The planets of the binary star Kepler-16

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The two stars in *Kepler-16* emit gravitational waves with a frequency of 563.5 nHz, which can be measured on Earth. The decoding of the phase modulations of the GW shows *eleven* companions. The orbital times fit very well with the predictions of Dermott's law, an improved version of the Titus-Bode rule. The physical interpretation of the result of the phase modulation is difficult, but allows the masses of the planets to be estimated.

1 Introduction

It has been known since 2011 that the planet *Kepler-16b* orbits a binary star system. The trio was discovered due to short-term fluctuations in the brightness of the stars [1]. Two stars $A1$ and $A2$ orbit each other every 41.07922 days and emit a gravitational wave (GW) of frequency 563.5 nHz. The measurement procedure is described in [2, 7] and is not repeated here. The long-term analysis of the GW over a period of twenty years shows several results:

- Eleven planets orbit the binary system. They can be distinguished because each causes a periodic Doppler shift of the GW with a different frequency.
- The modulation index a of all phase modulations is remarkably large.
- The frequency drift of the binary star system is measured precisely.

Electromagnetic wave observations [1] allow the calculation of f_{GW} in 2011, but not the frequency drift. The analysis of historical data from the year 2000 requires a reliable initial value for f_{GW}. At the expected frequency, the spectrum shows a clear maximum (figure 1). The aim of this work is to reduce the energy content of the sidebands and to increase the amplitude of the central spectral line.

2 The order of measurements

As we know the expected value $f_{GW} = 2f_{orbit}$, the signal is filtered with a bandwidth of 0.3 nHz in order to minimize interference from neighboring GWs. This dispenses with all energy components that are transported by sidebands (a modulation produces sidebands). Initially, we have to accept an inaccurate result due to the low amplitude and poor S/N. The aim of this first step is to assess the frequency stability of f_{GW} over a period of twenty years and to eliminate the *slow* modulations that may cause it.

- A curvature means a phase modulation (PM) with $f_{mod} > 10^{-9}$ Hz. From the curvature, we determine an approximate value for f_{mod}.

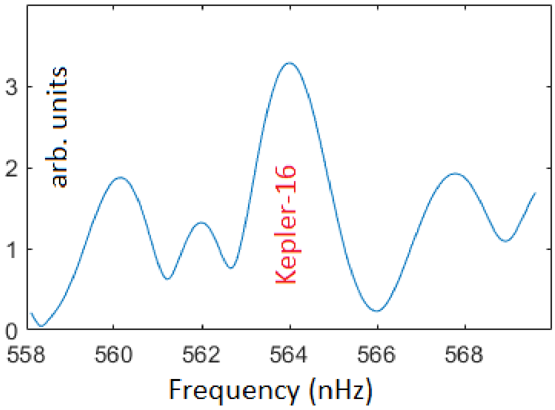

Figure 1): *Spectrum of the GW from Kepler-16. The central spectral line is easily recognizable because none of the many planets causes a PM with a critical modulation index. The environment contains the sidebands of all PM and the GW of other binaries. Compare figure 3.*

- A linear progression may be generated by a constant frequency drift or by a very low-frequency PM ($f_{mod} < 10^{-9}$ Hz) with a suitable phase. Then the solution requires a time-consuming iteration.

Before all other measurements, these possible modulations must be roughly eliminated. To achieve this, one iterates the modulation index and phase of the suspected low-frequency PM until the intervals between two zero-crossings of the sine wave match.

Next, one compensates the inevitable PM with $f_{orbit} = 31.8$ nHz generated by the Earth's orbit. The signal amplitude and the accuracy of the frequency measurement increase because more energy is concentrated on f_{GW}. In addition, we are informed about the direction from where the GW arrives. As with all higher-frequency PM, the modulation index and phase are iterated until the amplitude of f_{ZF} reaches a maximum.

These two steps are repeated several times in order to increase the signal amplitude as much as possible.

After this preliminary work, the detective work begins: Planets force the GW source to orbit the common center of gravity and each planet modulates f_{GW} at a different frequency (see discussion in section 5). Since the corresponding sidebands have a very low amplitude and lie outside the narrow bandwidth of the signal processing (BW$<$ 0.4 nHz), the modulation frequencies have to be guessed at. It's pointless to look for the individual sideband frequencies in the surrounding noise since there are no clues for frequency and amplitude. And if anyone spots suspicious lines, it would be even more difficult to prove that the frequencies found are part of the GW in terms of amplitude and phase. The MSH method [5] [7] does these tasks in a completely different way.

It is helpful to know the orbital data of *one* planet, because then Dermott's empirical law [4] provides clues for the orbital periods of other planets of the binary system. Although these estimates are quite rough, the *capture range* of the iteration is sufficient to determine the exact value. Vertical dips in figure 2 may occur when the additional planet corrects the frequency and/or phase of already known planetary orbits. The noise for n$>$500 arises because the calculations are performed with 64-bit floating-point numbers ($\approx$ 16 significant decimal digits precision). A more precise calculation of the linearity error is neither useful nor possible.

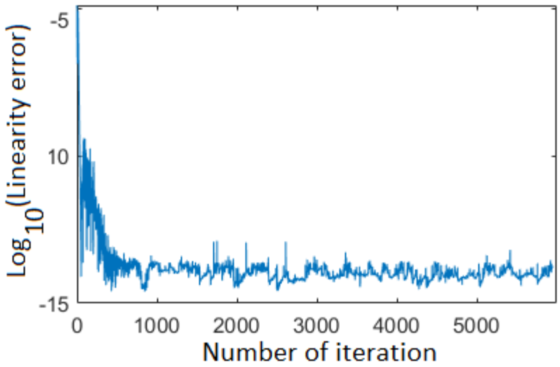

Figure 2): *Convergence behavior of the iteration of the MSH method. The vertical axis shows how the total error in calculating the orbital data of Kepler-16's seventh planet decreases after a few iterations.*

Previous studies have shown that binary stars often have many planets. Assuming that there are ten planets and that each causes a PM with the modulation index $a = 3$, the total energy of the GW is distributed over a total of about $10 \cdot 2 \cdot 3 + 1 \approx 60$ spectral lines in the vicinity of f_{GW}, which disappear in the noise because of their rather small amplitudes. Since the signal power is spread over a large bandwidth, the signal PSD is much lower – often significantly lower than the noise PSD – so that it may not be possible to determine whether the signal is present at all. Therefore, at an early stage of the analysis, it is pointless to look for conspicuous lines in the spectrum. This changes in the course of the analysis because the amplitude of f_{GW} increases with each detected planet. The energy of many sidebands is then transferred to the central spectral line.

3 Results

Assuming that all phase modulations are generated by planets, the binary system $A1-A2$ of the GW source *Kepler-16* has eleven planets.

- Planet B with the orbital period $P_B = 146.04$ days ($f_B = 79.252$ nHz). The parameters $a_B = 2.018$ and $\phi_B = 2.049$ are discussed from section 5 onwards.
- Planet C with $P_C = 227.9$ days ($f_C = 50.789$ nHz). $a_C = 3.297$ und $\phi_C = 3.815$. This is the only *Kepler-16* planet yet discovered using electromagnetic waves [1].
- Planet D with $P_D = 381.1$ days ($f_D = 30.3676$ nHz). $a_D = 4.866$ und $\phi_D = 3.757$.
- Planet E with $P_E = 602.7$ days ($f_E = 19.2044$ nHz). $a_E = 1.811$ und $\phi_E = 0.274$.
- Planet F with $P_F = 2.528$ years ($f_F = 12.536$ nHz). $a_F = 1.7729$ und $\phi_F = 1.132$.
- Planet G with $P_G = 3.765$ years ($f_G = 8.4174$ nHz). $a_G = 1.52$ und $\phi_G = 5.907$.
- Planet H with $P_H = 8.483$ years ($f_H = 3.7355$ nHz). $a_H = 1.826$ und $\phi_H = 5.065$.
- Planet J with $P_J = 11.428$ years ($f_J = 2.7727$ nHz). $a_J = 1.25$ und $\phi_J = 5.537$.
- Planet K with $P_K = 35.85$ years ($f_K = 884$ pHz). $a_K = 0.8597$ und $\phi_K = 4.372$.

- Planet L with $P_L = 53.70$ years ($f_L = 590$ pHz). $a_L = 0.158$ und $\phi_L = 3.1496$.
- Planet M with $P_M = 140.8$ years ($f_M = 225$ pHz). $a_M = 0.034$ und $\phi_M = 2.68$.

All results are reproducible and have been calculated several times with different values of important parameters such as bandwidth.

After compensation of all PM with the frequencies $f_B ... f_M$ mentioned above, the residual ripple of f_{ZF} is so low that the existence of further planets with $P < 500$ years is improbable. The short database of only 20 years does not allow to determine even longer time constants.

As expected, f_{GW} is also phase modulated with $f_{orbit} = 31.68754$ nHz. $a_{orbit} = 2.056$. From the phase angle $\phi_{orbit} = 3.179$ it follows that here on Earth, we receive maximum blueshift on every $365 \cdot \phi_{orbit}/2\pi$ =185th day of the year f_{GW}. According to [3], this should already take place on the 123th day of the year (error $\approx 17\%$).

On January 1, 2000, the frequency of the GW source was 563.5 nHz. The drift is $\dot{f}_{GW} = 83.3 \times 10^{-20}$ Hz/s, it has never been measured with electromagnetic waves.

In retrospect, it is confirmed that it is important to eliminate *all* PM: With PM, the Bessel function $J_0(a)$ is a measure for the amplitude of the carrier frequency f_{GW}. An unmodulated GW has an amplitude of 100%. With special values of the modulation index like $a_1 = 2.4$ or $a_2 = 5.52$ the amplitude of the carrier frequency decreases to extremely low values. In the case of *Kepler-16*, there is no planet that causes a PM with such an unfavorable modulation index. Therefore the spectral line at f_{GW} is well recognizable in the original data (figure 1). Compensating the PM with the MSH method allows the amplitude of the GW to rise back to 100%, improving the S/N significantly (figure 3). The vicinity of f_{ZF} is filled with the distorted spread spectra of previously undiscovered GWs of similar frequency.

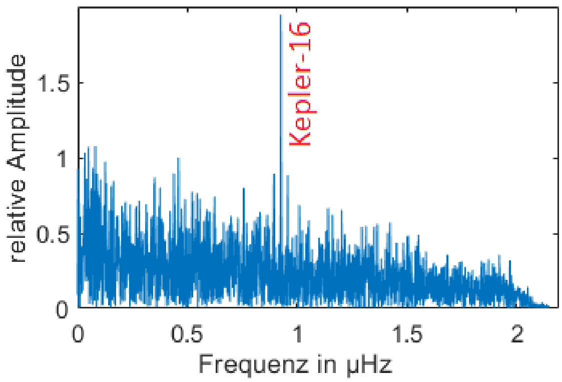

Figure 3): *Spectrum of the GW of Kepler-16 after changing the frequency to $f_{ZF} = 1/(300$ hours) and compensating the phase modulations of all eleven planets. The amplitude of the carrier frequency of the GW is very large because it now contains the energy content of all compensated sidebands (constructive interference).*

4 Dermott's Law

For a long time people have been looking for reasons for obvious connections between the orbital periods P of planets. The ansatz (1) comes from Dermott [4]

$$P_n = P_0 \cdot c^n \tag{1}$$

with $n = 1, 2, 3, 4...$ Figure 4 shows the best approximation with $P_0 = 100.42$ days and $c = 1.566 \pm 0.001$. For the relation of Dermott and the older Titius-Bode series there is no deeper justification. Dermott's law reliably provides good initial values when searching for unknown planets. At $n = 9, 10$ and 13, despite an intensive search, no PM were found that indicate planets with an appropriate orbital period.

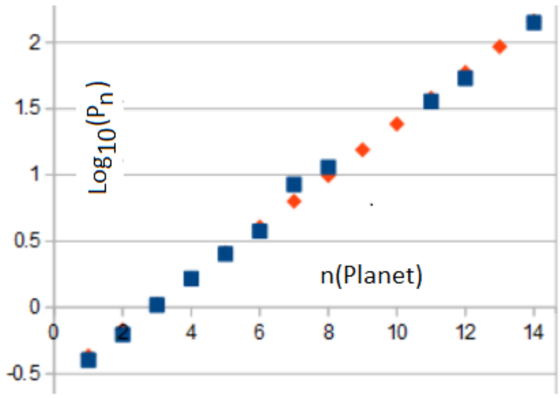

Figure 4): *The logarithm of the orbital period of the planets (in years) of Kepler-16 as a function of their order. The actual values (blue) hardly differ from Dermott's law (red). The discrepancy at $n = 7$ remains a mystery. A possible cause is the orbital resonance $P_H : P_J \approx 3 : 4$.*

5 Notes from an astronomical point of view

A word of caution: The MSH method measures phase modulations of f_{GW}. Now we assume that the Doppler effect caused by planets is the only reason for the PM of the GW source.

Translating the abstract results of the iteration (section 3) into astronomical terms, the following relationships apply: All time specifications refer to the beginning of the analyzed data chains on 2000-01-01 and apply under the condition that the corresponding celestial bodies describe circular orbits. The phase shift ϕ indicates at what later point in time the instantaneous frequency of the GW is blue-shifted to the maximum. Then one has to add the frequency deviation Δf produced by the Doppler effect to the average frequency f_{GW}. The results of the compilation given above may be evaluated independently of one another because all PM are linearly superimposed.

5.1 Earth orbit causes a PM

The Earth's orbit around the sun generates a phase modulation with the frequency $f_Y = 31.178328$ nHz. The largest blue shift of f_{GW} is measured at $365 \cdot \phi_Y / 2\pi = 185$th day of the year. The highest red shift is measured on the first day of the year; if the GW comes from the source *Kepler-16*, the redshift should reach its maximum value on day 123 [3]. One reason for the measurement error of 17% may be that the Earth's orbital velocity is only ±9000 m/s as seen by *Kepler-16*.

From the modulation index of a PM $a_Y = \Delta f_Y / f_Y$ follows $\Delta f_Y = 65.15$ nHz – an unexpectedly large frequency deviation that cannot be explained with previous assumptions. This value means that the instantaneous frequency of the GW oscillates between $f_{GW} - \Delta f_Y$ and $f_{GW} + \Delta f_Y$ over the course of the year.

It is commonly assumed, without proof, that any GW travels at the speed of light. Therefore, we expect for the maximum value of the Doppler shift:

$$\Delta f_Y = f_{GW} \cdot \left(\sqrt{\frac{c + v_{orbit}}{c - v_{orbit}}} - 1 \right) \approx f_{GW} \cdot \frac{v_{orbit}}{c} = f_{GW} \cdot 10^{-4} \tag{2}$$

The actually measured value Δf_Y is about 3850 times larger! A measurement error of this magnitude can be ruled out after careful examination. What is causing the discrepancy? The equations of the PM and the Doppler effect are well founded and confirmed a million times. What remains is the correction of the assumption, that GWs propagate at the speed of light. The calculation of the instantaneous frequency uses the longitudinal Doppler effect, in which the frequency is corrected relativistically. For maximum blueshift applies

$$f_{GW} + \Delta f_Y = f_{GW} \sqrt{1 - \left(\frac{v_{orbit}}{c} \right)^2} \cdot \frac{1}{1 - \frac{v_{orbit}}{v_{GW}}} \approx \frac{f_{GW}}{1 - \frac{v_{orbit}}{v_{GW}}} \tag{3}$$

The position of *Kepler-16* is far north of the ecliptic plane ($\delta = +51°$ 45' 27"). Therefore, the Earth is approaching this target with the maximum speed $v_{orbit} = 9000$ m/s. If we transform the equation (3), we get

$$\frac{v_{orbit}}{v_{GW}} = 1 - \frac{f_{GW}}{f_{GW} + \Delta f_Y} = 0.1036. \tag{4}$$

With this intermediate result, we calculate

$$v_{GW} = \frac{v_{orbit}}{0.1036} = 87 \times 10^3 \ \frac{m}{s} \approx \frac{1}{3500} c. \tag{5}$$

This result is much lower than the speed of light and is valid for $f_{GW} \approx 0.56$ μHz.

5.2 The Planet *C*

The planet C was detected with electromagnetic waves *and* with GW. Decoding the PM of the GW shows that C orbits the GW source $A1 - A2$ with the period $P_C = 227.89$ days. Assuming a circular orbit, it follows from the phase angle $\phi_B = 3.815$ that this companion produces maximum blueshift of f_{GW} on $227.89 \cdot \phi_C/2\pi$ =138th day after 2000-01-01. $P_C/2 = 114$ days later it caused maximum redshift. So planet C must have partially covered the GW source on the 309th day after 2000-01-01 (transit of the planet across both stars).

From the modulation index $a_C = \Delta f_C/f_C = 3.297$ of the PM, we calculate the maximum frequency deviation $\Delta f_C = 167.5$ nHz. This maximum value of the periodic frequency shift of f_{GW} should be the result of the Doppler effect, because the GW source rotates around the center of gravity of the system. It is difficult to explain this result with previous assumptions: The ratio $f_{GW}/\Delta f_C \approx 3.4$ contradicts the maximum value that results from previous assumptions of the theory of relativity (equation 2).

The calculation of the radial velocity is the task of classical astronomy. Considering the GW source $A1 - A2$ as a star and the planet C as a companion, Kepler's third law provides the orbital equation for the two-body system.

$$4\pi^2(r_A + r_C)^3 = GT^2(m_{A1} + m_{A2} + m_C) \tag{6}$$

The radii refer to the center of gravity of the trio and the center of gravity theorem $(m_{A1} + m_{A2})r_A = m_C r_C$ applies (we ignore other planets). With the assumed masses [1] $m_{A1} = 0.6897m_\odot$, $m_{A2} = 0.20255m_\odot$ and $m_C \approx 0.333m_{Jupiter}$ we calculate $r_A = 3.73 \times 10^7$ m and the orbital velocity $v_A = 11.91$ m/s (the orbital speed of C is about 2800 times higher and not measurable because the planet C does not emit any GW).

Analogous to the equations (4) and (5) we get

$$v_{GW} = \frac{11.91\ m\ s^{-1}}{0.2291} = 52\ \frac{m}{s} \approx \frac{1}{5.8 \times 10^6}c. \tag{7}$$

This result is ridiculous low and raises questions: Where is the source of the GW in a binary system? At the barycenter of the two stars $A1$ and $A2$ or closer to one of the massive stars? Does the Doppler effect decribe a GW in the vicinity of the source? In the immediate vicinity of the GW source, does one have to differentiate between the near field and the far field (as with electromagnetic waves)?

5.3 The Planets *B* and *D...M*

The result that the Doppler effect gives for the planet C does not correspond to the usual expectations for the propagation speed of GW. However, one can use the value $v_{GW} = 52$ m/s to classify the masses of the other planets of *Kepler-16*. Reference is $m_C = 0.333 \cdot m_{Jupiter}$ [1].

Substituting the results for P_n and a_n from Section 3 into equations 6 and 7 gives the following results for m_n:

Planet	B	D	E	F	G	H	J	K	L	M
$m_{Jupiter}$	0.277	0.359	0.117	0.088	0.059	0.042	0.023			
m_{Earth}	88.1	114	37.2	28	18.7	13.2	7.45	2.4	0.34	0.04

Table 1): *Estimated masses of the eleven Kepler-16 planets.*

6 Summary

From a communications point of view, decoding the phase modulations of f_{GW} is a standard task of digital signal processing. The signal has a good S/N (figure 3), the receiving antenna is insensitive to earthquakes. No assumptions are needed at any stage of decoding. There is no computationally intensive comparison with pre-calculated patterns (search templates) based on model assumptions.

The opposite is true for the interpretation of the results from an astronomical point of view: The high values for the frequency deviation (Δf) can only be explained by the assumption that gravitational waves at low frequencies around 10 μHz propagate significantly more slowly than the speed of light. The processing of this question is not yet complete.

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