Relative and Absolute Stellar Aberration

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Abstract

If we talk about Stellar Aberration, then we think of the form of Stellar Aberration that was first discovered and explained by Bradley. In addition to Bradley's Stellar Aberration, which can also be defined as Relative Stellar Aberration, we will define Absolute Stellar Aberration based on just one measurement. Here after we will refer to the Absolute Stellar Aberration as ASA. We will try to explain in a few words why it is necessary to measure and interpret Stellar Aberration in this way. Suppose we performed two measurements of the Doppler Effect within six months. If we don't know the results of those measurements, but only difference between them, then we cannot determine the radial velocities with which the observer moves with respect to the star. We will prove that similar reasoning can be applied in the case of Stellar Aberration as defined by Bradley. Knowing only the difference between the two measurements of the Stellar Aberration, we are not able to determine the transverse velocities the observer moves with respect to the line of sight, but only their difference. Using the results of ASA measurements, we will determine a Stationary Frame of Reference and after that derive formulas for Relative and Absolute Stellar Aberration.

Keywords: *Stellar Aberration, Doppler Effect, Stationary Frame of Reference, Transverse Velocity*

1. Introduction

This paper is not original but the author's attempt to write a shorter and simpler version of the paper [1]. We have considered some new ideas but we also copied and hopefully improved some parts from the paper [1]. We expect that with the construction and use of a new type of telescope, which we have already described in the paper [1], it would be possible to measure the ASA. This would allow us to determine the appropriate Stationary Frame of Reference and the transverse velocity of the sun relative to that frame.

2. Transformation Matrices for Single Rotation of Coordinate System

First we will review coordinate transformation in general. Changing from one Coordinate System to another can be achieved by using matrix method. We will consider the cases when two coordinate systems are related by single rotation through some arbitrary angle about one of the coordinate axis.

The transformation matrix $\mathbf{E}(x, \alpha)$ corresponding to a single rotation of the Coordinate System about the positive x axis through an angle α is is given by the equation:

$$\mathbf{E}(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \tag{1}$$

The matrix $\mathbf{E}^{-1}(x,\alpha)$ is inverse of the matrix $\mathbf{E}(x,\alpha)$.

$$\mathbf{E}(x,-\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\alpha) & \sin(-\alpha) \\ 0 & -\sin(-\alpha) & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \tag{2}$$

It is easy to prove that

$$\mathbf{E}^{-1}(x,\alpha) = \mathbf{E}^{T}(x,\alpha) = \mathbf{E}(x,-\alpha) \tag{3}$$

The transformation matrix $\mathbf{E}(y,\beta)$ corresponding to a single rotation of the Coordinate System about the positive y axis through an angle β is

$$\mathbf{E}(y,\beta) = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \tag{4}$$

The matrix $\mathbf{E}^{-1}(y,\beta)$ is inverse of the matrix $\mathbf{E}(y,\beta)$.

$$\mathbf{E}(y,-\beta) = \begin{bmatrix} \cos(-\beta) & 0 & -\sin(-\beta) \\ 0 & 1 & 0 \\ \sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix} = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \tag{5}$$

It is easy to prove that

$$\mathbf{E}^{-1}(y,\beta) = \mathbf{E}^{T}(y,\beta) = \mathbf{E}(y,-\beta) \tag{6}$$

The transformation matrix $\mathbf{E}(z,\gamma)$ corresponding to a single rotation of the Coordinate System about the positive z axis through an angle γ is

$$\mathbf{E}(z,\gamma) = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

The matrix $\mathbf{E}^{-1}(z,\gamma)$ is inverse of the matrix $\mathbf{E}(z,\gamma)$.

$$\mathbf{E}(z,-\gamma) = \begin{bmatrix} \cos(-\gamma) & \sin(-\gamma) & 0 \\ -\sin(-\gamma) & \cos(-\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

It is easy to prove that

$$\mathbf{E}^{-1}(z,\gamma) = \mathbf{E}^{T}(z,\gamma) = \mathbf{E}(z,-\gamma) \tag{9}$$

3. Coordinate Systems

In this section are given the descriptions of the four Coordinate Systems that will be used in a further discussion. Denote by (P) "The Heliocentric-Ecliptic Coordinate System" [Figure 1]. Its origin O_p is centered on the center of mass of the solar system, and the fundamental plane coincides with the *ecliptic* plane of the Earth's revolution about the sun. The line of intersection of the *ecliptic* plane and the earth's equatorial plane defines the $x_p - axis$. On the first day of Spring a line joining the center of the Earth and the center of the sun points in the direction of positive $x_p - axis$ [1].

Denote by (Q) "The Geocentric-Equatorial Coordinate System" Figure[1]. Its origin O_q is at the center of the Earth, the fundamental plane is the equator and the positive x_q points in the vernal equinox direction. The z_p points in the direction of the north pole. By the definition the Coordinate System (Q) is non-rotating with the respect to the stars [1].

Let $\varphi = 23.43693 * \Pi/180$ denotes Earth's axial tilt Figure[1]

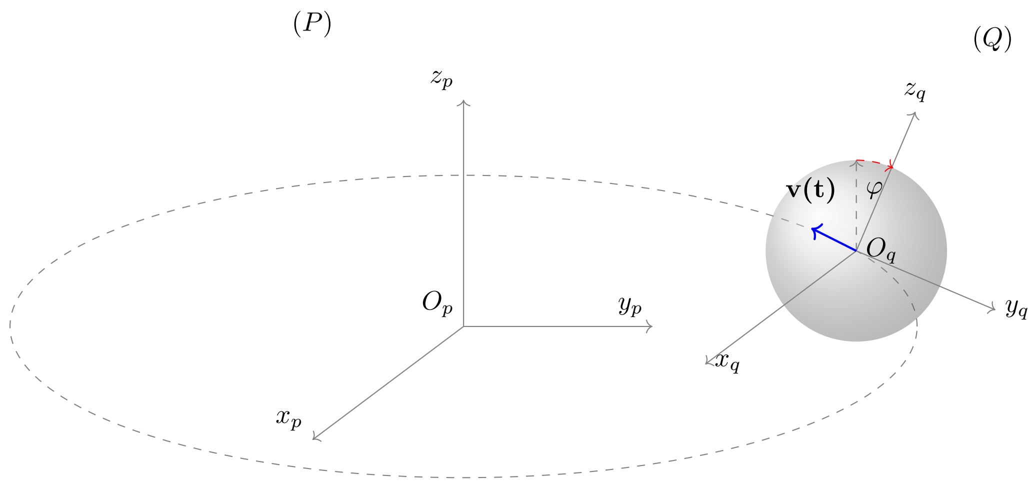

Figure 1: Ecliptic Coordinate System (P) and Equatorial Coordinate System (Q)

Conversion from Ecliptic Coordinates to Equatorial Coordinates is defined as follows

$$\mathbf{E}(x_p, -\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{bmatrix} \tag{10}$$

$$\mathbf{A}_1 = \mathbf{E}(x_p, -\varphi) \tag{11}$$

$$\begin{bmatrix} x_q \\ y_q \\ z_q \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \tag{12}$$

Using equation (12) we are able to convert the coordinates from the Ecliptic Coordinate System (P) to the Equatorial Coordinate System (Q).

The position of the star is determined by two angles called right ascension and declination Figure[2]. The right ascension α is measured eastward in the plane of equator from the vernal equinox direction. The declination δ is measured northward from the equator to the line of sight, we would say that is an angle between the plane of equator and the direction of the starlight [1].

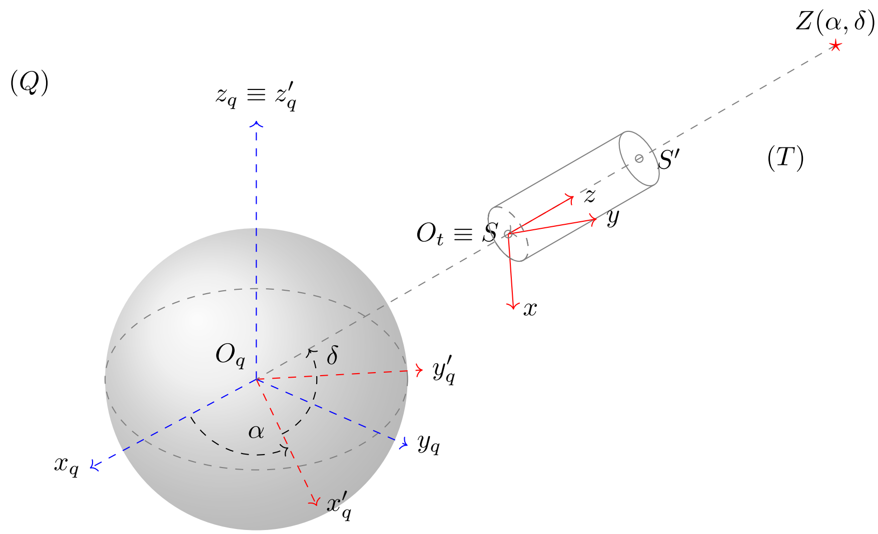

Figure 2: Equatorial Coordinate System (Q) and Telescope Coordinate System (T)

Conversion from equatorial coordinates to telescope coordinates will be implemented in two steps, Figure[2].

The transformation matrix $\mathbf{E}(z_q, \alpha)$ corresponding to a single rotation of the Equatorial Coordinate System (Q) about the positive z_q axis through an angle α.

$$\mathbf{E}(z_q,\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$\mathbf{A}_2 = \mathbf{E}(z_q,\alpha) \tag{14}$$

$$\begin{bmatrix} x'_q \\ y'_q \\ z'_q \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} x_q \\ y_q \\ z_q \end{bmatrix} \tag{15}$$

Using equation (15) we are able to convert the coordinates from the Ecliptic Coordinate System (Q) to the Coordinate System (Q').

The transformation matrix $\mathbf{E}(y'_q, -\left(\frac{\Pi}{2} - \delta\right))$ corresponding to a single rotation of the Coordinate System (Q') about the positive y'_q axis through an angle $-\left(\frac{\Pi}{2} - \delta\right)$

$$-\left(\frac{\Pi}{2} - \delta\right) = \delta - \frac{\Pi}{2} \tag{16}$$

$$\mathbf{E}\left(y'_q, \delta - \frac{\Pi}{2}\right) = \begin{bmatrix} \cos(\delta - \frac{\Pi}{2}) & 0 & -\sin(\delta - \frac{\Pi}{2}) \\ 0 & 1 & 0 \\ \sin(\delta - \frac{\Pi}{2}) & 0 & \cos(\delta - \frac{\Pi}{2}) \end{bmatrix} = \begin{bmatrix} -\sin(\delta) & 0 & \cos(\delta) \\ 0 & 1 & 0 \\ -\cos(\delta) & 0 & -\sin(\delta) \end{bmatrix} \tag{17}$$

$$\mathbf{A}_3 = \mathbf{E}\left(y'_q, \delta - \frac{\Pi}{2}\right) \tag{18}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}_3 \begin{bmatrix} x'_q \\ y'_q \\ z'_q \end{bmatrix} \tag{19}$$

Using equation (19) we are able to convert the coordinates from the the Coordinate System (Q') to the Telescope Coordinate System (T).

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}_3\mathbf{A}_2\mathbf{A}_1 \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \tag{20}$$

Using equation (20) we are able to convert the coordinates from the Ecliptic Coordinate System (P) to the Telescope Coordinate System (T).

4. Determining the Stationary Frame of Reference

Suppose we observe an arbitrarily chosen star, denoted by (Z) Figure [3]. At the instant t the photon (electromagnetic wave) hits in a perpendicular direction at the center of the top plane of the telescope noted by S'. At the instant t' the photon (electromagnetic wave) hits the bottom plane of the telescope noted by A. We define a Coordinate System $(T') \equiv (O', x', y', z')$ as it follows. Its origin is noted by the point O' $(O' \equiv S')$ and the positive z' coordinate is determined by direction $O'Z$. We will define (T') as a Stationary Coordinate System and assume that telescope is moving uniformly with velocity $\mathbf{U}$ regarding the (T'). The Telescope Coordinate System is denoted by $(T) \equiv (S, x, y, z)$. Its origin is noted by the point S and the positive z coordinate is determined by direction SS' Figure[3]. We will say that the Stationary Frame of Reference is determined by the Coordinate System (T').

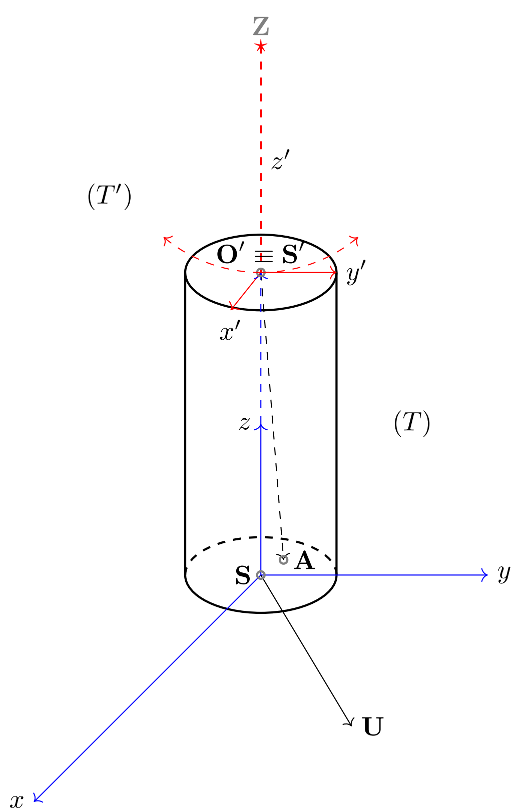

Figure 3: The telescope is moving uniformly regarding Coordinate System (T')

We will define velocities $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ in the following way:

(i) $\mathbf{u}$ - the velocity at which the solar system moves relative to the Coordinate System (T')

(ii) $\mathbf{v}$ - the velocity at which the Earth moves relative to the Sun
(iii) $\mathbf{w}$ - the velocity at which the telescope moves relative to the center of the Earth

We will assume that the velocities $\mathbf{v}$ and $\mathbf{w}$ are known.
Let $\mathbf{U}$ denotes the velocity with which the telescope moves relative to the Coordinate System (T').

$$\mathbf{U} = \mathbf{u} + \mathbf{v} + \mathbf{w} \tag{21}$$
$$\mathbf{U} = [U_x, U_y, U_z] \tag{22}$$
$$U_x = u_x + v_x + w_x \tag{23}$$
$$U_y = u_y + v_y + w_y \tag{24}$$
$$U_z = u_z + v_z + w_z \tag{25}$$

We can express the velocity $\mathbf{v}$ in the Ecliptic and the Telescope Coordinate Systems as it follows:

$$\mathbf{v} = [v_{x_p}, v_{y_p}, 0] \tag{26}$$
$$\mathbf{v} = [v_x, v_y, v_z] \tag{27}$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{A}_3\mathbf{A}_2\mathbf{A}_1 \begin{bmatrix} v_{x_p} \\ v_{y_p} \\ 0 \end{bmatrix} \tag{28}$$

We can express velocity $\mathbf{w}$ in the Equatorial and the Telescope Coordinate Systems as it follows:

$$\mathbf{w} = [w_{x_q}, w_{y_q}, 0] \tag{29}$$
$$\mathbf{w} = [w_x, w_y, w_z] \tag{30}$$

$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \mathbf{A}_3\mathbf{A}_2 \begin{bmatrix} w_{x_q} \\ w_{y_q} \\ 0 \end{bmatrix} \tag{31}$$

5. Determining a telescope's velocity

First we will determine the direction in which the photons (electromagnetic waves) move in relation to the transverse component of the telescope's velocity $\mathbf{U}$. In order to achieve that, let's assume that we are observing sunlight, Figure[4].

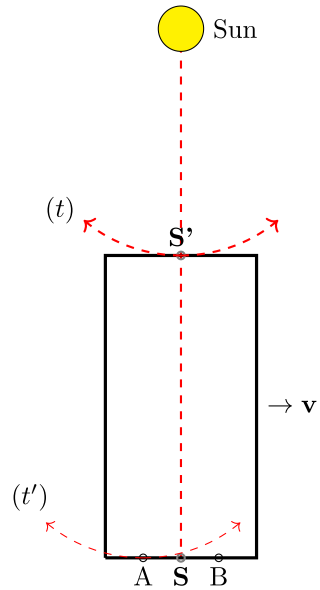

Figure 4: The telescope is moving uniformly regarding the sunlight. At the instant t the photon (electromagnetic wave) hits in a perpendicular direction at the center of the top plane of the telescope noted by S'. At the instant t' the photon (electromagnetic wave) hits the bottom plane of the telescope noted by A.

If we mark with point B the intersection between the bottom side of the telescope and the sun's ray, then the vector **SB** and the velocity **v** at which the earth moves around the sun have the same direction. If we mark with point A the intersection between the bottom side of the telescope and the sun's ray, then the vector **SA** and the velocity **v** at which the earth moves around the sun have the opposite direction. We will assume that the second possibility is correct. This would mean that the direction in which the photons(electromagnetic waves) move does not change and that the Stellar Aberration is caused by the movement of the telescope in relation to the sun's rays, Figure[4].

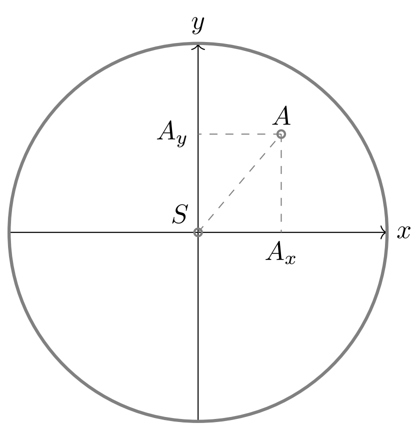

Figure 5: Due to the Stellar Aberration photons hit the bottom side of the telescope at point A

In the next step, we will determine the velocity **u**. The origin of the Telescope Coordinate System (T) is noted by S Figure[5]. Its axes are marked by x and y, A denotes the point where light hits the bottom plane of the telescope and the points A_x and A_y are the projections of the point A on the x and y axes, respectively.

$$l = SS' \quad (the\ length\ of\ the\ telescope) \tag{32}$$

$$\Delta t = \frac{l + U_z \Delta t}{c} \tag{33}$$

$$\Delta t = \frac{l}{c - U_z} \approx \frac{l}{c} \tag{34}$$

$$a_x = SA_x \tag{35}$$

$$a_y = SA_y \tag{36}$$

$$\Delta t\ U_x = -a_x \tag{37}$$

$$\Delta t\ U_y = -a_y \tag{38}$$

$$u_x + v_x + w_x = -\frac{a_x}{\Delta t} \tag{39}$$

$$u_y + v_y + w_y = -\frac{a_y}{\Delta t} \tag{40}$$

$$u_x = -\frac{a_x}{\Delta t} - v_x - w_x \tag{41}$$

$$u_y = -\frac{a_y}{\Delta t} - v_y - w_y \tag{42}$$

We will analyze two cases and assume that the Doppler Effect is measured relative to the same Stationary Frame of Reference as the ASA.

1) $\sqrt{u_x^2 + u_y^2} = 0$
This means that the velocity $\mathbf{u}$ does not affect the Stellar Aberration.

2) $\sqrt{u_x^2 + u_y^2} \neq 0$
We will analyze two subcases:

2.1) $\mathbf{u} = const$

We will choose any two cosmic objects from our Galaxy (constellations,Local Group galaxies,..) and by combining the results obtained from the two measurements, determine the velocity $\mathbf{u}$. The velocity $\mathbf{u}$ can be expressed in the Equatorial Coordinate systems (Q) as it follows.

$$\mathbf{u} = [u_{x_q}, u_{y_q}, u_{z_q}] \tag{43}$$

For the first star we will have the following equations.

$$\mathbf{u} = [u_x, u_y, u_z] \tag{44}$$

$$\mathbf{A}_3\mathbf{A}_2 \begin{bmatrix} u_{x_q} \\ u_{y_q} \\ u_{z_q} \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{45}$$

Since the component u_z is unknown, we will omit the third equation.

For the second star we will have the following equations.

$$\mathbf{u} = [u'_x, u'_y, u'_z] \tag{46}$$

$$\mathbf{A}'_3\mathbf{A}'_2 \begin{bmatrix} u_{x_q} \\ u_{y_q} \\ u_{z_q} \end{bmatrix} = \begin{bmatrix} u'_x \\ u'_y \\ u'_z \end{bmatrix} \tag{47}$$

Since the component u'_z is unknown, we will omit the third equation.

The components u_x, u_y, u'_x, u'_y are known. It remains to calculate the components $u_{x_q}, u_{y_q}, u_{z_q}$ from the three equations that we arbitrarily chose from the four remaining equations. If velocity $\mathbf{u}$ has a constant value this means the Stationary Frame of Reference (T') is not stationary with respect to the observed star. The ASA measurements are sufficient to determine the velocity $\mathbf{u}$. In order to determine the radial velocity ar which the "star" moves in relation to the Stationary Frame of Reference (T'), we apply the formula for the Doppler Effect given in [2].

f' - the frequency of the signal measured by the observer
f - the frequency of the signal measured by the sender
$\mathbf{v}_o$ - the velocity of the observer
$\mathbf{v}_s$ - the velocity of the sender

$$\mathbf{a} = [0, 0, 1] \tag{48}$$

$$f' = f\left(\frac{c + \mathbf{a}\cdot\mathbf{v}_o}{c + \mathbf{a}\cdot\mathbf{v}_s}\right) \tag{49}$$

$$\mathbf{a}\cdot\mathbf{v}_s = \frac{f}{f'}\ \mathbf{a}\cdot\mathbf{v}_o - \frac{\Delta\ f}{f'}\ c \tag{50}$$

We are still unable to determine the transverse velocity of the sender relative to the Stationary Frame of Reference (T').

2.2) $\mathbf{u} \neq const$

The velocity $\mathbf{u}$ is not equal to a constant value. We can say that in this case the velocity $\mathbf{u}$ is equal to the velocity with which the solar system moves in relation to the observed star. This means the Stationary Frame of Reference (T') is stationary with respect to the observed star. Using the Doppler Effect formula (50) we are able to determine the component U_z of the velocity $\mathbf{U}$.

$$\mathbf{v}_s = \mathbf{0} \tag{51}$$

$$U_z = \mathbf{a}\cdot\mathbf{v}_o \tag{52}$$

$$U_z = \frac{\Delta f}{f}c \tag{53}$$

$$U_z = u_z + v_z + w_z \tag{54}$$

$$u_z = U_z - v_z - w_z \tag{55}$$

We can express velocity $\mathbf{u}$ in Ecliptic Coordinate system (P) as it follows.

$$\mathbf{u} = [u_{x_p}, u_{y_p}, u_{z_p}] \tag{56}$$

$$\begin{bmatrix} u_{x_p} \\ u_{y_p} \\ u_{z_p} \end{bmatrix} = \mathbf{A}_1^{-1}\mathbf{A}_2^{-1}\mathbf{A}_3^{-1} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{57}$$

Combining the results of Stellar Aberration and Doppler Effect, we are able to determine the velocity at which the sun moves in relation to the observed star and vice versa. If we assume that the sun is stationary, then the star moves with velocity $-\mathbf{u}$ in relation to the heliocentric coordinate system.

6. Relative stellar aberration

Let's assume that we performed two measurements at instants T and T', Figure[6]. We will denote the corresponding sidereal times with t_s and t'_s. Let A and A' denote the intersection points between the light beam and the bottom plane of the telescope determined by point S, at the instants T and T' respectively. The coordinates are given in the Telescope Coordinate System, noted by (T).

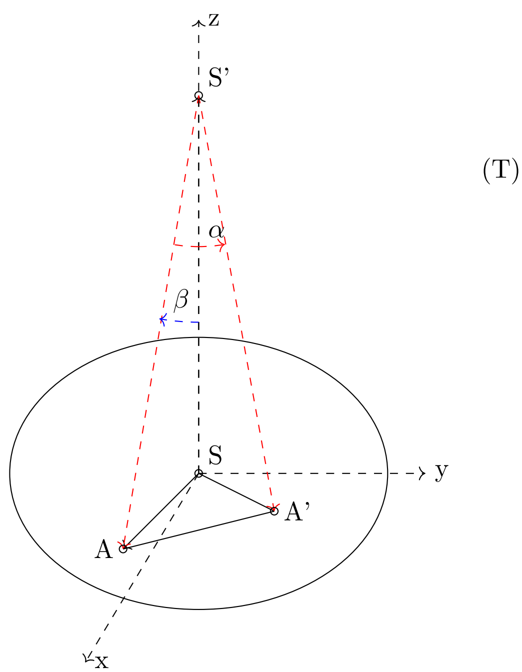

Figure 6: Relative and Absolute Stellar Aberration

Referring to the Figure[6] we have following equations.

$$\alpha = \angle\left(\mathbf{S'A}, \mathbf{S'A'}\right) \tag{58}$$
$$l = SS' \tag{59}$$
$$\Delta t = \frac{l}{c} \tag{60}$$
$$\mathbf{U}(T) = \mathbf{u}(T) + \mathbf{v}(T) + \mathbf{w}(t_s) \tag{61}$$
$$\mathbf{SA} = -\mathbf{U}(T)\Delta t \tag{62}$$
$$\mathbf{U}(T') = \mathbf{u}(T') + \mathbf{v}(T') + \mathbf{w}(t'_s) \tag{63}$$
$$\mathbf{SA'} = -\mathbf{U}(T')\Delta t \tag{64}$$
$$\mathbf{SA'} - \mathbf{SA} = \mathbf{U}(T)\Delta t - \mathbf{U}(T')\Delta t = \Delta\mathbf{U}(T)\Delta t \tag{65}$$
$$\Delta\mathbf{U}(T)\Delta t = \left(\mathbf{u}(T) - \mathbf{u}(T')\right)\Delta t + \left(\mathbf{v}(T) - \mathbf{v}(T')\right)\Delta t + \left(\mathbf{w}(t_s) - \mathbf{w}(t'_s)\right)\Delta t \tag{66}$$
$$\Delta\mathbf{U}(T)\Delta t = \Delta\mathbf{u}\Delta t + \Delta\mathbf{v}\Delta t + \Delta\mathbf{w}\Delta t \tag{67}$$
$$\mathbf{k} = (0, 0, 1) \tag{68}$$
$$\mathbf{SS'} = l\mathbf{k} = \Delta tc\mathbf{k} \tag{69}$$
$$\mathbf{S'A} = \mathbf{S'S} + \mathbf{SA} \tag{70}$$
$$\mathbf{S'A} = \mathbf{S'S} + \mathbf{SA} = -\Delta tc\mathbf{k} - \mathbf{U}(T)\Delta t = -\Delta tc\left(\mathbf{k} + \frac{\mathbf{U}(T)}{c}\right) = \left|\frac{\mid \mathbf{U}(T) \mid}{c} << 1\right| \approx -\Delta tc\mathbf{k} \tag{71}$$
$$\mathbf{S'A'} = \mathbf{S'S} + \mathbf{SA'} = -\Delta tc\mathbf{k} - \mathbf{U}(T')\Delta t = -\Delta tc\left(\mathbf{k} + \frac{\mathbf{U}(T')}{c}\right) = \left|\frac{\mid \mathbf{U}(T') \mid}{c} << 1\right| \approx -\Delta tc\mathbf{k} \tag{72}$$

We assumed that $\mathbf{SS'}(T) = \mathbf{SS'}(T')$, but actually we have that $\mathbf{SS'}(T') = \mathbf{SS'}(T) + \mathbf{E}$, where $|\mathbf{SS'}| >> |\mathbf{E}|$.

$$\mathbf{S'A} \times \mathbf{S'A'} = \left(\mathbf{S'S} + \mathbf{SA}\right) \times \left(\mathbf{S'S} + \mathbf{SA'}\right) = \left(\Delta tc\mathbf{k} + \mathbf{U}(T)\Delta t\right) \times \left(\Delta tc\mathbf{k} + \mathbf{U}(T')\Delta t\right) \tag{73}$$
$$\sin(\alpha) = \frac{\parallel \mathbf{S'A} \times \mathbf{S'A'} \parallel}{\parallel \mathbf{S'A} \parallel \parallel \mathbf{S'A'} \parallel} \approx \left|\left|\frac{\mathbf{k} \times \Delta\mathbf{U}(T)}{c} + \frac{\mathbf{U}(T)}{c} \times \frac{\mathbf{U}(T')}{c}\right|\right| \tag{74}$$
$$\mathbf{k} \times \Delta\mathbf{U}(T) = \mathbf{k} \times \left(\Delta U_x(T)\mathbf{i} + \Delta U_y(T)\mathbf{j} + \Delta U_z(T)\mathbf{k}\right) = -\Delta U_y(T)\mathbf{i} + \Delta U_x(T)\mathbf{j} \tag{75}$$
$$if \ \left(\left|\left|\frac{-\Delta U_y(T)\mathbf{i} + \Delta U_x(T)\mathbf{j}}{c}\right|\right| >> \left|\left|\frac{\mathbf{U}(T)}{c} \times \frac{\mathbf{U}(T')}{c}\right|\right|\right) \ then \tag{76}$$
$$\sin(\alpha) \approx \frac{\left|\left| - \Delta U_y(T)\mathbf{i} + \Delta U_x(T)\mathbf{j}\right|\right|}{c} = \frac{\left|\left| - (\Delta u_y + \Delta v_y + \Delta w_y)\mathbf{i} + (\Delta u_x + \Delta v_x + \Delta w_x)\mathbf{j}\right|\right|}{c} \tag{77}$$

Equation (77) gives the general formula for calculating the angle α associated with Relative Stellar Aberration. The angle α depends on the change in the velocity with which the telescope moves in relation to the Stationary Frame of Reference (T').

To simplify things, we will assume that

$$(t_s = t'_s) \Rightarrow (\mathbf{w}(t_s) = \mathbf{w}(t'_s)) \tag{78}$$

It follows that:

$$\sin(\alpha) \approx \frac{\left|\left| - (\Delta u_y + \Delta v_y)\mathbf{i} + (\Delta u_x + \Delta v_x)\mathbf{j}\right|\right|}{c} \tag{79}$$

Suppose $T' = T + 0.5 \ year$

$$\mathbf{v}(T) = -\mathbf{v}(T') \tag{80}$$

Then it follows that:

$$\Delta\mathbf{v}(T') = \mathbf{v}(T') - \mathbf{v}(T) = 2\mathbf{v}(T') \tag{81}$$

$$\mathbf{D} = -\left(\Delta u_y + 2v_y\right)\mathbf{i} + \left(\Delta u_x + 2v_x\right)\mathbf{j} \tag{82}$$

$$\mathbf{D}^2 = \left(\Delta u_y + 2v_y\right)^2 + \left(\Delta u_x + 2v_x\right)^2 = 4(v_x^2 + v_y^2) + (\Delta u_x^2 + \Delta u_y^2) + 4v_x\Delta u_x + 4v_y\Delta u_y \tag{83}$$

$$\sin(\alpha) \approx \frac{||\mathbf{D}||}{c} = \frac{\sqrt{4(v_x^2 + v_y^2) + (\Delta u_x^2 + \Delta u_y^2) + 4v_x\Delta u_x + 4v_y\Delta u_y}}{c} \tag{84}$$

Equation (84) gives the general formula for calculating the angle α associated with Bredley's Stellar Aberration.

$$if\ (\Delta v_x^2 + \Delta v_y^2) >> (\Delta u_x^2 + \Delta u_y^2)\ then\ \left(\mathbf{D}^2 \approx 4(v_x^2 + v_y^2) + 4v_x\Delta u_x + 4v_y\Delta u_y\right) \tag{85}$$

$$\frac{||\mathbf{D}||}{2\sqrt{v_x^2 + v_y^2}} \approx \sqrt{1 + \frac{v_x\Delta u_x + v_y\Delta u_y}{\sqrt{v_x^2 + v_y^2}}} \tag{86}$$

$$if\ \left(1 >> \frac{v_x\Delta u_x + v_y\Delta u_y}{\sqrt{v_x^2 + v_y^2}}\right)\ then\ \left(\frac{||\mathbf{D}||}{2\sqrt{v_x^2 + v_y^2}} \approx 1 + \frac{v_x\Delta u_x + v_y\Delta u_y}{2\sqrt{v_x^2 + v_y^2}}\right) \tag{87}$$

$$||\mathbf{D}|| \approx 2\sqrt{v_x^2 + v_y^2} + v_x\Delta u_x + v_y\Delta u_y \tag{88}$$

$$\sin(\alpha) \approx \frac{2\sqrt{v_x^2 + v_y^2} + v_x\Delta u_x + v_y\Delta u_y}{c} \tag{89}$$

Equation (89) gives the simplified form of the formula (84) for calculating the angle α associated with Bredley's Stellar Aberration.

7. Absolute Stellar Aberration

In this section we will calculate the angle β corresponding to the Absolute Stellar Aberration. Referring to the Figure[6] we have following equations.

$$\beta = \angle\left(\mathbf{S'A}, \mathbf{S'S}\right) \tag{90}$$

$$l = SS' \tag{91}$$

$$\Delta t = \frac{l}{c} \tag{92}$$

$$\mathbf{U}(T) = \mathbf{u}(T) + \mathbf{v}(T) + \mathbf{w}(t_s) \tag{93}$$

$$\mathbf{SA} = -\mathbf{U}(T)\Delta t \tag{94}$$

$$\mathbf{k} = (0, 0, 1) \tag{95}$$

$$\mathbf{SS'} = l\mathbf{k} = \Delta tc\mathbf{k} \tag{96}$$

$$\mathbf{S'A} = \mathbf{S'S} + \mathbf{SA} \tag{97}$$

$$\mathbf{S'A} \times \mathbf{S'S} = \left(\mathbf{S'S} + \mathbf{SA}\right) \times \mathbf{S'S} = \mathbf{SA} \times \mathbf{S'S} = (-\mathbf{U}(T)\Delta t) \times (-\Delta tc\mathbf{k}) = \Delta t^2 c\left(\mathbf{U}(T) \times \mathbf{k}\right) \tag{98}$$

$$\sin(\beta) = \frac{\parallel \mathbf{S'A} \times \mathbf{S'S} \parallel}{\parallel \mathbf{S'A} \parallel\parallel \mathbf{S'S} \parallel} \approx \frac{\Delta t^2 c\left|\left|\mathbf{U}(T) \times \mathbf{k}\right|\right|}{l^2} \approx \frac{\left|\left|\mathbf{U}(T) \times \mathbf{k}\right|\right|}{c} = \frac{\left|\left|U_y(T)\mathbf{i} - U_x(T)\mathbf{j}\right|\right|}{c} \tag{99}$$

$$\sin(\beta) \approx \frac{\sqrt{U_x^2(T) + U_y^2(T)}}{c} = \frac{\sqrt{(u_x(T) + v_x(T) + w_x(t_s))^2 + (u_y(T) + v_y(T) + w_y(t_s))^2}}{c} \tag{100}$$

or we can simply write that

$$\tan(\beta) \approx \frac{|SA|}{l} \tag{101}$$

Equations (100) and (101) give the formulas for calculating the angle β associated with Absolute Stellar Aberration.

8. Conclusion

We derived formula for the angle of Bradley's Stellar Aberration and proved that it depends on the change in velocity of the detector. In addition, we derived the formula for the angle of the ASA and proved that it depends on the magnitude of the velocity of the detector. It would also be interesting to perform the experiment so that instead of a distant star, we use a light source from the lab.

9. Conflicts of Interest Statement

The author has no conflicts of interest to disclose.

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