Inhomogeneous distribution of the universe's matter density as a physical basis for MOND's acceleration a_0

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Abstract

One of the most effective theories for dark matter is Milgrom's Modified Newtonian Dynamics, where a modified law of gravity based in a fixed acceleration scale a_0 is postulated that provides a correct description of the gravitational fields in galaxies. However, the significance of a_0 is unknown, and the whole theory is generally viewed as a mere phenomenological description of the observations. Based on Newton's gravitational law as applied to a uniform continuous mass we posit a non-homogeneous distribution of mass at cosmological scales that would give rise to a constant acceleration that agrees with MOND's a_0. The implications for MOND as a viable theory of dark matter and for the problem of dark energy are briefly discussed.

Modified Newtonian Dymanics (MOND) is a Newtonian-derived hypothetical model of gravity proposed 40 years ago by Mordehai Milgrom to explain the multiple gravitational anomalies observed in galaxies and galaxy clusters [1-3]. They are summarized and conventionally explained through the existence Dark Matter, an elusive new form of matter that interacts only gravitationally and is not included in the Standard Model of particle physics. While no such particles have yet been found, the search goes on and MOND usually plays a secondary role in the list of candidate explanations for dark matter. One of the reasons is that a_0, the distinctive element of MOND, does not correspond to any physical entity, and –it is argued- was postulated solely as a means to obtain a gravitational law that fits the observations. It is sometimes dubbed a phenomenological explanation.

While a_0 agrees (to within one order of magnitude) with the acceleration calculated at the border regions of the observable universe from the simple Newtonian formula, and it is also found to

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agree with the currently accepted values of Hubble's constant and with the square root of the cosmological constant Λ, in both cases multiplied by the speed of light *c*, no physical representation has yet been devised and most physicists would agree that it behaves as another constant of nature, whose role would be to relate fundamental gravitational phenomena in the low-acceleration regime.

The Newtonian ball model of gravity

A generally accepted assumption of all current astrophysical models is the Cosmological Principle, the idea that the universe at large scales is both homogeneous and isotropic. While it may still be isotropic and strong constraints have been set on the range of variation in matter density, the homogeneity condition has little theoretical supporting evidence. Based on a previous toy model of Newtonian gravity (* Article Toy Model of Gravity - Ref pending) that postulates that spacetime is a dynamical network of nodes joined by constantly changing and reformatting virtual force-vectors that can elongate and reorient in the presence of mass, we shall follow on original ideas due to Isaac Newton and argue that the universe can be modelled as a nearly homogeneous continuous distribution of mass that obeys simple dynamics derived from the Universal Law of Gravitation. As Newton amazingly found in the late 1600s [5], when a continuous distribution of mass with constant density is allowed to evolve according to such law, an acceleration appears that is null at the center and increases outwards in linear proportion to radial distance until it reaches, for a distance equal to the radius of the ball, the exact same value as predicted by conventional Newtonian gravity (Fig).

$F_B = G\, M\, m\, r / R^3$ as opposed to a point-mass gravitational field:

$F_N = G\, M\, m / R^2$

where F_B (force of Newtonian ball model) and F_N (Newton's conventional point-mass gravitational force) are the force on a test particle with mass m placed at distance r from the center of the R-ball, or at a distance R from the central point-mass M, respectively. The acceleration for the ball with mass M is then

$Acc_B = G\, M\, r / R^3$ and solving for G

$G = Acc_B\, R^3 / M\, r$

We now define G' as $4\pi G$ and substitute it for G above, following the ideas of the previously mentioned toy model (*). The resulting expression is mathematically equivalent, though it may facilitate the visualization of upcoming considerations.

$G' = (Acc_B\, 4\pi\, R^3) / (M\, r)$ $[G' := 4\pi G]$

And multiplying both parts of the right-hand fraction by a factor of three,

$$G' = 3\ Acc_B\ 4/3\ \pi\ R^3 / M\ r$$

and since $4/3\ \pi\ R^3 / M$ equals the inverse of the matter density for the spherical volume,

$$G' = 3\ (Acc_B / r) \cdot (1/\rho)$$

$$G' = 3\ Acc_B / r \cdot \rho \qquad (1)$$

where ρ is now the average, not necessarily constant matter density of the universe.

Looking at equation (1) we see that in such a ball model of the universe, if ρ is constant, then the quotient (Acc_B / r) must be constant, which is very nice and agrees with the Newtonian view but does not help us understand the existence of a constant acceleration pervading the whole universe that at the same time agrees with the Newtonian acceleration at its border regions, as MOND postulates and available evidence strongly suggests.

We therefore let ρ vary with radial distance, however small the constant of proportionality may be, and assume that it is the product in the denominator of Equation (1) ($r \cdot \rho$) that is constant. In other words, we let density to decay as the inverse of radial distance. We immediately see then that since both G' and the product ($r{\cdot}\rho$) are constant, so must be Acc_B, and this acceleration agrees with MOND's universal acceleration a_0 and with the calculated Newtonian acceleration at the border regions of the ball to whithin one order of magnitude, as can be easily checked. Indeed, feeding in the accepted values for the mass of the observable universe (10^{53} Kg), radial distance (10^{26} m) and G, it turns out that the acceleration perceived at the border regions of the observable universe is about $3.4 \cdot 10^{-10}\ m{\cdot}s^{-2}$, quite close to the reported value for a_0 ($1.2{\cdot}10^{-10}$). According to the Newtonian ball model and assuming $r{\cdot}\rho$ constant, this same aceleration would be present as a background curvature in the whole universe.

Since both G and G' are approximately of the order of Acc_B –assuming Acc_B equals a_0 and both lay around 10^{-10} in MKS units-- density ρ decays as 1/r with a constant of proportionaly of the order of 10^{-26}, the currently accepted value for the average density of the universe in Kg/m^3. (We will neglect here the difference between G and G').

Another supporting argument for the model would be the striking ressemblance of equation (1) with the Friedman equation. For flat space (k = 0), the Friedman equation can be expressed as

$G' = 4\pi G = 3/2 \cdot H^2 / \rho$ which certainly reminds us of Eq 1:

$G' = 3\ Acc_B / r \cdot \rho$ and since dimensions of Accel / r equals $1/T^2$ we have

$G' = 3\cdot (1 / t)^2 \cdot 1 / \rho$,

If we then interpret 1/t as the constant rate of expansion H_0,

$G' = 3H^2 / \rho$

which differs from the Firedman equation only by a factor of 2. The reason for the discrepancy we ignore, but it has happened in other realms of physics when a classical, non-relativistic approach has been later superseded by the appropriate relativistic version (for instance, in the old estimation of the bending of light from gravity before Einstein, which differed from the relativistic version by a factor of 2).

We therefore conclude that

1. In a modified Newtonian ball model of the universe, a continuously decreasing matter density that scales as 1/r, as opposed to the uniform distribution from the Cosmological Principle, gives rise to a constant universal physical acceleration that agrees with MOND's a_0.

2. This can provide a physical basis for MOND and support it as a viable interpretation of the dark matter problem.

3. The resulting matter density distribution is easy to describe mathematically from available data on the density of the cosmos, but may be hard to verify experimentally, for the densities involved, as well as the variations incurred might be extremely low.

Cosmological acceleration as a basis for the universe's expansion

We have discussed how a real constant universal acceleration a_0 (sometimes denoted a_Λ) can explain the presence of abnormal accelerations around galaxies and - following the lines of MOND – might account for the problem of dark matter. The observed accelerations below a certain threshhold turn out to be an average of the Newtonian and the background constant acceleration $a_{0.}$ This would be a real physical phenomenon, not only a mathematical construct, the details of which should be further discerned and worked out. We now turn our attention to the mysterious empirical relation observed between a_0 and the parameters that reflect the universe's expansion, H_0 and Λ.

Indeed, the numerical value of MOND's a_0 has been found to be approximately

$a_0 \sim (c / 2\pi) \cdot H_0 \sim (c^2 / 2\pi)\cdot \mathrm{SQRT}(\Lambda/3)$

Why is that? What is the intimate relation of a_0 to the accelerated expansion of the universe?

Let's take a look at the modified Newtonian ball model of gravity as applied to the whole universe. The postulated real universe (Fig, top) is made of a spacetime network (*Ref Toy model) with a constant acceleration (a_0), so that the separation between neighbouring nodes in the network decays linearly with radial distance, i.e., there is a constant gradient of the deformation in the radial direction. A test particle located at any point in this universe experiences a force toward the center that is numerically equal to the local gradient of the volume of the lelementary space units. This increase in volume of neighboring space units can be expressed as dx·dy·dr, but since no deformation occurs in the X and Y directions, but only in the radial dimension towards the center, that scales as 1/R, a test particles would experience a centripetal acceleration equal to this gradient, which we call a_0. Light emmitted from a star S in the universe would travel at speed *c*, but this is only in flat space. Light speed in a curved space is actually the number of elementary space units that are traversed per unit time, times their average size in the radial direction. An increase in light speed results when space units are elongated, in proportion to δ_R and a_0 ($c' = c \cdot a_0$). A corresponding increase in wavelenght also takes place and is observed.
For an observer unaware of this particular spacetime geometry and expecting space to be flat, who also assumes that speed of light is constant (Figure, bottom), the only way to interpret the observations is to assume that the light-emitting star S is moving with a velocity $H_0 \cdot R$ away from him. If the observed redshift that scales with radial distance is to be explained in the context of flat space and constant speed of light, the star S needs to move with an acceleration a_0 away from the observer. Then everything is consistent, space appears flat, light speed is *c* and the redshift can be interpreted as the star moving away at an accelerated speed, following the well established relation of velocity to redshift known and confirmed from nearer stellar bodies. Since this redshift is observed for all distant stars and galaxies, and indeed is proportional to the distance from the observer, the natural conclusion is that all stars are moving away from the observer with an accelerated speed proportional to distance and a spacetime expansion is taking place.

The concept of flattening or apparent normalization of spacetime curvature induced by an accelerated motion is analogous to the one embodied in Einstein's Equivalence Principle. When a point mass generates a deformation of spacetime around it, it does so by generating an elongation of all elementary space units in the R direction that decays as $1/R^2$ (See our paper on Toy Model of Newtonian gravity). Any test particle in the surroundings suffers a force towards the center proportional to the local gradient of the volume of the space units. The only way this particle can cease to experience a gravitational force is by moving toward the center with an accelerated motion, the acceleration being exactly the gravitational GM/R^2. When in free fall, all bodies experience no gravity because the space in which they move is effectively flat in their frame of reference. If we imagine them ticking at every transition from one space unit to the next, the ticks at rest or with slow motion show a lower time rate the closer the object is from the central mass, and it becomes constant only when the centripetal component of acceleration equals $GM/R^{2,}$ i.e., in free fall.

In our case with a universal curved space, we can imagine a particle moving in any direction and ticking at every transition from one spacetime unit to the next. Of course, in a direction perpendicular to radial distance from the center, the ticking is always constant because no acceleration is felt. But in the radial direction away or towards the center, the ticking rate varies in inverse proportion to the size of the local elementary space units, i.e., in proportion to radial distance, and the rate of increase or decrease is constant and equal to a_0. The only way a particle moving in the radial direction could give away a constant ticking rate, as it were in flat space, would be to by moving towards the center with accelerated motion equal to the universal acceleration a_0. In the case of distant galaxies, the ticking needs not become constant, but needs to agree with the real ticking rate that is happening to the emitted light, so that the resulting light speed is c. And this -assuming flat space- occurs only when the star or galaxy moves away from the observer with acceleration a_0. This is why a_0 matches almost exactly the Hubble parameter H_0 and scales with the cosmological constant Λ.

Note that this does not invalidate redshift as an accurate indicator of velocity for stellar bodies in general. A constant velocity in flat space translates into a uniformly increased (for bodies moving away) or decreased (for those moving closer) size of the unit-space traversed by light in its own frame of reference. The ticking is faster for bodies moving in the same direction as the emitted light, and slower for bodies moving in the opposite direction, and both are constant in time. A corresponding redshift is recorded that scales linearly with velocity and in general is not affected by the universal background acceleration. Only when very large distances and cosmological scales are involved will the discrepancy be noticeable. We remind that for short distances, spacetime is approximately flat in the universe. Thus, the discrepancies are negligible when redshift is used to measure velocities at galactic, sub-cosmological scales.

Discussion and Q&A

The first comment that comes to mind is how plausible a non-uniform distribution of matter is, given the fundamental character of homogeneity, as well as isotropy, in modern cosmology. The answer is that we don’t know, and it is not easy to either verify or rule it out. Even assuming that inhomogeneities in the mass distribution are constrained by some observations, including the CMB and the wide field observations of distant galaxies, small inhomogeneities that are still larger that the ones needed here cannot be presently ruled out by observations. As for the theoretical arguments, given that we can only record with certainty a minute fraction of the matter assumed as present in the universe, this highlights our limitations to determine theoretical estimates and boundaries for the mass density distribution.

On the other hand, a central concentration of mass density would intuitively make sense when one considers the behavior of a fluid-like mass governed only by gravity. We cannot confer much reliability to the postulate that there is no central point in the universe, that all points are equivalenty separated from the center, if any. Such reasoning, derived from special relativity, holds little water in a cosmological context of wide uncertainties and few proven quantitative facts. On the other hand, some theoretical models have already described a central predominant mass density that decays with radial distance in galaxies. Lastly, the extremely low values of mass density in the universe, as well as the predominance of gas and dust over stars when it comes to baryonic matter pose serious challenges to any direct measurement of density. We would conclude by saying that the assumption of inhomogeneity is as difficult to disprove as it is to measure and, in the absence of direct evidence, the agreement with the observed effects of a putative universal acceleration should count as an argument in its favor, if certainly provisional.

-Assuming that mass density varies with radial distance in a model that was meant and tuned to describe uniformly distributed masses, does it not disprove the argument ?

The Newtonian model for gravity in solid spheres is valid not only for spheres with uniform density, but for any sphere in which density depends only on radial distance, i.e, for any spherically symmetrical distribution of matter. Furthermore, density gradient in the universe is expected to be very low, so that matter density would be very likely approximately uniform. This might explain why astrophysical observations have been thought so far to support the cosmological principle of homogeneity.

-How well does the model support MOND as an effective theory for dark matter ?

MOND has been considered by most authors either as a modification of the laws of gravity that is awaiting proper justification, or as a mere phenomenological description based on the introduction of a new free variable, a_0. We will claim that it is neither.
Insofar as the main drawback of MOND has been its speculative nature and the arbitrary splitting of the gravitational law in two domains, corresponding to accelerations higher and lower than a_0 without a clear rationale, a theory proposing the natural occurrence of a_0 based on Newtonian laws of gravity would make it much more plausible, for it would explain not only a_0, but also the fact that MOND kicks in at a definite acceleration in the gravitational field. The way such background acceleration then interacts with gravity originating from point-like masses like galaxies would still need further consideration and research. Whether –as it was postulated in a recent paper (*)- such interactions would be resolved by an appropriate averaging function of both accelerations or otherwise, the fact is that MOND would become a more complete and correct interpretation of Newtonian gravity and -a fortiori- of General Relativity. If the model proposed here is proved right, MOND would stand among the viable, serious contenders to explain dark matter.

-How does this model affect other parts of the current ΛCDM cosmological framework such as dark energy and the Big Bang?

It is currently difficult to forsee the impact that a confirmation of the physical nature of a_0 and an inhomogeneous distribution of matter would bring about. As shown above, dark energy would certainly be one of the prime targets –or beneficiaries- for a revision. Though still in the minority, many physicists have stated the suspicion that dark matter and dark energy might be related, but no consistent hypothesis has so far been provided. The idea that the observed expansion of the universe may be due to a constant acceleration pervading it seems a reasonable hypothesis. Of course this leads to the disturbing conclusion that redshift, the observational cornerstone of Λ and of the whole idea of expansion, might be called into question. However, the basic significance of redshift for astrophysics should probably remain untouched. It is only that another source of redshift, that of a universally curved spacetime should be also held into account.

-How does this model compare to the previous published paper on fluid spacetime and dark matter?

In our previous paper [4] we postulated and equivalence mass-spacetime that allowed us calculate its precise equivalence rate, to devise a mechanism for Newtonian gravity [ ] and to figure out a way in which the anomalous 1/R accelerations might be generated in galaxies. While the original idea of mass-spacetime equivalence we still deem as correct, the exact mechanism for the abnormal accelerations was not. For one thing, we realized that the actual amount of spacetime generated per unit mass (approximately 1 mm^3 per Kg of mass) is too small. On the other hand, the process of liberation of spacetime from mass through nuclear reactions would likely be a violent one, with liberation of huge amount of energy that would destroy any curvature generated from the fluid spacetime. Finally, on theoretical and philosophical grounds, two different explanations for the same phenomenon is not what one should expect. For all these reasons, we are glad to recant parts of our Fluid Spacetime model. For instance, the mechanism involved in the Bullet cluster anomaly cannot be explained solely on the grounds of the gas being in Newtonian regime and the galaxies in MOND regime. It turns out that the galaxies are also for the most part in Newtonian regime. We still believe that the Bullet should be explained by gravitational effects, likely including some elements from the present discussion.

Lastly, it will be argued that this discussion is limited to the non-relativistic case, where time is absolute, and space is likewise treated quite naively. The observation would be pertinent, but we are not all that sure that absolute spacetime is not a real feature of the universe. Relativistic phenomena indeed are an accurate description of the universe we live in, but this might be only as long as we do not take actual deformations of spacetime into account. Einstein himself warned us in his famous Leiden address of 1920 that spacetime might in the end turn out to be a real entity, albeit possibly an unmeasurable one. And perhaps by ‘real’, in this context, he meant absolute. At any rate, the present discussion seeks to adress very real problems in the astrophysical realm that for the most part occur at sub-relativistic speeds. The approach is classical, intuitive and Newtonian because this is the only way our imagination can be put to work, but we expect that a precise formulation of the present ideas could be adapted to the postulates of Special and General Relativity, as Einstein seemed not unwilling to accept.

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