Special Relativity: Time Dilation Revisited

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Motion being a relative quantity, the question often asked in Special Relativity (SR) is: which observer is moving, and which one is at rest? How is this to be determined? There are many who have tried to answer this conundrum. Most answers fall into two categories: 1) the use of a Minkowski diagram (space-time coordinates) – but motion is still relative even on a space-time coordinate system; 2) by specifying which observer is accelerating – acceleration like velocity is still a relative quantity. None of these are satisfactory. In this paper we offer a solution. But to answer that question properly we must go back to the derivation of the time dilation equation and focus on what is really at stake. Out of this investigation we get a new look at time dilation as the equivalent of a Doppler effect.

Part 1: One Clock Versus Two Clocks

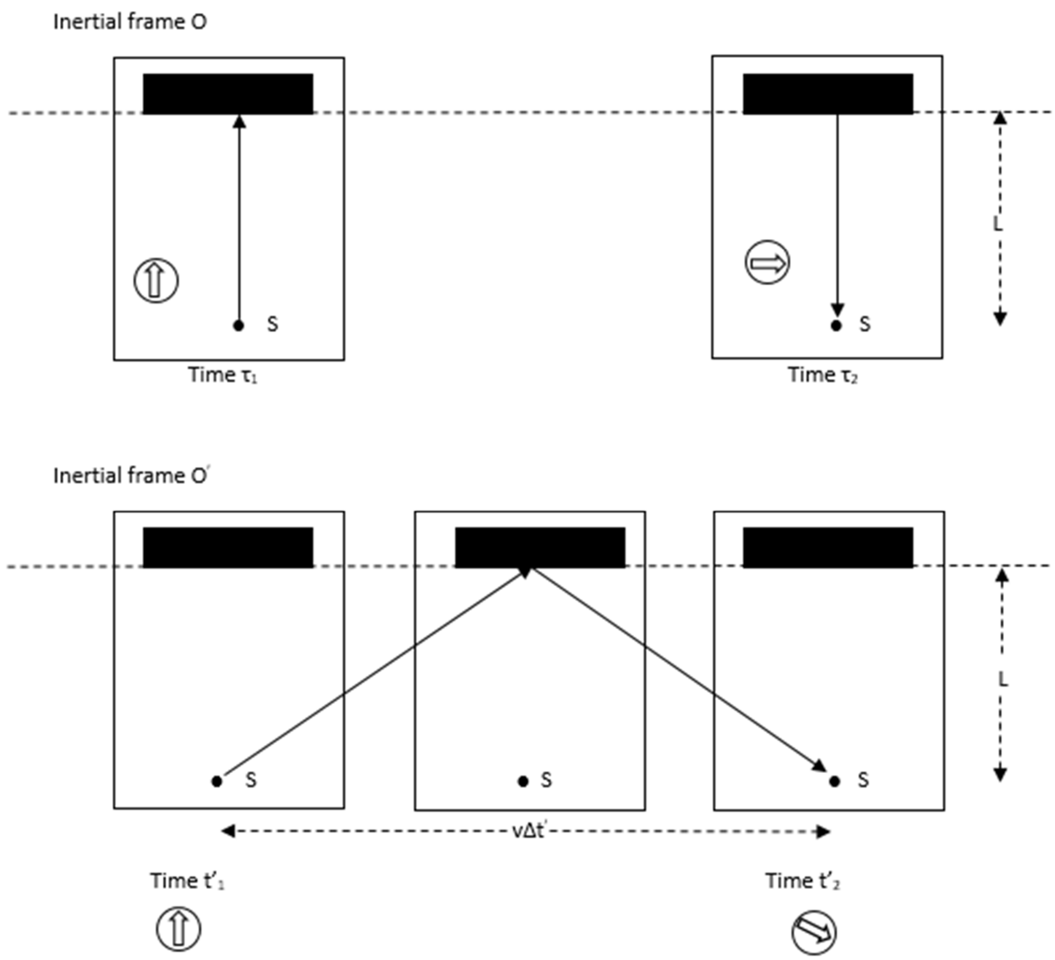

Figure A

We consider two inertial frames O and O'. In frame O (Figure A), we have one observer in a box who is considered to be at rest with respect to that box. The box is also equipped with a clock, a source S which upon emitting a beam of light at time τ_1 is then reflected by a mirror at a distance L. The beam reaches back to the source at a time τ_2. The total distance traveled by the beam of light is $2L$ in a time interval $\Delta\tau = \tau_2 - \tau_1$.That is, we have $c\Delta\tau = 2L$ or $L = c\Delta\tau/2$.

Note: in this frame, the first observer is at rest with the clock. But what is important is that this observer uses <u>one</u> clock to measure both events: the first event occurs when the light is emitted at τ_1, and the second when it returns at τ_2.

In the second inertial frame O', a second observer is outside the box, and sees the first inertial frame O moving to the right at a speed v. He observes the same two events – the emission of the beam of light at t'_1 and its return to the source at t'_2. He knows that because he has installed a clock at t'_1, and a second clock at t'_2.

The total distance travelled by the light in O' is $2L' = c\Delta t'$, where $\Delta t' = t'_2 - t'_1$.

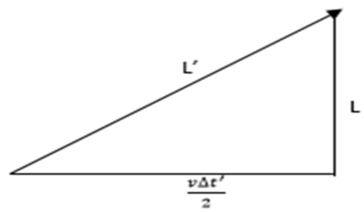

Figure B

However, the total distance is also $= 2(L^2 + \left(\frac{v\Delta t'}{2}\right)^2)^{½}$ (figure B).

Therefore in O', $c\Delta t' = 2\left(L^2 + \left(\frac{v\Delta t'}{2}\right)^2\right)^{½}$

$$= 2\left(\left(\frac{c\Delta\tau}{2}\right)^2 + \left(\frac{v\Delta t'}{2}\right)^2\right)^{½}$$

Square both sides, and the two terms inside the big bracket,

$$(c\Delta t')^2 = 4\left(\frac{c^2\Delta\tau^2}{4} + \frac{v^2\Delta t'^2}{4}\right) = c^2\Delta\tau^2 + v^2\Delta t'^2$$

Solving for $\Delta\tau$, we first rearrange:

$$c^2\Delta\tau^2 = c^2\Delta t'^2 - v^2\Delta t'^2$$

Or $$\Delta\tau^2 = \Delta t'^2 - \frac{v^2}{c^2}\Delta t'^2 = \left(1 - \frac{v^2}{c^2}\right)\Delta t'^2$$

Taking the square root: $\Delta\tau = \left(1 - \frac{v^2}{c^2}\right)^{½}\Delta t'$

Standard definition of the factor $\gamma = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}}$, with $\gamma \geq 1$

Finally we have, $$\Delta\tau = \Delta t'/\gamma.$$

The clock in O is moving with respect to this second observer in O'. It measures the *proper* time. At $v < c$, γ is greater than 1, and so the first observer in O will experience *time dilation,* which has given rise to the saying, "Moving clocks slow down."

Note: in the frame O', the second observer must use two clocks to measure these two events that are to be observed: one clock stationed when the light is emitted at the source at time t_1', and a second clock stationed at a different position when it returns to the source at time t_2'.

The rule is simple:

The clock which measures both events is the clock that measures the proper time. The observer in that inertial frame will experience time dilation.

With this new rule we resolve the question, "who is moving?" with the question, "who is using one clock to measure the two events to be observed?"

In the famous Twin Paradox[1], consider Alice is in a rocket ship about to undergo a trip to the nearest star to our sun, say planet Proxima Centauri in the Alpha Centauri star system – about 4.22 light-years away. Her twin brother Bob stays back home on planet Earth. In this case, the clock in Alice's ship will register the proper time. How do we come to that conclusion? Alice will register two events with one clock: her departure from Earth and her arrival on Proxima Centauri. Her clock is at rest with respect to Alice but not with respect to the two events – it has to move from event one (the departure from Earth) to the second event (the arrival on Proxima). On the other hand, Bob will need two clocks to measure the time taken by his sister of these two events: one clock on planet Earth, and another one on Proxima Centauri – he needs his good friend Cortney already on Proxima Centauri to observe Alice's arrival and register her arrival time. So we can see that Bob's clock is NOT the moving clock as it stays back home, registering just one of those two events.

Part 2: One Observer Versus Two Observers

Instead of looking at the clocks in Figure A, consider that in frame O, we have one observer but in frame O', we have two observers, each one is placed where the two clocks are standing. We must be reminded that light is a wave phenomenon and its speed c is independent of the speed of the source. As far as the first observer in O is concerned, the source S is at rest and he does not see anything unusual in the behavior of the light as it travels from the source S up to the mirror then back to the source.

However, in frame O', for the observer at t_1', he sees the light and the source S moving away from him. Hence a Doppler effect says that this observer will see the light red shifted. While the second observer stationed at t_2', who sees the light and the source coming towards him, will see the light blue shifted.

How can we account for these wildly different observations between observers in the two frames O and O'?

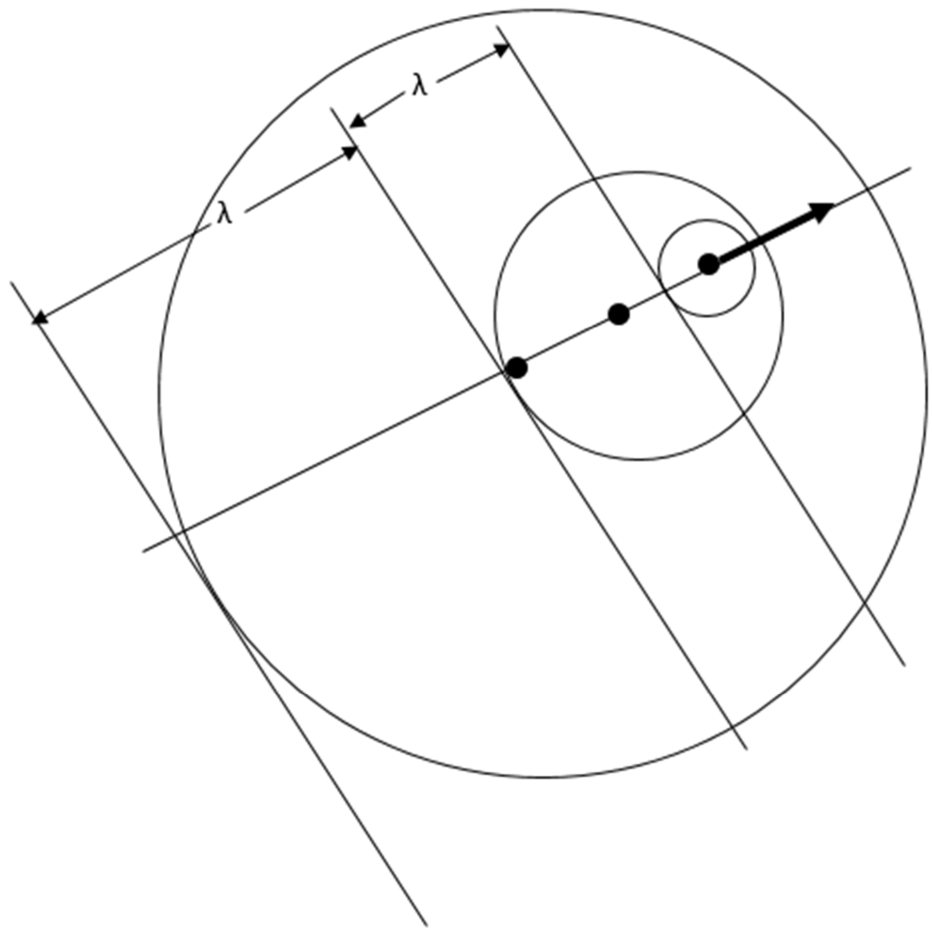

Figure C

In frame O’, the observer stationed at t_1' sees the light travelling away over a distance L', which is longer than L because the beam of light is moving not only up to the mirror but also to the right. In terms of the wave phenomenon, the wavelength λ is being stretched (Figure C). Hence it is red shifted.

One can say that observers in O will experience the slowing down of their clock (time dilation), or that observers in O’ are measuring different wavelengths (red shift at t_1' and blue shift at t_2', also known as a Doppler effect). Both are equivalent. Note: there is no dispute in the speed of light being constant in both frames. It’s a wave - its speed is independent of the speed of its source, as it should be.

[1] Many of the arguments used in this paper can be found in <u>The Real Nature of Time</u>, Joseph Palazzo, https://vixra.org/abs/2004.0278, 2020-04-12, and in <u>Everything is Matter Moving Through Space</u>, Joseph Palazzo, Authorhouse, 2018.