The Pionic Deuterium and the Pion Tetrahedron Vacuum Polarization

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Abstract: A double-well potential model is proposed for the pionic deuterium that enables calculating the energy split, the potential barrier and the pion tetrahedron edge length. We propose that pion tetrahedrons, $\pi^{Td} = u\tilde{d}d\tilde{u}$, mediate the strong force by enabling quark exchange reactions between protons and neutrons. A vacuum polarization Feynman diagram is proposed for the π^{Td} having two chains of gluons and fermion loops for the two valence quarks and anti-quarks. With a higher order vacuum polarization diagram, the two fermion loops are connected and flipped by additional gluons and the pion tetrahedron chiral symmetry is broken by the non-empty vacuum as expected by the QCD chiral perturbation theory.

Keywords: Pionic Deuterium (**π**D), Yukawa interaction, QCD vacuum, Double-well potential, Chiral Perturbation Theory, Vacuum polarization.

1. Yukawa Interaction

Yukawa interaction[1] describes the strong force between hadrons mediated by pions. The Yukawa interaction describes also the coupling between the Higgs field and massless quark and lepton fields, where through spontaneous symmetry breaking, the fermions acquire a mass proportional to the vacuum expectation value (VEV). This Higgs-fermion coupling was first described by Steven Weinberg to model lepton masses[2]. The Yukawa interaction term has a single coordinate r:

$$L_{Yukawa}(\Psi, \varphi) = - g \, \bar{\Psi}(r)\varphi(r)\,\Psi(r) \qquad (1)$$

Where g is a coupling constant, $\Psi(r)$ is the fermion field and $\varphi(r)$ is the pion field. The Yukawa classical potential is:

$$V(r) = - \frac{g^2 \, e^{-\mu r}}{4\pi r} \qquad (2)$$

Where μ is the Yukawa particle meson mass that determines the exponential decay of the attractive interaction between the hadrons. In the next section we review briefly the pionic hydrogen and pionic deuterium hydrogen like atoms before presenting the role of the pion tetrahedron.

2. Pionic Deuterium

A pionic hydrogen, $\pi^{-}p$, is an unstable hydrogen-like atom, where the electron is replaced with the negatively charged pion, π^{-} $(d\bar{u})$. The Bohr radius of the pionic hydrogen is about 200 femtometer which is shorter than the hydrogen atom Bohr radius but still significantly larger than proton radius and hence the QCD interaction between the proton and the charged pion is expected to be small[3]. The pionic hydrogen atom reveals the influence of the strong force by a negative shift and broadening of the low-lying atomic levels with respect to the pure electromagnetic interaction.

For the pionic deuterium[3], $\pi^{-}D$, the energy shift, ε_{1s}, is negative and is about -2.3 eV and the energy width, Γ_{1s} , is about 1 eV. The negative sign of the energy shift is due to repulsive interaction with the deuterium nucleus that screens the QED attraction between the negative charged pion and the positively charged proton, and the ground state level width is explained by the pion absorption and production by the nucleus.

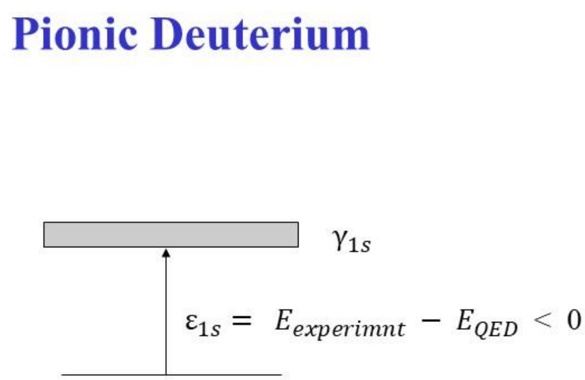

Figure 1 illustrates the ground state pionic deuterium shift and width of the order of -2.3 and 1eV, respectively. (Strauch et al[3], Figure 1 page 3).

In the next section we propose that pion tetrahedrons mediate the strong force by enabling quark exchange reactions between protons and neutrons. In addition, the pion tetrahedrons enable a pion exchange reaction with the deuterium nucleus that reduces the effective charge of the proton.

3. Pionic Deuterium and the Pion Tetrahedron

Inspired by the theory of Loop Quantum Gravity (LQG)[4], we proposed that exotic meson tetraquarks, $\pi^{Td} = u\tilde{d}d\widetilde{u}$, introduced in previous papers[5,6,7,8] condense to a tetrahedron geometry and may be part of the QCD ground state condensate. We noted that neutral pions, a superposition of $d\widetilde{d}$ and $u\widetilde{u}$ mesons, may condense into a $u\tilde{d}d\widetilde{u}$ tetrahedron geometry having two chiral states[9] as shown below in equation 3 and Figure 2.

$$d\widetilde{d} + u\widetilde{u} \rightarrow u\tilde{d}d\widetilde{u}\ (tetrahedron\ pion, \pi^{Td}_{L,R}) \qquad (3)$$

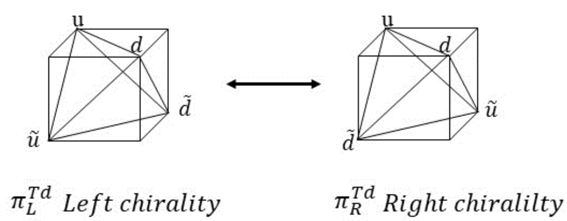

Figure 2 illustrates the $u\tilde{d}d\widetilde{u}$ pion tetrahedron with left π_L^{Td} and right π_R^{Td} chirality.

We propose here that in the case of the pionic deuterium, the Yukawa interaction may be more complex where a transition state may be generated where two pairs of quarks are exchanged coherently between protons and neutrons. Quark exchange reaction transforms the proton to a neutron and the neutron to a proton concurrently as shown below in Figure 3 and the Feynman diagram of Figure 7 further below.

The quark exchange reaction is triggered by the tetrahedron anti-quarks $\tilde{d}$ *and* $\tilde{u}$ that capture their quark pairs from the proton and the neutron d *and* u quarks and replace them with the tetrahedrons' u *and* d quarks (on the left-hand side). The quark exchange reaction on the right side occurs in the opposite direction transforming the neutron back to a proton and the proton to a neutron. The proposed quark exchange reaction is symmetric, where the reactants and products are the same.

$$uud + u\tilde{d}d\tilde{u} + udd \quad \rightarrow \quad udd + u\tilde{d}d\tilde{u} + uud \qquad (4)$$

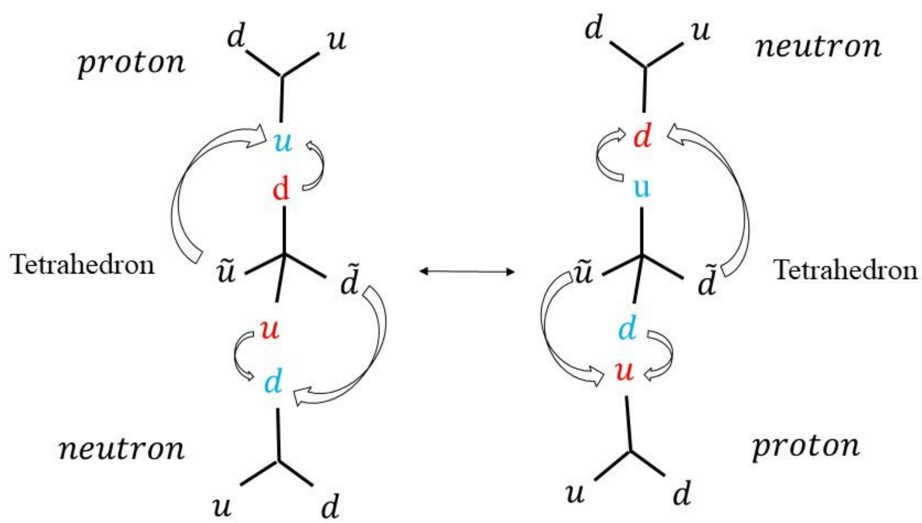

Figure 3 illustrates the double quark exchange reaction in deuterium nucleus mediated by a QCD tetrahedron pion π^{Td}.

The transition state complex includes the deuterium nucleus and the pion tetrahedron, overall 10 quarks and antiquarks. The transition state complex may have a specific geometry and symmetry where the reaction active quarks are the two anti-quarks. The potential surface is a many-body potential surface that however may be simplified by an effective one-dimensional reaction coordinate double well shape as shown in Figure 4 below. Thus, the pion tetrahedron (π^{Td}) plays the role of the Yukawa interaction meson mediator that provides here both the anti-quarks and the two quarks that are exchanged. Note that the quarks and antiquarks numbers are conserved in the exchange reaction (equation 4).

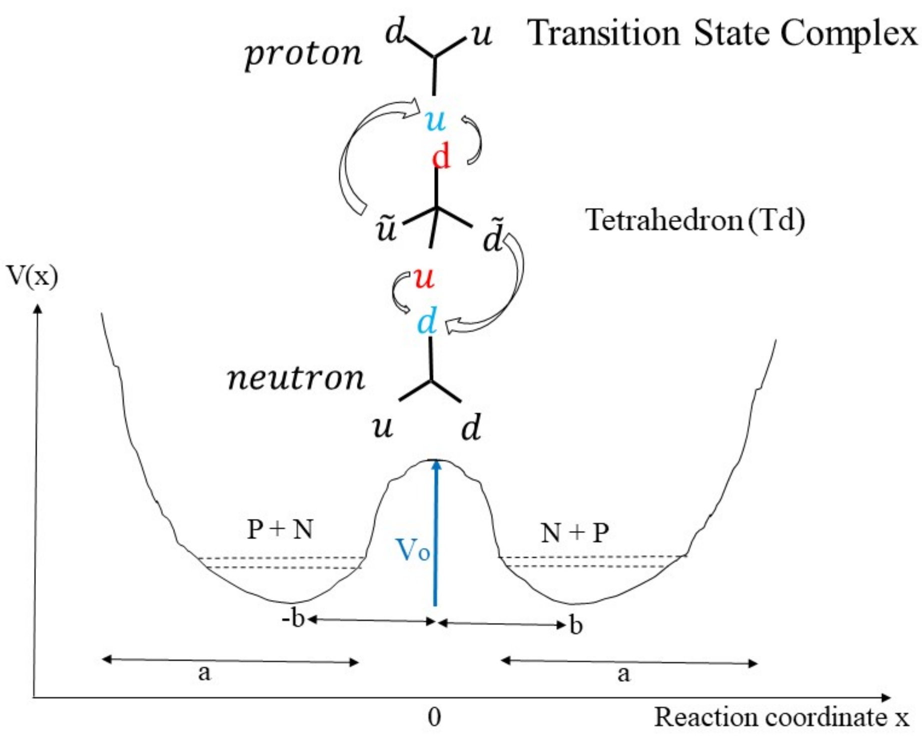

Figure 4 illustrates the double quark exchange reaction transforming protons (P) to neutrons (N) and vice versa mediated by a p[ion tetrahedron ($\pi_{L,R}^{Td}$) forming the symmetric double well potential.

In addition to the quark exchange reaction of equation 4, the charged pion π^- $(d\widetilde{u})$ is scattered by the nucleus and an exchange reaction occurs with the pion tetrahedron as shown in equation 5 -

$$uud + udd + u\tilde{d}d\widetilde{u} + d\widetilde{u} \rightarrow udd + uud + u\tilde{d}d\widetilde{u} + d\widetilde{u} \qquad (5)$$

The symmetric double well potential model, which was used to study the ammonia molecule inversion[10], splits the pionic deuterium energy levels including the ground state to symmetric and anti-symmetric doublets. The strong force coupling of the deuterium nucleus to the charged pion, π^- $(d\widetilde{u})$, shifts and broadens the measured ground state energy as shown in Figure 1 above. Strauch et al[3] estimated that the shift is negative, about -2.3 eV, reducing the QED attraction between the charged pion and the proton. The charged pion exchange reaction of equation 5 above generates a negatively charged cloud at the deuterium nucleus vicinity that reduces the effective charge of the proton. The proposed pion exchange reaction is an alternative mechanism to pion absorption and production by the nucleus at low energies[3].

The energy split of a double well potential model can be calculated numerically and it depends on 4 parameters, the potential barrier height, V_0, two length parameters, a and b, shown in Figure 4 above, and the particle mass m. We used $a = 800$ fm for the double well width and $b = \mathrm{b}_{\mathrm{QCD-Td}} + \mathrm{a}/2$, where $\mathrm{b}_{\mathrm{QCD-Td}}$ is the pion tetrahedron edge length of about 0.1 femtometer. Based on Strauch et al[3], we assume that Γ_{1s} value is 1 eV and is the double well potential split $E_0^a - E_0^s$, where E_0^s is the ground state symmetric energy and E_0^a is the anti-symmetric energy. The negative charge pion mass is m_{π^-}= 273 m_e, where m_e is the electron mass. We determined the potential barrier value V_0 with a given $\mathrm{b}_{\mathrm{QCD-Td}}$ $value$ (0.1 fm)from the numerical solution of the double well potential model that matches the experimental value for the energy split Γ_{1s}=1eV as shown in figure 5 below.

$$E_0^a - E_0^s = \Gamma_{1s} = 1 \text{ eV} \qquad (6)$$

The numerical solutions of two transcendental equations is obtained by solving the matching condition of the wavefunction values and derivatives at the barrier potential walls at x=b_{QCD-Td} and x=$-b_{QCD-Td}$ [10].

$$\tan(k_s a) = -\frac{k_s}{\sqrt{\alpha^2 - k_s{}^2}} \coth\left(\sqrt{\alpha^2 - k_s{}^2}\, b_{QCD-Td}\right) \quad (7a)$$

$$\tan(k_a \boldsymbol{a}) = -\frac{k_a}{\sqrt{\alpha^2 - k_a{}^2}} \tanh\left(\sqrt{\alpha^2 - k_a{}^2}\, b_{QCD-Td}\right) \quad (7b)$$

Where $\alpha^2 = \frac{2m_\pi - V_0}{\hbar^2}$, $k_s{}^2 = \frac{2m_\pi - E_0^s}{\hbar^2}$ and $k_a{}^2 = \frac{2m_\pi - E_0^a}{\hbar^2}$.

Figure 5 shows the numerical solution of equations 7a-b . The difference between the symmetric and anti-symmetric solution, e.g. the ground state energy split is **Δ**E=1.0 eV with V_0~2.04 GeV. The calculated deuterium ground state energy in the double-well potential model is E_0^s=2150.943 eV.

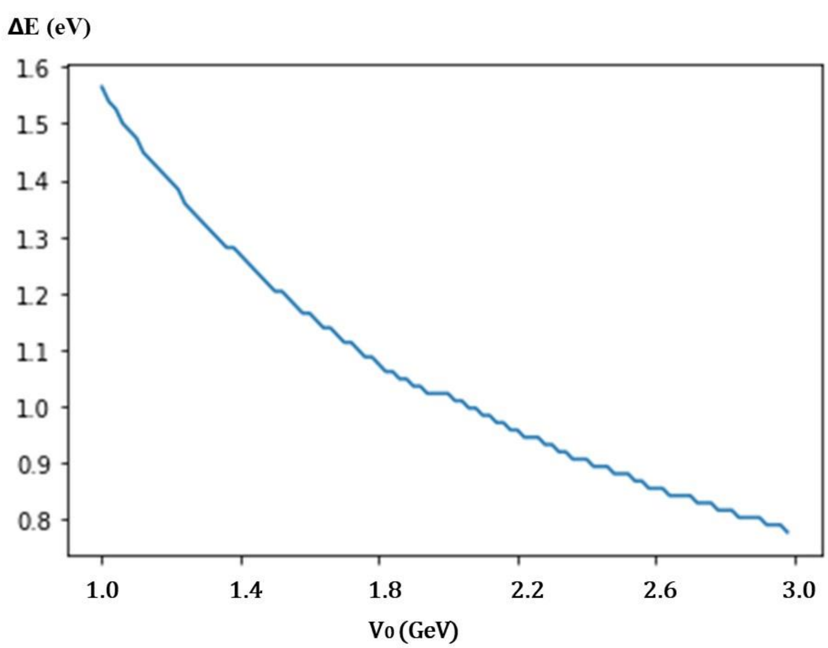

Figure 5 illustrates the ground state energy split **Δ**E as a function of V0 with $b_{QCD-Td} = 0.1\ fm$.

Figure 6 shows the potential barrier height V_0 as a function of the barrier width, b_{QCD-Td}, which models the tetrahedron edge length and with **Δ**E=1.0 eV. The minimum value of the barrier potential V_0 is 1.66 GeV at the tetrahedron edge length b_{QCD-Td} of 0.175 fm. Hence the calculated pion tetrahedron π^{Td} edge length is smaller, 0.175 fm, but of the same order of magnitude of the proton diameter of about 0.83 fm.

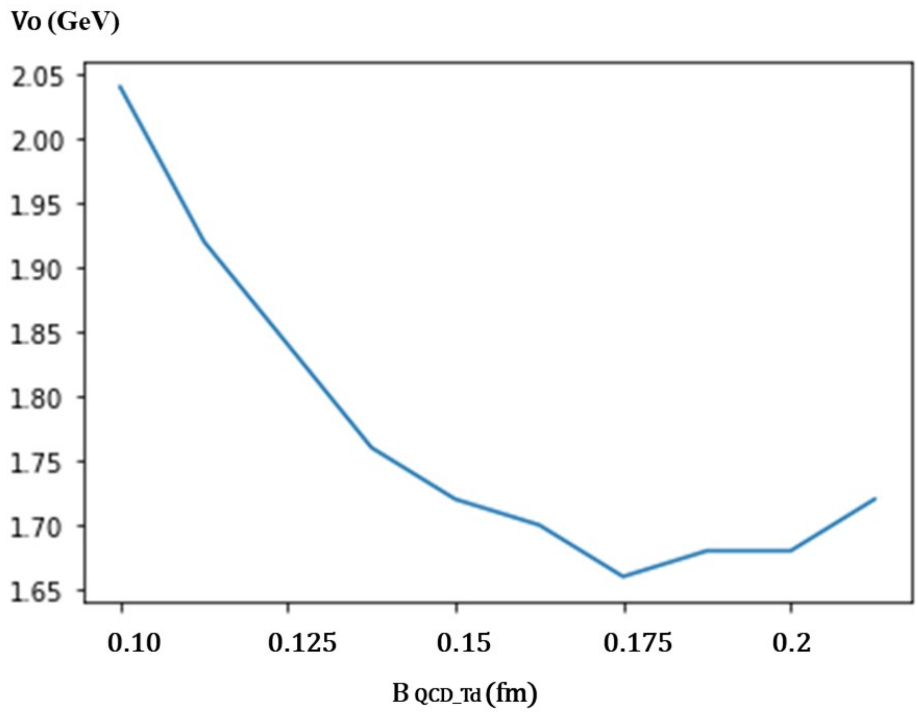

Figure 6 illustrates the calculated potential barrier height V0 as a function of the barrier width, b_{QCD-Td}, which models the pion tetrahedron $\pi_{L,R}^{Td}$ edge length.

4. Pion Tetrahedron and the Vacuum Polarization

The mass of the protons and neutrons that comprises most of the mass of the visible universe is not the sum of masses of their valence quarks that contribute only about 9% of their total ~1 GeV mass per particle[11]. The masses of the valence quarks in the proton are estimated to be ~3 MeV per quark while the total proton mass is 938 MeV. According to lattice QCD calculation, the quark kinetic energy and glue field energy contribute 33% and 37% respectively, the trace anomaly gives a 23% contribution, and the u, d, and s quark scalar condensates contribute about 9%[12].

The QCD Lagrangian is -

$$L_{QCD} = \sum_{j=u,d,s,...} \bar{q}_j \left[i\, \gamma^{\mu}\, D_{\nu} - m_j\right] q_j - \frac{1}{4}\, G^{a}_{\mu\nu}\, G^{a\mu\nu} \qquad (8)$$

q_j and $\bar{q}_j$ are the quark and ant-quark field Dirac spinors, the QCD 8 gluon fields that carry the strong force are described by the vector potential $A^{a}_{\mu},\ a = 1, \dots, 8$ that enters the QCD Lagrangian through the covariant derivative D_{ν} and the tensors $G^{a}_{\mu\nu}$ -

$$D_{\nu} = \partial_{\nu} + i\, g\, \frac{1}{2}\, \lambda^{a}\, A^{a}_{\mu} \qquad (9)$$

$$G^{a}_{\mu\nu} = \partial_{\mu}\, A^{a}_{\nu} + \partial_{\nu}\, A^{a}_{\mu} + i\, g\, f^{abc} A^{b}_{\mu}\, A^{c}_{\nu} \qquad (10)$$

g is the strong force coupling constant, λ^{a} are the generators of the SU(3) color gauge group and f^{abc} are the SU(3) adjoint group representation coefficients.

The quark exchange reactions of nucleons with the QCD pion tetrahedrons, $\pi^{Td}_{L,R}$, may be seen as gluon exchanges. For example, the symmetric proton to neutron quark exchange reaction of equation 4, $uud + u\tilde{d}d\tilde{u} + udd \rightarrow udd + u\tilde{d}d\tilde{u} + uud,$ may be illustrated by the following Feynman diagram where quark exchange reactions between the proton and the neutron via the pion tetrahedron gluonic transition state complex occurs via two gluons.

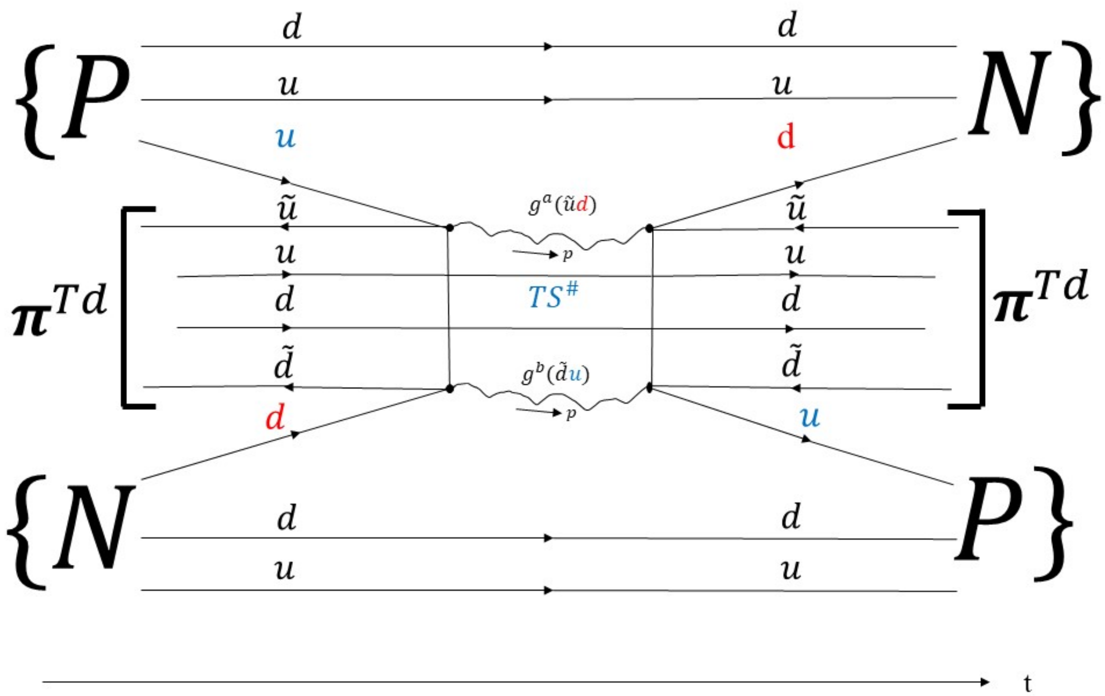

Figure 7 illustrates the proton and neutron symmetric quark exchange reactions enabled by the pion tetrahedron $\pi_{L,R}^{Td}$ assumed to occur with very high frequency in the vacuum.

The proton's quarks may perform similar quark exchange reactions with the pion tetrahedrons, $\pi_{L,R}^{Td}$ cooling the protons by carrying away part of the momentum -

$$uud + u\tilde{d}d\widetilde{u} \rightarrow u\tilde{d}d\widetilde{u} + uud \qquad (11)$$

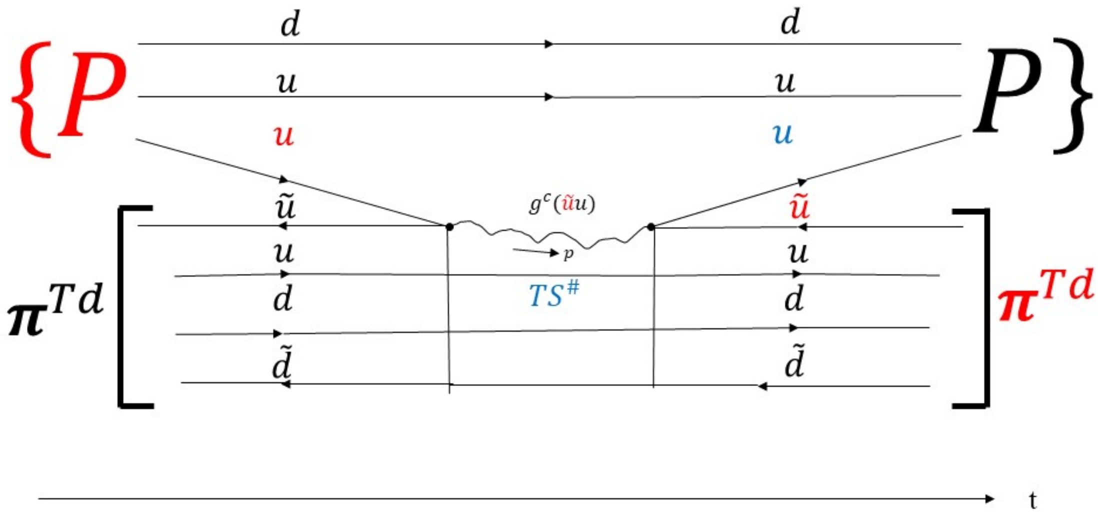

Figure 8 illustrates the proton scattering quark exchange reactions enabled by the pion tetrahedron $\pi_{L,R}^{Td}$ that may cool the accelerated proton.

Vacuum polarization Feynman integral diverges but it can be renormalized with QED. The QED vacuum polarization integral is[13]

$$\pi_2^{\mu\nu}(p^2) = -(-ie)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{(p-k)^2-m^2} \frac{i}{k^2-m^2} Tr\big(\gamma^\mu (k-p+m)\gamma^\nu (k+m)\big) \quad (12)$$

And the renormalized electron charge is given by –

$$e_R^2(p^2) = e^2 - e^4 \pi_2^{\mu\nu}(p^2) + \ldots \quad (13)$$

Alternatively, the vacuum polarization may be handled by effective running permittivity $\varepsilon_0(r)$ of the vacuum instead of renormalizing the electric charge[14].

Below we propose a π^{Td} vacuum polarization Feynman diagram, which includes two quark and anti-quark loops, one for the u *and* $\tilde{u}$ and a second for the d *and* $\tilde{d}$ of the pion tetrahedron connected by gluon chains, $g^b(\tilde{u}u)$ and $g^a(\tilde{d}d)$ respectively. The gluons increase the π^{Td}energy and mass beyond the bare masses of the four valence quarks and anti-quarks $\tilde{u}u\tilde{d}d$.

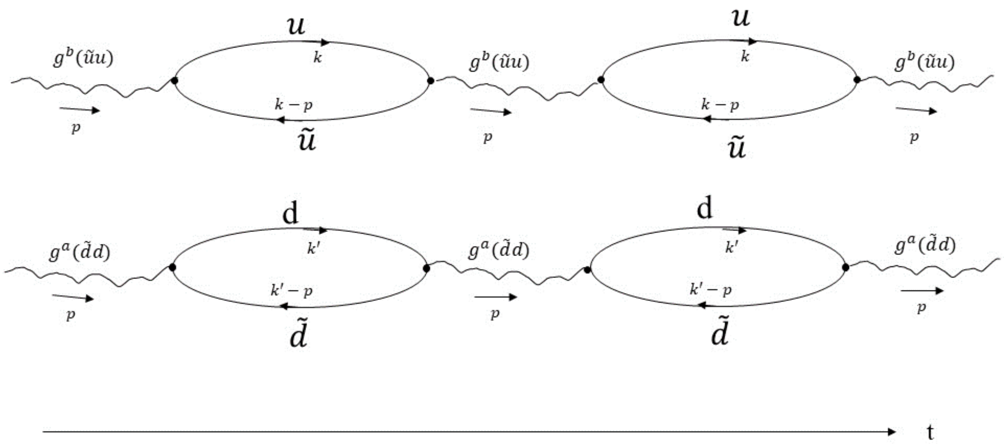

Figure 9 illustrates the vacuum polarization Feynman diagram for the pion tetrahedron $\pi^{Td}$with gluon chains.

The proposed π^{Td} vacuum polarization is a sum of the two loop amplitudes, each loop can be re-normalized as presented by Andela on page 57 [15] –

$$\pi_2^{Td}\,(p^2) = (-ie)^2 \int \frac{d^4k}{(2\pi)^4}\; Tr\left(ig\gamma^{\mu}\, T^a \frac{i\,(k-p+m_u)}{(p-k)^2 - m_u^2}\, ig\gamma^{\nu}\, T^b\; \frac{i\,(k+m_u)}{k^2-m_u^2}\right) +$$

$$(-ie)^2 \int \frac{d^4k\prime}{(2\pi)^4}\; Tr\left(ig\gamma^{\nu}\, T^b \frac{i\,(k\prime-p+m_d)}{(p-k\prime)^2 - m_d^2}\, ig\gamma^{\mu}\, T^a \frac{i\,(k\prime+m_d)}{k\prime^2-m_d^2}\right) \qquad (14)$$

The Figure below illustrates a higher order vacuum polarization Feynman diagram including additional π^{Td} and gluons that exchange between the u and d fermion loops. The exchanges are symmetric symmetric and the result is pion tetrahedrons with mixed right and left chiral tetrahedrons that breaks the tetrahedron chiral symmetry. The symmetric quark exchange reaction is -

$$\tilde{u}\,u\tilde{d}d + \tilde{u}\,u\tilde{d}d \rightarrow \tilde{d}d\tilde{u}u + \tilde{d}d\tilde{u}u \qquad (15)$$

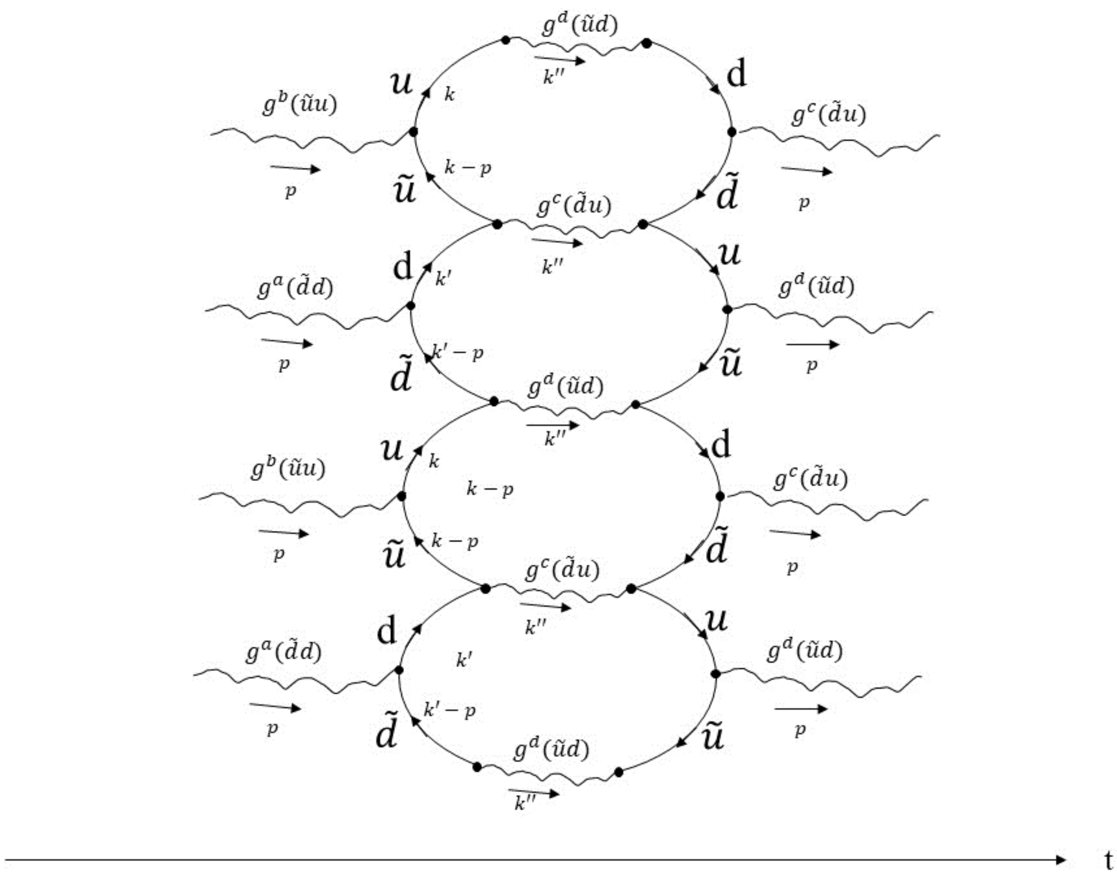

Figure 10 illustrates a higher order vacuum polarization Feynman diagram.

The π^{Td} decay rate is proportional to the square of the Feynman diagram amplitude -

$$\Gamma_{\pi^{Td}} \sim \frac{4\,\pi^2\,(\pi_2^{Td})^2}{\mathrm{h}} \qquad (16)$$

However, the pion tetrahedron π^{Td} may be stabilized by the pion condensate and the vacuum polarization diagram may be periodic as shown above.

The properties of the pions are intimately related to the QCD chiral symmetry and the ground state vacuum. In the chiral limit, the quark masses m_u and m_d vanish, the chiral symmetry becomes exact, the massless Dirac Lagrangian being invariant under the group $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$. The chiral symmetry is assumed to be spontaneously broken to the isospin subgroup $\mathrm{SU}(2)_V$ by the vacuum condensate. The pions represent the corresponding Goldstone bosons and the vacuum pion condensate is assumed to be the leading order parameter of the spontaneously broken symmetry $\langle 0|u\tilde{u} + d\tilde{d}|0\rangle$[16]. The QCD chiral perturbation theory is based on the massless Dirac Lagrangian symmetry, however, it does not determine explicitly what is the QCD vacuum content and how the vacuum breaks the chiral symmetry. The proposed pion tetrahedrons have two chiral states, $\pi_{L,R}^{Td}$, determined by the relative positions of the two quarks and the two anti-quarks in the tetrahedron vertices and they may be a realization of the QCD Lagrangian chiral symmetry.

5. Pionium $A_{2\pi}$ and the Vacuum Expectation Value

A bound state of π^+ and π^-, $A_{2\pi}$, has been observed in 1993[17]. The pionium has a Bohr radius of about 400 femtometers[18] and lifetime of 2.9 ± 0.1 10^{-15} seconds[19]. The DIRAC lifetime measurement method is based on production of pionium π+ π− ($A_{2\pi}$) atoms in a thin Ni target by accelerated protons and subsequent detection of highly correlated π+π− split pairs leaving the target as a result of a breakup (ionization due to electromagnetic interaction with the Ni target) of a part of the π+π− atoms which did not decay within the target.

The pionium $A_{2\pi}$ decay is dominated by the charge exchange annihilation/condensation reaction that leads to the neutral pions π^0 -

$$A_{2\pi} \rightarrow \pi^0 + \pi^0 \qquad (17)$$

However, the main competing reaction to the $A_{2\pi}$ decay is the breakup by ionization of the two charged pions that the DIRAC experiment was built to measure -

$$A_{2\pi} \rightarrow \pi^+ + \pi^- \qquad (18)$$

The $A_{2\pi}$ lifetime is related to the free path before it is scattered with acattering lengths a_0 and a_2 , the S-wave pion-pion scattering lengths for isospin 0 and 2, respectively. The partial decay width of the atomic ground state (principal quantum number n = 1, orbital quantum number l = 0) depends on the difference between the scattering lengths a_0 and a_2

$$\Gamma_{1s} = \frac{1}{\tau_{1s}} = \frac{2}{9}\,\alpha^3\,p\,|a_0 - a_2|^2(1 + \delta) \qquad (19)$$

Where τ_{1s} is the lifetime of the atomic ground state, α the fine-structure constant, p the π^0 momentum in the atomic rest frame, δ = 0.058 ± 0.012, is a QED and QCD correction.

The measured scattering lengths are –

$$a_0 = \frac{7\,{M_\pi}^2}{32\,\pi\,{F_\pi}^2} = 0.22 \quad a_2 = -\frac{{M_\pi}^2}{16\,\pi\,{F_\pi}^2} = -0.044 \quad a_0 - a_2 = 0.265 \pm 0.004 \qquad (20)$$

And the pion mass is -

$${M_\pi}^2 = \frac{(m_u + m_d)\left\langle 0\middle|u\tilde{u} + d\tilde{d}\middle|0\right\rangle}{{F_\pi}^2} \qquad (21)$$

The pion mass and the scattering lengths difference a_0- a_2 is related to the structure of the QCD vacuum expectation value (VEV), $\langle 0|u\tilde{u} + d\tilde{d}|0\rangle$[20]. The pion condensate VEV may be replaced by the proposed pion tetrahedron vacuum polarization π_2^{Td} and the pion mass may then be given by -

$${M_\pi}^2 \sim \frac{(m_u + m_d)\,|\pi_2^{Td}|}{{F_\pi}^2} \qquad (22)$$

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