Exploring dark energy through gravity and its relationship with quantum mechanics

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Abstract

Modern physics requires a new gravity theory to explain relationship about gravity and quantum effects. However, quantum mechanics have an unsolved problem about the interpretation. Also, theory of relativity cannot explain dark energy. Currently, there is no theory that can comprehensively solve these problems. Here, as a result of reinterpreting the concept of reduced mass, this paper shows that quantum superposition is a collection of spacetimes for quanta according to gravitational potential. Spacetime which is absolute in relativity theory is relative in this theory, and it can be explained why the speed of light is

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independent from the inertial system. Additionally, because the classical mechanical energy conservation law is relative, as in the Casimir effect, a new conservation law for a quantum version is proposed. Dark energy can be explained by the theory that rest mass changes according to gravitational potential. Dark energy is an action along the dimension of gravitational potential, and its size increases as that of rest energy decreases. Dark energy affects quantum superposition and the asymptotic freedom of strong interaction can be explained similarly.

1. Introduction

Modern physics requires a new gravity theory[1]. Einstein resolved a gap between Newtonian mechanics and Maxwell's equations[2]. and he argued that the rest mass of a photon has to be zero according to relativity theory. On the other hand, according to the Yang Mills mass gap hypothesis, there is a minimum mass that cannot be reduced even though the mass is not zero[3]. In addition, according to Modern physics, it is argued that it is theoretically possible that photon mass is not zero[4]. However, many theories based on the assumption that the photon's mass is zero(like relativity theory) have explained phenomena well. To solve this problem, this paper proposes a new axiom about mass. Before that, some concepts are needed. Newton's law of universal gravitation explains the relationship between mass, gravitational potential, and distance[5]. And, there is a concept of reduced mass that easily expresses the central force motions[6]. A new relative velocity formula is created by using the fact that the relative speed of light is always the same and maximum in spacetime(4-dimensions) but is infinite in a spatial component of 4-vector which means the speed measured in proper time according to relativity theory[7]. Just as the speed of light is the maximum value that speed can have in the relativity theory, this paper proposes the axiom that a certain minimum mass exists and that minimum mass always be zero in time-space(4-dimensions) but has the same reduced mass in 5-dimensions. If this axiom is introduced, many predictions can be obtained.

2. A new gravity theory and consequences

2.1 A metric using the new axiom

First, for simplicity of discussion, only two dimensions are dealt with. And since the addition law of reduced mass is originally the sum of reciprocal numbers, in this paper, the mass is made the reciprocal number and the reciprocal number of the distance is defined as a character representing one-dimensional scalar curvature of a circle[8]. In addition, the gravitational constant is multiplied by the curvature to define the gravitational curvature. Through this, a new gravitational potential space metric corresponding to the Minkowski metric in the special relativity theory introduced[9]:

$$\phi = -\frac{Gm}{r} \Rightarrow \frac{1}{m}(-\phi) = \kappa \Rightarrow \frac{1}{m}\Phi = \kappa \ . \qquad (1)$$

($\kappa \equiv \frac{G}{r}$, $\Phi \equiv -\phi$, For convenience of calculation, the minus sign is omitted.)

$$ds^2 = -(cdt)^2 + (dr)^2 \text{ (Minkowski metric) (c is the speed of light.)}$$

$$\frac{1}{G}dr = d(\frac{r}{G}) \Rightarrow (d(\frac{s}{G}))^2 = -(c\frac{dt}{G})^2 + (d(\frac{r}{G}))^2$$

$$\Rightarrow ds^2 = dr^2 \text{ (if dt = 0) (space-like separated}^{10}\text{)}. \qquad (2)$$

(Assuming that there is no time change in Minkowski space-time, it becomes the same.)

$$\Rightarrow\ (d(\frac{G}{s}))^2\ =\ (d(\frac{G}{r}))^2\ =\ (d\kappa)^2\ \Rightarrow\ dK^2\ =\ -(\frac{1}{q}d\Phi)^2\ +\ (d\kappa)^2\ . \qquad (3)$$

(the new gravitational potential space metric)

(‘q’ is the minimum mass an object can have.)

(K is a scalar curvature of potential space dimension.)

2.2 A new transformation using the metric

The classical mechanics propose the principle of relativity about velocity[11]. Likewise, this paper suggests a principle of relativity about reduced mass. From now on, various theories will be developed, but since the logical process is mathematically the same as that presented in the relativity theory, the process of deriving formulas will be omitted. This is to avoid unnecessary duplication as the mathematical solution process in this paper is the same as that of the relativity theory. And, as Lorentz presented a new coordinate transformation different from the Galilean transformation[12], a new mass transformation system is created:

(Special relativity)

$$x' = \gamma(x - \beta ct),\ ct' = \gamma(ct - \beta x)\ (\beta = \frac{v}{c},\ \gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}})$$

$$\Rightarrow\ v_x\ =\ \frac{v' + v}{1 + \frac{v'v}{c^2}}\ (v_x\ =\ v'\ +\ v\text{: the classical law}), \qquad (4)$$

(In this paper)

$$\kappa' = \alpha(\kappa - \beta\ \frac{1}{q}\Phi),\ \frac{1}{q}\Phi' = \alpha(\frac{1}{q}\Phi\ - \beta\kappa)\ (\beta = \frac{q}{m},\ \alpha = \frac{1}{\sqrt{1-(\frac{q}{m})^2}}) \qquad (5)$$

$$\Rightarrow\ m_x\ =\ \frac{m'm+q^2}{m'+m} \qquad (6)$$

$(m_x = \frac{m'm}{m'+m}$: the original reduced mass addition law).

Through this(5), various physical predictions can be provided.

① New reduced mass formula

This paper created a reduced mass formula(6) that maintains the invariance of the minimum reduced mass.

② Expansion of potential difference

In the relativity theory, the faster the speed, the more time delay occurs[13]:

$$d\tau = \gamma^{-1}dt\ ,\ (d\tau^2 \equiv -\frac{ds^2}{c^2})\ (\tau \text{ is the proper time.})\ \tau = \gamma t\ . \tag{7}$$

Also, in this paper, potential difference expands as the mass of an object becomes lighter:

$$dU = \alpha^{-1}d\Phi,\ dU^2 \equiv -(qdK)^2 \ \ (\text{U is the proper potential in this paper.}). \tag{8}$$

This can be interpreted as violating the classical mechanical energy conservation law, but according to quantum mechanics, there is an experimental result which is called Casimir effect that looks like violating the classical energy conservation law in special cases[14].

③ curvature decrease

In this paper, the lighter the mass, the smaller the curvature of the object:

(Special relativity)

$$L^2 = (L')^2 - (c\Delta t)^2,\ t = \frac{vx}{c^2}\ (t' = \gamma(t - \frac{vx}{c^2}),\ t' = 0),\ \Delta t = \frac{vL'}{c^2} \Rightarrow L = L'\sqrt{1-\frac{v^2}{c^2}} \tag{9}$$

(Length contraction[15]),

(In this paper)

$$\kappa = \kappa'\sqrt{1-\frac{q^2}{m^2}}\ \text{(curvature decrease).} \tag{10}$$

Here, curvature means space-time, so space-time is relative. According to the relativity theory, space-time itself is absolute, but in this paper, space-time is also relative. In quantum mechanics, there is the many-world interpretation that makes a claim that spacetime splits into many branches[16] which means that there is no single space-time.

2.3 A reason for an absolute speed of light in relativity theory

Also, in the gravity theory, the acceleration due to gravity can be expressed as ($g = -\frac{GM}{r^2} = -\frac{GM}{s^2} = -G^{-1}M(\kappa)^2$) ($dt = 0$)[17], and for an observer with a large mass, the acceleration of the minimum mass due to gravity becomes zero, because of this curvature decrease(10). And according to the principle of equivalence of general relativity, since gravity and inertial force are essentially the same[18], it can be seen that the object with the minimum mass is not affected by the inertial force. Also, the object with the minimum mass appears to go straight because the curvature is zero. However, according to the general relativity theory, light can appear curved, which means that this curvature is flat based on a given space-time in a situation created by gravity, so space-time may not be flat when viewed by other observers.

2.4 Using a four-vector idea in this paper

The four-vector idea in special relativity can be utilized in this paper as well. For simplicity, it is used in only two dimensions[19]:

(Special relativity)

$$\text{Four-displacement vector: dx} = (\mathrm{dt}, \mathrm{dx}), \tag{11}$$

$$\text{Four-velocity: } \mathrm{u} = (\mathrm{u}^t,\ \mathrm{u}^x) = (\frac{\mathrm{dt}}{\mathrm{d\tau}}, \frac{\mathrm{dx}}{\mathrm{d\tau}}) = (\gamma, \gamma\mathrm{v}), \tag{12}$$

(In this paper)

$$\frac{1}{\mathrm{w}} = ((\frac{1}{\mathrm{w}})^{\Phi}, (\frac{1}{\mathrm{w}})^{\kappa}) = (\frac{\mathrm{d}\Phi}{\mathrm{dU}}, \frac{\mathrm{d}\kappa}{\mathrm{dU}}) = (\alpha, \frac{\alpha}{\mathrm{m}}) \tag{13}$$

$$\Rightarrow \frac{1}{\mathrm{w}} \cdot \frac{1}{\mathrm{w}} = -\mathrm{q}^{-2}\alpha^2 + (\frac{\alpha}{\mathrm{m}})^2 = -(\frac{\alpha}{\mathrm{q}})^2(1-(\frac{\mathrm{q}}{\mathrm{m}})^2) = -\mathrm{q}^{-2}(\text{transformation invariance}). \tag{14}$$

Since the potential metric(3) dealt with in this paper does not assume a change in time(2), the spatial dimension, the curvature dimension, can be viewed as the same as the 4-dimensional space-time dimension. ($(\mathrm{d}\kappa)^2 = (\mathrm{d}(\frac{\mathrm{G}}{\mathrm{s}}))^2$, s is the four-dimensional distance in special relativity.) Since the mass of a photon converges to 0 in this dimension ($\alpha = (1 - (\frac{\mathrm{q}}{\mathrm{m}})^2)^{-(\frac{1}{2})} = \infty \Rightarrow \frac{\alpha}{\mathrm{m}} = \infty$), the interpretation that the minimum mass is 0 was derived from the relativity theory dealing with only 4-dimensions(time-space).

3. Generalization of the theory

3.1 A new interpretation of Mass-Energy Equivalence

For the case where the mass changes, another assumption corresponding to the mass equivalence principle in the relativity theory is needed[20]. In gravity, mass is the source of change in the velocity of an object. Similarly, this theory requires a source that changes mass. Einstein's $E^2=(mc^2)^2$ represents the relationship between mass and energy, where E means the binding energy of matter[21]. In this equation, energy can determine the degree of mass.

For generalization of the theory, it has to be understood further the meaning of $E^2=(mc^2)^2$ mentioned above. For that, Einstein's field equation is needed. According to Einstein's field equation ($G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$)[22], which the left side (Einstein tensor: $G_{\mu\nu}$) represents the degree of curvature of space, and the Energy stress tensor of the right side ($T_{\mu\nu}$) represents the distribution of energy. The two are linked by Einstein's gravitational constant ($\frac{8\pi G}{c^4}$). In other words, Einstein's gravitational constant represents the degree to which space is curved per unit energy.

Also, universal gravitation has to be looked at. Of course, the law requires two masses. However, it is described as if there is one mass by introducing a scalar potential from universal gravitation and creating a field equation. In addition, the escape velocity formula for gravity can be known as ($v^2 = \frac{2GM}{r} = -2\phi$)[23].

Likewise, $E^2=(mc^2)^2$ describes the interaction of two energies as if there is one energy. The above equation will be used in this paper to express the relationship between mass and energy, which corresponds to the relationship between velocity and mass in the escape velocity formula.

More precisely, velocity corresponds to the reciprocal of mass, and gravitational mass corresponds to the reciprocal of the square of the binding energy. What corresponds to the distance is the curvature. Escape velocity refers to the minimum velocity at which the distance can be infinitely far, and $E^2=(mc^2)^2$ means the mass at which the curvature can increase infinitely. An infinite increase in curvature means the same thing as closer infinitely, and this means that two objects can be considered as one object. Therefore, it can be said that this equation corresponds to the escape velocity formula. Also, the curvature is used as $\frac{8\pi G}{c^4}$, because $\frac{8\pi G}{c^4}$ is used as the Einstein gravitational constant in Einstein's field equation. In the relativity theory, since the speed of light is constant, this unit curvature is constant, but in this paper, it can be seen that this curvature is the same as the curvature(κ) in the previous metric, because the speed of light can vary according to the relationship between binding energy and potential energy of this new action. In cosmology, recession velocities of expanding universe which are faster than the speed of light do not violate special relativity[24].

It can be known that the square of the velocity is -2 times the scalar potential of gravitation. That is, the scalar potential could be obtained through the square of the velocity. Through this, the scalar potential of the action that causes the change in mass (this paper will explain it as the Energy force for convenience) is as follows. **(**$\left(\frac{1}{m}\right)^2 = \frac{c^4}{E^2} \equiv -2V$, V is the scalar potential of Energy force.**)**

Just as the Poisson equation for the gravitational field was created by taking the Laplacian of the scalar potential of universal gravitation[25], it must be also created the corresponding field equation for the classical Energy force. Expressed as an expression, it is:

$$V = -\frac{1}{2E^2}c^4 = -\frac{4\pi G(E^{-2})}{\kappa} \ \left(\kappa \equiv \frac{8\pi G}{c^4}\right). \tag{15}$$

$$\text{(In gravity, } \phi = -\frac{GM}{r}, \quad \nabla\ \phi = -\frac{GM}{r^2}) \ (\nabla = \frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}), \tag{16}$$

$$\nabla^2\phi = 4\pi G\mu \ (\mu\text{: mass density}) \tag{17}$$

($dV = r^2 \sin\theta \, dr d\theta d\varphi$) (V: Volume expressed using radius r).

$$(\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\ ^{26}.) \quad (18)$$

$$(\text{Likewise, } V = -\frac{4\pi G(E^{-2})}{\kappa},\ \nabla V = -\frac{4\pi G(E^{-2})}{(\kappa)^2}\ ) \quad (19)$$

$$(\nabla = \frac{\partial}{\partial\kappa} + \frac{1}{\kappa}\frac{\partial}{\partial\theta} + \frac{1}{\kappa\sin\theta}\frac{\partial}{\partial\varphi}),$$

$$\nabla^2 V = 16\pi^2 G\varepsilon \ (\varepsilon\text{: density of } E^{-2} \text{ using curvature } \kappa). \quad (20)$$

$$(dL = \kappa^2 \sin\theta\, d\kappa d\theta d\varphi)\ (L\text{: Volume expressed using curvature } \kappa.) \quad (21)$$

$$(\nabla^2 = \frac{1}{\kappa^2}\frac{\partial}{\partial\kappa}(\kappa^2\frac{\partial}{\partial\kappa}) + \frac{1}{\kappa^2 \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{\kappa^2 \sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\ .) \quad (22)$$

This Energy force is the repulsive force of the energy itself, and this physical action seems to be related to a phenomenon called dark energy, the cause of which was unknown in the existing cosmology[27]. This is because Energy force gets stronger while curvature gets smaller(Time-space gets bigger) like dark energy which is accelerating expansion. Plus, this repulsive force can affect superluminal phenomena like dark energy. In this paper, this Energy force can be helpful for understanding quantum mechanics, because Energy force can change a mass and create an orbit of space-like separated masses through gravitational potential in the same time(dt = 0). This can be related to quantum superposition in quantum mechanics[28]. Just as the change in space with time is small when the speed is slow, the change in space-time according to gravitational(mechanical) potential is small when the mass is large, so this superposition is insignificant. It is interpreted that this phenomenon is related to Schrödinger's cat in quantum mechanics[29]. Because of the difference in strength of the repulsive forces depending on the magnitude of bind energy(E), this phenomenon can be an answer to the Cosmological constant problem which is about the gap of strength between dark energy in cosmology and vacuum energy in quantum mechanics[30]. Also, this repulsive force is assumed to be related to a repulsive force of electric charge[31].

3.2 New equivalence theorem

In the gravity theory, the gradient of gravitational potential equals the gravitational acceleration($\nabla\ \phi = -\ \frac{GM}{r^2} = g = a = \frac{dv}{dt}$). Likewise, the gradient of the new action above can be said to be equal to the change in a reciprocal of mass with respect to gravitational potential in that action($\nabla V = -\frac{4\pi G(E^{-2})}{(\kappa)^2} = \frac{dm^{-1}}{d\Phi}$). In addition, as momentum and kinetic energy are defined in classical mechanics, new physical quantities corresponding to them can be defined. Likewise, just as the mechanical work of gravity is obtained from the difference in gravitational potential energy, the concept of new work can be defined by the difference in potential energy of the new force:

$$(p = mv,\ E_k = \frac{1}{2}mv^2 \Rightarrow Q \equiv \frac{1}{E^2}\frac{1}{m}\ ,\ A_k \equiv \frac{1}{2}\frac{1}{E^2}(\frac{1}{m})^2)$$

$$(W = U - U_0 \Rightarrow W_e \equiv U_e - U_{e_0}), \tag{23}$$

$$(U = -\frac{GMm}{r} : \text{gravitational potential energy})$$

$$(U_e \equiv -\frac{4\pi G(E_1{}^{-2})(E_2{}^{-2})}{\kappa} : \text{Energy force potential energy}). \tag{24}$$

(The meaning of these new physical quantities will be explained later in this paper.)

However, just as mass in classical mechanics is inertial mass and is different from gravitational mass, the energy contained in these new physical quantities is also different from the binding energy in $E^2=(mc^2)^2$. So, the meaning of energy in physical quantities is needed to know about.

In the gravity theory, there is E in the classical elliptical orbital equation of universal gravitation ($\frac{1}{r} = \frac{1}{r_0} + \cos\theta \sqrt{\frac{1}{r_0^2} + \frac{2\mu E}{l^2}}$, r_0 is a radius of circular orbit, μ is a reduced mass, and l is a constant angular momentum.)[32], and this E (mechanical energy) determines the shape of the orbit[33]. This paper proposes a new energy equivalence principle that the previous two energies are indistinguishable from each other. Corresponding to the velocity in the relativity theory is the reciprocal of mass in this theory, and corresponding to mass is the reciprocal of the square of energy. The reason why the logic is developed by using the square of the energy is that the mechanical energy must be negative for the two masses to maintain the orbit in the universal gravitational elliptical orbit equation. (When the energy is or exceeds 0, a parabolic or hyperbolic trajectory is formed, so a periodic motion is not formed.)

As it is impossible to distinguish between gravity and inertial force in the equivalence principle of relativity theory, a new inertial action corresponding to the new force is needed in this paper. Just as an object can experience pseudo-gravity through the acceleration of a system(frame) to which the object belongs, this thesis suggests that a phenomenon similar to a new force can be experienced through a change in the mass of a system. As an example of the classic inertial force, if the speed of a bus moving at constant velocity is reduced, the passengers inside the bus feel a fictitious force called inertial force. Likewise, if a bus with a certain mass grows, the passengers on the bus will think that they have become smaller. This is the new inertial action proposed in this paper. Through this, a new physical quantity corresponding to the force of classical mechanics is presented, which is as follows: $F_e \equiv \frac{1}{E^2}\frac{dm^{-1}}{d\Phi}$. Also, this new physical quantity has the following relationship with the other physical quantities mentioned above:

$$F_e \equiv \frac{dQ}{d\Phi},\ F_e \equiv \frac{dA_k}{d\kappa}\ (\kappa\text{: gravitational curvature}(\frac{G}{r})). \qquad (25)$$

(Q is named as mass-orbital quantity in this paper.) (A is named mass-orbital energy in this paper.)

Looking more closely at Q(23), Q corresponds to the momentum of classical mechanics, and the reason why momentum is defined is because velocity is a term that is difficult to explain without the concept of an object with mass. Likewise, in this paper, mass is a term that is hard to define without energy. This is because mass is determined by gravitational potential and curvature, and potential energy is difficult to explain without the concept of mechanical energy. Just as Einstein's general relativity theory presented a new explanation for gravity through the equivalence about mass, this paper explains a new theory that replaces $E^2 = (mc^2)^2$ using these new axioms. First of all, this paper proposes a law corresponding to mass-energy equivalence in the special relativity theory. As Einstein defined the 4-dimensional momentum and kinetic energy for discussion[34], new physical quantities corresponding to quantities of classical mechanics must be defined in this paper as well.

$$Q = \frac{1}{E^2}\frac{1}{m} = \frac{1}{E^2}\frac{1}{\mu},\ A_k = \frac{1}{2}\frac{1}{E^2}(\frac{1}{m})^2 = \frac{1}{2}(\frac{1}{mE})^2 = \frac{1}{2}(\frac{1}{\mu E})^2$$

(In $\frac{1}{r} = \frac{1}{r_0} + \cos\theta\sqrt{\frac{1}{r_0^2} + \frac{2\mu E}{l^2}}$, μE is in the shape of the orbit in $\frac{1}{r} = \frac{1}{r_0} + \cos\theta\sqrt{\frac{1}{r_0^2} + \frac{2\mu E}{l^2}}$.)

(μ is m in this paper because m obeys the reduced mass addition law.)

This new physical quantity Q corresponds to the momentum in the special relativity theory:

(a momentum in special relativity)

$$u = (u^t,\ u^x),\ p \equiv mu,\ \ p\ = (\gamma m,\ \gamma mv) = (p^t,\ p^x), \tag{26}$$

(mass-orbital quantity in this paper)

$$Q = (Q^{\Phi},\ Q^{\kappa}) = (\frac{\alpha}{E^2},\ \frac{\alpha}{mE^2}). \tag{27}$$

In special relativity, the closer an object's speed to the speed of light, the greater its momentum. In relativity, this is interpreted as an increase in mass. Likewise, in this paper, as the mass approaches the minimum mass, Q increases. Thus, E^{-2} increases. As said previously, there is an experimental result that the classical energy conservation law may not hold under certain circumstances, so this paper can also use the conclusion that energy can be relative. E^2 stands for the binding energy, and when the binding energy is weakened, it means that the substance is decomposed. However, since the minimum mass cannot get any lighter, E^{-2} of the minimum mass is 0 in 5-dimensions. Also, the new equivalence can be obtained through the same mathematical process in which the energy formula was derived by taking the Lorentz invariance of the momentum in the relativity theory for obtaining mass-energy equivalence:

$$Q = (Q^{\Phi},\ Q^{\kappa}),$$

$$Q \cdot Q = \frac{1}{wE^2} \cdot \frac{1}{wE^2} = (\frac{\alpha}{E^2},\ \frac{\alpha}{mE^2}) \cdot (\frac{\alpha}{E^2},\ \frac{\alpha}{mE^2}),$$

$$\alpha = \left(1 - (\frac{q}{m})^2\right)^{-(\frac{1}{2})} \approx 1 + \frac{1}{2}(\frac{q}{m})^2 + \cdots\ (q << m),$$

$$Q^{\Phi} = \frac{1}{E^2} + \frac{1}{2}\frac{1}{E^2}(\frac{q}{m})^2 + \cdots \approx q^2(\frac{1}{(qE)^2} + \frac{1}{2(mE)^2}) = q^2(A_0 + A_k) = q^2 A_{total}\ ,$$

$$Q^{\kappa} = \frac{1}{mE^2} + \frac{1}{2}\frac{1}{mE^2}(\frac{q}{m})^2 + \cdots \approx \frac{1}{mE^2} = Q,$$

$$Q \cdot Q = -q^{-2}(q^2 A_{total})^2 + Q^2 = -(\frac{1}{qE^2})^2 \Rightarrow A^2 = (\frac{1}{E^2})^2(\frac{1}{q})^4 + (\frac{1}{q}Q)^2$$

$$\Rightarrow \ A^2 = (\frac{1}{E^2})^2(\frac{1}{q})^4 \ \ (\text{if } \ m^{-1} \approx 0). \tag{28}$$

(The mass is extremely large compared to the smallest mass, so m^{-1} can be considered zero.)

$E = mc^2$ means that mass itself (without velocity) can be converted into energy that can do mechanical work such as work by gravity. Likewise, $A^2 = (\frac{1}{E^2})^2(\frac{1}{q})^4$ means that Energy itself(E^{-2}) (with too large mass to move, compared to photon) can be converted into mass-orbital energy(A) that can be turned into work by Energy force.

3.3 Mathematical derivation of general theory

In general relativity, Einstein devised the Einstein tensor using non-Euclidean geometry and created the Einstein Field Equation[35]. In this paper, a field equation for the Energy force must also be created, and since the mathematical logic process of this paper is the same as that of the general relativity theory, Einstein tensor can be used. However, Einstein expressed the spatial dimension in a Cartesian coordinate system, but in this paper, the spatial(time-space) dimension is expressed in a spherical coordinate system, and the dimension representing the distance in the spherical coordinate system is expressed as a gravitational curvature. (A spherical coordinate system expresses a position in terms of the radius of a sphere and two angles. However, in the spherical coordinate system, even if the curvature of a circle, which is its reciprocal, is used instead of the radius, there is no problem in displaying the position, so it is essentially the same as the original spherical coordinate system.) In addition, the gravitational curvature(1) is equal to the Einstein gravitational constant. Unlike the Minkowski metric, which expresses space-time in the relativity theory, here, potential(positive) is used instead of time in the Einstein tensor because it expresses the metric of universal gravitational potential and space(time-space). Even if the coordinate system is expressed differently, the derived physical phenomena are the same, so there is no theoretical problem. (Derivations of mathematical theorization (such as Einstein tensor, Geodesic) are omitted in this paper. Mathematical derivations of General relativity used in this paper is in the reference ([35]~[46]).):

$$\text{General relativity: } G_{\mu\nu}(t, x, y, z) = G_{\mu\nu}(t, r, \theta, \varphi), \tag{29}$$

$$\text{This paper: } G_{\mu\nu}(\Phi, \kappa, \theta, \varphi) \equiv C_{\mu\nu} \text{ (about Energy force)}, \tag{30}$$

$$\text{Einstein gravitational field equation: } G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} , \tag{31}$$

$$T_{\mu\nu}\text{: Energy-Momentum Tensor } (T_{tt} = \rho c^2\text{: energy density})^{36}, \tag{32}$$

ρ: mass density.

Einstein extended the Cauchy stress tensor to 4-dimensions to create a 4-dimensional energy-momentum tensor. To this end, terms of energy density, momentum density, and energy flux were added to the Cauchy tensor. (At this time, the momentum density and the energy flux are the same, because energy-momentum tensor is symmetric[37].) In this paper, a new tensor(30) corresponding to this tensor can be created. However, unlike Einstein who introduced the Cartesian coordinate system, this paper uses a spherical coordinate system and uses gravitational potential and curvature instead of time and distance. As new concepts were introduced in Einstein's tensor extension, there are corresponding concepts in the new tensor in this paper: mass-orbital energy density, mass-orbital energy flux, mass-orbital quantity density. Also, Mass-Orbital-Energy-Momentum Tensor is also symmetric like energy-momentum tensor in general relativity theory.

From this, the field equation for the Energy force is:

$$C_{\mu\nu} = kH_{\mu\nu} , \quad (33)$$

$H_{\mu\nu}$: Mass-Orbital-Energy-Momentum Tensor,

$$H_{\Phi\Phi} = \varepsilon q^{-2}\text{: mass-orbital energy density} \quad (34)$$

(ε: E^{-2} density using curvature κ (20))

Just as Einstein found the constant that mediates the two terms through the classical limit, the constant must be obtained through a similar process in this paper. Einstein assumed a special case to find a constant[38], and this paper also adopts a fairly similar assumption:

General relativity:

$$① \ \frac{dx^t}{d\tau} >> \frac{dx^i}{d\tau} \ (i = r, \ \theta, \ \varphi) \quad ② \ \frac{\partial g_{\mu\nu}}{\partial x^t} \approx 0$$

$$③ \ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ (h_{\mu\nu} << 1, \ \eta_{\mu\nu}\text{: Minkowski metric tensor}), \quad (35)$$

This paper:

$$① \ \frac{dx^\Phi}{dU} >> \frac{dx^i}{dU} \ (i = \kappa, \ \theta, \ \varphi), \quad (36)$$

$$② \ \frac{\partial g_{\mu\nu}}{\partial x^\Phi} \approx 0 , \quad (37)$$

$$③\ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

(38)

($h_{\mu\nu} \ll 1$, $\eta_{\mu\nu}$: the new flat gravitational potential space metric),

$$-\frac{1}{2}\nabla^2 h_{\Phi\Phi} = \frac{1}{2}kH_{\Phi\Phi},\ h_{\Phi\Phi} = -2q^2V\ , \tag{39}$$

$$q^2\nabla^2V = \frac{1}{2}kH_{\Phi\Phi}\ (\nabla^2V = 16\pi^2G\varepsilon,\ H_{\Phi\Phi} = \varepsilon q^{-2}),$$

$$k = 32\pi^2Gq^4\ , \tag{40}$$

$$C_{\mu\nu} = 32\pi^2Gq^4H_{\mu\nu}\ . \tag{41}$$

4. Applications of the general theory

4.1 New metric using general theory

Now, just as Einstein's field equation was solved in the general relativity theory assuming Schwarzschild metric which is spherically symmetric[39], this equation will be solved through the same mathematical solution process as Schwarzschild's:

Schwarzschild metric:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + (\sin\theta)^2d\varphi^2)$$

$$= -c^2(1-\frac{2GM}{rc^2})dt^2 + (1-\frac{2GM}{rc^2})^{-1}dr^2 + r^2(d\theta^2 + (\sin\theta)^2d\varphi^2), \tag{42}$$

($r_s = \frac{2GM}{c^2}$:Schwarzschild radius[40])

Spherical symmetric potential space metric:

$$dK^2 = -B(\kappa)d\Phi^2 + A(\kappa)d\kappa^2 + (\kappa)^2(d\theta^2 + (\sin\theta)^2d\varphi^2)$$

$$= -q^{-2}(1-\frac{8\pi Gq^2}{\kappa E^2})d\Phi^2 + (1-\frac{8\pi Gq^2}{\kappa E^2})^{-1}d\kappa^2 + (\kappa)^2(d\theta^2 + \tag{43}$$

$$(\sin\theta)^2 d\varphi^2).$$

$$(\kappa_S \equiv \frac{8\pi G q^2}{E^2})$$

In this way, a potential metric similar to the Schwarzschild metric was obtained, and various physical facts can also be inferred through this. In the field created by weak binding energy, potential difference expands and curvature decreases. This is mathematically described as follows[41]:

$$\text{General relativity: } d\tau = dt\sqrt{1-\frac{2GM}{rc^2}}\ , \ L = L'\sqrt{1-\frac{2GM}{rc^2}}\ , \tag{44}$$

$$\text{This paper: } dU = d\Phi\sqrt{1-\frac{8\pi G q^2}{\kappa E^2}}\ , \ \kappa = \kappa'\sqrt{1-\frac{8\pi G q^2}{\kappa E^2}}\ . \tag{45}$$

4.2 A geodesic equation and its applications

In this paper, mathematical geodesic equations in the Energy force can be obtained. This is as follows[42]:

$$\text{General relativity: } \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0,$$

$$\text{This paper: } \frac{d^2x^\mu}{dU^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dU}\frac{dx^\beta}{dU} = 0\ . \tag{46}$$

In addition, if the spherical symmetric potential geodesic equation is obtained through the same mathematical process as obtaining the geodesic equation in the Schwarzschild metric[43], it is as follows:

Schwarzschild geodesic equation solution in general relativity:

$$\varphi= \pm\int \frac{dr\sqrt{(1-\frac{2GM}{rc^2})^{-1}}}{r^2}\frac{1}{\sqrt{\frac{1}{J^2\{-c^2(1-\frac{2GM}{rc^2})\}}-\frac{E}{J^2}-\frac{1}{r^2}}}\ , \tag{47}$$

$$(J = r^2\frac{d\varphi}{dt}, -E = (1-\frac{2GM}{rc^2})^{-1}(\frac{dr}{dt})^2 + \frac{J^2}{r^2} - \frac{1}{\{-c^2(1-\frac{2GM}{rc^2})\}}) \tag{48}$$

(J, E: constants),

geodesic equation solution in this paper:

$$\varphi= \pm\int \frac{d\kappa\sqrt{(1-\frac{8\pi Gq^2}{\kappa E^2})^{-1}}}{(\kappa)^2}\frac{1}{\sqrt{\frac{1}{J^2\{-q^{-2}(1-\frac{8\pi Gq^2}{\kappa E^2})\}}-\frac{A}{J^2}-\frac{1}{(\kappa)^2}}}\ . \tag{49}$$

$$(J = (\kappa)^2\frac{d\varphi}{d\Phi}, -A = (1 - \frac{8\pi Gq^2}{\kappa E^2})^{-1}(\frac{d\kappa}{d\Phi})^2 + \frac{J^2}{(\kappa)^2} - \frac{1}{\{-q^{-2}(1-\frac{8\pi Gq^2}{\kappa E^2})\}}) \tag{50}$$

(J, A: constants).

The general relativity theory explains the sustained orbits of matter slower than the speed of light and the orbits of light that are not confined by gravity of ordinary matter except a black hole[44](like precession of Mercury). Similarly, in this paper, the maintained orbit of a material with a mass larger than that of a photon and the orbit of a photon that escapes the influence of repulsive Energy force are introduced:

To find the angle of a bound orbit,

(General relativity)

$$\Delta\varphi = \frac{3\pi r_s}{(1-e^2)a} = \frac{6\pi GM}{(1-e^2)ac^2} = \frac{6\pi GM}{c^2L} \tag{51}$$

(e: eccentricity, a: semi-major axis),

The semi latus rectum L

$$\frac{1}{L} \equiv \frac{1}{2}\left(\frac{1}{r_+} + \frac{1}{r_-}\right) = \frac{1}{(1-e^2)a} \quad (r_+ = (1+e)a \ , \ r_- = (1-e)a), \tag{52}$$

(This paper)

$$\Delta\varphi = \frac{3\pi\kappa_s}{(1-e^2)a} = \frac{24\pi^2 Gq^2}{(1-e^2)aE^2} = \frac{24\pi^2 Gq^2}{E^2 K} \tag{53}$$

(e: eccentricity, a: semi-major axis),

$$\frac{1}{K} \equiv \frac{1}{2}\left(\frac{1}{\kappa_+} + \frac{1}{\kappa_-}\right) = \frac{1}{(1-e^2)a} \tag{54}$$

$$(\kappa_+ = \frac{8\pi G}{(c_+)^4} = (1+e)a \ , \ \kappa_- = \frac{8\pi G}{(c_-)^4} = (1-e)a).$$

To find the angle of an unbound orbit of light,

(General relativity)

$$\Delta\varphi = \frac{2r_s}{r_0} = \frac{4GM}{r_0 c^2} \tag{55}$$

(r_0: the closest distance of light to an object, r_s: Schwarzschild radius of the object),

(This paper)

$$\Delta\varphi = \frac{2\kappa_s}{\kappa_0} = \frac{16\pi Gq^2}{\kappa_0 E^2} \tag{56}$$

(κ_0:the smallest curvature(inverse to distance) of photon to an object,

κ_s:Curvature limit of the object).

4.3 Curvature limit

Meanwhile, the Schwarzschild radius can be obtained from the Schwarzschild metric in General relativity, and the corresponding curvature (or radius) can also be obtained from the spherical symmetric potential metric(43). Expressed as an expression, it is:

$$\kappa_S = \frac{8\pi G q^2}{E^2} = \frac{8\pi G}{c_S{}^4} \Rightarrow c_S = \sqrt{\frac{|E|}{q}} \ . \qquad (57)$$

This curvature is proportional to E^{-2}, and as the energy increases, the curvature decreases. In other words, the radius increases, which is a formula that expresses the limit to which space can expand. And it is a formula that can determine the value of the speed of light. However, when E^{-2} becomes 0, the radius can grow infinitely, which means that the volume can expand infinitely. In other words, if space expands infinitely, it is due to the binding energy of matter with a mass that can no longer be reduced(the binding energy of photon). And, it can be seen that the speed of light at that time is infinite. (This is like the logic that the mass must be zero for the Schwarzschild radius to be zero in general relativity.)

4.4 Energy force wave

In general relativity, the existence of gravitational waves is derived. With the same mathematical logic development, the change in space created by energy can derive a similar pseudo-wave equation of gravitational potential and time-space. Mathematically expressed[45]:

(Gravitational wave equation in general relativity)

$$\Box^2 h_{\mu\nu} = 0, \quad \frac{\partial}{\partial x^\mu} h^\mu{}_\nu = \frac{1}{2}\frac{\partial}{\partial x^\nu} h^\mu{}_\mu \ \left(\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\ ^{46}\right) \qquad (58)$$

$$(\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}),$$

(pseudo-wave equation in this paper)

$$\Box^2 h_{\mu\nu} = 0, \quad \frac{\partial}{\partial x^\mu} h^\mu{}_\nu = \frac{1}{2}\frac{\partial}{\partial x^\nu} h^\mu{}_\mu \quad (\Box^2 = \nabla^2 - q^2 \frac{\partial^2}{\partial \Phi^2}). \tag{59}$$

$$(\nabla^2 = \frac{1}{\kappa^2}\frac{\partial}{\partial \kappa}(\kappa^2 \frac{\partial}{\partial \kappa}) + \frac{1}{\kappa^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial}{\partial \theta}) + \frac{1}{\kappa^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2} .) \tag{60}$$

5. New quantum-scale laws in the 5-dimensional world

5.1 Theoretical tools for deriving new equations

Physicists derived the concept of gravito-electromagnetism in a weak gravitational field. Then, an equation describing gravity and gravitational magnetism was obtained, which is as follows in the CGS(Centimetre-gram-second) system[47]:

$$\mathrm{div} E_g = \nabla \cdot E_g = 4\pi G \rho_g, \ \mathrm{div} B_g = 0,$$

$$\mathrm{rot}\ E_g = \nabla \times E_g = -\frac{1}{c}\frac{\partial B_g}{\partial t}, \ \mathrm{rot}\ B_g = \frac{4\pi G \xi}{c} J_g + \frac{1}{c}\frac{\partial E_g}{\partial t} , \tag{61}$$

E_g: gravito-electric field intensity,

B_g: gravito-magnetic induction,

ρ_g: mass density,

ξ: gravito-magnetic permeability of the medium,

J_g: mass current density,

c: speed of light (maximum speed).

$$(\nabla \cdot \mathrm{E}_g = \frac{1}{\mathrm{r}^2}\frac{\partial(\mathrm{r}^2\mathrm{E}_{g_r})}{\partial \mathrm{r}} + \frac{1}{\mathrm{r}\sin\theta}\frac{\partial}{\partial\theta}(\mathrm{E}_{g_\theta}\sin\theta) + \frac{1}{\mathrm{r}\sin\theta}\frac{\partial \mathrm{E}_{g_\varphi}}{\partial\varphi}, \quad (62)$$

$$\nabla \times \mathrm{E}_g = \frac{1}{\mathrm{r}\sin\theta}\left(\frac{\partial}{\partial\theta}(\mathrm{E}_{g_\varphi}\sin\theta) - \frac{\partial \mathrm{E}_{g_\theta}}{\partial\varphi}\right)\hat{\mathrm{r}}$$

$$+ \frac{1}{\mathrm{r}}\left(\frac{1}{\sin\theta}\frac{\partial \mathrm{E}_{g_r}}{\partial\varphi} - \frac{\partial}{\partial \mathrm{r}}(\mathrm{r}\mathrm{E}_{g_\varphi})\right)\hat{\theta} + \frac{1}{\mathrm{r}}\left(\frac{\partial}{\partial \mathrm{r}}(\mathrm{r}\mathrm{E}_{g_\theta}) - \frac{\partial \mathrm{E}_{g_r}}{\partial\theta}\right)\hat{\varphi} \text{ [48].)} \quad (63)$$

This is very similar to Maxwell's equations in the CGS system[49]:

$$\text{div E} = \frac{4\pi\rho}{\varepsilon}, \text{ div B} = 0, \text{ rot E} = -\frac{1}{c}\frac{\partial \mathrm{B}}{\partial t}, \text{ rot B} = \frac{4\pi\mu}{c}\mathrm{J} + \frac{\varepsilon}{c}\frac{\partial \mathrm{E}}{\partial t}, \quad (64)$$

E: electric field intensity,

B: magnetic induction,

ρ: electric charge density,

ε: permittivity of the medium is equal to 1 for the vacuum in the CGS system,

μ: permeability of the medium is equal to 1 for the vacuum in the CGS system,

J: electric current density.

Maxwell's equations are invariant under Lorentz transformations, but the GEM (gravito-electromagnetism) equations are not[50]. This is because ρ_g and J_g cannot construct a four-current.

5.2 Derivation of equations for new forces

Since this paper has the same mathematical process as the general relativity theory, the new repulsive force(Energy force)(20) in this paper can be organized as a simultaneous equation like the GEM equation(61) above. Expressing this mathematically, it is as follows in the CGS system:

$$\nabla \cdot \mathrm{E}_e = 16\pi^2 \mathrm{G}\rho_e, \ \nabla \cdot \mathrm{B}_e = 0,$$

$$\nabla \times \mathrm{E}_e = -\mathrm{q}\frac{\partial \mathrm{B}_e}{\partial \Phi}, \ \nabla \times \mathrm{B}_e = 16\pi^2 \mathrm{Gq}\xi_e \mathrm{J}_e + \mathrm{q}\frac{\partial \mathrm{E}_e}{\partial \Phi} \ , \tag{65}$$

($\nabla^2 \mathrm{V} = 16\pi^2 \mathrm{G}\varepsilon$ (20))

$$(\nabla \cdot \mathrm{E}_e \equiv \frac{1}{\kappa^2}\frac{\partial(\kappa^2 \mathrm{E}_{e_\kappa})}{\partial \kappa} + \frac{1}{\kappa \sin\theta}\frac{\partial}{\partial\theta}(\mathrm{E}_{e_\theta}\sin\theta) + \frac{1}{\kappa\sin\theta}\frac{\partial \mathrm{E}_{e_\varphi}}{\partial\varphi}, \tag{66}$$

$$\nabla \times \mathrm{E}_e \equiv \frac{1}{\kappa\sin\theta}(\frac{\partial}{\partial\theta}(\mathrm{E}_{e_\varphi}\sin\theta) - \frac{\partial \mathrm{E}_{e_\theta}}{\partial\varphi})\hat{\kappa}$$

$$+ \frac{1}{\kappa}(\frac{1}{\sin\theta}\frac{\partial \mathrm{E}_{e_\kappa}}{\partial\varphi} - \frac{\partial}{\partial\kappa}(\kappa \mathrm{E}_{e_\varphi}))\hat{\theta} + \frac{1}{\kappa}(\frac{\partial}{\partial\kappa}(\kappa \mathrm{E}_{e_\theta}) - \frac{\partial \mathrm{E}_{e_\kappa}}{\partial\theta})\hat{\varphi}) \tag{67}$$

E_e: Energyforce-electric field intensity,

B_e: Energyforce -magnetic induction,

ρ_e: density of E^{-2} using curvature κ ($\mathrm{dL} = \kappa^2 \sin\theta \, \mathrm{d}\kappa \mathrm{d}\theta \mathrm{d}\varphi$) (L: Volume expressed using radius κ),

ξ_e: Energyforce-magnetic permeability of the medium,

J_e: E^{-2} current density using curvature κ and gravitational scalar potential Φ,

q: mass of photon (minimum mass).

Also, these equations(65) are not invariant(14) under transformations(3) of this paper (the new gravitational potential space metric) because these equations and transformation in the same way as the mathematical process of the relativity theory.

Therefore, New quantum equations which is invariant(14) under transformations(3) of this paper are proposed in this paper. The equations are similar to the above Energy-force equations(65) and Maxwell’s equations(64). Also, those describe the 5-dimensional world:

$$\nabla \cdot \mathrm{E}_n = \frac{16\pi^2 \rho_n}{\varepsilon_n}, \ \nabla \cdot \mathrm{B}_n = 0,$$

$$\nabla \times \mathrm{E}_n = -\mathrm{q}\frac{\partial \mathrm{B}_n}{\partial \Phi}, \ \nabla \times \mathrm{B}_n = 16\pi^2 \mu_n \mathrm{q}\, \mathrm{J}_n + \mathrm{q}\varepsilon_n \frac{\partial \mathrm{E}_n}{\partial \Phi}, \quad (68)$$

E_n: Newforce-electric field intensity,

B_n: Newforce-magnetic induction,

ρ_n: Newforce-electric charge density using curvature κ,

ε_n: Newforce-permittivity of the medium,

μ_n: Newforce-permeability of the medium,

J_n: Newforce-electric current density using curvature κ and gravitational scalar potential Φ.

In these equations(68), a new direction corresponding to attractive force can be established, just as charges of different signs attract each other. In addition, according to modern physics, the strong interaction is similar to this. Strong interaction attracts particles, and the strength of the force increases as the distance increases[51]. This is called asymptotic freedom. In addition, just as the new quantum equations were obtained in a similar way to Maxwell's equations, new theories of quantum mechanics can be obtained by analogy with the theories of electromagnetism. More formulas can be derived from many analogies in the appendices of references[52].

6. Conclusions

In this paper, to solve the gap between the relativity theory and quantum mechanics, three new axioms and the new gravity theory were created with the same mathematical development used in the relativity theory. And with this theory, it was possible to know the cause of dark energy, and that the infinitely accelerating expansion universe must have the binding energy of photon.

It was also possible to know the cause of the invariance and linearity of the speed of light, and to understand the process by which the quantum effects occur. It was possible to explore the principle of movement in the microscopic world without using electric charge or color.

Data availability

The author declares that the data supporting the findings of this study are available within the paper.

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There is one author in this paper.

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The author declares no competing interests.