (N) of QHOs in the ES is large.

   It is the prevalent opinion among psychoanalysts that the number of ideas residing either in the ego or the *Ucs* is large.

The fifth and final property in the abstract definition of ES is also met in the theory of classical psychoanalysis. This implies that the ES is the only thermodynamic system that could describe the psychic apparatus. All that is left is not to disprove this assertion.

3.3 Definition of the psychical Temperature.

I consider two cases:

a. Temperature referring, macroscopically, to the entire ego.

   Candidate thermodynamic models of the ego can only be those whose extensive parameters are solely energy and the number of particles. Additionally, the particles (ideas) are non-interacting and there is no need for potential energy beyond vibrational kinetic energy. These arise from the requirements described in the ‘Project’ and the entire psychoanalytic theory. Additionally, it appears natural to define the average kinetic energy ($\langle E_{kin} \rangle$), of the particles that comprise the candidate thermodynamic model of the ego as the measure of the excitation in the ego. From the equipartition theorem, Pathria and Beale (2022), we have that, $d \cdot kT/2 = \langle E_{kin} \rangle$, where d is the number of degrees of freedom and T is the temperature. (The equipartition theorem is valid both classically and quantum mechanically for the translational kinetic energy. It is also accurate classically and in the high-temperature limit quantum mechanically for the vibrational kinetic energy). Since in the case we examine, the temperature is unambiguously related to, and as a result, represents the average kinetic energy, ego’s temperature has a meaning and can be defined by this relation.

Another simple and reasonable definition of the psychical temperature would be the fraction of the total amount of the cathected libido over the ideas of the ego, by the number of the ideas of the ego. This way, psychical temperature expresses the average cathexis/tension/excitation per idea. It is directly proportional to the total amount of the cathected libido and inversely proportional to the number of ideas, residing in the ego, at a given time. I.e., equal to T' defined according to Eq. $(5.4.3)$. The identification of the two definitions for the temperature is another vote for the ES as a model of the psychic apparatus.

The equilibrium value of the temperature, in the grand canonical ensemble, is imposed by the reservoir.

b. Temperature referring individually to an idea.

It is defined again from the equipartition theorem. For the ES in thermodynamic equilibrium, in the case of the average-energy-QHO, in the high-temperature limit, it is: $kT = nhf \Leftrightarrow n = \frac{q}{N}$ Eqs. (5.5.8), (5.5.9), where T is the temperature of the ES.

3.4 The first law.

The total amount of energy of an isolated system is conserved.

In the case of the ES, in the high-temperature limit, the average energy solely depends on the number of energy quanta, Eq. $(5.4.9)$. In the System, the total number of energy quanta remains constant.

The same applies to the total number of QHOs.

3.5 Entropy.

Entropy is the measure of a system’s thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordered molecular motion, the amount of entropy is also a measure of the molecular disorder or randomness or uncertainty about the state of a system. For an isolated thermodynamic system such as the System, entropy is the fundamental physical quantity for the description of its evolution and derivation of its further thermodynamic properties. For an isolated ES, the Boltzmann entropy is defined as: $S = k ln\Omega$. Ω is called the multiplicity of the ES. It is defined as the total number of the possible arrangements of q indistinguishable energy quanta, among N distinguishable QHOs. Each such arrangement describes (and defines) a unique microstate of the ES. The physical system is always in a microstate. The set of all microstates for given N and q defines a macrostate of the ES. Macrostates are observable, while microstates are indistinguishable by (human) observers.

3.6 The second law.

> a. 'The non-equilibrium entropy $\hat{S}$ of an isolated system averaged over the thermodynamic time scale t_{TD}, which we denote $\tilde{S}$, increases with time and reaches a maximum value $\bar{S}$ when the system arrives at equilibrium', Mishin (2015f).
>
> b. 'The individual values of $\hat{S}$ fluctuate but can never exceed the equilibrium value S', Mishin (2015f).

Thermodynamic equilibrium can be attained at a time when there would be no available energy from which we can extract work. In a simplified version, there would be no place or no process in a thermodynamic system by which we can create pressure, temperature, or chemical potential (thermodynamic forces') differences. If there is no such difference, excluding fluctuations, work cannot be further extracted.

Uniform temperature and (equivalently in the case of the ES Eq. (5.1.13)) chemical potential distribution throughout the ES can be achieved when the number of energy quanta residing in each QHO is the same, Eq. (5.5.9). If the ES correctly describes the ego this provides the definition of mental equilibrium in the theory of psychoanalysis. I.e., libido is equally distributed among the ideas of the ego.

It is a fact that Freud defines the principles of Constancy and Nirvana in such a way that they define the maintenance of libido at a constant level as one of the equilibrium conditions for the psychic apparatus.

3.7 Reality principle.

To be able to decide about reality, or equivalently the actual state of a physical system, we need to establish an observer, the observed system, and (optionally) a measuring device, Hemmo and Shenker (2012a). Measurement is closely related to the concept of information and its acquisition on behalf of the observer, Hemmo and Shenker (2012a). I assume a strict correspondence between internal and external reality when the thermodynamic system that arguably represents the psyche is at equilibrium. This assumption appears reasonable since according to Jean Piaget cognitive equilibrium is defined as a state of balance between individuals' mental schemata, or frameworks, and their environment. It is the prevalent opinion among psychoanalysts that the information that the subject possesses about the external reality is due to 'a case of' self-observation. As a result, the consequent information acquisition that follows the measurement performed by the ego over the conscious part of the psyche, will represent accurate information about the external world as well.

The reality principle is defined in psychoanalysis as a temporary postponement of pleasure according to the conditions imposed by the outside world. The maximum exploitability of energy dictates the way in our best interest to achieve an aim (or pleasure). Therefore, the Reality principle can equivalently be defined as an attempt to achieve maximum exploitability of energy when the subject is about to act in the environment to satisfy its needs or achieve pleasure. 'We can better exploit the energy in systems around us whenever we have more accurate information about their microstates. And it is precisely this that we gain by measurement', Hemmo and Shenker (2012b).

Since the Reality principle is closely related to measurement it allows for defying the second law in the thermodynamic modeling of the mental apparatus: 'Measurement can decrease the entropy of the entire universe', Hemmo and Shenker (2012b). This fact explains and allows for phenomena of deviation from equilibrium. However, measurement does not always decrease entropy.

The tendency of life instincts is to create and maintain even greater unities instead of leading to disintegration, or decay. This is true for the immediate environment where the subject acts physically. An increase in order and decrease in randomness, in a thermodynamic system, is represented by a decrease in its entropy. Therefore, we may infer that in a physical sense, life instincts lead to physical actions that tend to defy the second law and imply intelligence and sublimation.

I must note that the decrease of the entropy of a thermodynamic system in the grand canonical ensemble (as the ES is) due to equilibrium fluctuations in the extensive parameters (Eq. (5.3.12) in the case of the ES) does not imply a violation of the second law. That is because of the coupling to the reservoir. Additionally, it is questionable whether and how an observer, the observed system, and the operation of measurement, with the attributes defined in Hemmo and Shenker (2012a), can be configured in the grand canonical ensemble and particularly for the mental apparatus as defined in the theory of psychoanalysis.

3.8 Nirvana principle and the second law.

> 'The Nirvana principle is a term to describe the tendency of the psychical apparatus to reduce the quantity of excitation in itself, whether of internal or of external origin, to zero-or, failing that, to as low a level as possible'. Freud formulates the Nirvana principle

> as a tendency expressing 'the effort to reduce, to keep constant or to remove internal tension due to stimuli.' (Laplance and Pontalis, 1988c)

From what has been explicated in the definition of the second law, we can infer that the Nirvana principle (and additionally the principle of Constancy, Laplance and Pontalis (1988b)) as the definition of mental equilibrium, should be equivalent to keeping the psychical temperature and the rest of the thermodynamic forces (chemical potential in the case of the ES) uniform throughout the ego. In the case of the ES uniform temperature is equivalent to the equal distribution of energy quanta (libido) to the QHOs (ideas), Eq. (5.5.9). Since the chemical potential is solely dependent on temperature, Eq. (5.1.13), we conclude that for the ego equal distribution of quota of affect to ideas implies the second law. This is the thermodynamically valid definition of the Nirvana and Constancy principles.

The death instinct corresponds to the Nirvana principle. This can be derived from what Freud stresses once again in 'The Economic Problem of Masochism' (1924): 'The Nirvana principle expresses the trend of the death instinct'. 'The latter (death instinct) was a new concept: it referred to a 'daemonic force' (p. 35), which searched for psychosomatic quiescence and, at its deepest core, sought to reduce the animate to its original inanimate status', Akhtar and O'Neil (2011). In an isolated system when the maximum entropy is reached no more work can be extracted from the system. The spontaneous increase of entropy in nature is the most general frame responsible for the degradation and decomposition of living species. See also the 'heat death' hypothesis. Therefore, by a first examination, it would be irrational to argue that the death instinct, or the Nirvana principle, should contradict the second law. The Nirvana principle appears to be a special case of the second law.

I examine the case where the Nirvana principle applies to the mental apparatus. The second law of thermodynamics dictates the way to equilibrium. Quiescence could be interpreted as psychical equilibrium attainment. The Nirvana principle dictates the direction to quiescence. Therefore, it could be argued that it entails the second law in the thermodynamic modeling of the mental apparatus.

However, if we do not explicitly define 'quiescence' and 'the direction to it', we cannot conclude whether the Nirvana principle agrees with the second law.

In case we equate the Nirvana principle with the tendency of the psyche to keep the excitation (temperature) constant throughout the ego, then the Nirvana principle is in accordance with the second law.

The Nirvana principle may also be defined as the tendency of the psyche to decrease ego's temperature (excitation) to zero, as each candidate model of the mental apparatus (a thermodynamic system) spontaneously approaches equilibrium. The examination of this case is a subject of non-equilibrium statistical thermodynamics and consequently not within the scope of this article. However, we must note that the temperature of the reservoir is always a positive number and never zero. In the case that we examine in this article, i.e., for equilibrium fluctuations of the constituents of the ES $(\Delta N, \Delta E)$ the temperature (T) remains constant and equal to that of the reservoir. The stability of the System, Eq. $(5.3.15)$, does not depend on temperature.

In case the Nirvana principle refers to the progression and aging of the human body then we have: $S_{animate} = S_{ordered} \leq S_{disordered} = S_{inanimate}$, so the Nirvana principle consents to the second law. (S, is the entropy in each case).

3.9 Pleasure principle.

The pleasure principle is the innate drive of the id to seek immediate gratification from its impulses and desires, without regard for external constraints or consequences. The pleasure principle refers to the id's instinctive drive to seek pleasure and avoid pain. Since the pleasure principle primarily refers to the *Ucs* (reservoir) and in this paper we are not examining the primary processes or the state of the id, we must refer to the influence of the pleasure principle on the ego (ES).

'In the second theory of the psychical apparatus, it is the id, as the instinctual pole of the personality, which is seen as the origin of all cathexes, and the other agencies draw their energy from this primary source', Laplance and Pontalis (1988a). Since the *Ucs* supplies libido to the ego through drives, then the fraction of quota of affect by the number of ideas residing in the ego, i.e., (T'), may change. This implies that the *Ucs* imposes on the ego its equilibrium temperature exactly as the reservoir does to the ES. We must note here that since we are referring to the grand canonical ensemble, the temperature (T') will be considered fixed. For equilibrium fluctuations of the constituents of the ES $(\Delta N, \Delta E)$ the temperature remains constant.

According to Freud, the ego is the part of the psyche that mediates between the impulses of the id and the demands of the external world. The ego's role in managing these impulses in a way that is appropriate to the demands of reality. When the ego is unable to satisfy these impulses in a way that is acceptable to both the id and the external world, tension can arise.

In the case that tension refers to a shorter than the ego group of ideas then an attempt for gratification occurs. Gratification aims to lower the level of cathexis of the overcharged group or single ideas. Consequently, the purpose of gratification is to keep the excitation (temperature) uniform throughout the ego. I.e., to lead to thermodynamic equilibrium.

The succession of microstates within a given macrostate of the ES describes the variations of the absolute magnitude of cathexis for each specific idea in the ego. Whenever gratification occurs, all the possible/resulting arrangements of libido among the ideas of the ego are considered in the multiplicity Ω and the Boltzmann entropy of the ES.

In response to tension, the ego may seek to find alternative ways to satisfy the id's desires that are more acceptable to external reality. This is where the reality principle comes in, as the ego seeks to find ways to meet the id's needs and desires in a way that is realistic and socially acceptable.

Thus, while the pleasure principle can create tension in the ego, it can also serve as a motivator for the ego to find creative solutions to meet the needs of the id in a way that is adaptive and effective.

3.10 Mapping of the reservoir to the Ucs.

The Ego combined with the *Ucs* is treated as an isolated system.

Primary processes refer to the *Ucs*. There is no mention, or even definition, of 'processes' taking place in the reservoir in the science texts. Therefore, only secondary processes are considered by the macrostate/microstate of the ego that each process produces. Ideas may belong to the *Ucs* or the ego exactly as QHOs may belong to the ES or the reservoir. The total energy and the number of QHOs in the System are conserved. This implies that ideas or libido quanta can be transferred from the ego to the *Ucs* and vice versa, but their total number in the System is required to be constant. I assume that the mental apparatus is isolated and as a result, no interaction with the physical surroundings (or the model of the living vesicle) is considered.

> 'The forces which we assume to exist behind the tensions caused by the needs of the id are called instincts. They represent the somatic demands upon the mind. Though they are the ultimate cause of all activity, they are of a conservative nature', Freud (1940)

Consequently, only (an instinct's) ideational representative and quota of affect are considered in this model.

There is no creation or annihilation of QHOs or energy in the System. Therefore, drives are not supposed to create new ideas or libido in the mental apparatus.

Propositions for the mapping of the *Ucs* to the reservoir:

a. According to the theory of psychoanalysis, there is more to the unconscious than the repressed, Akhtar and O'Neil (2013), i.e., the 'reservoir approximation', in thermodynamics. The reservoir contains a finite but larger number of QHOs and energy quanta, than the ES: 'The matrix elements Λ_{ij}^{r} scale as $1/N^{r}$. When the size of the reservoir increases to infinity, the quadratic terms in Eq. (164) vanish and we return to the generalized canonical distribution, Eq. (72)', Mishin (2015e). 'In the second theory of the psychical apparatus, it is the id, as the instinctual pole of the personality, which is seen as the origin of all cathexes, and the other agencies draw their energy from this primary source', Laplance and Pontalis (1988a). Apparently, the amount of libido residing in the *Ucs.*, is defined in the Freudian texts as much larger than that of the ego.

b. The reservoir imposes constant temperature and chemical potential. ‘The role of the reservoir is only to impose the thermodynamic forces F_i^0’, Mishin (2015g). From what we discussed in the subsection ‘Pleasure principle’, we conclude that in equilibrium, the *Ucs* imposes on the ego the values of the thermodynamic forces conjugate to the fluctuating parameters.

c. ‘We assume that the exchange of the quantities X_i occurs slowly enough that we can consider both the system and the reservoir as quasi-equilibrium systems’, Mishin (2015c).

d. The reservoir respects the conservation of energy and the number of particles (QHOs) when coupled to the ES.

e. The concept of entropy applies to the reservoir.

f. The reservoir has an infinite heat capacity, Mishin (2015b).

g. The system *Ucs* operates in ways that are different from the system *Cs*. ‘Because the Boltzmann reservoir is not based on a Hamiltonian, there is no reason to believe that it models any real system’, Leff (2000). I.e., the reservoir is not an ES.

h. No *Pcs* allowed. In the System, there is either ES or reservoir. Consequently, there can be no unconscious part of the ego.

i. Free energy in the *Ucs*. Energy and QHOs are independent in the reservoir.

4. Findings and Discussion

The one-to-one correspondence between the fundamental constituents of the psyche and those of the ES was successful. At this stage of examination, the ES was approved as the only possible thermodynamic model of the mental apparatus. All that was left was for this statement not to be disproved. Therefore, I examined the laws of thermodynamics as applied to the ES and have shown that they agree with the principles of the economic point of view, Reality, Pleasure, Nirvana, and Constancy, under conditions. The Pleasure principle is in accordance with the second law if it leads to gratification. For the Reality principle I have shown that it is closely related to measurement and as such it allows for defying the second law in the thermodynamic modeling of the mental apparatus. Measurement does not always decrease the entropy of the observed system.

Because of the above reasoning, the System is arguably the model of the mental apparatus from the economic point of view.

Additionally, I have concluded that the Nirvana principle cannot imply a decrease of the excitation to zero as an equilibrium condition.

I have also referred to the topographical point of view and argued for the identification of the reservoir with the *Ucs*.

Boltzmann entropy was defined for the ES, and it defines the equilibrium condition for the psyche.

Psychical temperature was defined as a measure of the excitation in the ego. I have asserted that psychical temperature coincides conceptually with the temperature of the thermodynamic system that models the ego. For the ES the chemical potential solely depends on temperature by Eq. (5.1.13). Consequently, uniform temperature distribution implies uniform chemical potential distribution throughout the ES. This is a necessary and sufficient condition for thermodynamic equilibrium.

The principles of Constancy (and a special case of the Nirvana principle as I have outlined) dictate the equal distribution of libido to all the ideas of the ego. I have defined psychical temperature to refer to a single idea, at equilibrium. For the ES this is provided by Eqs. (5.5.8) and (5.5.9). The equal distribution of libido is a necessary and sufficient condition for equal temperature distribution throughout the ES. As discussed in the previous paragraph this is the equilibrium condition. That is another advantage of the ES as a model of the psyche.

Temperature is related to entropy by Eq. (5.3.4).

The fluctuations in the missing information (ΔI) in accordance with the fluctuations in the number of QHOs in the ES are given from Eq. (5.3.13). For the negative value of ΔI, there is information production about the state of the ES (ego). The value of the missing information and the corresponding Figure 2 is according to Eq. (5.3.14).

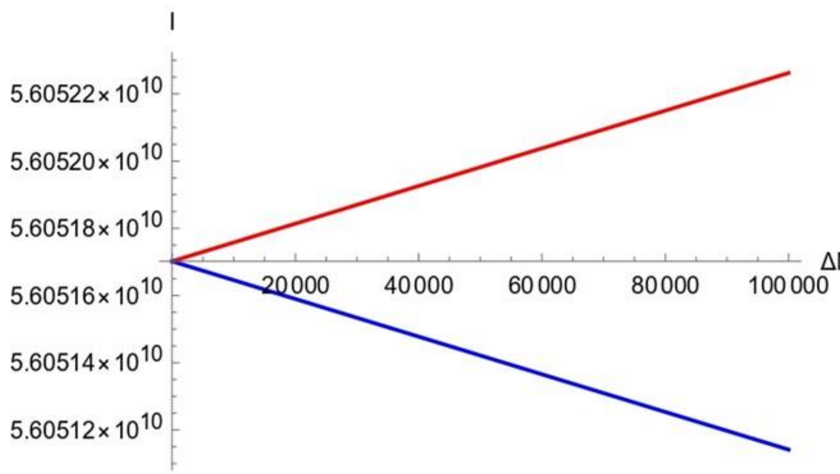

Fig. 2. Equation (5.3.14). The variation of the missing information I, as a function of the fluctuating parameter N. I have set $\bar{N} = 10^{10}$ and $\bar{q} = 10^{12}$, arbitrarily.

Next to the adoption of the ES, the stability condition for the mental apparatus is the single most important of the findings. The stability of a thermodynamic system is defined by the relative fluctuations in the extensive parameters and entropy. According to Eq. (5.3.15) the stability of the System (mental apparatus) depends solely on the number of QHOs (ideas), residing in the ES (ego) at a given time. The higher the number of ideas in the ego, the more stable the mental apparatus becomes. This finding consents with the psychoanalytic theory and the axiom that the more ideas that belong to the ego the more stable the psychic apparatus becomes. The stability of the System does not depend on temperature (T).

5. Appendices

5.1 The Chemical Potential.

We first start with the definition of the multiplicity of an ES. We have from Schroeder (2000a):

$$\Omega = \binom{N+q-1}{q} = \frac{(N+q-1)!}{N!\,(q-1)!} = \frac{(N+q)!}{N!\,q!} \cdot \frac{q}{N+q} \tag{5.1.1}$$

From Stirling's approximation we have:

$$N! \approx N^N e^{-N} \sqrt{2\pi N} \tag{5.1.2}$$

Applying (5.1.2) we get:

$$(N+q)! \approx (N+q)^{(N+q)} e^{-(N+q)} \sqrt{2\pi(N+q)} \tag{5.1.3}$$

$$q! \approx q^q e^{-q} \sqrt{2\pi q} \tag{5.1.4}$$

From (5.1.3) and (5.1.4), equation (5.1.1) becomes:

$$\Omega \approx \frac{(N+q)^{(N+q)}e^{-(N+q)}\sqrt{2\pi(N+q)}}{N^N e^{-N}\sqrt{2\pi N}\cdot q^q e^{-q}\sqrt{2\pi q}}\cdot\frac{N}{N+q} \Rightarrow$$

$$\Rightarrow \Omega = \left(\frac{q+N}{N}\right)^N \left(\frac{q+N}{q}\right)^q \sqrt{\frac{N}{2\pi q(q+N)}} \qquad (5.1.5)$$

We now have for entropy S:

$$S = k\ln\Omega \qquad (5.1.6)$$

Substituting (5.1.5) to (5.1.6) and since the logarithm is additive, we have:

$$S = k\ln\left(\frac{q+N}{N}\right)^N + k\ln\left(\frac{q+N}{q}\right)^q + k\ln\sqrt{\frac{N}{2\pi q(q+N)}} =$$

$$= kN\ln\left(\frac{q+N}{N}\right) + kq\ln\left(\frac{q+N}{q}\right) + k\ln\sqrt{\frac{N}{2\pi q(q+N)}} \qquad (5.1.7)$$

We can clearly notice that the last logarithmic term in (5.1.7) is negligible compared to the first two, multiples of N and q, respectively. As such we can omit it. As a result, (5.1.7) becomes:

$$S \approx k\ln\left(\frac{q+N}{N}\right)^N + k\ln\left(\frac{q+N}{q}\right)^q \qquad (5.1.8)$$

By exponentiating (5.1.8) we get for the multiplicity $\Omega = S/k$:

$$\Omega \approx \left(\frac{q+N}{N}\right)^N \cdot \left(\frac{q+N}{q}\right)^q \qquad (5.1.9)$$

By applying the First Law of Thermodynamics to the ES, we get:

$$dU = TdS - PdV + \mu dN$$

For Entropy:

$$dS = \left(\frac{\partial S}{\partial U}\right)_{N,V} dU + \left(\frac{\partial S}{\partial V}\right)_{U,N} dV + \left(\frac{\partial S}{\partial N}\right)_{U,V} dN \qquad (5.1.10)$$

For the temperature T:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} \tag{5.1.11}$$

Substituting $U = qhf$ and $(5.1.8)$ in $(5.1.11)$ we have:

$$\frac{hf}{kT} = \ln\left(1 + \frac{N}{q}\right) \Rightarrow$$

$$\Rightarrow \frac{q}{N} = \frac{1}{e^{\frac{hf}{kT}} - 1} \tag{5.1.12}$$

The definition of the chemical potential μ:

$$\mu \equiv -T\left(\frac{\partial S}{\partial N}\right)_{U,V} =$$

$$= -kT\ln\left(1 + \frac{q}{N}\right) \tag{5.1.13}$$

5.2 Information content.

Based on Katz (1967).

Let us assume that we are faced with a number n of possibilities. Let us further assume that these possibilities are mutually exclusive, equally probable, and complete in the sense that all together imply certainty about the outcome of the experiment. Since there is a set of possibilities, there is missing information about the outcome. We denote this amount of missing information as

$$I(n) \tag{5.2.1}$$

$I(n)$ is a real number and n is a non-negative integer. We naturally expect that the greater the number of possibilities, the greater the amount of missing information. So $I(n)$ is a strictly increasing function:

$$\mathrm{I(n)} > \mathrm{I(m)},\ \mathrm{n} > \mathrm{m} \tag{5.2.2}$$

In the special case of one scenario, we expect that there is no information missing, so:

$$I(1) = 0 \tag{5.2.3}$$

Let us now assume that we have two independent experiments, one with n possibilities about its outcome and another with m possibilities. Since the experiments are independent the total number of possibilities about their outcome is their Cartesian product:

$$n \cdot m \tag{5.2.4}$$

We denote the total missing information as:

$$I(n \cdot m) = I(nm) \tag{5.2.5}$$

In this case, the total missing information is the sum of the ignorance about the outcome of the first experiment, plus the ignorance about the second:

$$I(nm) = I(n) + I(m) \tag{5.2.6}$$

So far, we have chosen the number of possibilities to be a non-negative integer. We will generalize this statement to include positive rational numbers. We define:

$$I\left(\frac{n}{m}\right) \equiv I(n) - I(m) \tag{5.2.7}$$

From (5.2.2) and (5.2.7) we can extend $I(x)$ to all positive real numbers, x. We can also prove that $I(x)$, is continuous. Finally, we can prove, from the above, that:

$$I(x) = \rho \log(x), \rho \equiv I(e) > 0 \tag{5.2.8}$$

In the case of a probability distribution, such that the probabilities P_α are non-negative numbers:

$$P_\alpha \geq 0, \qquad \alpha = 1, \ldots, N \tag{5.2.9}$$

and the probabilities sum to one:

$$\sum_{\alpha=1}^{N} P_\alpha = 1 \tag{5.2.10}$$

then

$$I = -\rho \sum_{\alpha=1}^{N} P_\alpha \log P_\alpha \tag{5.2.11}$$

In the case of an isolated system at equilibrium, we have for the probability P_α of each microstate:

$$P_\alpha = P = constant, \qquad \alpha = 1, \dots, N \tag{5.2.12}$$

In the case of an isolated Einstein Solid, with multiplicity Ω:

$$I = -\rho \sum_{\alpha=1}^{\Omega} P_\alpha \log P_\alpha == -\rho \sum^{\Omega} \frac{1}{\Omega} \log \frac{1}{\Omega} \Rightarrow$$

$$\Rightarrow I = \rho \log \Omega \tag{5.2.13}$$

We will be using by convention $\ln\Omega$, instead of $\log\Omega$. As a result, $(B.13)$ becomes:

$$I = \ln\Omega \tag{5.2.14}$$

The entropy, divided by Boltzmann's constant, is identical to the missing information at equilibrium, Schroeder (2000a).

5.3 Stability of the System.

I consider the case where:

$$|(\Delta N)| \approx \sqrt{N} \tag{5.3.1}$$

For an Einstein Solid with $q \gg N$, we have from Schroeder (2000b):

$$\Omega \approx \left(\frac{eq}{N}\right)^N \tag{5.3.2}$$

$$S = k\ln\Omega = kN\ln\left(\frac{eq}{N}\right) \tag{5.3.3}$$

Using (5.4.3) we have that:

$$S = kN\ln(eT') \tag{5.3.4}$$

In (5.3.4) we note the dependence of the Boltzmann entropy on the temperature T'.

We can calculate the entropy variation by a 2^{nd} degree Taylor expansion of S, for fluctuations of E and N.

$$\begin{aligned}\Delta S \equiv S(E \pm \Delta E, N \pm \Delta N) - S(E, N) \\ = \pm(\Delta E)\left(\frac{\partial S}{\partial E}\right)_N \pm (\Delta N)\left(\frac{\partial S}{\partial N}\right)_E + \frac{1}{2}(\Delta E)^2\left(\frac{\partial^2 S}{\partial E^2}\right)_N + \frac{1}{2}(\Delta N)^2\left(\frac{\partial^2 S}{\partial N^2}\right)_E \\ + \frac{1}{2}(\Delta E)(\Delta N)\left(\frac{\partial^2 S}{\partial E\,\partial N}\right) + \frac{1}{2}(\Delta E)(\Delta N)\left(\frac{\partial^2 S}{\partial N\,\partial E}\right)\end{aligned}$$

Using Eqs. (5.1.11), (5.1.12), (5.1.13) and (5.4.9) and for $q \gg N$ we have: $\bar{E} = U = q\epsilon, \Delta E = \epsilon\Delta q, \epsilon = hf$,

$$F_N \equiv \left(\frac{\partial S}{\partial N}\right)_E = -\frac{\mu}{T} = k\ln\left(\frac{q}{N}\right) \Rightarrow$$

$$F_N(\Delta N) = (\Delta N)k\ln\left(\frac{q}{N}\right) \tag{5.3.5}$$

$$F_E \equiv \left(\frac{\partial S}{\partial E}\right)_N = \frac{1}{T} = \frac{kN}{\epsilon q} \Rightarrow F_E(\Delta E) = \frac{kN(\Delta q)\epsilon}{\epsilon q} \Rightarrow$$

$$F_E(\Delta E) = kN\left(\frac{\Delta q}{q}\right) \tag{5.3.6}$$

From Anagnostopoulos (2014):

$$\Delta N = \sqrt{N} \sim \frac{E}{\Delta E} = \frac{q}{\Delta q} \tag{5.3.7}$$

From (5.3.6) and (5.3.7) we have:

$$F_E(\Delta E) = kN\frac{1}{\sqrt{N}} = k\sqrt{N} = k(\Delta N) \tag{5.3.8}$$

For the quadratic terms, considering (5.3.5) for $S = k\ln\Omega$ we have:

$$S = kN\ln\left(\frac{eq}{N}\right) \Rightarrow \frac{\partial^2 S}{\partial N\,\partial E} = \frac{1}{\epsilon}\frac{\partial^2 S}{\partial N\,\partial q} = \frac{1}{\epsilon}\frac{\partial^2 S}{\partial q\,\partial N} = \frac{k}{q\epsilon} \Rightarrow$$
$$\frac{1}{\epsilon}\frac{\partial^2 S}{\partial q\,\partial N}(\Delta q)(\Delta N)\epsilon = k\left(\frac{\Delta q}{q}\right)\Delta N \approx k\left(\frac{1}{\sqrt{N}}\right)\sqrt{N} \approx k \tag{5.3.9}$$

$$S = kN\ln\left(\frac{eq}{N}\right) \Rightarrow \frac{\partial^2 S}{\partial E^2} = \frac{1}{\epsilon^2}\frac{\partial^2 S}{\partial q^2} = -\frac{kN}{\epsilon^2 q^2} \Rightarrow \frac{\partial^2 S}{\partial E^2}(\Delta E)^2 \approx -k \tag{5.3.10}$$

$$S = kN\ln\left(\frac{eq}{N}\right) \Rightarrow \frac{\partial^2 S}{\partial N^2} = -\frac{k}{N} \Rightarrow \frac{\partial^2 S}{\partial N^2}(\Delta N)^2 \approx -k \tag{5.3.11}$$

We conclude that the sum of the quadratic terms in the Taylor expansion cancels. Considering only the case of equilibrium and by using Eqs. (5.3.1), (5.3.2) and (5.3.3) we have for the variation of the entropy:

$$\Delta S = \pm F_N(\Delta N) \pm F_E(\Delta E) = \pm k(\Delta N)\ln\left(\frac{eq}{N}\right) \tag{5.3.12}$$

From Lemons (2014) and Anagnostopoulos (2014) we have:

$$\Delta I = \frac{\Delta S}{k} = \pm(\Delta N)\ln\left(\frac{eq}{N}\right) \tag{5.3.13}$$

$$I = \frac{S}{k} = N\ln\left(\frac{eq}{N}\right) \pm (\Delta N)\ln\left(\frac{eq}{N}\right) \tag{5.3.14}$$

For the stability of the system at equilibrium, we have:

$$\boxed{\frac{|\Delta \mathrm{N}|}{\mathrm{N}} = \frac{|\Delta \mathrm{E}|}{\mathrm{E}} = \frac{|\Delta \mathrm{S}|}{\mathrm{S}} = \frac{|(\Delta \mathrm{N})\,|\ln\left(\frac{eq}{N}\right)}{\mathrm{N}\ln\left(\frac{\mathrm{eq}}{\mathrm{N}}\right)} \approx \frac{1}{\sqrt{\mathrm{N}}}} \tag{5.3.15}$$

From (5.3.15) we have that the system becomes more stable as N increases. The stability of the system depends solely on the number of QHOs of the ES.

5.4 The average energy in the grand canonical ensemble.

We only consider the case of high temperatures, i.e.,

$$q \gg N \tag{5.4.1}$$

From (5.1.13) and (5.4.1) we have:

$$\mu = kT\ln\left(\frac{N}{q}\right) \tag{5.4.2}$$

From (5.1.12) and (5.4.1) we have:

$$T' \equiv \frac{kT}{hf} = \frac{q}{N} \Leftrightarrow kT = \frac{q}{N}hf \tag{5.4.3}$$

The canonical partition function for one oscillator, neglecting the ground state energy is:

$$Z_1 = \frac{1}{1 - e^{\frac{-1}{T'}}} \tag{5.4.4}$$

$$Z_N = Z_1^N \tag{5.4.5}$$

From Bellac et al. (2004), we obtain the equations $(3.127) - (3.129)$:

$$\alpha = \frac{\mu}{kT} \Rightarrow e^{\alpha} = \frac{N}{q} = \frac{1}{T'} \qquad (3.128)$$

The grand partition function Q is:

$$Q_{(\alpha,\beta)} = \sum_{N=0}^{\infty} e^{\alpha N} Z_N(\alpha,\beta) \qquad (3.127)$$

$$\Rightarrow Q = \frac{1}{1 - \left(\frac{1}{T'}\right)\left(\frac{1}{1 - e^{-\frac{1}{T'}}}\right)} \qquad (5.4.6)$$

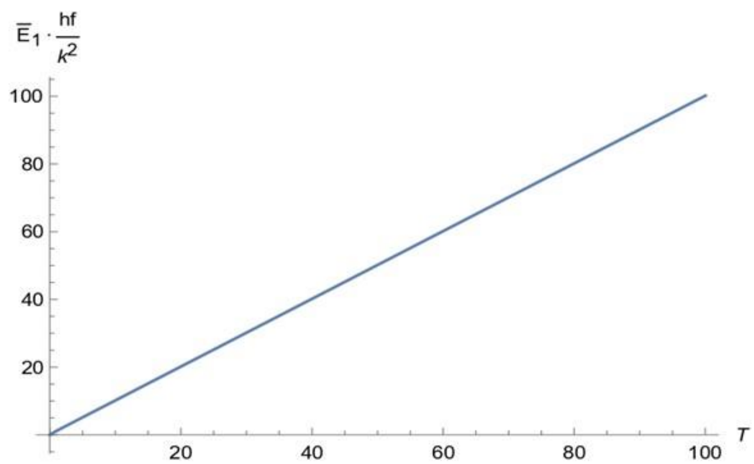

Fig. 3: Equation (5.4.7). The average energy of one oscillator $\bar{E}_1$, as a function of T.

From (3.129), we have for the average energy $\bar{E}_1$ of one oscillator:

$$\bar{E}_1 = -\left(\frac{\partial \ln Q}{\partial T'} \cdot \frac{\partial T'}{\partial T}\right) / \frac{\partial \beta}{\partial T} \qquad (5.4.7)$$

Equation (5.4.7). The average energy of one oscillator $\bar{E}_1$, as a function of T.

The Laurent series expansion of the average energy of one oscillator, for high temperatures, is:

$$\bar{E}_1 \cdot \frac{hf}{k^2} = T + \frac{1}{6} - \frac{1}{36T} - \frac{1}{270T^2} + O\left[\frac{1}{T}\right]^3 \qquad (5.4.8)$$

Only the first term survives for high temperatures. For N oscillators we multiply $\bar{E}_1 \frac{hf}{k}$ by N to find the total average energy $\bar{E}$, of the ES. It is:

$$\bar{E} = N\bar{E}_1 \frac{hf}{k} = NkT = N\frac{q}{N}hf = qhf \qquad (5.4.9)$$

This agrees with the result for the internal energy of the ES, in the microcanonical ensemble.

In the calculations and figures, Wolfram Mathematica has been used.

5.5 Energy eigenvalues of the QHO.

The Hamiltonian of the QHO is:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \tag{5.5.1}$$

Where m is the QHO's mass, k is the force constant, $\omega = 2\pi f = \sqrt{k/m}$ is the angular frequency and $\hat{x}$, $\hat{p}$ are the position and momentum operators respectively.

The time independent Schrödinger's equation is:

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \tag{5.5.2}$$

We define the ladder operators α and its adjoint $\alpha^\dagger$ as

$$\alpha = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right) \tag{5.5.3}$$

$$\alpha^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right) \tag{5.5.4}$$

From (5.5.3) and (5.5.4), (5.5.1) becomes:

$$\hat{H} = hf\left(\alpha^\dagger\alpha + \frac{1}{2}\right) \tag{5.5.5}$$

The energy eigenstates $|n\rangle$ have the property:

$$\alpha^\dagger\alpha|n\rangle = n|n\rangle \tag{5.5.6}$$

Consequently, the energy eigenvalues are:

$$E_n = hf\left(n + \frac{1}{2}\right) \tag{5.5.7}$$

From (5.4.8) and (5.5.7) and at the high-temperature limit we have that:

$$kT = nhf \tag{5.5.8}$$

(5.5.8) defines the relation between the temperature T of the ES and the number n of energy quanta that the QHO contains. In (5.5.8) k is the Boltzmann constant. From (5.4.3) we have:

$$n = \frac{kT}{hf} = \frac{q}{N} \tag{5.5.9}$$

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