The eight planets of the Kepler-47 star system

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Eight planets of the binary star Kepler-47 modulate the gravitational wave at 3.1 μHz. Three of them are already known. The measured orbital times fit very well with the predictions of Dermott's rule, an improved version of the Titus-Bode rule. The phase modulation measurement technique is explained in detail.

1 Introduction

The two Kepler-47 stars Aa and Ab orbit each other in a period of 7.45 days and emit a gravitational wave (GW) with a frequency of 3.107725 μHz. This GW is easily measurable because we are approximately in the plane of rotation of the system Aa–Ab ("edge on", inclination $\approx 90^o$). Despite their low mass, the planets B, C,.. produce a periodic frequency wobble of f_{GW}, a phase modulation. The long-term analysis of the GW shows several results:

- The three planets known so far with the orbital periods $P_B \approx 49$ days, $P_C \approx 187$ days and $P_D \approx 303$ days [1, 2] are confirmed, the times specified.
- There are five additional planets causing GW Doppler shifts. The orbital period and mass of each planet can be estimated from the measurements of the phase modulation.
- The frequency drift of the binary star system can be measured precisely.

How to receive GW? Studies of the natural resonances of the earth have shown that long-term recordings of barometers contain signals of unknown origin [3]. Identical spectral lines with properties of gravitational waves are found in the records of weather stations that are far apart. This study assumes that the atmosphere – just like any other object – responds to GW and is therefore a suitable antenna for receiving GW. The quality of the signals is sufficient for precise measurements in the frequency range around 3 μHz.

In order to achieve a frequency resolution Δf better than 1 nHz, data for Fourier analysis (FFT) must be collected during a long period of time. Filters of this low bandwidth take a long time to settle down. According to Küpfmüller [8],

$$T_{min} \cdot \Delta f \geq 0.5 \tag{1}$$

holds. The German weather service DWD stores air pressure data [5], which can be used after some preparatory work. In order to reduce the influence of local peculiarities and data gaps of individual weather stations, the records of as many barometers as possible, which are distributed all over Germany and have been in operation almost without gaps for at least ten years, are added. In the period 2000 to 2009, 64 data chains were found, in the period 2010 to 2019 only 51 data chains. Since the wavelength of the GW exceeds the mutual distances between the barometers by a factor of at least 10^6, all instruments react to the GW in phase. This coherent addition significantly improves the S/N of the signals and shows spectral lines that disappear in the noise when analyzing a single data chain. The time span of twenty years allows a spectral resolution of about 0.8 nHz (equation (1)). The very small signal amplitude requires extremely narrow-band filters in order to suppress the interfering surroundings. Windowed-sinc filters with 10^6 nodes and 0.5 nHz bandwidth deliver good results. IIR filters are not very suitable because they produce phase distortions.

Nearby celestial bodies also excite the earth and the atmosphere to vibrate, the frequencies are listed in the Tidal Potential Catalog HW95 [6]. According to this table, the moon causes weak spectral lines at 2.98085 μHz and at 3.11096 μHz, which are *not* detectable in the air pressure records. Both frequencies differ from f_{GW} and could be identified because their drift is zero. Apparently, the GW of nearby binary systems affect the atmosphere stronger than planets.

2 Identification of f_{GW} and signal processing

The signal flow of the data and the demodulation of the phase modulation (PM) are described in detail in [4]. The modulation index with the definition $a = \Delta f / f_{mod}$ identifies a PM. Δf is the frequency deviation, i.e. the largest difference between the instantaneous frequency and the average f_{GW} as a result of the Doppler effect. $f_{mod} = 1/P$ is the orbital frequency of a planet causing the phase modulation. Astronomers prefer the orbital period P. A GW may be phase modulated with several frequencies that are linearly superimposed and can be examined independently.

Analysis of the GW of the Algol trio [10] provides evidence that GW – at least at low frequencies – do not propagate at the speed of light. For this reason, a transformation of the measured values into the solar system barycentre is dispensed within this study. This transformation requires the value of v_{GW} – which has never been verified experimentally. Without this transformation, f_{GW} is phase-modulated with $f_{orbit} = 31.69$ nHz (in addition to all other PM).

The data analysis starts with a reduction of the signal frequency f_{GW} by 1.7 μHz (heterodyne method, see Figure 2) and subsequent decimation by a factor of 40. This reduces the sampling frequency to $f_s = 1/(40 \text{ hours})$ and shortens the file length to be processed. Further data reduction is the subject of the following section.

f_{GW} in the year 2023 can be calculated from observations with electromagnetic waves [1, 2], but not the frequency drift. Analysis of historical data from the year 2000 requires a reliable initial value for f_{GW}. At the expected frequency, the spectrum (Figure 1) shows *no* maximum. Maybe, this is a result of the PM: If only one of the planets causes an unfavorable modulation index a, the amplitude of the spectral line at f_{GW} is almost zero.

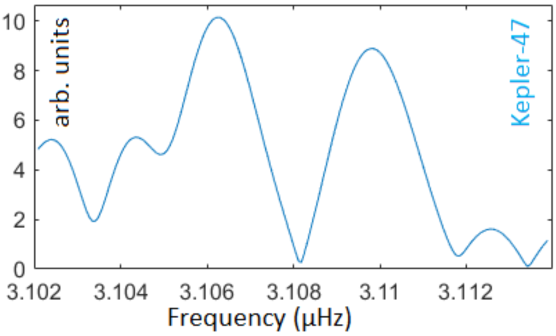

Figure 1): *The binary system in the center of Kepler-47 generates a GW at $f_{GW} = 3.1078\ \mu Hz$. The double peak instead of a maximum is probably the result of the phase modulation of the GW with an unfavorable modulation index ($a \approx 2.4$).*

Then, the whole energy of the GW is transfered into the sidebands. In the absence of more detailed information, we have to assume the "worst case" and start with the initial value $f_{GW} = 3.108\ \mu$Hz. The absence of a prominent spectral line is not a problem for the MSH method because it is insensitive to signal amplitude and amplitude modulation [4].

3 MSH method and slow PM measurements

The GW source of the Kepler-47 star system is phase modulated at several relatively high frequencies (inner planets). Their sidebands are much further away from f_{GW} than the spectral resolution of the database of about 0.8 nHz. If these sidebands are suppressed, the MSH method should deliver a constant intermediate frequency. Measurements with different bandwidths show a ripple that is obviously generated by extremely low-frequency phase modulations and that cannot be suppressed by changing any parameters.

Example: If Kepler-47 has a planet with an orbital period of 50 years, its orbit will generate a phase modulation with $f_{planet} \approx 0.63$ nHz. This value roughly corresponds to the frequency resolution Δf of the base data (equation (1)) and cannot be separated from f_{GW} by filters. The MSH method is used to determine extremely low modulation frequencies (Figur 2):

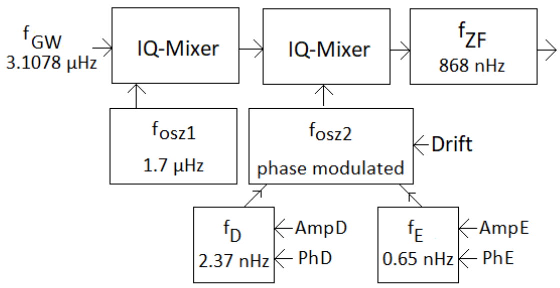

Figure 2): *The MSH method: f_{osz1} reduces the signal frequency without affecting the modulation. f_{osz2} mimics all properties of the signal so f_{ZF} is constant. f_D and f_E are the modulation frequencies. The period of oscillation of f_{ZF} is determined at the output at the top right.*

An oscillator with the starting value $f_E \approx 0.65$ nHz phase-modulates oscillator 2 of the heterodyne method (f_{osz2}). Its parameters f_E, a_E and ϕ_E are iterated with the aim of making all slow frequency fluctuations (also the frequency drift) of f_{GW} disappear. This is checked by counting the period of f_{ZF}. The accuracy increases if you choose the value $f_{sampling}/8$ or $f_{sampling}/10$ for f_{ZF}. If the measurements show that f_{GW} is modulated with additional low frequencies, add appropriate oscillators. The frequency of Oscillator-2 is calculated using the equation

$$f_{osz2} = f_{GW} - f_{osz1} + f_{ZF} + t \cdot \dot{f}_{GW} + \sum_{i=B}^{Z} a_i \cdot sin(2\pi t f_i + \phi_i) \tag{2}$$

The parameters have the following meaning:
f_{GW} is the frequency of the suspected GW.
f_{ZF} is basically arbitrary. A special value is chosen here, where the oscillation period is an integer in relation to the sampling period.
$\dot{f}_{GW}$ is the frequency drift of f_{GW} (initial value=0 Hz/s)
B, C,...Z denote the possible planets of *Kepler-47*.
a_i is the modulation index of the PM assuming that the i–th planet describes a circular orbit around the GW source.
f_i is the orbital frequency of the assumed i–th planet.
From the phase ϕ_i it follows when the i–th planet is in front of or behind the GW source from Earth's point of view.

The aim is to determine the values of the parameters in equation (2) in such a way that f_{ZF} is constant. Then the intermediate frequency is unmodulated. In section 8 it is examined whether planets with very long orbital periods produce these PM.

4 Measuring the fast PM

Planet B orbits the GW source every 49 days. Therefore f_{GW} is phase modulated with $f_B \approx 236$ nHz. Assuming $a_B \approx 2.5$ (the exact value is still unknown), the data must be processed with the Carson bandwidth $2f_B \cdot (a_B + 1) \approx 1660$ nHz in order not to distort the phase modulation, which would lead to erroneous results. This broad range is filled with seven spectral lines (split from f_{GW}), lots of noise and lines that other, still unknown star systems produce.

It is hopeless to identify the different spectral lines of f_{GW} in this mixture and to prove that they belong together. The contrast to the methodology in the previous section could not be greater, where we use a much lower bandwidth of 0.8 nHz.

The MSH method finds the spectral lines in the broad total spectrum and regenerates a single and *stronger* line at f_{ZF} by in-phase addition. All other lines and noise are ignored even without narrow-band filters.

The MSH method indirectly proves the phase modulation of a GW: The energy of a phase-modulated GW is distributed over several spectral lines (sidebands), each with a low amplitude. If the PM is compensated, the sidebands disappear and the energy is concentrated in the single line (f_{GW}). The measurable increase in amplitude is considered evidence of PM because noise does not interfere constructively, at least not over the entire

20-year period. The procedure is described in [10] and is similar to the scheme in figure 2 with the changes: At the output (top right in the picture) one does not measure the period of oscillation, but the amplitude of f_{ZF}. And the modulation frequencies at the bottom of the screen change and are now f_B and f_{Earth}.

5 The tortuous path to the result

This measurement was full of surprises. The original goal was to prove the GW of the double star *Aa–Ab* in Kepler-47. The sidebands caused by the three known planets B, C and D are far away from f_{GW} because of the short orbital times and may be easily removed using narrow-band filters. Then, in the first step, f_{GW} and the frequency drift $\dot{f}_{GW}$ should be measurable without interference.

First problem: Despite the high filter effort, an unacceptable residual ripple of f_{GW} is annoying. In the period covered by the database (20 years), the frequency fluctuates in a peculiar way that has never been observed in previous measurements. A phase modulation with a frequency mixture was assumed to be the cause. Time-consuming measurements and experiments with the MSH method solved the problem: Kepler-47's f_{GW} changes in the rhythm of *three* very low frequencies (current designations: f_G, f_H and f_J). These are so low that the associated sidebands cannot be separated from f_{GW}. The elimination of this PM greatly reduces the frequency fluctuations.

2. Problem: The filter width influences the measured frequencies because the amplitude of the GW hardly exceeds the noise level. After compensating for the high-frequency PM caused by the known planets B, C and D, the amplitude of f_{GW} increases noticeably, the S/N improves. Side effect: The sidebands (suppressed by filters in the previous step) carry information about frequency and drift. The MSH method transfers this information to the carrier frequency f_{GW}. Therefore, all frequencies change as all PM are progressively compensated. This forces iteration in tiny steps.

3. Problem: The now known six orbital periods of the planets are strikingly different: $\max(P_B, P_C, P_D) < 1$ year and $\min(P_G, P_H, P_J) > 19$ years. A logarithmic representation suggests a gap and a regularity: After inserting two spaces f_E and f_F, all measuring points lie on a straight line. Details follow in section 7, as does the method for setting the initial values for f_E and f_F.

6 Results

The GW source Kepler-47 (the binary system *Aa–Ab*) has eight planets.

- A planet B with the orbital period $P_B = 49.504$ days ($f_B = 233.8001$ nHz). This value is almost exactly as large as in [2]. The parameter $a_B = 3.7350$ is discussed in section 8. The phase angle $\phi_B = 3.8012$ tells us that $49.5 \cdot \phi_B/2\pi = 30$ days after January 1st, 2000, the blueshift caused by Planet B was at its maximum.
- A planet C with $P_C = 196.80$ days ($f_C = 58.810$ nHz). P_C is 5% larger than in [2]. $a_C = 5.3665$ and $\phi_C = 5.094$.

- A planet D with $P_D = 294.25$ days ($f_D = 39.335$ nHz). P_D is 3% smaller than in [2]. $a_D = 1.0644$ and $\phi_D = 5.110$. The $P_D/P_C = 3/2$ resonance is striking.
- A planet E with $P_E = 2.2676$ years ($f_E = 13.974$ nHz). $a_E = 0.7261$ and $\phi_E = 2.2038$.
- A planet F with $P_F = 7.2653$ years ($f_F = 4.3615$ nHz). $a_F = 1.9675$ and $\phi_F = -0.2397$.
- A planet G with $P_G = 19.118$ years ($f_G = 1.6575$ nHz). $a_G = 2.2960$ and $\phi_G = 0.8511$.
- A planet H with $P_H = 50.477$ years ($f_H = 0.62776$ nHz). $a_H = 2.8326$ and $\phi_H = -0.5018$.
- A planet J with $P_J = 126.706$ years ($f_J = 0.25009$ nHz). $a_J = 8.8546$ and $\phi_J = 5.7007$.
- After compensation of all PM with the frequencies $f_B ... f_J$ mentioned above, the residual ripple of f_{ZF} is so low that the existence of further planets with $P < 10^4$ years can be excluded. The short database of only 20 years does not allow the determination of even longer time constants.
- As expected, f_{GW} is also phase modulated with $f_{orbit} = 31.68754$ nHz. $a_{orbit} = 3.3287$. From the phase angle $\phi_{orbit} = 0.7571$ it follows that the earth on each $365 \cdot \phi_{orbit}/2\pi = 44$th day of the year f_{GW} with maximum blueshift receives. According to [7], this should not take place until April 10 (measurement error $\approx 16\%$).
- On January 1, 2000, the frequency of the GW source was 3.10784 μHz. The drift of the GW is $\dot{f}_{GW} = 4.3469 \times 10^{-18}$ Hz/s and has not been measured with electromagnetic waves so far.

In retrospect, it is confirmed that it is important to eliminate *all* PM: With PM, the Bessel function $J_0(a)$ is a measure for the amplitude of the carrier frequency f_{GW}. An unmodulated GW has an amplitude of 100%. With special values of the modulation index such as $a_1 = 2.4$ or $a_2 = 5.52$ the amplitude of the carrier frequency drops to extremely low values. Some planets cause a PM with such an unfavorable modulation index that the spectral line at f_{GW} is hardly recognizable in the original data (see figure 1). Compensating the PM with the MSH method increases the amplitude of the GW back to 100% and significantly improves the S/N. The area around f_{ZF} is filled with the distorted spectra of previously undiscovered GWs of similar frequency (figure 3).

7 Dermott's Law

For a long time people have been looking for reasons for obvious relations between the orbital periods P_n of planets. Dermott [9] proposes the approach

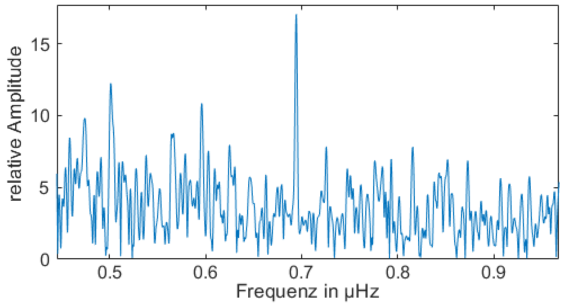

Figure 3): *Spectrum of the GW after frequency change to $f_{ZF} = 1/(400$ hours) and compensation of all phase modulations. The amplitudes of the numerous sidebands were reduced, the amplitude of the carrier frequency was increased by constructive interference.*

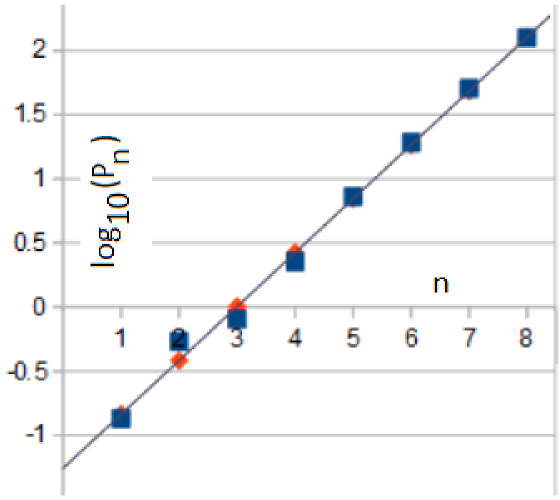

Figure 4): *The logarithm of the orbital period of the planets (in hours) of Kepler-47 as a function of their order. The actual values (blue) hardly differ from Dermott's predictions (red). The reason for the deviations at $n = 2$ and $n = 3$ could be that the strong resonance $P_C : P_D = 2 : 3$ forced an orbit change.*

$$P_n = P_0 \cdot c^n \tag{3}$$

with $n = 1, 2, 3, 4...$ If we add blanks for $n = 4$ and $n = 5$ for two hypothetical planets E and F (which were still unknown at this stage of the investigation), we get figure 4 with $P_0 = 20.28$ days and $c = 2.63$. Taking initial values P_E and P_F from this diagram and inserting them into the iteration loop, we started a new search for planets. In fact, two additional planets E and F of the star system Kepler-47 can be found almost exactly at $n = 4$ and $n = 5$.

For the relation of Dermott and the older Titius-Bode series $r_n = 0.4 + 0.3 \cdot 2^n$ there is no deeper justification. After a few conversions and with an additional correction factor, we get exactly the same results for the TB series as with Dermott's rule, but a total of four factors have to be adjusted empirically. That makes TB less believable. Here we do not philosophize about the two equations, but confirm that these equations obviously provide good initial values when searching for unknown planets.

8 Notes from an astronomical point of view

A word of caution: The MSH method measures phase modulations of f_{GW}. We assume that the PM are generated in the GW source itself or by surrounding planets.

If one translates the abstract results of the iteration of equation (2) determined above into astronomical terms, the following relationships apply: All times refer to the beginning of the analyzed data chains on 2000-01-01 and apply under the condition that that the corresponding celestial bodies describe circular orbits. The phase shift ϕ indicates at what later point in time the measurable instantaneous frequency is blue-shifted at most. Then one has to add the frequency shift Δf produced by the Doppler effect to the average frequency f_{GW}. The results of the compilation given above can be evaluated independently of one another because all PM are linearly superimposed.

All of the above results are reproducible. The following information about the mass of the planets is *preliminary* and is based on a (still) uncertain assumption about the propagation speed of GW.

$m_B \approx 24 \cdot m_{Jupiter}$, $m_C \approx 16 \cdot m_{Jupiter}$, $m_D \approx 2.6 \cdot m_{Jupiter}$, $m_E \approx 0.9 \cdot m_{Jupiter}$, $m_F \approx 1.1 \cdot m_{Jupiter}$, $m_G \approx 0.7 \cdot m_{Jupiter}$, $m_H \approx 0.44 \cdot m_{Jupiter}$, $m_J \approx 0.75 \cdot m_{Jupiter}$.

9 Summary

From a communications point of view, decoding the phase modulations of f_{GW} is a standard task of digital signal processing. The signal has a good S/N, the receiving antenna is insensitive to earth movements. No assumptions are needed at any stage of decoding. There is no computationally intensive comparison with pre-calculated patterns (search templates) based on model assumptions. The existence of the eight planets is assured.

The opposite is true for the interpretation of the PM from an astronomical point of view: The high values for the frequency deviation (Δf) can only be explained by the assumption that gravitational waves at low frequencies around 3 μHz are considerably slower than the speed of light. The processing of this question is not yet complete. Nevertheless, these results may help to clarify open questions about the stability of planetary systems [12].

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