The Pionic Deuterium and the QCD Tetrahedron

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Abstract: We propose that QCD tetrahedrons mediate the strong force between hadrons in the nucleus by enabling double quark exchange reaction between protons and neutrons splitting the pionic deuterium ground state energy level to a doublet. The QCD tetrahedron generates with the deuterium a transition state complex that further performs quark exchange reactions with the charged pion π^- $(d\bar{u})$ that shifts and broadens the pionic deuterium ground state energy. The proposed charged pion exchange reactions generate a negatively charged cloud at the deuterium nucleus vicinity that reduces the effective charge of the proton. The proposed pion exchange reaction is an alternative mechanism to pion absorption and production by the nucleus at low energies.

Keywords: Pionic Deuterium ($\boldsymbol{\pi}$D), Yukawa interaction, QCD Tetrahedrons, Double-well potential.

1. The Yukawa Interaction

The Yukawa interaction[1] describes the strong force between hadrons mediated by pseudoscalar mesons pions. A Yukawa interaction describes the coupling between the Higgs field and massless quark and lepton fields. Through spontaneous symmetry breaking, the fermions acquire a mass proportional to the vacuum expectation value of the Higgs field. This Higgs-fermion coupling was first described by Steven Weinberg to model lepton masses[2]. The Yukawa interaction term has a single coordinate r that represents the distance between the interacting hadrons:

$$L_{Yukawa}(\Psi,\varphi) = -\,g\,\bar{\Psi}\,(r)\varphi(r)\,\Psi(r) \qquad (1)$$

Where g is a coupling constant, $\Psi(r)$ is the fermion field and $\varphi(r)$ is the pion field. The Yukawa classical potential is:

$$V(r) = -\,\frac{g^2\,e^{-\mu r}}{4\pi r} \qquad (2)$$

Where μ is the Yukawa particle mass that determines the exponential decay of the attractive interaction between the hadrons. In the next section we review briefly the pionic hydrogen and pionic deuterium systems before presenting the role of the QCD tetrahedron.

2. Pionic Deuterium Shift and Broadening

A pionic hydrogen system, $\pi^{-}p$, is an unstable hydrogen like atom, where the electron is replaced with the negatively charged pion, π^{-} $(d\bar{u})$. The Bohr radius of the pionic hydrogen is about 200 femtometer which is shorter than Bohr radius but still significantly larger than proton radius and hence the QCD interaction between the proton and the charged pion is expected to be small[3]. The pionic hydrogen system reveals the influence of the strong force by a negative shift and broadening of the low-lying atomic levels with respect to the pure electromagnetic interaction (QED).

For the pionic deuterium[3], the energy shift, ε_{1s}, is negative and is about -2.3 eV and the energy width, Γ_{1s} , is about 1 eV. The negative sign of the energy shift is explained as a repulsive interaction with the deuterium nucleus that reduces the QED attraction between the negative charged pion and the positively charged proton, and the width is explained by the pion absorption and production at low energies by the nucleus.

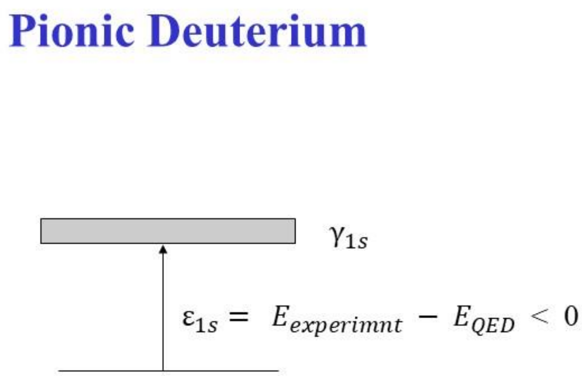

Figure 1 illustrates the ground state pionic deuterium shift and width of the order of -2.3 and 1eV, respectively. (Strauch et al[3], Figure 1 page 3).

In the next section we propose that QCD tetrahedrons mediate the strong force between hadrons in the nucleus by enabling double quark exchange reaction between protons and neutrons splitting the pionic deuterium ground state energy. In addition, the QCD tetrahedrons enable a pion exchange reaction with the deuterium nucleus that shifts and broadens the pionic deuterium ground state energy.

3. Pionic Deuterium and the QCD Tetrahedron

Inspired by the theory of Loop Quantum Gravity (LQG)[4], we proposed that the QCD exotic meson tetraquarks $u\tilde{d}d\tilde{u}$ introduced in previous papers[5,6,7,8] has a tetrahedron geometry and may populate the QCD ground state. We noted that pion π^0 comprised of a superposition of $d\tilde{d}$ and $u\tilde{u}$ mesons, may condense into a $u\tilde{d}d\tilde{u}$ tetrahedron geometry and having two chiral states[9] as shown below in equation 3 and Figure 2.

$$d\tilde{d} + u\tilde{u} \rightarrow u\tilde{d}d\tilde{u}\,(tetrahedron) \tag{3}$$

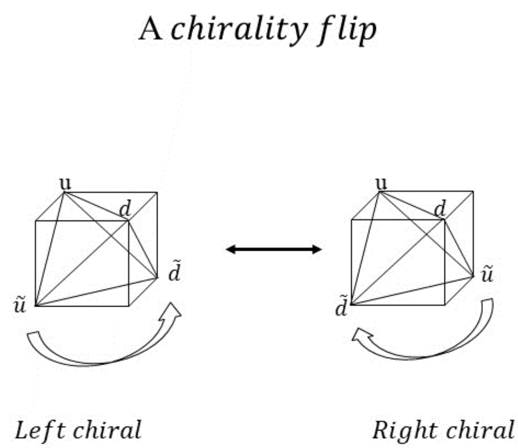

Figure 2 illustrates the $u\tilde{d}d\tilde{u}$ QCD tetrahedron with left and right chirality.

We propose here that in the case of the pionic deuterium, the Yukawa interaction is more complex and that a transition state complex is generated where 10 quarks take part and quarks are exchanged coherently. A double quark exchange reaction transforms the proton to a neutron and the neutron to a proton as shown below in Figure 3.

The quark exchange reaction is triggered by the tetrahedron anti-quarks $\tilde{d}$ *and* $\tilde{u}$ ($colored\ in\ black\ below$) that capture their quark pairs from the proton and the neutron d *and* u quarks and replace them with the tetrahedrons' d *and* u quarks (on the left hand side). The double exchange reaction on the right side occurs in the opposite direction transforming the neutron back to a proton and the proton to a neutron. The double exchange reaction is symmetric where the reactants and products are the same and only the four quarks exchanged their partners.

$$uud + u\tilde{d}d\tilde{u} + udd \quad \rightarrow \quad udd + u\tilde{d}d\tilde{u} + uud \qquad (4$$

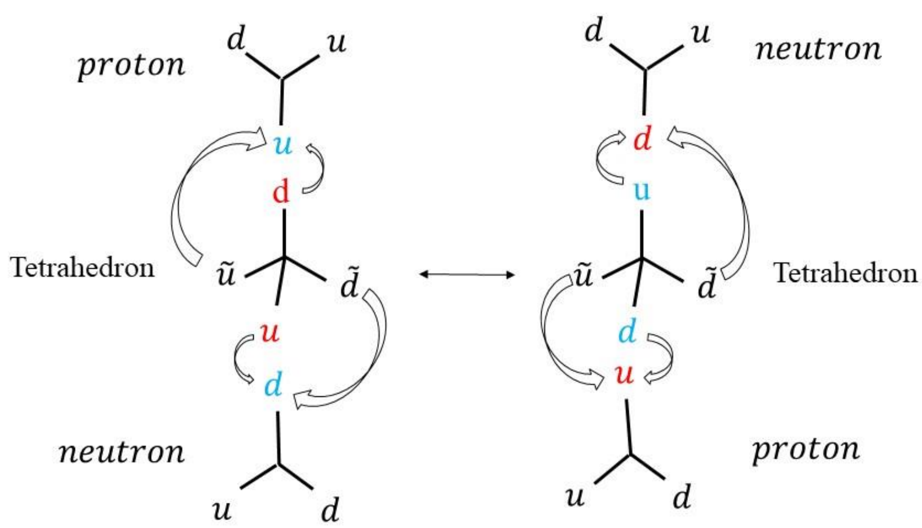

Figure 3 illustrates the double quark exchange reaction in deuterium nucleus mediated by a QCD tetrahedron.

The transition state complex includes the deuterium nucleus and the tetrahedron, overall 10 quarks and antiquarks. The transition state complex may have a specific geometry where the active reaction sites are the two anti-quarks. The potential surface is a many body potential that may be simplified by an effective one dimensional reaction coordinate double well shape as shown in Figure 4 below. Thus, the QCD tetrahedron (Td) plays the role pf the Yukawa interaction meson mediator that provides both the anti-quarks reagents and the two quarks that are exchanged. Note that the quarks and antiquarks numbers are conserved in the exchange reaction.

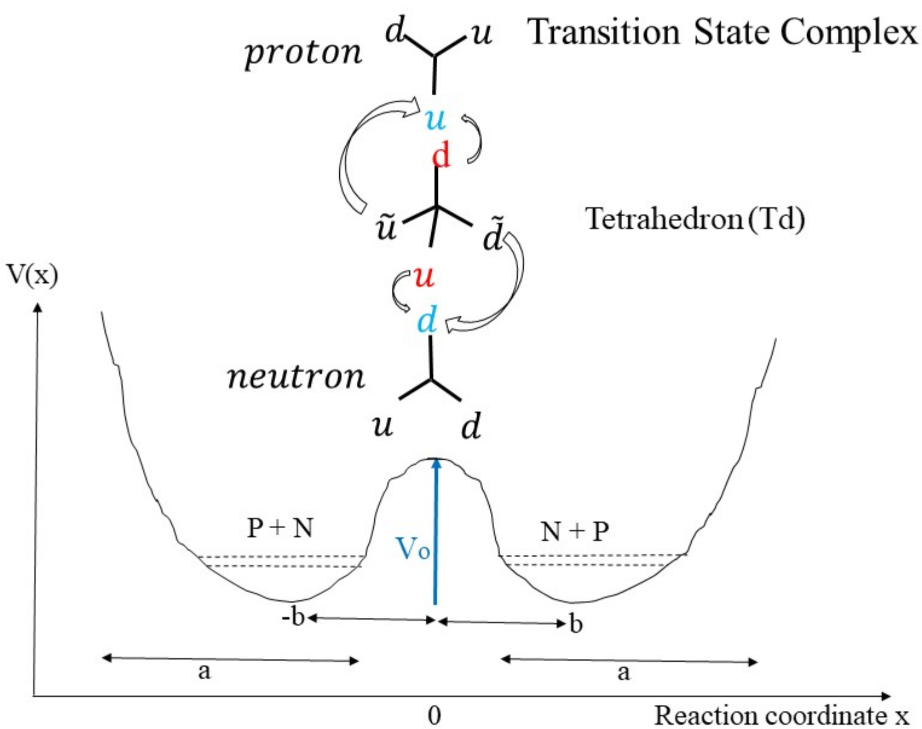

Figure 4 illustrates the double quark exchange reaction transforming protons (P) to neutrons (N) and vice versa mediated by a QCD tetrahedron (Td) forming the symmetric double well potential.

In addition to the double quark exchange reaction of equation 4, the charged pion π^{-} $(d\widetilde{u})$ is scattered by the nucleus and an exchange reaction occurs with the QCD tetrahedron, which is part of the pionic deuterium nucleus as shown below in equation 5-

$$uud + udd + u\tilde{d}d\widetilde{u} + d\widetilde{u} \rightarrow udd + uud + u\tilde{d}d\widetilde{u} + d\widetilde{u} \qquad (5)$$

The symmetric double well potential model, which was used to study the ammonia molecule inversion[10], splits the pionic deuterium energy levels including the ground state to symmetric and anti-symmetric doublets. The strong force coupling of the deuterium nucleus to the charged pion, $\pi^- \, (d\widetilde{u})$, shifts and broadens the measured ground state energy as shown in Figure 1 above. Strauch et al[3] estimated that the shift is negative, about -2.3 eV, reducing the QED attraction between the charged pion and the proton. The charged pion exchange reaction of equation 5 above generates a negatively charged cloud at the deuterium nucleus vicinity that reduces the effective charge of the proton. The proposed pion exchange reaction is an alternative mechanism to pion absorption and production by the nucleus at low energies[3].

The energy split of a double well potential model can be calculated numerically and it depends on 4 parameters, the potential barrier height, $\mathrm{V_0}$, two length parameters, a and b, shown in Figure 4 above, and the particle mass m. We used $a = 800$ fm for the well width and $b = \mathrm{b_{QCD-Td}} + \mathrm{a}/2$, where $\mathrm{b_{QCD-Td}}$ is the tetrahedron edge length of about 0.1 femtometer. Based on Strauch et al[3], we assume that Γ_{1s} value is 1 eV and is the double well potential split $E_0^a - E_0^s$, where E_0^s is the ground state symmetric energy and E_0^a is the anti-symmetric energy. The negative charge pion mass is m_π= 273 m_e, where m_e is the electron mass. We determined the potential barrier value $\mathrm{V_0}$ from the numerical solution of the double well potential model that matches the experimental value for the energy split Γ_{1s}=1eV as shown in figure 5 below.

$$E_0^a - E_0^s = \Gamma_{1s} = 1\ \mathrm{eV} \qquad (6)$$

The numerical solutions of two transcendental equations is obtained by solving the matching condition of the wavefunction values and derivatives on the barrier potential walls at x=b_{QCD-Td} and x=$-b_{QCD-Td}$ [10].

$$\tan(k_s a) = -\frac{k_s}{\sqrt{\alpha^2 - k_s{}^2}} \coth\left(\sqrt{\alpha^2 - k_s{}^2}\, b_{QCD-Td}\right) \quad (7a)$$

$$\tan(k_a \boldsymbol{a}) = -\frac{k_a}{\sqrt{\alpha^2 - k_a{}^2}} \tanh\left(\sqrt{\alpha^2 - k_a{}^2}\, b_{QCD-Td}\right) \quad (7b)$$

Where $\alpha^2 = \frac{2m_\pi V_0}{\hbar^2}$, $k_s{}^2 = \frac{2m_\pi E_0^s}{\hbar^2}$ and $k_a{}^2 = \frac{2m_\pi E_0^a}{\hbar^2}$.

Figure 5 shows the numerical solution of equations 7a-b . The difference between the symmetric and anti-symmetric solution, e.g. the ground state split is **Δ**E=1.0 eV with V_0~2.04 GeV. The calculated deuterium ground state energy in the double-well potential model is E_0^s=2150.943 eV.

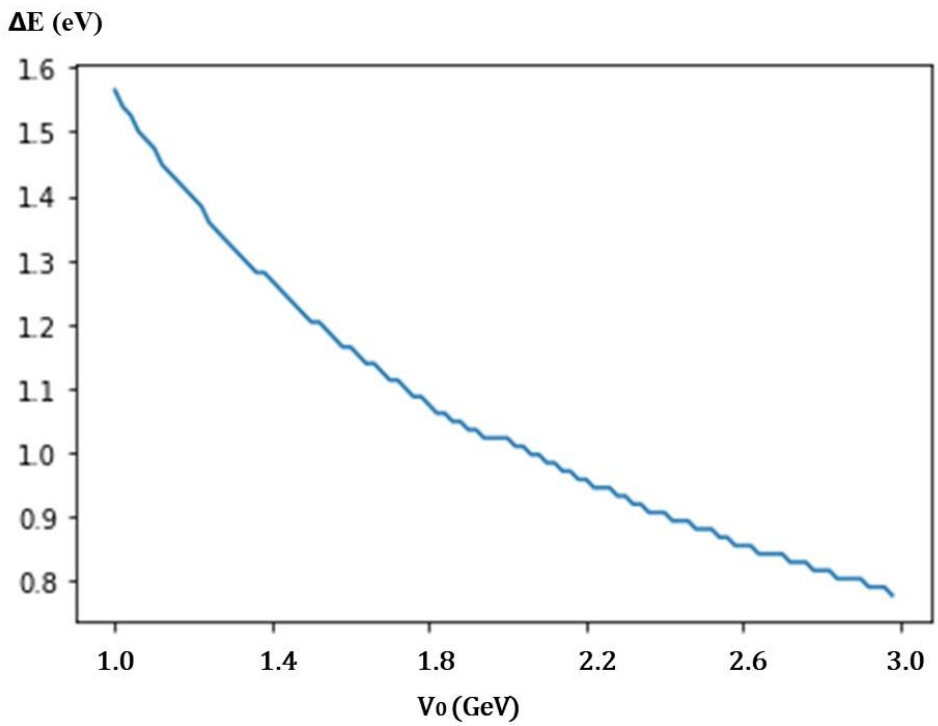

Figure 5 illustrates the double well potential energy split calculated numerically for each V0 value.

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