Solving particle-antiparticle and cosmological constant problems

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Abstract

Following the results of our publications, we argue that fundamental objects in particle theory are not elementary particles and antiparticles but objects described by irreducible representations (IRs) of the de Sitter (dS) algebra. One might ask why, then, experimental data give the impression that particles and antiparticles are fundamental and there are conserved additive quantum numbers (electric charge, baryon quantum number and others). The matter is that, at the present stage of the universe, the contraction parameter R from the dS to the Poincare algebra is very large and, in the formal limit $R \to \infty$, one IR of the dS algebra splits into two IRs of the Poincare algebra corresponding to a particle and its antiparticle with the same masses. The problem why the quantities $(c, \hbar, R)$ are as are does not arise because they are contraction parameters for transitions from more general Lie algebras to less general ones. Then the baryon asymmetry of the universe problem does not arise. At the present stage of the universe (when semiclassical approximation is valid), the phenomenon of cosmological acceleration (PCA) is described without uncertainties as an inevitable kinematical consequence of quantum theory in semiclassical approximation. In particular, it is not necessary to involve dark energy the physical meaning of which is a mystery. In our approach, background space and its geometry (metric and connection) are not used, R has nothing to do with the radius of dS space, but, in semiclassical approximation, the results for PCA are the same as in General Relativity if $\Lambda = 3/R^2$, i.e., $\Lambda > 0$ and there is no freedom in choosing the value of Λ.

Keywords: irreducible representations; cosmological acceleration;
baryon asymmetry of the universe

Contents

Chapter 1

General principles of quantum theory

1.1 Problem with space-time background in quantum theory

Modern fundamental particle theories (QED, QCD and electroweak theory) are based on the concept of particle-antiparticle. Historically, this concept has arisen as a consequence of the fact that the Dirac equation has solutions with positive and negative energies. The solutions with positive energies are associated with particles, and the solutions with negative energies - with corresponding antiparticles. And when the positron was found, it was treated as a great success of the Dirac equation. Another great success is that in the approximation $(v/c)^2$ the Dirac equation reproduces the fine structure of the hydrogen atom with a very high accuracy.

However, now we know that there are problems with the physical interpretation of the Dirac equation. For example, in higher order approximations, the probabilistic interpretation of non-quantized Dirac spinors is lost because the coordinate description implies that they are described by representations induced from non-unitary representations of the Lorenz algebra. Moreover, this problem exists not only for Dirac spinors but for any functions described by relativistic covariant equations (Klein-Gordon, Dirac, Rarita-Schwinger and others). In general, as shown by Pauli [1], in the case of fields with an integer spin it is not possible to define a positive-definite charge operator while in the case of fields with a half-integer spin it is not possible to define a positive-definite energy operator. It is also known that the description of the electron in the external field by the Dirac spinor is not accurate (e.g., it does not take into account the Lamb shift).

Another fundamental problem in the interpretation of the Dirac equation is as follows. One of the key principles of quantum theory is the principle of superposition. This principle states that if ψ_1 and ψ_2 are possible states of a physical system then $c_1\psi_1 + c_2\psi_2$, when c_1 and c_2 are complex coefficients, also is a possible state. The Dirac equation is the linear equation, and, if $\psi_1(x)$ and $\psi_2(x)$ are solutions of the equation, then $c_1\psi_1(x)+c_2\psi_2(x)$ also is a solution, in agreement with the principle of superposition. In the spirit of the Dirac equation, there should be no separate particles the electron and the positron. It should be only one particle which can be called electron-positron such that electron states are the states of this particle with positive energies, positron states are the states of this particle with negative energies and, in general, the superposition of electron and positron states should not be prohibited. However, in view of charge conservation, baryon number conservation and lepton numbers conservation, the superposition of a particle and its antiparticle is prohibited.

Modern particle theories are based on Poincare (relativistic) symmetry. In these theories, elementary particles are described by irreducible representations (IRs) of the Poincare algebra. Such IRs have a property that energies in them can be either strictly positive or strictly negative but there are no IRs where energies have different signs. The objects described by positive-energy IRs are called particles, and objects described by negative-energy IRs are called antiparticles, and energies of both, particles and antiparticles become positive after second quantization. In this situation, there are no elementary particles which are superpositions of a particle and its antiparticle, and as noted above, this is not in the spirit of the Dirac equation.

In particle theories, only quantized Dirac spinors $\psi(x)$ are used. Here, by analogy with non-quantized spinors, x is treated as a point in Minkowski space. However, $\psi(x)$ is an operator in the Fock space for an infinite number of particles. Each particle in the Fock space can be described by its own coordinates (in the approximation when the position operator exists — see e.g., [2]). **In view of this fact, the following natural question arises: why do we need an extra coordinate x which does not have any physical mean-**

ing because it does not belong to any particle and so is not measurable? Moreover, I can ask the following seditious question: in quantum theory, do we need Minkowski space at all?

When there are many bodies, the impression may arise that they are in some space but this is only an impression. In fact, background space-time (e.g., Minkowski space) is only a mathematical concept needed in classical theory. For illustration, consider quantum electromagnetic theory. Here we deal with electrons, positrons and photons. As noted above, in the approximation when the position operator exists, each particle can be described by its own coordinates. The coordinates of the background Minkowski space do not have a physical meaning because they do not refer to any particle and therefore are not measurable. However, in classical electrodynamics we do not consider electrons, positrons and photons. Here the concepts of the electric and magnetic fields $(\mathbf{E}(x), \mathbf{B}(x))$ have the meaning of the mean contribution of all particles in the point x of Minkowski space.

This situation is analogous to that in statistical physics. Here we do not consider each particle separately but describe the mean contribution of all particles by temperature, pressure etc. Those quantities have a physical meaning not for each separate particle but for ensembles of many particles.

A justification of the presence of x in quantized Dirac spinors $\psi(x)$ is that in quantum field theories (QFT) the Lagrangian density depends on the four-vector x, but this is only the integration parameter which is used in the intermediate stage. The goal of the theory is to construct the S-matrix, and, when the theory is already constructed, one can forget about Minkowski space because no physical quantity depends on x. This is in the spirit of the Heisenberg S-matrix program according to which in relativistic quantum theory it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to \infty$.

The fact that the theory gives the S-matrix in momentum representation does not mean that the coordinate description is excluded. In typical situations, the position operator in momentum representation exists not only in the nonrelativistic case but in the relativistic case as well. In the latter case, it is known, for example, as the Newton-Wigner position operator [3] or its modifications. However, as pointed out even in textbooks on quantum theory, the coordinate description of elementary particles can work only in some approximations. In particular, even in most favorable scenarios, for a massive particle with the mass m its coordinate cannot be measured with the accuracy better than the particle Compton wave length $\hbar/mc$.

Space-time background is the basic element of QFT which has no analogs in the history of science. There is no branch of science where so impressive agreements between theory and experiment have been achieved. However, those successes have been achieved only in perturbation theory while it is not known how the theory works beyond perturbation theory. Also, the level of mathematical rigor in QFT is very poor and, as a result, QFT has several known difficulties and inconsistencies.

One of the key inconsistencies of QFT is the following. It is known (see e.g. the textbook [5]) that quantum interacting local fields can be treated only as operatorial distributions. A known fact from the theory of distributions is that the product of distributions at the same point is not a correct mathematical operation. Physicists often ignore this problem and use such products because, in their opinion, it preserves locality (although the operator of the quantity x does not exist). As a consequence, the representation operators of interacting systems constructed in QFT are not well defined and the theory contains anomalies and infinities. While in renormalizable theories the problem of infinities can be somehow circumvented at the level of perturbation theory, in quantum gravity infinities cannot be excluded even in lowest orders of perturbation theory. As noted above, in spite of such mathematical problems, QFT is very popular since it has achieved great successes in describing many experimental data.

In the present paper, we consider particle-antiparticle and cosmological constant problems. In our approach, for solving those problems there is no need to involve space-time background and the problems can be solved using only rigorous mathematics.

1.2 Symmetry at quantum level

In the literature, symmetry in QFT is usually explained as follows. Since Poincare group is the group of motions of Minkowski space, the system under consideration should be described by unitary representations of this group. This approach is in the spirit of the Erlangen Program proposed by Felix Klein in 1872 when quantum theory did not yet exist.

However, as noted in Sec. 1.1, background space is only a mathematical concept: in quantum theory, each physical quantity should be described by an operator but there are no operators for the coordinates of background space. There is no law that every physical theory must contain a background space. For example, it is not used in nonrelativistic quantum mechanics and in irreducible representations (IRs) describing elementary particles. In particle theory, transformations from the Poincare group are not used because, according to the Heisenberg S-matrix program, it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$. In this theory, systems are described by observable physical quantities — momenta and angular momenta. So, symmetry at the quantum level is defined not by a background space and its group of motions but by a representation of a Lie algebra A by self-adjoint operators (see [2, 4] for more details).

Then each elementary particle is described by an IR of A and a system of N noninteracting particles is described by the tensor product of the corresponding IRs. This implies that, for the system as a whole, each momentum operator is a sum of the corresponding single-particle momenta, each angular momentum operator is a sum of the corresponding single-particle angular momenta, and *this is the most complete possible description of this system*. In particular, nonrelativistic symmetry implies that A is the Galilei algebra, relativistic symmetry implies that A is the Poincare algebra, de Sitter (dS) symmetry implies that A is the dS algebra so(1,4) and anti-de Sitter (AdS) symmetry implies that A is the AdS algebra so(2,3).

In his famous paper "Missed Opportunities" [6] Dyson notes that:

- a) Relativistic quantum theories are more general (fundamental) than nonrelativistic quantum theories even from pure mathematical considerations because Poincare group is more symmetric than Galilei one: the latter can be obtained from the former by contraction $c \to \infty$.

- b) dS and AdS quantum theories are more general (fundamental) than relativistic quantum theories even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one: the latter can be obtained from the former by contraction $R \to \infty$ where R is a parameter with the dimension *length*, and the meaning of this parameter will be explained below.

- c) At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

As noted above, symmetry at the quantum level should be defined by a Lie algebra, and in [2], the statements a)-c) have been reformulated in terms of the corresponding Lie algebras. It has also been shown that the fact that quantum theory is more general (fundamental) than classical theory follows even from pure mathematical considerations because formally the classical symmetry algebra can be obtained from the symmetry algebra in quantum theory by contraction $\hbar \to 0$. For these reasons, the most general description in terms of ten-dimensional Lie algebras should be carried out in terms of quantum dS or AdS symmetry. However, as explained below, in particle theory, dS symmetry is more general than AdS one.

The definition of those symmetries is as follows. If M^{ab} $(a, b = 0, 1, 2, 3, 4,\ M^{ab} = -M^{ba})$ are the angular momentum operators for the system under consideration, they should satisfy the commutation relations:

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \tag{1.1}$$

where $\eta^{ab} = 0$ if $a \neq b$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{44} = \mp 1$ for the dS and AdS symmetries, respectively.

Although the dS and AdS groups are the groups of motions of dS and AdS spaces, respectively, the description in terms of relations (1.1) does not involve those groups and spaces at all, and *those relations can be treated as a definition of dS and AdS symmetries at the quantum level* (see the discussion in [2, 4]). In QFT, interacting particles are described by field functions defined on Minkowski, dS and AdS spaces. However, since we consider only noninteracting bodies and describe them in terms of IRs, at this level we don't need these fields and spaces.

The procedure of contraction from dS or AdS symmetry to Poincare one is defined as follows. If we *define* the momentum operators P^μ as $P^\mu = M^{4\mu}/R$ $(\mu = 0, 1, 2, 3)$ then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (1.1) become the commutation relations for the Poincare algebra (see e.g., [2, 4]). Here *R is a parameter which has nothing to do with the dS and AdS spaces.* As seen from Eqs. (1.1), quantum dS and AdS theories do not involve the dimensionful parameters $(c, \hbar, R)$ at all because (kg, m, s) are meaningful only at the macroscopic level.

In particle theories, the quantities c and $\hbar$ typically are not involved and it is said that the units $c = \hbar = 1$ are used. Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8 m/s$ and $\hbar \approx 1.054 \cdot 10^{-34} kg \cdot m^2/s$ and those values are as are simply because people want to describe velocities in m/s and angular momenta in $kg \cdot m^2/s$. At the same time, physicists usually believe that physics should derive the value of Λ and that the solution to the dark energy problem depends on this value.

At the classical level, Λ is the curvature of the background space and equals $\pm 3/R^2$ for the dS and AdS spaces, respectively, where R is the radius of those spaces. As noted below, in semiclassical approximation, R is the same as the parameter R in quantum theory where this parameter is only the coefficient of proportionality between $M^{4\mu}$ and P^μ. As follows from the above discussion, at the level of contraction parameters, the quantity R is fundamental to the same extents as c and $\hbar$. Here the question why R is as is does not arise simply because the answer is: because people want to describe distances in meters. There is no guaranty that the values of $(c, \hbar, R)$ in (kg, m, s) will be the same during the whole history of the universe.

Chapter 2

Solving particle-antiparticle problem

2.1 Concepts of particles and antiparticles in standard quantum theory

Standard particle theories are based on Poincare symmetry, and here the concepts of particles and antiparticles are considered from the point of view of two approaches which we call ApproachA and ApproachB.

ApproachA is based on the fact that in irreducible representations (IRs) of the Poincare algebra by self-adjoined operators in Hilbert spaces, the energy spectrum can be either ≥ 0 or ≤ 0, and there are no IRs where the energy spectrum contains both, positive and negative energies. The objects described by the corresponding IRs are called elementary particles and antiparticles, respectively.

When we consider a system consisting of particles and antiparticles, the energy signs for both of them should be the same. Indeed, consider, for example a system of two particles with the same mass, and let their momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ be such that $\mathbf{p}_1 + \mathbf{p}_2 = 0$. Then, if the energy of particle 1 is positive, and the energy of particle 2 is negative then the total four-momentum of the system would be zero what contradicts experimental data. By convention, the energy sign of all particles and antiparticles in question is chosen to be positive. For this purpose, the procedure of second quantization is defined such that after the second quantization the energies of antiparticles become positive. Then the mass of any particle is the minimum value of its energy in the case when the momentum equals zero.

Suppose now that we have two particles such that particle 1 has the mass m_1, spin s_1 and is characterized by some additive quantum numbers (e.g., electric charge, baryon quantum number etc.), and particle 2 has the mass m_2, spin $s_2 = s_1$ and all additive quantum numbers characterizing particle 2 equal the corresponding quantum numbers for particle 1 with the opposite sign. A question arises when particle 2 can be treated as an antiparticle for particle 1. Is it necessary that m_1 should be exactly equal m_2 or m_1 and m_2 can slightly differ each other? In particular, can we guarantee that the mass of the positron exactly equals the mass of the electron, the mass of the proton exactly equals the mass of the antiproton etc.? If we work only in the framework of ApproachA then we cannot answer this question because here, IRs for particles 1 and 2 are independent on each other and there are no limitations on the relation between m_1 and m_2.

On the other hand, in ApproachB, $m_1 = m_2$ but, as explained below, this is achived at the expense of losing probabilistic interpretation. Here, a particle and its antiparticle are elements of the same field state $\psi(x)$ with positive and negative energies, respectively, where x is a vector from Minkowski space and $\psi(x)$ satisfies a relativistic covariant field equation (Dirac, Clein-Gordon, Rarita-Schwinger and others).

As noted in Sec. 1.1, historically, the particle-antiparticle concept has arisen as a consequence of the fact that the Dirac equation has solutions with both, positive and negative energies. In this section, we have also described problems with the physical interpretation of this equation and noted that similar problems exist in the interpretation of any local relativistic covariant equation.

A usual phrase in the literature is that in QFT, the fact that $m_1 = m_2$ follows from the CPT theorem which is a consequence of locality since, *by construction*, states described by local covariant equations are direct sums of IRs for a particle and its antiparticle with equal masses. However, as noted above, since at the quantum level there are problems with the physical interpretation of covariant fields and the quantity x, the very meaning of locality at the quantum level is problematic.

Also, a question arises what happens if locality is only an approximation: in that case the equality of masses is exact or approximate?

Consider a simple model when electromagnetic and weak interactions are absent. Then the fact that the proton and the neutron have equal masses has nothing to do with locality; it is only a consequence of the fact that the proton and the neutron belong to the same isotopic multiplet. In other words, they are simply different states of the same object—the nucleon.

Since the concept of locality is not formulated in terms of selfadjoint operators, this concept does not have a clear physical meaning, and this fact has been pointed out even in known textbooks (see e.g., [5]). Therefore, QFT does not give a rigorous physical proof that $m_1 = m_2$. Note also that in Poincare invariant quantum theories, there can exist elementary particles for which all additive quantum numbers are zero. Such particles are called neutral because they coincide with their antiparticles.

2.2 Problems with the definition of particles and antiparticles in dS quantum theories

As noted in Sec. 1.2, dS and AdS quantum symmetries are more general than Poincare symmetry. For this reason, it is necessary to investigate how particles and antiparticles are described in the framework of those symmetries for which the descriptions are considerably different, and in this section we consider the case of dS symmetry.

In this case, all the operators $M^{\nu 4}$ ($\nu = 0,1,2,3$) are on equal footing. Therefore, M^{04} can be treated as the Poincare analog of the energy only in the approximation when R is rather large. In the general case, the sign of M^{04} cannot be used for the classification of IRs.

In his book [7] Mensky describes the implementation of dS IRs when the representation space is the three-dimensional unit sphere in the four-dimensional space (see also [2]). In this implementation, there exist one-to-one relations between the northern hemisphere and the upper Lorentz hyperboloid with positive Poincare energies and between the southern hemisphere and the lower Lorentz hyperboloid with negative Poincare energies, while points on the equator correspond to infinite Poincare energies. However, the operators of IRs are not singular in the vicinity of the equator and, since the equator has measure zero, the properties of wave functions on the equator are not important.

Since the number of states in dS IRs is twice as big as the number of states in IRs of the Poincare algebras, one might think that each IR of the dS algebra describes a particle and its antiparticle simultaneously. However, a detailed analysis in [2] shows that states described by dS IRs cannot be characterized as particles or antiparticles in the usual meaning.

For example, let us call states with the support of their wave functions on the northern hemisphere as particles and states with the support on the southern hemisphere as their antiparticles. Then states which are superpositions of a particle and its antiparticle obviously belong to the representation space under consideration, i.e., they are not prohibited. However, this contradicts the superselection rule that the wave function cannot be a superposition of states with opposite electric charges, baryon and lepton quantum numbers etc. Therefore, in the dS case there are no superselection rules which prohibit superpositions of states with opposite electric charges, baryon quantum numbers etc. In addition, in this case it is not possible to define the concept of neutral particles, i.e., particles which coincide with their antiparticles (e.g., the photon). This question will be discussed in Chap. 4.

As noted in Sec. 1.2, dS symmetry is more general than Poincare one, and the latter can be treated as a special degenerate case of the former in the formal limit $R \to \infty$. This means that, with any desired accuracy, any phenomenon described in the framework of Poincare symmetry can be also described in the framework of dS symmetry if R is chosen to be sufficiently large, but there also exist phenomena for explanation of which it is important that R is finite and not infinitely large (see [2]).

As shown in [2, 8], dS symmetry is broken in the formal limit $R \to \infty$ because one IR of the dS algebra splits into two IRs of the Poincare algebra with positive and negative energies and with equal masses. Therefore, the fact that the masses of particles and their corresponding antiparticles are equal to each other, can be explained as a consequence of the fact that observable properties of elementary particles can be described not by exact Poincare symmetry but by dS symmetry with a very large (but finite) value of R. In contrast to QFT, for combining a particle and its antiparticle into one object, there in no need to assume locality and involve local field functions because a particle and its antiparticle already belong to the same IR of the dS algebra (compare with the above remark about the isotopic symmetry in the proton-neutron system).

The fact that dS symmetry is higher than Poincare one is clear even from the fact that, in the framework of the latter symmetry, it is not possible to describe states which are superpositions of states on the upper and lower hemispheres. Therefore, breaking the IR into two independent IRs defined on the northern and southern hemispheres obviously breaks the initial symmetry of the problem. This fact is in agreement with the Dyson observation (mentioned above) that dS group is more symmetric than Poincare one.

When $R \to \infty$, standard concepts of particle-antiparticle, electric charge and baryon and lepton quantum numbers are restored, i.e., in this limit superpositions of particle and antiparticle states become prohibited according to the superselection rules. Therefore, those concepts arise as a result of symmetry breaking at $R \to \infty$, i.e., they are not universal.

2.3 Particles and antiparticles in AdS quantum theories

In theories where the symmetry algebra is the AdS algebra, the structure of IRs is known (see e.g., [2, 9]). The operator M^{04} is the AdS analog of the energy operator. Let W be the Casimir operator $W = \frac{1}{2}\sum M^{ab}M_{ab}$ where a sum over repeated indices is assumed. As follows from the Schur lemma, the operator W has only one eigenvalue in every IR. By analogy with Poincare quantum theory, we will not consider AdS tachyons and then one can define the AdS mass μ such that $\mu \geq 0$ and μ^2 is the eigenvalue of the operator W.

As noted in Sec. 1.2, the procedure of contraction from the AdS algebra to the Poincare one is defined such that if R is a parameter with the dimension *length* then $M^{\nu 4} = RP^{\nu}$. This procedure has a physical meaning only if R is rather large. In that case the AdS mass μ and the Poincare mass m are related as $\mu = Rm$, and the relation between the AdS and Poincare energies is analogous. Since AdS symmetry is more general (fundamental) then Poincare one then μ is more general (fundamental) than m. In contrast to the Poincare masses and energies, the AdS masses and energies are dimensionless. As noted in Sec. 3.4, at the present stage of the universe R is of the order of $10^{26}m$. Then the AdS masses of the electron, the Earth and the Sun are of the order of 10^{39}, 10^{93} and 10^{99}, respectively. The fact that even the AdS mass of the electron is so large might be an indication that the electron is not a true elementary particle. In addition, the present accepted upper level for the photon mass is $10^{-17}ev$. This value seems to be an extremely tiny quantity. However, the corresponding AdS mass is of the order of 10^{16}, and so, even the mass which is treated as extremely small in Poincare invariant theory might be very large in AdS invariant theory.

In the AdS case, there are IRs with positive and negative energies, and they belong to the discrete series [2, 9]. Therefore, one can define particles and antiparticles. If μ_1 is the AdS mass for a positive energy IR, then the energy spectrum contains the eigenvalues $\mu_1, \mu_1+1, \mu_1+2, ...\infty$, and, if μ_2 is the AdS mass for a negative energy IR, then the energy spectrum contains the eigenvalues $-\infty, ... -\mu_2-2, -\mu_2-1, -\mu_2$. Therefore, the situation is pretty much analogous to that in Poincare invariant theories, and, without involving local AdS invariant equations there is no way to conclude whether the mass of a particle equals the mass of the corresponding antiparticle.

2.4 dS vs. AdS and baryon asymmetry of the universe problem

In this chapter we have discussed how the concepts of particles and antiparticles should be defined in the cases of Poincare, dS and AdS symmetries. In the first and third cases, the situations are similar: IRs where the energies are ≥ 0 are treated as particles, and IRs where the energies are ≤ 0 are treated as antiparticles. Then a problem arises how to prove that the masses of a particle and the corresponding antiparticle are the same. As noted in Secs. 2.1 and 2.3, without involving local covariant equations there is no way to conclude whether it is the case. Since the concept of locality is not formulated in terms of selfadjoint operators, in the framework of Poincare and AdS symmetries, QFT does not give a rigorous proof that the masses of a particle and the corresponding antiparticle are the same.

As described in Sec. 2.2, in the case of dS symmetry, the approach to the concept of particle-antiparticle is radically different from the approaches in the cases of Poincare and AdS symmetries. Here, the fundamental objects are not particles and antiparticles, but the objects that are described by IRs of the dS algebra. One might ask why, then, experimental data in particle physics give the impression that particles and antiparticles are fundamental. As explained in Sec. 2.2, the matter is that, at this stage of the universe, the contraction parameter R from the dS to Poincare algebra is very large and, in the formal limit $R \to \infty$, one IR of the dS algebra splits into two IRs of the Poincare algebra corresponding to a particle and its antiparticle with the same masses. In this case, for proving the equality of masses there is no need to involve local covariant fields and the proof is given fully in terms of well defined selfadjoint operators. As noted in Sec. 1.1, in the spirit of the Dirac equation, there should not be separate particles, the electron and positron but there should be one particle which can be called electron-positron. In the case of dS symmetry, this idea is implemented exactly in this way. It has been also noted that in this case there are no conservation laws for additive quantum numbers: from the experiment it seems that such conservation laws take place, but in fact, these laws are only approximate because, at the present stage of the universe the parameter R is very large. **Thus, we can conclude that dS symmetry is more fundamental than Poincare and AdS symmetries.**

We now discuss the dS vs. AdS problem from the point of view whether standard gravity can be obtained in the framework of a free theory. In standard nonrelativistic approximation, gravity is characterized by the term $-Gm_1m_2/r$ in the mean value of the mass operator. Here m_1 and m_2 are the particle masses and r is the distance between the particles. Since the kinetic energy is always positive, the free nonrelativistic mass operator is positive definite and therefore there is no way to obtain gravity in the framework of a free theory. Analogously, in Poincare invariant theory, the spectrum of the free two-body mass operator belongs to the interval $[m_1 + m_2, \infty)$ while the existence of gravity necessarily requires that the spectrum should contain values less than $m_1 + m_2$.

As explained in Sec. 2.3, in theories where the symmetry algebra is the AdS algebra, for positive energy IRs, the AdS Hamiltonian has the spectrum in the interval $[m, \infty)$ and m has the meaning of the mass. Therefore the situation is pretty much analogous to that in Poincare invariant theories. In particular, the free two-body mass operator again has the spectrum in the interval $[m_1 + m_2, \infty)$ and therefore there is no way to reproduce gravitational effects in the free AdS invariant theory.

In contrast to the situation in Poincare and AdS invariant theories, the free mass operator in dS theory is not bounded below by the value of m_1+m_2. The discussion in Sec. 2.2 shows that this property by no means implies that the theory is unphysical. In the dS case, there is no law prohibiting that in the nonrelativistic approximation, the mean value of the mass operator contains the term $-Gm_1m_2/r$. Therefore if one has a choice between Poincare, AdS and dS symmetries then the only chance to describe gravity in a free theory is to choose dS symmetry, and, as discussed in [2], a possible nature of gravity is that gravity is a kinematical effect in a quantum theory based not on complex numbers but on a finite ring or field. **This is an additional argument in favor of dS vs. AdS.**

We now apply this conclusion to the known problem of baryon asymmetry of the universe. This problem is formulated as follows. According to modern particle and cosmological theories, the numbers of baryons and antibaryons in the early stages of the universe were the same. Then, since the baryon number is the conserved quantum number, those numbers should be the same at the present stage. However, at this stage, the number of baryons is much greater than the number of antibaryons.

However, as noted above, it seems to us that the baryon quantum number is conserved because at this stage of the evolution of the universe, the value of R is enormous. As noted in Sec. 1.2, it is reasonable to expect that R changes over time, and as noted in Sec. 3.3, in semiclassical approximation, R coincides with the radius of the universe. However, according to cosmological theories, at early stages of the Universe, R was much less that now. At such values of R, the concepts of particles, antiparticles and baryon number do not have a physical meaning. **So, the statement that at early stages of the universe the numbers of baryons and antibaryons were the same, also does not have a physical meaning, and, as a consequence, the baryon asymmetry problem does not arise.**

Chapter 3

Solving cosmological constant problem

3.1 Introduction

At the present stage of the universe (when semiclassical approximation is valid), in the phenomenon of cosmological acceleration (PCA), only nonrelativistic macroscopic bodies are involved, and one might think that here there is no need to involve quantum theory. However, ideally, the results for every classical (i.e., non-quantum) problem should be obtained from quantum theory in semiclassical approximation. We will see that, considering PCA from the point of view of quantum theory sheds a new light on understanding this problem.

In PCA, it is assumed that the bodies are located at large (cosmological) distances from each other and sizes of the bodies are much less than distances between them. Therefore, interactions between the bodies can be neglected and, from the formal point of view, the description of our system is the same as the description of N free spinless elementary particles.

However, in the literature, PCA is usually considered in the framework of dark energy and other exotic concepts. In Sec. 3.2 we argue that such considerations are not based on rigorous physical principles. In Sec. 1.2 we have explained how symmetry should be defined at the quantum level, and in Sec. 3.3 we describe PCA in the framework of our approach.

3.2 History of dark energy

This history is well-known. First Einstein introduced the cosmological constant Λ because he believed that the universe was stationary and his equations can ensure this only if $\Lambda \neq 0$. But when Friedman found his solutions of equations of General Relativity (GR) with $\Lambda = 0$ and Hubble found that the universe was expanding, Einstein said (according to Gamow's memories) that introducing $\Lambda \neq 0$ was the biggest blunder of his life. After that, the statement that Λ must be zero was advocated even in textbooks.

The explanation was that, according to the philosophy of GR, matter creates a curvature of space-time, so when matter is absent, there should be no curvature, i.e., space-time background should be the flat Minkowski space. That is why when in 1998 it was realized that the data on supernovae could be described only with $\Lambda \neq 0$, the impression was that it was a shock of something fundamental. However, the terms with Λ in the Einstein equations have been moved from the left-hand side to the right-hand one, it was declared that in fact $\Lambda = 0$, but the impression that $\Lambda \neq 0$ was the manifestation of a hypothetical field which, depending on the model, was called dark energy or quintessence. In spite of the fact that, as noted in wide publications (see e.g., [10] and references therein), their physical nature remains a mystery, the most publications on PCA involve those concepts.

Several authors criticized this approach from the following considerations. GR without the contribution of Λ has been confirmed with a high accuracy in experiments in the Solar System. If Λ is as small as it has been observed, it can have a significant effect only at cosmological distances while for experiments in the Solar System the role of such a small value is negligible. The authors of [11] titled "Why All These Prejudices Against a Constant?" note that it is not clear why we should think that only a special case $\Lambda = 0$ is allowed. If we accept the theory containing the gravitational constant G which is taken from outside, then why can't we accept a theory containing two independent constants?

Let us note that currently there is no physical theory which works under all conditions. For example, it is not correct to extrapolate nonrelativistic theory to cases when speeds are comparable to c, and it is not correct to extrapolate classical physics for describing energy levels of the hydrogen atom. GR is a successful classical (i.e., non-quantum) theory for describing macroscopic phenomena where large masses are present, but extrapolation of GR to the case when matter disappears is not physical. One of the principles of physics is that a definition of a physical quantity is a description of how this quantity should be measured. As noted in Sec. 2.1, the concepts of space and its curvature are pure mathematical. Their aim is to describe the motion of real bodies. But the concepts of empty space and its curvature should not be used in physics because nothing can be measured in a space which exists only in our imagination. Indeed, in the limit of GR when matter disappears, space remains and has a curvature (zero curvature when $\Lambda = 0$, positive curvature when $\Lambda > 0$ and negative curvature when $\Lambda < 0$) while, since space is only a mathematical concept for describing matter, a reasonable approach should be such that in this limit space should disappear too.

A common principle of physics is that, when a new phenomenon is discovered, physicists should try to first explain it proceeding from the existing science. Only if all such efforts fail, something exotic can be involved. But for PCA, an opposite approach was adopted: exotic explanations with dark energy or quintessence were accepted without serious efforts to explain the data in the framework of existing science.

Although the physical nature of dark energy and quintessence remains a mystery, there exists a wide literature where the authors propose quantum field theory (QFT) models of them. For example, as noted in [12], there are an almost endless number of explanations for dark energy. While in most publications, only proposals about future discovery of dark energy are considered, the authors of [10] argue that dark energy has already been discovered by the XENON1T collaboration. In June 2020, this collaboration reported an excess of electron recoils: 285 events, 53 more than expected 232 with a statistical significance of 3.5σ. However, in July 2022, a new analysis by the XENONnT collaboration discarded the excess [13].

Several authors (see e.g., [12, 14, 15]) proposed approaches where some quantum fields manifest themselves as dark energy at early stages of the universe, and some of them are active today. However, as shown in our publications and in the present paper, at least at the present stage of the universe (when semiclassical approximation is valid), PCA can be explained without uncertainties proceeding from universally recognized results of physics and without involving models and/or assumptions the validity of which has not been unambiguously proved yet.

3.3 Explanation of cosmological acceleration

Standard particle theories involve IRs of the Poincare algebra by self-adjoint operators. They are described even in textbooks and do not involve Minkowski space. Therefore, when Poincare symmetry is replaced by more general dS or AdS one, dS and AdS particle theories should be based on IRs of the dS or AdS algebras by self-adjoint operators, respectively. However, physicists usually are not familiar with such IRs because they believe that dS and AdS quantum theories necessarily involve quantum fields on dS or AdS spaces, respectively.

The mathematical literature on unitary IRs of the dS group is wide but there are only a few papers where such IRs are described for physicists. For example, the excellent Mensky's book [7] exists only in Russian. At the same time, to the best of our knowledge, IRs of the dS algebras by self-adjoint operators have been described from different considerations only in [2, 8, 16, 17].

In the framework of our approach, the explanation of cosmological acceleration consists of the following steps. First, instead of the angular momentum operators $M^{4\mu}$ we work with the momentum operators $P^\mu = M^{4\mu}/R$, and, in the approximation when R is very large, different components of P^μ commute with each other. Then we use the explicit expressions for the operators M^{ab} of IRs of the dS algebra — see e.g., Eqs. (3.16) in [2], Eqs. (17) in [8] or Eqs. (3) in [17]. Those operators act in momentum representation and *at this stage, we have no spatial coordinates yet.* For describing the motion of particles in terms of spatial coordinates, we must define the position operator. A question: is there a law defining this operator?

The postulate that the coordinate and momentum representations are related by the Fourier transform was taken at the dawn of quantum theory by analogy with classical electrodynamics, where the coordinate and wave vector representations are related by this transform. But the postulate has not been derived from anywhere, and there is no experimental confirmation of the postulate beyond the nonrelativistic semiclassical approximation. Heisenberg, Dirac, and others argued in favor of this postulate but, for example, in the problem of describing photons from distant stars, the connection between the coordinate and momentum representations should be not through the Fourier transform, but as shown in [2]. However, since, PAC involves only nonrelativistic bodies then, as follows from the above remarks, the position operator in momentum representation can be defined as usual, i.e., as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$ where $\mathbf{p}$ is the momentum. Then in semiclassical approximation, we can treat $\mathbf{p}$ and $\mathbf{r}$ as usual vectors.

The next step is to take into account that the representation describing a free N-body system is the tensor product of the corresponding single-particle IRs. It means that every N-body operator M^{ab} is a sum of the corresponding single-particle operators. Then one can calculate the internal mass operator for any two-body subsystem of the N-body system, and the result is given by Eq. (3.68) in [2], Eq. (61) in [8] or Eq. (17) in [17]. Now, as follows from the Hamilton equations, in any two-body subsystem of the N-body system, the relative acceleration in semiclassical approximation is given by

$$\mathbf{a} = \mathbf{r}c^2/R^2 = \frac{1}{3}c^2\Lambda\mathbf{r} \tag{3.1}$$

where $\mathbf{a}$ and $\mathbf{r}$ are the relative acceleration and relative radius vector of the bodies, respectively, and $\Lambda = 3/R^2$. The fact that the relative acceleration of noninteracting bodies is not zero does not contradict the law of inertia, because this law is valid only in the case of Galilei and Poincare symmetries, and in the formal limit $R \to \infty$, $\mathbf{a}$ becomes zero as it should be.

Let us note the following. Since c is the contraction parameter for the transition from Poincare invariant theory to Galilei invariant one, the results of the latter can be obtained from the former in the formal limit $c \to \infty$, and Galilei invariant theories do not contain c. Then one might ask why Eq. (3.1) contains c although we assume that the bodies in PCA are nonrelativistic. The matter is that Poincare invariant theories do not contain R but we work in dS invariant theory and assume that, although c and R are very large, they are not infinitely large, and the quantity c^2/R^2 in Eq. (3.1) is finite.

As noted in Sec. 2.4, dS symmetry is more fundamental than AdS one. Formally, an analogous calculation using the results of Chap. 8 of [2] on IRs of the AdS algebra gives that, in the AdS case, $\mathbf{a} = -\mathbf{r}c^2/R^2$, i.e., we have attraction instead of repulsion. The experimental facts that the bodies repel each other confirm that dS symmetry is indeed more fundamental than AdS one.

The relative accelerations given by Eq. (3.1) are the same as those derived from GR if the curvature of dS space equals $\Lambda = 3/R^2$, where R is the radius of this space. *However, the crucial difference between our results and the results of GR is as follows. While in GR, R is the radius of the dS space and can be arbitrary, in quantum theory, R is the coefficient of proportionality between $M^{4\mu}$ and P^μ, this coefficient is fundamental to the same extent as c and $\hbar$, and a question why R is as is does not arise.* **Therefore, our approach gives a clear explanation why Λ is as is.**

In GR, the result (3.1) does not depend on how Λ is interpreted, as the curvature of empty space or as the manifestation of dark energy. However, in quantum theory, there is no freedom of interpretation. Here R is the parameter of contraction from the dS Lie algebra to the Poincare one, it has nothing to do with the radius of the background space and with dark energy and it must be finite because dS symmetry is more general than Poincare one.

3.4 Discussion

We have shown that, at the present stage of universe (when semiclassical approximation is valid), the phenomenon of cosmological acceleration is simply a consequence of quantum theory in semiclassical approximation, and this conclusion has been made without involving models and/or assumptions the validity of which has not been unambiguously proved yet.

The concept of the cosmological constant Λ has been originally defined in GR which is the purely classical (i.e., not quantum) theory. Here Λ is the curvature of space-time background which, as noted in Secs. 1.1 and 2.1 is a purely classical concept. Our consideration in Sec. 3.3 does not involve GR, and the contraction parameter R from dS invariant to Poincare invariant theory has nothing to do with the radius of dS space.

However, in QFT, Λ is interpreted as vacuum energy density, and the cosmological constant problem is described in a wide literature (see e.g. [18] and references therein). Usually, this problem is considered in the framework of Poincare invariant QFT of gravity on Minkowski space. This theory contains only one phenomenological parameter — the gravitational constant G, and Λ is defined by the vacuum expectation value of the energy-momentum tensor. The theory contains strong divergencies which cannot be eliminated because the theory is not renormalizable. The results can be made finite only with a choice of the cutoff parameter. Since G is the only parameter in the theory, the usual choice of the cutoff parameter in momentum space is $\hbar/l_P$ where l_P is the Plank length. Then, if $\hbar = c = 1$, G has the dimension $length^2$ and Λ is of the order of $1/G$. This value is more than 120 orders of magnitude greater than the experimental one. It is discussed in a wide literature how the discrepancy with experiment can be reduced, but the problem remains.

As explained above, in quantum theory, Poincare symmetry is a special degenerate case of dS symmetry in the formal limit $R \to \infty$ where R is a parameter of contraction from the dS algebra to the Poincare one. This parameter is fundamental to the same extent as c and $\hbar$, it has nothing to do with the relation between Minkowski and dS spaces and the problem why R is as is does not arise by analogy with the problem why c and $\hbar$ are as are. As noted in Sec. 3.3, the result for cosmological acceleration in our approach and in GR is given by the same expression (3.1) but the crucial difference between our approach and GR is as follows. While in GR, R is the radius of the dS space and can be arbitrary, in our approach, R is defined uniquely because it is a parameter of contraction from the dS algebra to the Poincare one. Therefore, in our approach, the problem why the cosmological constant is as is does not arise.

Therefore, at the present stage of the universe (when semiclassical approximation is valid), the phenomenon of cosmological acceleration has nothing to do with dark energy or other artificial reasons. This phenomenon is an inevitable kinematical consequence of quantum theory in semiclassical approximation and the problem of cosmological constant does not arise.

Since 1998, it has been confirmed in several experiments [19] that $\Lambda > 0$, and $\Lambda = 1.3 \cdot 10^{-52}/m^2$ with the accuracy 5%. Therefore, *at the current stage of the universe*, R is of the order of $10^{26}m$. Since Λ is very small and the evolution of the universe is the complex process, cosmological repulsion does not appear to be the main effect determining this process, and other effects (e.g., gravity, microwave background and cosmological nucleosynthesis) may play a much larger role.

Chapter 4

Open problems

As noted by Dyson in his fundamental paper [6], nonrelativistic theory is a special degenerate case of relativistic theory in the formal limit $c \to \infty$ and relativistic theory is a special degenerate case of dS and AdS theories in the formal limit $R \to \infty$ and, as shown in Sec. 2.4, dS symmetry is more general than AdS one.

The paper [6] appeared in 1972, i.e., more than 50 years ago, and, in view of Dyson's results, a question arises why general particle theories (QED, electroweak theory and QCD) are still based on Poincare symmetry and not dS one. Probably physicists believe that, since, at least at the present stage of the universe, R is much greater than even sizes of stars, dS symmetry can play an important role only in cosmology and there is no need to use it for description of elementary particles.

We believe that this argument is not consistent because usually more general theories shed a new light on standard concepts. It is clear from the discussion in Sec. 2.4 that the construction of dS theory will be based on considerably new concepts than the construction of standard quantum theory because in dS theory, the concepts of particles, antiparticles and additive quantum numbers (electric charge, baryon quantum number and others) can be only approximate.

Another problem discussed in a wide literature is that supersymmetric generalization exists in the AdS case but does not exist in the dS one. It may be a reason why supersymmetry has not been discovered yet.

In [2] we have proposed a criterion when theory A is more general (fundamental) than theory B:

Definition: *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that with any desired accuracy theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken then one cannot return back to theory A and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.*

As shown in [2], by using this **Definition** one can prove that: a) nonrelativistic theory is a special degenerate case of relativistic theory in the formal limit $c \to \infty$; b) classical theory is a special degenerate case of quantum theory in the formal limit $\hbar \to 0$; c) relativistic theory is a special degenerate case of dS and AdS theories in the formal limit $R \to \infty$; d) standard quantum theory (SQT) based on complex numbers is a special degenerate case of finite quantum theory (FQT) based on finite mathematics with a ring or field of characteristic p in the formal limit $p \to \infty$.

As noted in Sec. 1.2, the properties a)-c) take place in SQT, and below we will discuss the property d). As described in Secs. 2.2 and 2.3, in IRs of the AdS algebra, the energy spectrum of the energy operator can be either positive or negative while in the dS case, the spectrum necessarily contains energies of both signs. As explained in Sec. 2.4, for this reason, the dS case is more physical than the AdS one. We now explain that in the FQT analog of the AdS symmetry the situation is analogous to that in the dS case of SQT. For definiteness, we consider the case when p is odd.

By analogy with the construction of positive energy IRs in SQT, in FQT we start the construction from the rest state, where the AdS energy is positive and equals μ. Then we act on this state by raising operators and gradually get states with higher and higher energies, i.e., $\mu+1, \mu+2, ...$ However, in contrast to the situation in SQT, we cannot obtain infinitely large numbers. When we reach the state with the energy $(p-1)/2$, the next state has the energy $(p-1)/2+1 = (p+1)/2$ and, since the operations are modulo p, this value also can be denoted as $-(p-1)/2$ i.e., it may be called negative. When this procedure is continued, one gets the energies $-(p-1)/2+1 = -(p-3)/2, -(p-3)/2+1 = -(p-5)/2, ...$ and, as shown in [2], the procedure finishes when the energy $-\mu$ is reached.

Therefore the spectrum of energies contains the values $(\mu, \mu +$

$1,...,(p-1)/2)$ and $(-\mu,-(\mu+1),...,-(p-1)/2)$ and in the formal limit $p\to\infty$, this IR splits into two IRs of the AdS algebra in SQT for a particle with the energies $\mu,\mu+1,\mu+2,...\infty$ and antiparticle with the energies $-\mu,-(\mu+1),-(\mu+2),...-\infty$ and both of them have the same mass μ. We conclude that in FQT all IRs necessarily contain states with both, positive and negative energies and the mass of a particle automatically equals the mass of the corresponding antiparticle. This is an example when FQT can solve a problem which standard quantum AdS theory cannot. By analogy with the situation in the standard dS case, for combining a particle and its antiparticle together, there is no need to involve additional coordinate fields because a particle and its antiparticle are already combined in the same IR.

Since the AdS case in FQT satisfies all necessary physical conditions, it is reasonable to investigate whether this case has a supersymmetric generalization. We first note that representations of the standard Poincare superalgebra are described by 14 operators. Ten of them are the representation operators of the Poincare algebra—four momentum operators and six operators of the Lorentz algebra, and in addition, there are four fermionic operators. The anticommutators of the fermionic operators are linear combinations of the momentum operators, the commutators of the fermionic operators with the Lorentz algebra operators are linear combinations of the fermionic operators and the fermionic operators commute with the momentum operators. However, the latter are not bilinear combinations of fermionic operators.

From the formal point of view, representations of the AdS superalgebra osp(1,4) are also described by 14 operators — ten representation operators of the so(2,3) algebra and four fermionic operators. There are three types of relations: the operators of the so(2,3) algebra commute with each other as in Eqs. (1.1), anticommutators of the fermionic operators are linear combinations of the so(2,3) operators and commutators of the latter with the fermionic operators are their linear combinations. However, representations of the osp(1,4) superalgebra can be described exclusively in terms of the fermionic operators. The matter is that anticommutators of four operators form ten independent linear combinations. Therefore, ten bosonic operators can be expressed in terms of fermionic ones. This is not the case for the Poincare superalgebra since it is obtained from the so(2,3) one by contraction. One can say that the representations of the osp(1,4) superalgebra is an implementation of the idea that supersymmetry is the extraction of the square root from the usual symmetry (by analogy with the treatment of the Dirac equation as a square root from the Klein-Gordon one). From the point of view of the osp(1,4) supersymmetry, only four fermionic operators are fundamental, in contrast to the case when in dS and AdS symmetries there are ten fundamental operators.

As noted in Sec. 2.2, in the approach when a particle and its antiparticle belong to the same IR, it is not possible to define the concept of neutral particles. For example, a problem arises whether the photon is the elementary particle. In Standard Model (based on Poincare invariance) only massless particles are treated as elementary. However, as shown in the seminal paper by Flato and Fronsdal [20] (see also [21]), in standard AdS theory, each massless IR can be constructed from the tensor product of two singleton IRs and, as noted in [2], this property takes place also in FQT. The concept of singletons has been proposed by Dirac in his paper [22] titled "A Remarkable Representation of the 3 + 2 de Sitter group", and, as discussed in [2], in FQT this concept is even more remarkable than in SQT. As noted in Sec. 2.3, even the fact that the AdS mass of the electron is of the order of 10^{39} poses a problem whether the known elementary particles are indeed elementary. In [2] we discussed a possibility that only Dirac singletons are true elementary particles.

As explained in [2], in FQT, physical quantities can be only finite, divergences cannot exist in principle, and the concepts of particles, antiparticles, probability and additive quantum numbers can be only approximate if p is very large. The construction of FQT is one of the most fundamental (if not the most fundamental) problems of quantum theory.

The above discussion indicates that fundamental quantum theory has a very long way ahead (in agreement with Weinberg's opinion [23] that a new theory may be centuries away).

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