The planets of the binary star R Canis Majoris (R CMa, HD 57167)

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The two stars in *R Canis Majoris* emit gravitational waves with a frequency of 20.33 μHz, which may be measured here on Earth. The decoding of the phase modulations of the GW shows *nine* companions – one was suspected using electromagnetic waves. The orbital times fit well with the predictions of Dermott's law. The physical interpretation of the result of the phase modulation is difficult, but allows the masses of the planets to be estimated.

1 Introduction

Since 2014 it is assumed that one planet orbits the binary star system *R Canis Majoris* [1,2]. Two stars $A1$ and $A2$ orbit each other every 1.1359 days and emit a gravitational wave (GW) of frequency 20.37793 μHz. The measurement method is described in [3,8] and not repeated here. The long-term analysis of the GW over a period of twenty years shows several results:

- Nine planets orbit the binary system. They may be distinguished because each causes a characteristic periodic Doppler shift of the GW.

- The modulation index a of all phase modulations is remarkably large and may be explained with the assumption $v_{GW} \ll c$ (in the examined frequency range).

- The frequency drift of the binary star system is measured precisely.

Electromagnetic wave observations allow the calculation of f_{GW} in 2014, but not the frequency drift. The analysis of historical data requires a reliable initial value for f_{GW} in the year 2000. At the expected frequency, the spectrum shows a clear maximum.

GWs are always phase modulated (PM) and each PM produces sidebands. The number of individual frequencies and their amplitudes cannot be predicted because the number of planets and their masses are initially unknown. Since each planet causes a PM with its orbital frequency, one has to reckon with a typical solar system that the total energy of the GW is distributed over about 50 spectral lines, each with a low amplitude. This may be desirable in technical applications because the bundle of many weak spectral lines is often mistaken for noise and remains undetected. Whether there is a signal at all can only be determined once the PM has been eliminated. The aim of this work is to reduce the sidebands and to increase the amplitude of the central spectral line (figure 1).

2 The order of measurements

As we know that the *R CMa* binary emits a GW of frequency $f_{GW} = 2f_{orbit}$, the signal is filtered with a bandwidth of 0.3 nHz in order to minimize interference from neighboring

GWs. This dispenses with all energy components that are transported by sidebands (a modulation produces sidebands). Initially, we have to accept an inaccurate result due to the poor S/N and the neglected sidebands. The aim of this first step is to assess the frequency stability of f_{GW} over a period of twenty years and to eliminate the *slow* modulations that may cause it.

- A curvature means a phase modulation (PM) with $f_{mod} > 10^{-9}$ Hz. From the curvature, we determine an approximate value for f_{mod}.

- A linear progression may be generated by a constant frequency drift or by a very low-frequency PM ($f_{mod} < 10^{-9}$ Hz) with a suitable phase. Then the solution requires a time-consuming iteration.

These possible low-frequency modulations must be eliminated first. To achieve this, one iterates the modulation index and phase of the suspected low-frequency PM until the intervals between two zero-crossings of the sine wave match (MSH-procedure).

Next, one compensates the inevitable PM with $f_{orbit} = 31.8$ nHz generated by the Earth's orbit. The signal amplitude and the accuracy of the frequency measurement increase because more energy is concentrated on f_{GW}. In addition, we are informed about the direction from where the GW arrives. As with all higher-frequency PM, the modulation index and phase are iterated until the amplitude of f_{ZF} reaches a maximum. These two steps are repeated several times in order to increase the signal amplitude as much as possible.

After this preliminary work, the detective work begins: Planets force the GW source to orbit the common center of gravity and each planet modulates f_{GW} at a different frequency (see discussion in section 5). Since the corresponding sidebands have a very low amplitude and lie outside the narrow bandwidth of the signal processing (BW< 0.4 nHz), the modulation frequencies have to be guessed at. It's pointless to look for the individual sideband frequencies in the surrounding noise since there are no clues for frequency and amplitude. And if anyone spots suspicious lines, it would be even more difficult to prove that the frequencies found are parts of a single GW in terms of amplitude and phase. The MSH method [6,8] does these tasks in a completely different way.

It is helpful to know the orbital data of at least one planet, because then Dermott's empirical law [5] provides clues for the orbital periods of other planets of the binary system. Although these estimates are quite rough, the *capture range* of the iteration is sufficient to determine the exact value.

Previous studies have shown that all binary stars have planets [6–15]. If *R CMa* has nine planets and each one causes a PM with the modulation index $a \approx 2$, the total energy of the GW is distributed over a total of about $9 \cdot 2 \cdot 2 + 1 \approx 40$ spectral lines in the vicinity of f_{GW}, which disappear in the noise because of their rather small amplitudes. Since the signal power is spread over a large bandwidth, the signal PSD is low – often significantly lower than the noise PSD – so that it may not be possible to determine whether the signal is present at all. Therefore, at an early stage of the analysis, it is pointless to look for conspicuous lines in the spectrum. This changes in the course of the analysis because

the amplitude of f_{GW} increases with each detected planet. Finally, the energy of many sidebands is accumulated in the central spectral line.

3 Results

Assuming that all phase modulations are generated by planets, the binary system $A1 - A2$ of the GW source $R\ CMa$ has nine planets.

- Planet B with the orbital period $P_B = 185.51$ days, $f_B = 62.39$ nHz. The parameters $a_B = 0.228$ and $\phi_B = 3.75$ are discussed from section 5 onwards.

- Planet C with $P_C = 303.22$ days, $f_C = 38.17$ nHz. $a_C = 1.62$ and $\phi_C = 2.71$.

- Planet D with $P_D = 473.57$ days, $f_D = 24.4403$ nHz. $a_D = 1.46$ and $\phi_D = 2.19$.

- Planet E with $P_E = 3.368$ years, $f_E = 9.4085$ nHz. $a_E = 1.85$ and $\phi_E = 2.69$.

- Planet F with $P_F = 5.761$ years, $f_F = 5.50$ nHz. $a_F = 0.84$ and $\phi_F = 0.56$.

- Planet G with $P_G = 9.320$ years, $f_G = 3.40$ nHz. $a_G = 1.51$ and $\phi_G = 4.24$.

- Planet H with $P_H = 37.10$ years, $f_H = 854.2$ pHz. $a_H = 2.164$ and $\phi_H = 5.632$.

- Planet J with $P_J = 79.22$ years, $f_J = 400$ pHz. $a_J = 0.52$ and $\phi_J = 4.03$. $P_J : P_H \approx 2 : 1$ applies to the orbital periods.

- Planet K with $P_K = 109.6$ years, $f_K = 289$ pHz. $a_K = 0.95$ and $\phi_K = 4.268$. $P_K : P_H \approx 3 : 1$ applies to the orbital periods. [2] assumes an orbital period of $P \approx 101.5$ years.

After compensation of all PM with the frequencies $f_B...f_K$ mentioned above, the residual ripple of f_{ZF} is so low that the existence of further planets with $P < 2000$ years is improbable. The short database of only 20 years does not allow to determine even longer time constants.

As expected, f_{GW} is also phase modulated with $f_{orbit} = 31.68754$ nHz. $a_{orbit} = 2.358$ (see section 5.1). From the phase angle $\phi_{orbit} = -0.2$ it follows that here on Earth, we receive maximum blueshift on every $365 \cdot \phi_{orbit}/2\pi = 353$th day of the year f_{GW}. According to [4], this should take place on the 295th day of the year (error $\approx 16\%$).

On January 1, 2000, the frequency of the GW source was 20.37793 μHz. The drift is $\dot{f}_{GW} = 122 \times 10^{-20}$ Hz/s and was never measured with electromagnetic waves.

In retrospect, it is confirmed that it is important to eliminate *all* PM: Compensating the PM with the MSH method allows the amplitude of the GW to rise back to 100%, improving the S/N significantly (figure 1).
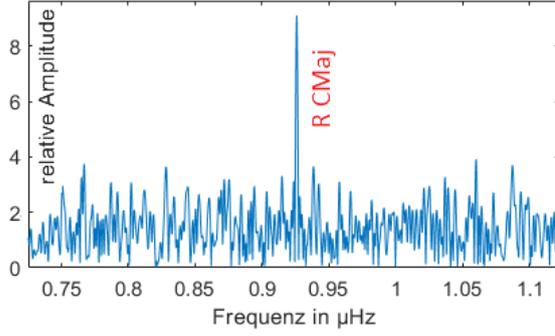
Figure 1): *Spectrum of the GW of R CMa after changing the frequency to $f_{ZF} = 1/(300 \ hours)$ and compensating the phase modulations. This increases the amplitude of the carrier frequency of the GW significantly. The vicinity of f_{ZF} is filled with the distorted spread spectra of previously undiscovered GWs of similar frequency.*

4 Dermott's Law

For a long time people have been looking for reasons for obvious connections between the orbital periods P of planets. The ansatz (1) comes from Dermott [5]

$$P_n = P_0 \cdot c^n \tag{1}$$

with $n = 1, 2, 3, 4...$ Figure 2 shows the best approximation with $P_0 = 66.8$ days and $c = 1.638 \pm 0.001$. For the relation of Dermott and the older Titius-Bode series there is no deeper justification. Dermott's law reliably provides good initial values when searching for unknown planets.
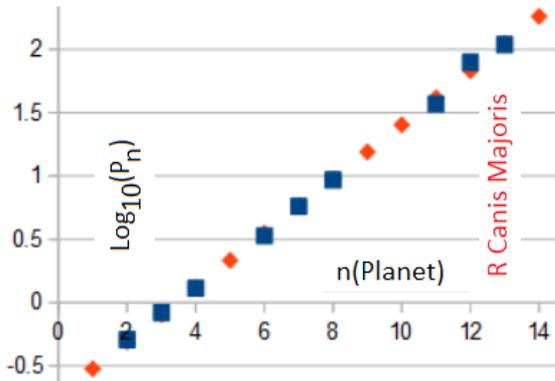

Figure 2): *The logarithm of the orbital period of the planets (in years) of R CMa as a function of their order. The actual values (blue) hardly differ from Dermott's law (red). Despite an intensive search, the PM of the "missing" planets could not be detected.*

5 Notes from an astronomical point of view

A word of caution: The MSH method measures and removes phase modulations from f_{GW}. Now we assume that the Doppler effect caused by planets is the only reason for the PM of the GW source.

Translating the abstract results of the iteration (section 3) into astronomical terms, the following relationships apply: All time specifications refer to the beginning of the analyzed data chains on 2000-01-01 and apply under the condition that the corresponding celestial bodies describe circular orbits. The phase shift ϕ indicates at what later

point in time the instantaneous frequency of the GW is blue-shifted to the maximum. Then one has to add the frequency deviation Δf produced by the Doppler effect to the average frequency f_{GW}. The results of the compilation given above may be evaluated independently of one another because all PM are linearly superimposed.

5.1 The Earth orbit causes a PM

From the modulation index $a_{orbit} = 2.358 = \Delta f_{orbit}/f_{orbit}$ follows $\Delta f_{orbit} = 74.72$ nHz. This frequency deviation cannot be explained with the assumption that any GW travels at the speed of light. According to RT, we expect a maximum Doppler shift of

$$\Delta f_{orbit} = f_{GW} \cdot \left(\sqrt{\frac{c + v_{orbit}}{c - v_{orbit}}} - 1 \right) \approx f_{GW} \cdot \frac{v_{orbit}}{c} = f_{GW} \cdot 10^{-4}. \tag{2}$$

The position of $R\ CMa$ is south of the ecliptic plane. Therefore, the Earth approaches this target with the maximum speed $v_{orbit} = 23000$ m/s. The maximum frequency deviation Δf_{orbit} should be smaller than 1562 pHz (equation (2)). The actually measured value is about 48 times larger! A measurement error of this magnitude can be ruled out after careful examination. What is causing the discrepancy? The equations of the PM and the Doppler effect are well founded and confirmed a million times. What remains is the correction of the assumption, that GWs propagate at the speed of light. The calculation of the instantaneous frequency uses the longitudinal Doppler effect, in which the frequency is corrected relativistically. For maximum blueshift applies

$$f_{GW} + \Delta f_{orbit} = f_{GW} \sqrt{1 - \left(\frac{v_{orbit}}{c}\right)^2} \cdot \frac{1}{1 - \frac{v_{orbit}}{v_{GW}}} \approx \frac{f_{GW}}{1 - \frac{v_{orbit}}{v_{GW}}} \tag{3}$$

If we transform the equation (3), we get

$$\frac{v_{orbit}}{v_{GW}} = 1 - \frac{f_{GW}}{f_{GW} + \Delta f_{orbit}} = 3.65 \times 10^{-3}. \tag{4}$$

With this intermediate result, we calculate

$$v_{GW} = \frac{v_{orbit}}{3.65 \times 10^{-3}} = 6.3 \times 10^6 \ \frac{m}{s} \approx \frac{1}{48}c. \tag{5}$$

This result is much lower than the speed of light and is valid for $f_{GW} \approx 20 \ \mu$Hz. The comparison of the results of previous measurements of binary systems reveals a certain trend. This topic will be deepened in another paper.

5.2 Are there Planets?

It has long been suspected that *R CMa* has planets [1, 2]. Although the binary star system is only 44 pc away, no planets have been seen using electromagnetic waves.

The investigation with GW could change that: The GW of the double star $A1 - A2$ is phase modulated with discrete frequencies and not with a broadband noise. The frequencies that can be measured precisely may be explained most simply by assuming that the binary system is orbited by several planets. The mass of the planets may be calculated from the individual frequency deviations of the PM.

Considering the GW source $A1 - A2$ as a star and the planet B as a companion, Kepler's third law provides the orbital equation for the two-body system.

$$4\pi^2 (r_A + r_B)^3 = GT^2 (m_{A1} + m_{A2} + m_B) \tag{6}$$

The radii refer to the center of gravity of the trio and the center of gravity theorem

$$(m_{A1} + m_{A2})r_A = m_B r_B \tag{7}$$

applies (we ignore other planets). With the assumed masses [1] $m_{A1} = 1.67 m_\odot$ and $m_{A2} = 0.22 m_\odot$ and the equations (3)...(7), we get an almost linear relationship between m_B and v_{GW}. If one knew the speed v_{GW} with which GW propagates in the immediate vicinity of the binary system $A1 - A2$, one could calculate the mass of each planet. Previous investigations [9–15] resulted in surprisingly low values around $v_{GW} \approx 50$ m/s. Using this value, we obtain the following estimates for the masses of the nine planets:

Planet	B	C	D	E	F	G	H	J	K
P_{orbit} (years)	0.508	0.830	1.297	3.368	5.761	9.32	37.10	79.22	109.65
m_{planet}/m_{Earth}	0.48	2.44	1.64	1.10	0.35	0.45	0.26	0.038	0.055

Table 1): *The orbital periods and the estimated masses of the nine planets of R CMa.*

6 Summary

From a communications point of view, decoding the phase modulations of f_{GW} is a standard task of digital signal processing. The signal has a good S/N (figure 1), the receiving antenna is insensitive to earthquakes. No assumptions are needed at any stage of decoding. We need no computationally intensive comparisons with pre-calculated patterns (search templates) based on model assumptions.

The opposite is true for the interpretation of the results from an astronomical point of view: The high values for the frequency deviation (Δf) may be explained by the assumption that gravitational waves at low frequencies around 20 μHz propagate significantly more slowly than the speed of light.

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