An Algebrologist in Wonderland

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Abstract. By imposing a requirement for spatial isotropy, it is possible to find an algebra with a subalgebra structure having a pattern matching that of the bosons and three families of fermions of the standard model.

Contents

1. Introduction

This paper documents a pattern found for the subloops of a loop (quasi-group) labeled U_L which is the embedded loop for an algebra, labelled $\mathbb{U}$, obtained as the product of the trigintaduonion algebra with that of 4×4 complex matrices . $\mathbb{U}$ is the same size as $Cl_{1,9}(R)$, the Clifford algebra which can represent multivectors for the dimensionality used in string theories. Its subalgebra structure is explored by considering isomorphisms of the subloops of its embedded loop, $U_L \cong T_L \otimes M_L$, where T_L is a loop embedded in $\mathbb{T}$ and M_L is a loop embedded in the algebra of 4×4 complex matrices, isomorphic to the complexified space-time Clifford algebra. The structure has features that correspond to those of the standard model.

As reported in a previous paper[1], T_L has an asymmetrical sub-loop structure, for which correspondences with the standard model can be identified. The correspondences distinguish between fermions and bosons, between left and right handed particles, between leptons and quarks, account for three families of fermions and identify SU(3) symmetry for color and broken symmetry for flavor. This paper modifies and extends that analysis.

The analysis presented is based on symmetry structures found by inspection of isomorphisms between subloops of T_L when subjected to a requirement for isotropy when aligned with the the unit elements for a complexified space-time Clifford algebra multivector.

1.1. Acknowledgements

Having made extensive use of the Loops package[2] for GAP4[3], I give thanks to the creators of the programs. I thank members of the physics community who have been responsive or helpful in the past, those who post informative articles on the internet.

1.2. Notation

1.2.1. M_L the loop of unit elements for $M_4(C)$, isomorphic to $Cl_4(C)$. Unit elements for the complexified space-time Clifford algebra $Cl_4(C)$ have been labeled as shown in table 1. They can be represented using the 4×4 matrices labeled as shown in table 2 together with their imaginary counterparts. For this labeling scheme, unit elements related by spatial rotation are labeled using sets of three sequential letters. Their Cayley table is set out in Appendix A.

TABLE 1. Notation for unit elements of $Cl_4(C)$ and $M_4(C)$

Signature	Blade	Scalar	Spatial bivectors	Pseudo scalar	Space/time bivectors	Spatial trivector	Spatial vectors	Time vector	Space/time trivectors
	Real	e_o	e_{yz}, e_{zx}, e_{xy}	e_{xyzt}	e_{xt}, e_{yt}, e_{zt}	e_{xyz}	e_x, e_y, e_z	e_t	$e_{yzt}, e_{zxt}, e_{xyt}$
(+++ -)	$Cl(3,1)$	S	L,M,N	V	D,E,F	U	X,Y,Z	T	P,Q,R
(+ - - -)	$Cl(1,3)$	S	L,M,N	V	D,E,F	iU	iX,iY,iZ	iT	iP,iQ,iR
	Imaginary	ie_o	$ie_{yz}, ie_{zx}, ie_{xy}$	ie_{xyzt}	$ie_{xt}, ie_{yt}, ie_{zt}$	ie_{xyz}	ie_x, ie_y, ie_z	ie_t	$ie_{yzt}, ie_{zxt}, ie_{xyt}$
(+++ -)	$Cl(3,1)$	iS	iL,iM,iN	iV	iD,iE,iF	iU	iX,iY,iZ	iT	iP,iQ,iR
(+ - - -)	$Cl(1,3)$	iS	iL,iM,iN	iV	iD,iE,iF	U	X,Y,Z	T	P,Q,R

TABLE 2. 4×4 unit matrices representing real elements of M_L

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$[S, L, M, N]$ and $[S, T, U, V]$ can be used to represent unit elements for right and left isoclinic quaternion algebras as used by Van Elfrinkhof[4].

1.2.2. T_L**, the loop of unit elements of** $\mathbb{T}$**.** For T_L labels have been assigned so that, for global subscript changes, $\iota \leftrightarrow \jmath \leftrightarrow \kappa$, subloop isomorphisms are maintained. Labels such as $\sigma_{o\iota\jmath\kappa}$ are used for subscripted sets of 4 elements for the same letter.

TABLE 3. Notation for T_L, the loop of unit elements of $\mathbb{T}$

$\pm$	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
$\pm$	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
$\pm$	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
$\pm$	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ

1.2.3. Sedenionic subloops of T_L**.** Cawagas[5] labelled the isomorphism types for the 31 subloops of T_L of order 32 as S_γ, S_α, S_β, S_L. In this paper these have been labelled using uppercase greek letters with subscripts: Γ_0, $A_{0..6}$, $B_{0..6}$, $\Sigma_{0..15}$. Elements of T_L for positive elements of these loops are shown in table 4.

TABLE 4. Positive elements for sedenionic subloops of T_L

		σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
1	Γ_0	■	■	■	■	■	■	■	■																	■	■	■	■	■	■	■	■
2	A_0	■	■	■	■					■	■	■	■									■	■	■	■					■	■	■	■
3	B_0	■	■	■	■									■	■	■	■					■	■	■	■	■	■	■	■				
4	A_1	■	■			■	■			■	■			■	■					■	■			■	■			■	■			■	■
5	A_2	■		■		■		■		■		■		■		■			■		■		■		■		■		■		■		■
6	A_3	■			■	■			■	■			■	■			■		■	■			■	■			■	■			■	■	
7	B_1	■	■			■	■					■	■			■	■			■	■			■	■	■	■			■	■		
8	B_2	■		■		■		■			■		■		■		■		■		■		■		■	■		■		■		■	
9	B_3	■			■	■			■		■	■			■	■			■	■			■	■		■			■	■			■
10	A_4	■	■					■	■	■	■					■	■			■	■	■	■					■	■	■	■		
11	A_5	■		■			■		■	■		■			■		■		■		■	■		■			■		■	■		■	
12	A_6	■			■		■	■		■			■		■	■			■	■		■			■		■	■		■			■
13	B_4	■	■					■	■			■	■	■	■					■	■	■	■			■	■					■	■
14	B_5	■		■			■		■		■		■	■		■			■		■	■		■		■		■			■		■
15	B_6	■			■		■	■			■	■		■			■		■	■		■			■	■			■		■	■	
16	Σ_0	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■																
17	Σ_1	■	■	■	■	■	■	■	■									■	■	■	■	■	■	■	■								
18	Σ_2	■	■	■	■					■	■	■	■					■	■	■	■					■	■	■	■				
19	Σ_3	■	■	■	■									■	■	■	■	■	■	■	■									■	■	■	■
20	Σ_4	■	■			■	■			■	■			■	■			■	■			■	■			■	■			■	■		
21	Σ_5	■		■		■		■		■		■		■		■		■		■		■		■		■		■		■		■	
22	Σ_6	■			■	■			■	■			■	■			■	■			■	■			■	■			■	■			■
23	Σ_7	■	■			■	■					■	■			■	■	■	■			■	■					■	■			■	■
24	Σ_8	■		■		■		■			■		■		■		■	■		■		■		■			■		■		■		■
25	Σ_9	■			■	■			■		■	■			■	■		■			■	■			■		■	■			■	■	
26	Σ_{10}	■	■					■	■	■	■					■	■	■	■					■	■	■	■					■	■
27	Σ_{11}	■		■			■		■	■		■			■		■	■		■			■		■	■		■			■		■
28	Σ_{12}	■			■		■	■		■			■		■	■		■			■		■	■		■			■		■	■	
29	Σ_{13}	■	■					■	■			■	■	■	■			■	■					■	■			■	■	■	■		
30	Σ_{14}	■		■			■		■		■		■	■		■		■		■			■		■		■		■	■		■	
31	Σ_{15}	■			■		■	■			■	■		■			■	■			■		■	■			■	■		■			■

1.2.4. Octonionic subloops. There are 155 subloops of T_L of order 16, 50 true octonionic loops labelled O_L, and 105 quasi-octonionic loops labelled $\widetilde{O}_L$.

1.2.5. Notation for products. The product of $\mathbb{T}$ with $M_4(C)$generates:
$\mathbb{U} = \mathbb{T} \otimes M_4(C) \cong \mathbb{T} \otimes Cl_4(C)$ with loop of unit elements $U_L = T_L \otimes M_L$
$\circledast$ is used to denote multiplication based on the Cayley table for $\mathbb{T}$.

2. Subloop structure and the standard model

An algebraic basis for the standard model must account for all features of the standard model. The subloop structure of T_L when isotropically aligned with the complexified Clifford algebra multivector for space-time may do this.

2.1. Isotropic alignment

The structure of the standard model is invariant with respect to spatial orientation. The Cayley tables for $\mathbb{T}$ and $Cl_4(C)$ (tables for positive elements of T_L and M_L) can be aligned so that, if the signs of products are ignored, they are identical. For some such alignments, T_L can be rotated with respect to the elements aligned with the M_L elements assigned to Space-Time Algebra (STA) unit spatial vectors without changing the pattern of isomorphisms for T_L subloops, making these alignments isotropic.

The isotropy of an alignment can be assessed by considering participation of elements of T_L and M_L in their subloops. There are only seven sets of quaternionic unit elements of T_L elements that can be isotropically aligned with the quaternionic set of spatial bivectors $[e_{yz}e_{zx}e_{xy}]$:
$[\sigma_\iota\sigma_\jmath\sigma_\kappa]$, $[\sigma_\iota\lambda_\jmath\lambda_\kappa]$, $[\sigma_\jmath\lambda_\iota\lambda_\kappa]$, $[\sigma_\kappa\lambda_\iota\lambda_\jmath]$, $[\sigma_\iota\lambda_o\lambda_\iota]$, $[\sigma_\jmath\lambda_o\lambda_\jmath]$, $[\sigma_\kappa\lambda_o\lambda_\kappa]$
Examples of isotropic alignments are shown in table 5.

TABLE 5. Participation by unit elements of T_L and M_L in their subloops of order 16

Gap4 ID	S	L	M	N	V	D	E	F	U	X	Y	Z	T	P	Q	R
16 10	15	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
16 11	45	6	6	6	6	15	15	15	6	15	15	15	6	15	15	15
16 12	15	6	6	6	6	1	1	1	6	1	1	1	6	1	1	1
16 13	80	20	20	20	20	16	16	16	20	16	16	16	20	16	16	16
Cl(3,1)	e_o	e_{yz}	e_{zx}	e_{xy}	e_{xyzt}	e_{xt}	e_{yt}	e_{zt}	e_{xyz}	e_x	e_y	e_z	e_t	e_{yzt}	e_{zxt}	e_{xyt}
Cl(1,3)	e_o	e_{yz}	e_{zx}	e_{xy}	e_{xyzt}	e_{xt}	e_{yt}	e_{zt}	ie_{xyz}	ie_x	ie_y	ie_z	ie_t	ie_{yzt}	ie_{zxt}	ie_{xyt}
1 per $\sigma_{\iota\jmath\kappa}$	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
3 per $\sigma_{\iota\jmath\kappa}$	σ_o	σ_ι	$\lambda_\jmath$	λ_κ	λ_o	λ_ι	$\sigma_\jmath$	σ_κ	μ_o	μ_ι	$\nu_\jmath$	ν_κ	ν_o	ν_ι	$\mu_\jmath$	μ_κ
3 per $\sigma_{\iota\jmath\kappa}$	σ_o	σ_ι	λ_o	λ_ι	$\sigma\jmath$	σ_κ	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	ν_o	ν_ι	$\mu_\jmath$	μ_κ	$\nu_\jmath$	ν_κ
O_L	50	14	14	14	14	14	14	14	14	10	10	10	10	10	10	10
$\widetilde{O}_L$	105	21	21	21	21	21	21	21	21	25	25	25	25	25	25	25

Gap4 ID	iS	iL	iM	iN	iV	iD	iE	iF	iU	iX	iY	iZ	iT	iP	iQ	iR
16 10	15	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
16 11	0	15	15	15	15	6	6	6	15	6	6	6	15	6	6	6
16 12	0	1	1	1	1	6	6	6	1	6	6	6	1	6	6	6
16 13	20	16	16	16	16	20	20	20	16	20	20	20	16	20	20	20
Cl(3,1)	ie_o	ie_{yz}	ie_{zx}	ie_{xy}	ie_{xyzt}	ie_{xt}	ie_{yt}	ie_{zt}	ie_{xyz}	ie_x	ie_y	ie_z	ie_t	ie_{yzt}	ie_{zxt}	ie_{xyt}
Cl(1,3)	ie_o	ie_{yz}	ie_{zx}	ie_{xy}	ie_{xyzt}	ie_{xt}	ie_{yt}	ie_{zt}	e_{xyz}	e_x	e_y	e_z	e_t	e_{yzt}	e_{zxt}	e_{xyt}
1 per $\sigma_{\iota\jmath\kappa}$	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
3 per $\sigma_{\iota\jmath\kappa}$	α_o	α_ι	$\beta_\jmath$	β_κ	β_o	β_ι	$\alpha_\jmath$	α_κ	γ_o	γ_ι	$\delta_\jmath$	δ_κ	δ_o	δ_ι	$\gamma_\jmath$	γ_κ
3 per $\sigma_{\iota\jmath\kappa}$	α_o	α_ι	β_o	β_ι	$\alpha_\jmath$	α_κ	$\beta_\jmath$	β_κ	γ_o	γ_ι	δ_o	δ_ι	$\gamma_\jmath$	γ_κ	$\delta_\jmath$	δ_κ
O_L	35	7	7	7	7	7	7	7	7	11	11	11	11	11	11	11
$\widetilde{O}_L$	0	28	28	28	28	28	28	28	28	24	24	24	24	24	24	24

2.2. Phase

The unit imaginary for M_L, ie_0, commutes with all elements of U_L, so can be associated with phase for excitations associated with its subloops.

2.3. Spin

For isotropic alignments, α_o has to be aligned with ie_0. As ie_0 is assigned to phase, this suggests assigning α_o to an extension of phase. ie_0 commutes with all elements of U_L, whereas α_o anti-commutes with imaginary elements of T_L. This suggests that excitations associated with subloops can be classified as spin 0, 1/2 or 1 if they include both, one or none of e_0 and α_0 respectively.

2.4. Fermions and bosons

For subloops of T_L of order 32 (refer to table 4), 16 exclude, and 15 include, α_o. For isotropic alignments, α_o must be aligned with the unit imaginary for $Cl_4(C)$. Subloops of order 32 that exclude α_o can be aligned with unit elements for the STA multivector. Subloops of order 32 that include α_o cannot be aligned with unit elements for the STA multivector, but can be aligned with a complexified three dimensional manifold corresponding to the light-cone. The former are identified as being fermionic, and the latter as being bosonic. The Σ_0 subloop is a special case, being fermionic, but also isomorphic to all the bosonic subloops. Any subloop of order 32 that includes α_o is a true sedenion subloop, and any subloop of order 16 that includes α_o is a true octonion subloop.

2.5. Color

The sets of elements aligned with spatial bivectors for isotropic alignments correspond to automorphisms for the octonion with unit elements $[\sigma_{o\iota\jmath\kappa}\lambda_{o\iota\jmath\kappa}]$. If one unit imaginary is fixed, its automorphism group is $SU(3)$ Associating the fixed unit imaginary with the Space Time Algebra (STA) pseudoscalar, this identifies color as being generated by relations between orientations of the octonion with respect to spatial bivectors and space-time bivectors.

2.6. Temporal alignment, three generations of each fermion family

The spatial bivectors for the STA, $[e_{yz}e_{zx}e_{xy}]$, generated as the products of $[e_xe_ye_z]$ are also obtained as the products of $[e_{xt}e_{yt}e_{zt}]$ and of $[e_{xyt}e_{yzt}e_{zxt}]$. In table 5, depending on signature the matrix T or iT is assigned to e_t.

For the assignment of $[LMN]$ to spatial bivectors, isomorphisms are unchanged if V or U for(+++-) or iV or iU for (- - - +) are used. For the alignment of $[\sigma_\iota\sigma_\jmath\sigma_\kappa]$ with spatial bivectors, (but not for a mixed $[\sigma\lambda]$ alignment), any of $[\lambda_o\mu_o\delta_o]$ or $[\nu_o\beta_o\gamma_o]$ may be chosen to be aligned with e_t. $[\sigma_{o\iota\jmath\kappa}\lambda_{o\iota\jmath\kappa}]$, $[\sigma_{o\iota\jmath\kappa}\mu_{o\iota\jmath\kappa}]$, $[\sigma_{o\iota\jmath\kappa}\delta_{o\iota\jmath\kappa}]$ are true octonionic subloops, whereas $[\sigma_{o\iota\jmath\kappa}\nu_{o\iota\jmath\kappa}]$, $[\sigma_{o\iota\jmath\kappa}\beta_{o\iota\jmath\kappa}]$, $[\sigma_{o\iota\jmath\kappa}\gamma_{o\iota\jmath\kappa}]$ are quasi-octonionic.

This suggests that realignments for $\mathbb{T}$ corresponding to different choices of the unit element aligned with e_t, such as $\lambda_o \leftrightarrow \mu_o \leftrightarrow \delta_o$ are associated with three flavors for fermions. In the rest frame of a particle, a difference in flavor associated with realignment of $\mathbb{T}$ with respect to the STA can be regarded as a realignment of space-time with respect to the particle. As the increase in observed mass for a particle moving with respect to an observer can be regarded as a rotation of the temporal dimension into a spatial dimension, this suggests that a similar effect generates differences in mass for different flavors.

2.7. Quark and lepton families

For octonionic subloops of order 16, such as $[\sigma_{o\iota\jmath\kappa}\lambda_{o\iota\jmath\kappa}]$, re-orientations with respect to spatial bivectors and space-time bivectors include ones between colored and uncolored subloops of order 16, suggesting that lepto-quark transformations would occur. However, once the subloops are extended to order 32, these transformations are suppressed.

The subloops of order 32 identified as fermionic have four isomorphism types. A_0, B_0, Γ_0 and Σ_0 are uncolored, so can be identified with components for lepton families. $A_{1...6}$ and $B_{1...6}$ are colored and can be identified with components for quark families. The number of components is twice that of the associated particles. The components can be assigned to ones which interact with $W^{\pm}$ bosons and those that do not. Identification of correspondence with the Brout-Englert-Higgs mechanism[6][7] results in assignment of B_0, $A_{1...3}$ and $B_{4...6}$ to fermionic components that do not interact with $W^{\pm}$ bosons. As electrons are more like quarks than neutrinos are, A_0 and B_0 are assigned to electron families. Γ_0 and Σ_0 are assigned to neutrino families, but because it is isomorphic to $\Sigma_{1...15}$, the Σ_0 subloop is used in the Brout-Englert-Higgs mechanism, suppressing the existence of a sterile neutrino family.

2.8. Gluons

Colored bosonic subloops, $\Sigma_{4...15}$ are assigned to gluon components. Any two of either $[\Sigma_4, \Sigma_5, \Sigma_6]$, $[\Sigma_7, \Sigma_8, \Sigma_9]$, $[\Sigma_{10}, \Sigma_{11}, \Sigma_{12}]$ or $[\Sigma_{13}, \Sigma_{14}, \Sigma_{15}]$ share a true octonionic subloop which includes α_o. The interaction of the unshared colored cosets for such a combination with colored fermionic subloops $[A_{1...6}B_{1...6}]$ with the coset colors leaves its elements subscripted o unchanged, whilst transforming those subscripted ι, $\jmath$ or κ by interchanges: $\iota \leftrightarrow \jmath$ or $\jmath \leftrightarrow \kappa$ or $\kappa \leftrightarrow \iota$.
For example, for a gluon associated with $\pm\Sigma_5 \mp \Sigma_4$, elements subscripted ι would be switched to those subscripted $\jmath$ and vice versa, leaving those subscripted o and κ unchanged, so for $B_1 \leftrightarrow B_2$:
$[\sigma_o\sigma_\iota\lambda_o\lambda_\iota\mu_\jmath\mu_\kappa\nu_\jmath\nu_\kappa\alpha_\jmath\alpha_\kappa\beta\jmath\beta_\kappa\gamma_o\gamma_\iota\delta_o\delta_\iota] \leftrightarrow [\sigma_o\sigma_\jmath\lambda_o\lambda_\jmath\mu_\iota\mu_\kappa\nu_\iota\nu_\kappa\alpha_\iota\alpha_\kappa\beta\iota\beta_\kappa\gamma_o\gamma_\jmath\delta_o\delta_\jmath]$

Gluons are assembled as su(3) color and anti-color combinations for components assigned to two subloops from the same set of three - $[\Sigma_4, \Sigma_5, \Sigma_6]$, $[\Sigma_7, \Sigma_8, \Sigma_9]$, $[\Sigma_{10}, \Sigma_{11}, \Sigma_{12}]$ or $[\Sigma_{13}, \Sigma_{14}, \Sigma_{15}]$. This suggests that the subloop's shared octonionic subloop acts as a common octonionic component with respect which to its coset components, one from each loop, undergo sinusoidal oscillation.undergo sinusoidal oscillation.

If components from different sets of three that share the same color are combined, unshared cosets complement each other to form a colorless combination, similar to that for combinations of $\Sigma_{1...3}$, so their interactions are electroweak. If components from different sets of three that do not share the same color are combined, the static loop of shared elements is not colorless, so do not assemble gluons.

2.9. Electroweak bosons and the Brout-Englert-Higgs mechanism

If the subloops of T_L displayed isotropy for alignments of $[\lambda_o\mu_o\nu_o]$ with permutations of $[e_{yz}e_{zx}e_{xy}]$, the electroweak symmetry group would be $SU(3)$, but that is not the case. Examining table 5, if only elements subscripted o are considered, for subloops of order 16 there is a symmetry between $\lambda_o\beta_o$ and $\mu_o\gamma_o$. If only elements subscripted $\iota\jmath\kappa$ are considered, for subloops of order 16 there is a symmetry between $\mu_{\iota\jmath\kappa}\gamma_{\iota\jmath\kappa}$ and $\nu_{\iota\jmath\kappa}\delta_{\iota\jmath\kappa}$. Any combination of scalar elements subscripted o is color neutral, as is a balanced combination of elements subscripted $\iota\jmath\kappa$. This suggests a basis for electroweak symmetry and the Brout-Englert-Higgs mechanism[6][7]. The need to separate elements subscripted o from elements subscripted $\iota\jmath\kappa$ suggests a basis for the Weinberg angle. as a re-orientation for $\mu_o\gamma_o$ separately from $\mu_{\iota\jmath\kappa}\gamma_{\iota\jmath\kappa}$. As for gluons, there is a static true octonion subloop of shared elements $[\sigma_{o\iota\jmath\kappa}\alpha_{o\iota\jmath\kappa}]$ with respect to which coset components could undergo sinusoidal oscillation.

To match the way in which combinations of two subloops from $[\Sigma_4, \Sigma_5, \Sigma_6]$, $[\Sigma_7, \Sigma_8, \Sigma_9]$, $[\Sigma_{10}, \Sigma_{11}, \Sigma_{12}]$ or $[\Sigma_{13}, \Sigma_{14}, \Sigma_{15}]$ are used for gluons, consider combinations of Σ_1, Σ_2 and Σ_3, the bosonic subloops with matching orientation with respect to $[\sigma_{o\iota\jmath\kappa}\alpha_{o\iota\jmath\kappa}]$. This symmetry is broken by the asymmetrical participation of elements in subloops of order 16 and 32. With reference to table 5, a partial symmetry of $\lambda_o\beta_o$ with $\mu_o\gamma_o$ (participation rates [14/7]) can be extracted from the combination of Σ_1 and Σ_2, and a partial symmetry of $\mu_{\iota\jmath\kappa}\gamma_{\iota\jmath\kappa}$ with $\nu_{\iota\jmath\kappa}\delta_{\iota\jmath\kappa}$ (participation rates [10/11]) can be extracted from the combination of Σ_2 and Σ_3. The combination of Σ_1 and Σ_3 does not offer a partial symmetry. For the partial symmetries, and matching the way in which color/anticolor gluons change quark color:

$\pm\Sigma_1 \mp \Sigma_2$ leaves $[\sigma_{o\iota\jmath\kappa}\alpha_{o\iota\jmath\kappa}]$ unchanged, whilst interchanging $[\lambda_{o\iota\jmath\kappa}\beta_{o\iota\jmath\kappa}]$ with $[\mu_{o\iota\jmath\kappa}\gamma_{o\iota\jmath\kappa}]$. This has no effect on $A_{1\ldots3}$, $B_{4\ldots5}$, Σ_0 and converts $A_{1\ldots3}$, $B_{4\ldots5}$ to A_0, $B_{1\ldots3}$ to $A_{4\ldots6}$ and vice-versa suggesting that it is to associated with W_1 and W_2 and $W^{\pm}$ bosons. The product of $[\lambda_{o\iota\jmath\kappa}\beta_{o\iota\jmath\kappa}]$ with $[\mu_{o\iota\jmath\kappa}\gamma_{o\iota\jmath\kappa}]$ generates $[\nu_{o\iota\jmath\kappa}\delta_{o\iota\jmath\kappa}]$. This can be disposed of by interaction with the $[\nu_{o\iota\jmath\kappa}]$ component of Σ_0 factored by the $\sigma_o\alpha_o i\sigma_o i\alpha_o$ complex doublet.

$\pm\Sigma_2 \mp \Sigma_3$ leaves $[\sigma_{o\iota\jmath\kappa}\alpha_{o\iota\jmath\kappa}]$ unchanged, whilst interchanging $[\mu_{o\iota\jmath\kappa}\gamma_{o\iota\jmath\kappa}]$ with $[\nu_{o\iota\jmath\kappa}\delta_{o\iota\jmath\kappa}]$. This has no effect on $B_0, A_{1\ldots3}, B_{1\ldots3}$, converts A_0 to B_0, $A_{4\ldots6}$ to $B_{4\ldots6}$ and vice-versa and converts Γ_0 to Σ_0 and vice versa, making it logical to associate it with W_3 and B and with photons and Z^0 bosons. The product of $[\mu_{o\iota\jmath\kappa}\gamma_{o\iota\jmath\kappa}]$ with $[\nu_{o\iota\jmath\kappa}\delta_{o\iota\jmath\kappa}]$ generates $[\mu_{o\iota\jmath\kappa}\gamma_{o\iota\jmath\kappa}]$. This can be disposed of by interaction with the $[\mu_{o\iota\jmath\kappa}]$ component of Σ_0 factored by the $\sigma_o\alpha_o i\sigma_o i\alpha_o$ complex doublet.

Interactions with Σ_0 suppress the interaction $\Gamma_0 \leftrightarrow \Sigma_0$ at low energies.

2.10. Higgs particle

The Higgs field and particle are scalar, so will involve alignments of, for T_L; $[\sigma_o\lambda_o\mu_o\nu_o\alpha_o\beta_o\gamma_o\delta_o]$ with for M_L: $[e_0e_te_{xyz}e_{txyz}ie_0ie_tie_{xyz}ie_{txyz}]$. Assigning the unit imaginary for M_L and α_o from T_L to phase, consider alignments of $[\sigma_o\lambda_o\mu_o\nu_o]$ with for M_L: $[e_0e_te_{xyz}e_{txyz}]$. Assign $[\sigma_oe_0]\times[\sigma_{o\imath\jmath\kappa}\alpha_{o\imath\jmath\kappa}]$ to the Higgs and $[\lambda_oe_0]\times[\sigma_{o\imath\jmath\kappa}\alpha_{o\imath\jmath\kappa}]$, $[\mu_oe_0]\times[\sigma_{o\imath\jmath\kappa}\alpha_{o\imath\jmath\kappa}]$, $[\nu_oe_0]\times[\sigma_{o\imath\jmath\kappa}\alpha_{o\imath\jmath\kappa}]$,to Goldstone bosons giving mass to the $W^{\pm}$ and Z^0. The mexican hat potential for the Higgs field could arise from properties of 4×4 matrices - refer to Appendix B.

2.11. Fermions and the Dirac equation

Γ_0, A_0, B_0, Σ_0 subloops have been identified with color neutral fermion families but with Σ_0 suppressed at low energies. $A_{1\ldots6}$, $B_{1\ldots6}$ have been identified with colored fermion families. As bosons are assembled using combinations of two bosonic subloops, this suggests assembling electron and quark families using combinations of A subloops with B subloops, assigning the Γ subloop to the neutrino family, and the Σ_0 to a sterile neutrino family that is not observed at low energies.

TABLE 6. A_0, B_0 orientations for three generations of the electron family

$Cl_4(C)$ Blades	Scalar	Bivectors						Pseudo	Vectors				Trivectors				Chirality/mass
Unit elements	e_o	e_{yz}	e_{zx}	e_{xy}	e_{xt}	e_{yt}	e_{zt}	e_{xyzt}	e_x	e_y	e_z	e_t	e_{yzt}	e_{zxt}	e_{xyt}	e_{xyz}	interaction
Dirac $\gamma^{0..3,5}$								$\sigma_o\gamma^5$	γ^1	γ^2	γ^3	γ^0					$\gamma^5+\alpha_o\gamma^5$
A_0	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\beta_\imath$	$\beta_\jmath$	β_κ	β_o	$\delta_\imath$	$\delta_\jmath$	δ_κ	μ_o	$\mu_\imath$	$\mu_\jmath$	μ_κ	δ_o	$\beta_o+\lambda_o$
	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\beta_\imath$	$\beta_\jmath$	β_κ	β_o	$\mu_\imath$	$\mu_\jmath$	μ_κ	δ_o	$\delta_\imath$	$\delta_\jmath$	δ_κ	μ_o	$\beta_o+\lambda_o$
B_0	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\beta_\imath$	$\beta_\jmath$	β_κ	β_o	$\nu_\imath$	$\nu_\jmath$	ν_κ	γ_o	$\gamma_\imath$	$\gamma_\jmath$	γ_κ	ν_o	$\beta_o+\lambda_o$
	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\beta_\imath$	$\beta_\jmath$	β_κ	β_o	$\gamma_\imath$	$\gamma_\jmath$	γ_κ	ν_o	$\nu_\imath$	$\nu_\jmath$	ν_κ	γ_o	$\beta_o+\lambda_o$
A_0	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\mu_\imath$	$\mu_\jmath$	μ_κ	μ_o	$\beta_\imath$	$\beta_\jmath$	β_κ	δ_o	$\delta_\imath$	$\delta_\jmath$	δ_κ	β_o	$\mu_o+\gamma_o$
	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\mu_\imath$	$\mu_\jmath$	μ_κ	μ_o	$\delta_\imath$	$\delta_\jmath$	δ_κ	β_o	$\beta_\imath$	$\beta_\jmath$	β_κ	δ_o	$\mu_o+\gamma_o$
B_0	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\gamma_\imath$	$\gamma_\jmath$	γ_κ	γ_o	$\beta_\imath$	$\beta_\jmath$	β_κ	ν_o	$\nu_\imath$	$\nu_\jmath$	ν_κ	β_o	$\gamma_o+\mu_o$
	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\gamma_\imath$	$\gamma_\jmath$	γ_κ	γ_o	$\nu_\imath$	$\nu_\jmath$	ν_κ	β_o	$\beta_\imath$	$\beta_\jmath$	β_κ	ν_o	$\gamma_o+\mu_o$
A_0	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\delta_\imath$	$\delta_\jmath$	δ_κ	δ_o	$\mu_\imath$	$\mu_\jmath$	μ_κ	β_o	$\beta_\imath$	$\beta_\jmath$	β_κ	μ_o	$\delta_o+\nu_o$
	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\delta_\imath$	$\delta_\jmath$	δ_κ	δ_o	$\beta_\imath$	$\beta_\jmath$	β_κ	μ_o	$\mu_\imath$	$\mu_\jmath$	μ_κ	β_o	$\delta_o+\nu_o$
B_0	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\nu_\imath$	$\nu_\jmath$	ν_κ	ν_o	$\gamma_\imath$	$\gamma_\jmath$	γ_κ	β_o	$\beta_\imath$	$\beta_\jmath$	β_κ	γ_o	$\nu_o+\delta_o$
	σ_o	$\sigma_\imath$	$\sigma_\jmath$	σ_κ	$\nu_\imath$	$\nu_\jmath$	ν_κ	ν_o	$\beta_\imath$	$\beta_\jmath$	β_κ	γ_o	$\gamma_\imath$	$\gamma_\jmath$	γ_κ	β_o	$\nu_o+\delta_o$

2.12. Signature, matter and antimatter imbalance

As shown in table 5, for M_L there is a distinction between participation rates for elements in subloops of order 16, which are isomorphic to four groups, GAP4 IDs: [16,10], [16,11], 16 12] and [16 13]. When identified with unit elements for the Space Time Algebra (STA), going from $(+++-)$ to $(+---)$ signature, the signatures for vector and tri-vector blades are reversed. In combination with distinctions between participation rates by elements of T_L in its subloops of order 16 this may account for the imbalance between matter and anti-matter observed in the universe.

2.13. Measurement and Quantum ambiguity

Quantum field theory calculations generate probabilities for future events. Measurement collapses those probabilities into the observed event, with parameters subject to the uncertainty principle. Interpretations for the process between events differ. Interpretations based on an absence of reality between events, or the generation of multiple realities for every event, or a discontinuous collapse of a smeared all have their proponents. Super-determinism and retro-causality allow for unambiguous reality between events, but introduce other counter-intuitive features.

There have been many approaches to the use of non-associative algebras to account for quantum ambiguity, as documented by Liebmann et al[9], and elsewhere[10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][25][26][27][28][29].

$\mathbb{U} \cong M_4(C) \otimes \mathbb{T}$ includes, as sub-algebras, some of the non-associative algebras considered. As non-associativity can generate indefinite results for calculations if the order of multiplications varies, this suggests the postulate that events must be consistent, to within the parameters of uncertainty, with end points of trajectories (measurement points) based on perspectives for all other events that constitute reality, but that, between points, different perspectives may cause different observers to infer different trajectories.

2.14. Dark matter and dark energy

Dark matter composed of gluons has been proposed and was revisited by Carenza et al[30][31]. In section 2.8, it is postulated that gluon combinations can be assembled using color and anti-color combinations for components using two subloops from the same set of three - [Σ_4, Σ_5, Σ_6], [Σ_7, Σ_8, Σ_9], [Σ_{10}, Σ_{11}, Σ_{12}] or [Σ_{13}, Σ_{14}, Σ_{15}]. The set [Σ_4, Σ_5, Σ_6] differs from the other sets in that its subloops are flavor neutral. This suggests the possibility that glueballs assembled from gluon components from this set may be stable and not subject to confinement, making them candidates for dark matter.

An apparent feature of dark energy is the non-conservation of energy. An approach that might reconcile its properties with conservation of energy could be to identify our universe as a three dimensional wavefront propagating in four dimensions in accordance with Huygens principle, with a retarded wave propagating backwards in time. If that wake takes energy backwards, it could balance the energy generated forwards by the expansion of space-time. $M_4(C) \cong Cl_{4,1}(R) \cong Cl_{2,3}(R)$, so can be used to represent the Clifford algebra of space-time for either signature, but the signature of the added dimension would be space-like for one space-time signature or time-like for the other. If one possibility is consistent with relativity[32], but not the other, this would identify the signature of space-time, and could account for the matter-antimatter asymmetry in the universe.

2.15. Gauging the symmetries

The loops package for GAP4 reports automorphism groups of:
U_L : C2xC2x((((C2xC2xC2xC2):A6):(C2xC2))x((C2xC2xC2).PSL(3,2)))
T_L : C2 x C2 x ((C2 x C2 x C2) . PSL(3,2))
M_L : ((C2 x C2 x C2 x C2) : A6) : (C2 x C2)
With a spacetime subalgebra of $M_4(C)$ assigned as a principal bundle, $\mathbb{T} \otimes \mathbb{C}$ as a fibre bundle, particle components as sections of the fibre bundle, the structure found suggests that a subgroup of the automorphism group for U_L would correspond to the $SU(3) \times SU(2 \times U(1)$ symmetry group of the standard model.

2.16. Comparison with unification approaches

The $M_4(C)$ subalgebra of $\mathbb{U}$ can be used to represent space-time with an added fifth spatial dimension, a feature of the original Kaluza-Klein model[33][34]. Five real dimensions can be represented by $M_4(C)$ which can then be complexified using a imaginary unit elements from T_L, suggesting similarities with $SU(5)$ unification[35]. $M_4(C)$ can be complexified by quaternionic unit elements from T_L, suggesting similarities with $SU(4) \times SU(2) \times SU(2)$ unification[36].

The connection betweeen the trigintaduonions and the 31-sphere suggests a connection with SO32 heterotic string theory. The assembly of fundamental particles using pairs of sedenionic and quasi-sedenionic subloops of T_L suggests identification with the closed string of SO32 heterotic string theory[37] with a static circle in a 31-sphere, and instead of identifying particles with excitations of circle, identifying them as spherical harmonics with that circle remaining static.

$\mathbb{U} \cong Cl_{3,1}(R) \times \mathbb{C} \circledast \mathbb{H} \circledast \mathbb{O}$ has similarities to the algebra $\mathbb{R} \times \mathbb{C} \times \mathbb{H} \times \mathbb{O}$, used by G. Dixon in his model[19][20].

3. Conclusion

The pattern of fundamental particles of the standard model displays a blend of symmetry and asymmetry which is hard to replicate using direct association of particles with sub-algebras of a Clifford algebra. A similarity between that and the symmetry and asymmetry can be found in the subalgebra structure loop, T_L, embedded in $\mathbb{T}$ when a requirement for spatial isotropy is imposed by aligning it with a Clifford algebra. This paper describes that similarity. Deeper analysis is required to establish whether that similarity can be congruence. The algebrology suggests that it might be, but congruence with established physics is extremely constraining, so the similarity may be coincidental. As a non-physicist, I can only wonder about the possibility, knowing that the chances of such a naive approach being correct are low.

Appendix A. Cayley tables for $\mathbb{T}$ and $M_4(C)$

TABLE 7. Cayley table for $\mathbb{T}$ unit elements, T_L

	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
σ_o	$+\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$
σ_ι	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$
$\sigma_\jmath$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$
σ_κ	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$
λ_o	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$
λ_ι	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$
$\lambda_\jmath$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$
λ_κ	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$
μ_o	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$
μ_ι	$+\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$
$\mu_\jmath$	$+\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$
μ_κ	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$
ν_o	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$
ν_ι	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$
$\nu_\jmath$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$
ν_κ	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$
α_o	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$
α_ι	$+\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$
$\alpha_\jmath$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$
α_κ	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$
β_o	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$
β_ι	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$
$\beta_\jmath$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$
β_κ	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$
γ_o	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$
γ_ι	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$
$\gamma_\jmath$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$
γ_κ	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$
δ_o	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$
δ_ι	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$
$\delta_\jmath$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$
δ_κ	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$

TABLE 8. Cayley table for $M_4(C)$ unit elements, M_L

Label	S	L	M	N	V	D	E	F	iU	iX	iY	iZ	iT	iP	iQ	iR	iS	iL	iM	iN	iV	iD	iE	iF	U	X	Y	Z	T	P	Q	R
S	$+S$	$+L$	$+M$	$+N$	$+V$	$+D$	$+E$	$+F$	$+iU$	$+iX$	$+iY$	$+iZ$	$+iT$	$+iP$	$+iQ$	$+iR$	$+iS$	$+iL$	$+iM$	$+iN$	$+iV$	$+iD$	$+iE$	$+iF$	$+U$	$+X$	$+Y$	$+Z$	$+T$	$+P$	$+Q$	$+R$
L	$+L$	$-S$	$+N$	$-M$	$-D$	$+V$	$-F$	$+E$	$+iX$	$-iU$	$+iZ$	$-iY$	$-iP$	$+iT$	$-iR$	$+iQ$	$+iL$	$-iS$	$+iN$	$-iM$	$-iD$	$+iV$	$-iF$	$+iE$	$+X$	$-U$	$+Z$	$-Y$	$-P$	$+T$	$-R$	$+Q$
M	$+M$	$-N$	$-S$	$+L$	$+E$	$-F$	$-V$	$+D$	$-iY$	$+iZ$	$+iU$	$-iX$	$-iQ$	$+iR$	$+iT$	$-iP$	$+iM$	$-iN$	$-iS$	$+iL$	$+iE$	$-iF$	$-iV$	$+iD$	$-Y$	$+Z$	$+U$	$-X$	$-Q$	$+R$	$+T$	$-P$
N	$+N$	$+M$	$-L$	$-S$	$-F$	$-E$	$+D$	$+V$	$-iZ$	$-iY$	$+iX$	$+iU$	$+iR$	$+iQ$	$-iP$	$-iT$	$+iN$	$+iM$	$-iL$	$-iS$	$-iF$	$-iE$	$+iD$	$+iV$	$-Z$	$-Y$	$+X$	$+U$	$+R$	$+Q$	$-P$	$-T$
V	$+V$	$-D$	$+E$	$-F$	$-S$	$+L$	$-M$	$+N$	$+iT$	$-iP$	$+iQ$	$-iR$	$-iU$	$+iX$	$-iY$	$+iZ$	$+iV$	$-iD$	$+iE$	$-iF$	$-iS$	$+iL$	$-iM$	$+iN$	$+T$	$-P$	$+Q$	$-R$	$-U$	$+X$	$-Y$	$+Z$
D	$+D$	$+V$	$+F$	$+E$	$+L$	$+S$	$+N$	$+M$	$+iP$	$+iT$	$+iR$	$+iQ$	$+iX$	$+iU$	$+iZ$	$+iY$	$+iD$	$+iV$	$+iF$	$+iE$	$+iL$	$+iS$	$+iN$	$+iM$	$+P$	$+T$	$+R$	$+Q$	$+X$	$+U$	$+Z$	$+Y$
E	$+E$	$+F$	$-V$	$-D$	$-M$	$-N$	$+S$	$+L$	$-iQ$	$-iR$	$+iT$	$+iP$	$+iY$	$+iZ$	$-iU$	$-iX$	$+iE$	$+iF$	$-iV$	$-iD$	$-iM$	$-iN$	$+iS$	$+iL$	$-Q$	$-R$	$+T$	$+P$	$+Y$	$+Z$	$-U$	$-X$
F	$+F$	$-E$	$-D$	$+V$	$+N$	$-M$	$-L$	$+S$	$-iR$	$+iQ$	$+iP$	$-iT$	$-iZ$	$+iY$	$+iX$	$-iU$	$+iF$	$-iE$	$-iD$	$+iV$	$+iN$	$-iM$	$-iL$	$+iS$	$-R$	$+Q$	$+P$	$-T$	$-Z$	$+Y$	$+X$	$-U$
iU	$+iU$	$+iX$	$-iY$	$-iZ$	$-iT$	$-iP$	$+iQ$	$+iR$	$+S$	$+L$	$-M$	$-N$	$-V$	$-D$	$+E$	$+F$	$-U$	$-X$	$+Y$	$+Z$	$+T$	$+P$	$-Q$	$-R$	$-iS$	$-iL$	$+iM$	$+iN$	$+iV$	$+iD$	$-iE$	$-iF$
iX	$+iX$	$-iU$	$-iZ$	$+iY$	$+iP$	$-iT$	$-iR$	$+iQ$	$+L$	$-S$	$-N$	$+M$	$+D$	$-V$	$-F$	$+E$	$-X$	$+U$	$+Z$	$-Y$	$-P$	$+T$	$+R$	$-Q$	$-iL$	$+iS$	$+iN$	$-iM$	$-iD$	$+iV$	$+iF$	$-iE$
iY	$+iY$	$-iZ$	$+iU$	$-iX$	$-iQ$	$+iR$	$-iT$	$+iP$	$-M$	$+N$	$-S$	$+L$	$+E$	$-F$	$+V$	$-D$	$-Y$	$+Z$	$-U$	$+X$	$+Q$	$-R$	$+T$	$-P$	$+iM$	$-iN$	$+iS$	$-iL$	$-iE$	$+iF$	$-iV$	$+iD$
iZ	$+iZ$	$+iY$	$+iX$	$+iU$	$+iR$	$+iQ$	$+iP$	$+iT$	$-N$	$-M$	$-L$	$-S$	$-F$	$-E$	$-D$	$-V$	$-Z$	$-Y$	$-X$	$-U$	$-R$	$-Q$	$-P$	$-T$	$+iN$	$+iM$	$+iL$	$+iS$	$+iF$	$+iE$	$+iD$	$+iV$
iT	$+iT$	$-iP$	$-iQ$	$+iR$	$+iU$	$-iX$	$-iY$	$+iZ$	$+V$	$-D$	$-E$	$+F$	$+S$	$-L$	$-M$	$+N$	$-T$	$+P$	$+Q$	$-R$	$-U$	$+X$	$+Y$	$-Z$	$-iV$	$+iD$	$+iE$	$-iF$	$-iS$	$+iL$	$+iM$	$-iN$
iP	$+iP$	$+iT$	$-iR$	$-iQ$	$-iX$	$-iU$	$+iZ$	$+iY$	$+D$	$+V$	$-F$	$-E$	$-L$	$-S$	$+N$	$+M$	$-P$	$-T$	$+R$	$+Q$	$+X$	$+U$	$-Z$	$-Y$	$-iD$	$-iV$	$+iF$	$+iE$	$+iL$	$+iS$	$-iN$	$-iM$
iQ	$+iQ$	$+iR$	$+iT$	$+iP$	$+iY$	$+iZ$	$+iU$	$+iX$	$-E$	$-F$	$-V$	$-D$	$-M$	$-N$	$-S$	$-L$	$-Q$	$-R$	$-T$	$-P$	$-Y$	$-Z$	$-U$	$-X$	$+iE$	$+iF$	$+iV$	$+iD$	$+iM$	$+iN$	$+iS$	$+iL$
iR	$+iR$	$-iQ$	$+iP$	$-iT$	$-iZ$	$+iY$	$-iX$	$+iU$	$-F$	$+E$	$-D$	$+V$	$+N$	$-M$	$+L$	$-S$	$-R$	$+Q$	$-P$	$+T$	$+Z$	$-Y$	$+X$	$-U$	$+iF$	$-iE$	$+iD$	$-iV$	$-iN$	$+iM$	$-iL$	$+iS$
iS	$+iS$	$+iL$	$+iM$	$+iN$	$+iV$	$+iD$	$+iE$	$+iF$	$-U$	$-X$	$-Y$	$-Z$	$-T$	$-P$	$-Q$	$-R$	$-S$	$-L$	$-M$	$-N$	$-V$	$-D$	$-E$	$-F$	$+iU$	$+iX$	$+iY$	$+iZ$	$+iT$	$+iP$	$+iQ$	$+iR$
iL	$+iL$	$-iS$	$+iN$	$-iM$	$-iD$	$+iV$	$-iF$	$+iE$	$-X$	$+U$	$-Z$	$+Y$	$+P$	$-T$	$+R$	$-Q$	$-L$	$+S$	$-N$	$+M$	$+D$	$-V$	$+F$	$-E$	$+iX$	$-iU$	$+iZ$	$-iY$	$-iP$	$+iT$	$-iR$	$+iQ$
iM	$+iM$	$-iN$	$-iS$	$+iL$	$+iE$	$-iF$	$-iV$	$+iD$	$+Y$	$-Z$	$-U$	$+X$	$+Q$	$-R$	$-T$	$+P$	$-M$	$+N$	$+S$	$-L$	$-E$	$+F$	$+V$	$-D$	$-iY$	$+iZ$	$+iU$	$-iX$	$-iQ$	$+iR$	$+iT$	$-iP$
iN	$+iN$	$+iM$	$-iL$	$-iS$	$-iF$	$-iE$	$+iD$	$+iV$	$+Z$	$+Y$	$-X$	$-U$	$-R$	$-Q$	$+P$	$+T$	$-N$	$-M$	$+L$	$+S$	$+F$	$+E$	$-D$	$-V$	$-iZ$	$-iY$	$+iX$	$+iU$	$+iR$	$+iQ$	$-iP$	$-iT$
iV	$+iV$	$-iD$	$+iE$	$-iF$	$-iS$	$+iL$	$-iM$	$+iN$	$-T$	$+P$	$-Q$	$+R$	$+U$	$-X$	$+Y$	$-Z$	$-V$	$+D$	$-E$	$+F$	$+S$	$-L$	$+M$	$-N$	$+iT$	$-iP$	$+iQ$	$-iR$	$-iU$	$+iX$	$-iY$	$+iZ$
iD	$+iD$	$+iV$	$+iF$	$+iE$	$+iL$	$+iS$	$+iN$	$+iM$	$-P$	$-T$	$-R$	$-Q$	$-X$	$-U$	$-Z$	$-Y$	$-D$	$-V$	$-F$	$-E$	$-L$	$-S$	$-N$	$-M$	$+iP$	$+iT$	$+iR$	$+iQ$	$+iX$	$+iU$	$+iZ$	$+iY$
iE	$+iE$	$+iF$	$-iV$	$-iD$	$-iM$	$-iN$	$+iS$	$+iL$	$+Q$	$+R$	$-T$	$-P$	$-Y$	$-Z$	$+U$	$+X$	$-E$	$-F$	$+V$	$+D$	$+M$	$+N$	$-S$	$-L$	$-iQ$	$-iR$	$+iT$	$+iP$	$+iY$	$+iZ$	$-iU$	$-iX$
iF	$+iF$	$-iE$	$-iD$	$+iV$	$+iN$	$-iM$	$-iL$	$+iS$	$+R$	$-Q$	$-P$	$+T$	$+Z$	$-Y$	$-X$	$+U$	$-F$	$+E$	$+D$	$-V$	$-N$	$+M$	$+L$	$-S$	$-iR$	$+iQ$	$+iP$	$-iT$	$-iZ$	$+iY$	$+iX$	$-iU$
U	$+U$	$+X$	$-Y$	$-Z$	$-T$	$-P$	$+Q$	$+R$	$-iS$	$-iL$	$+iM$	$+iN$	$+iV$	$+iD$	$-iE$	$-iF$	$+iU$	$+iX$	$-iY$	$-iZ$	$-iT$	$-iP$	$+iQ$	$+iR$	$-S$	$-L$	$+M$	$+N$	$+V$	$+D$	$-E$	$-F$
X	$+X$	$-U$	$-Z$	$+Y$	$+P$	$-T$	$-R$	$+Q$	$-iL$	$+iS$	$+iN$	$-iM$	$-iD$	$+iV$	$+iF$	$-iE$	$+iX$	$-iU$	$-iZ$	$+iY$	$+iP$	$-iT$	$-iR$	$+iQ$	$-L$	$+S$	$+N$	$-M$	$-D$	$+V$	$+F$	$-E$
Y	$+Y$	$-Z$	$+U$	$-X$	$-Q$	$+R$	$-T$	$+P$	$+iM$	$-iN$	$+iS$	$-iL$	$-iE$	$+iF$	$-iV$	$+iD$	$+iY$	$-iZ$	$+iU$	$-iX$	$-iQ$	$+iR$	$-iT$	$+iP$	$+M$	$-N$	$+S$	$-L$	$-E$	$+F$	$-V$	$+D$
Z	$+Z$	$+Y$	$+X$	$+U$	$+R$	$+Q$	$+P$	$+T$	$+iN$	$+iM$	$+iL$	$+iS$	$+iF$	$+iE$	$+iD$	$+iV$	$+iZ$	$+iY$	$+iX$	$+iU$	$+iR$	$+iQ$	$+iP$	$+iT$	$+N$	$+M$	$+L$	$+S$	$+F$	$+E$	$+D$	$+V$
T	$+T$	$-P$	$-Q$	$+R$	$+U$	$-X$	$-Y$	$+Z$	$-iV$	$+iD$	$+iE$	$-iF$	$-iS$	$+iL$	$+iM$	$-iN$	$+iT$	$-iP$	$-iQ$	$+iR$	$+iU$	$-iX$	$-iY$	$+iZ$	$-V$	$+D$	$+E$	$-F$	$-S$	$+L$	$+M$	$-N$
P	$+P$	$+T$	$-R$	$-Q$	$-X$	$-U$	$+Z$	$+Y$	$-iD$	$-iV$	$+iF$	$+iE$	$+iL$	$+iS$	$-iN$	$-iM$	$+iP$	$+iT$	$-iR$	$-iQ$	$-iX$	$-iU$	$+iZ$	$+iY$	$-D$	$-V$	$+F$	$+E$	$+L$	$+S$	$-N$	$-M$
Q	$+Q$	$+R$	$+T$	$+P$	$+Y$	$+Z$	$+U$	$+X$	$+iE$	$+iF$	$+iV$	$+iD$	$+iM$	$+iN$	$+iS$	$+iL$	$+iQ$	$+iR$	$+iT$	$+iP$	$+iY$	$+iZ$	$+iU$	$+iX$	$+E$	$+F$	$+V$	$+D$	$+M$	$+N$	$+S$	$+L$
R	$+R$	$-Q$	$+P$	$-T$	$-Z$	$+Y$	$-X$	$+U$	$+iF$	$-iE$	$+iD$	$-iV$	$-iN$	$+iM$	$-iL$	$+iS$	$+iR$	$-iQ$	$+iP$	$-iT$	$-iZ$	$+iY$	$-iX$	$+iU$	$+F$	$-E$	$+D$	$-V$	$-N$	$+M$	$-L$	$+S$

Appendix B. The Brout-Englert-Higgs mechanism

This appendix is part of a previous paper by this author[**?**].

The Brout-Englert-Higgs mechanism acts on a complex doublet and involves scalar fields. For $M_4C \otimes \mathbb{T}$ a scalar subalgebra can be assembled as the product: $[\sigma_o S, \sigma_o iS, \alpha_o S, \alpha_o iS] \otimes [\sigma_o S, \sigma_o T, \sigma_o V, \sigma_o U] \otimes [\sigma_o S, \lambda_o S, \mu_o S, \nu_o S]$.

$[\sigma_o S, \sigma_o T, \sigma_o V, \sigma_o U] \otimes [\sigma_o S, \lambda_o S, \mu_o S, \nu_o S]$ is isomorphic to $\mathbb{H} \otimes \mathbb{H}$ and to $M_4(R)$. Its unit elements can be relabeled as matrices from table 1 as follows:
$[\sigma_o S] \cong [S]$, $[\sigma_o T, \sigma_o V, \sigma_o U] \cong [TVU]$, $[\lambda_o S, \mu_o S, \nu_o S] \cong [LMN]$
$[\lambda_o T, \mu_o T, \nu_o T] \cong [PQR]$, $[\lambda_o V, \mu_o V, \nu_o V] \cong [DEF]$, $[\lambda_o U, \mu_o U, \nu_o U] \cong [XYZ]$

The Brout-Englert-Higgs mechanism is based on a scalar field with a mexican hat potential. It is possible to find subalgebras of $M_4(R)$, and thus of $[\sigma_o S, \sigma_o T, \sigma_o V, \sigma_o U] \otimes [\sigma_o S, \lambda_o S, \mu_o S, \nu_o S]$, with this property
Subalgebras of $M_4(R)$ for which the scalar component (unit matrix $[S]$), is associated with a mexican hat potential, can be found by considering unitary abelian subgroups of $M_4(R)$. Unitary abelian subgroups of $M_4(R)$ can be represented by diagonal 4×4 matrices.

$$\begin{bmatrix} e^{\theta_1} & 0 & 0 & 0 \\ 0 & e^{\theta_2} & 0 & 0 \\ 0 & 0 & e^{\theta_3} & 0 \\ 0 & 0 & 0 & e^{\theta_4} \end{bmatrix}$$

where $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 0$, allowing it to be rewritten:

$$\begin{bmatrix} e^{a} & 0 & 0 & 0 \\ 0 & e^{b} & 0 & 0 \\ 0 & 0 & e^{c} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The product of two elements of this type with parameters a, b, c and a', b', c' has parameters $a + a', b + b', c + c'$. A subgroup of the Heisenberg group H(5) shares this property:

$$\begin{bmatrix} 1 & a & b & c + ab \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix has determinant = 1, and the commuting products of the form:

$$\begin{bmatrix} 1 & a + a^{'} & b + b^{'} & c + c^{'} + (a + a^{'}) \times (b + b^{'}) \\ 0 & 1 & 0 & b + b^{'} \\ 0 & 0 & 1 & a + a^{'} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix can be written in terms of unit elements of $M_4(R)$ as:
$[S] + a/2[V + Y] + b/2[M + F] + (c + ab)/4[E + U + N + P]$.

There are other combinations of unit elements of $M_4(R)$ with similar properties. These can be found using a 6×6 array having anti-commuting basis matrices and the identity in each row/column:

$$\begin{bmatrix} S & V & T & X & Y & Z \\ V & S & U & P & Q & R \\ T & U & S & D & E & F \\ X & P & D & S & N & M \\ Y & Q & E & N & S & L \\ Z & R & F & M & L & S \end{bmatrix}$$

Interchanging rows and matching columns preserves group properties and commutation relationships with respect to position in the array. For example, rows and columns 1 and 2 can be interchanged to make the array:

$$\begin{bmatrix} S & V & U & P & Q & R \\ V & S & T & X & Y & Z \\ U & T & S & D & E & F \\ P & X & D & S & N & M \\ Q & Y & E & N & S & L \\ R & Z & F & M & L & S \end{bmatrix}$$

Inspecting this array to assign unit matrices for an equivalent H5 subgroup group, they would be:
$[S] + a/2[V + Q] + b/2[M + F] + (c + ab)/4[E + N + T + X]$
This combination has the same properties. Interchanging rows and columns 1 and 2 has not changed the signatures of the matrices allocated to each position.

If a further interchange is made that does affect the signatures, e.g interchanging rows and columns 1 and 4, to generate:

$$\begin{bmatrix} S & Q & U & P & V & R \\ Q & S & E & N & Y & L \\ U & E & S & D & T & F \\ P & N & D & S & X & M \\ V & Y & E & N & S & L \\ R & L & F & M & Z & S \end{bmatrix}$$

For the combination:
$[S] + a/2[Y + Q] + b/2[M + F] + (c + ab)/4[P + U + T + X]$
The determinant is no longer 1. To make this combination generate a unitary matrix, a factor has to be applied to $[S]$. That factor is $\sqrt{(} \pm 1 \pm 2(a/2)^2)$, provided that the factor is real and not imaginary.

For the resulting matrix, there are four plus/minus permutations, for which the possible components for $[S]$ are :

$$\begin{bmatrix} \sqrt(1+a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(1+a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(1+a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(1+a^2/2) \end{bmatrix}$$

Which always has real entries, and determinant $= 1 + a^2 + a^4/4$

$$\begin{bmatrix} \sqrt(-1-a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(-1-a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(-1-a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(-1-a^2/2) \end{bmatrix}$$

Which never has real entries, and determinant $= 1 + a^2 + a^4/4$

$$\begin{bmatrix} \sqrt(1-a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(1-a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(1-a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(1-a^2/2) \end{bmatrix}$$

Which has real entries for $a^2/2 \leq 1$, and determinant $= 1 - a^2 + a^4/4$

$$\begin{bmatrix} \sqrt(-1+a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(-1+a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(-1+a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(-1+a^2/2) \end{bmatrix}$$

Which has real entries for $a^2/2 \geq 1$, and determinant $= 1 - a^2 + a^4/4$

The function $f(a) = 1 - a^2 + a^4/4$ has the form of a mexican hat potential.
For the assignment of unit elements of $\mathbb{T} \otimes Cl_{3,1} \otimes \mathbb{C}$ to matrices:

$[\sigma_o e_0] = [S]$, $[\sigma_o e_t, \sigma_o e_{xyzt}, \sigma_o e_{xyz}] = [TVU]$, $[\lambda_o e_0, \mu_o e_0, \nu_o e_0] = [LMN]$

$[\lambda_o e_t, \mu_o e_t, \nu_o e_t] = [PQR]$, $[\lambda_o e_{xyzt}, \mu_o e_{xyzt}, \nu_o e_{xyzt}] = [DEF]$, $[\lambda_o e_{xyz}, \mu_o e_{xyz}, \nu_o e_{xyz}] = [XYZ]$

The group represented by a plus/minus choice for:

$\sqrt(\pm 1 \pm a^2/2)[S] + a/2[Y + Q] + b/2[M + F] + (c + ab)/4[P + U + T + X]$

is isomorphic to that for the same plus/minus choice for:

$\sqrt(\pm 1 \pm a^2/2)[\sigma_o e_0] + a/2[\mu_o e_{xyz} + \mu_o e_t] + b/2[\mu_o e_0 + \nu_o e_{xyzt}]$

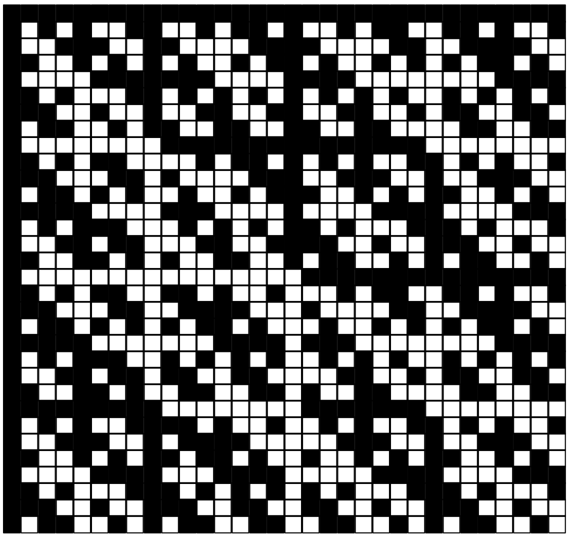
$+ (c + ab)/4[\lambda_o e_t + \sigma_o e_{xyz} + \sigma_o e_t + \lambda_o e_{xyz}]$

This features only scalar unit elements of $\mathbb{U}$.

Appendix C. Clifford algebras and Cayley-Dickson-type algebras

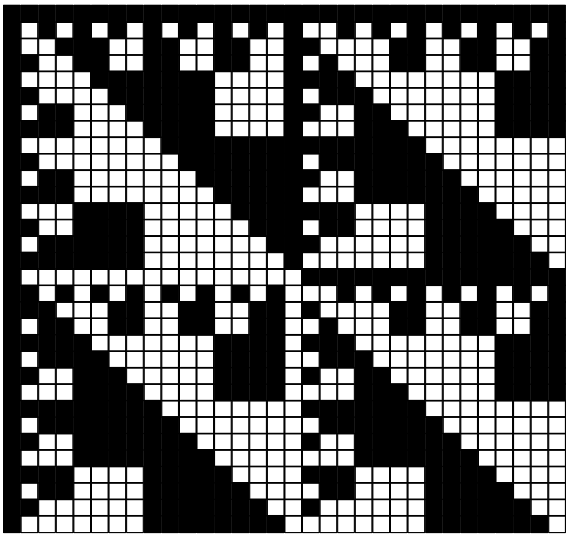
As reported in a previous paper [1], there are parallels between the left and right quaternion subalgebras of $M_4(R)$ and particular octonionic subalgbras of $\mathbb{T}$.

$\mathbb{T}$ is generated using the Cayley Dickson construction, with the product:
$(a, b)(c, d) = (ac - db^*, a^*d + cb)$
Its subalgebra structure has been analysed by Cawagas et al[**?**]. Cayley-Dickson type algebras have also been analysed by J.W.Bales[38]. He arranges Cayley tables for their unit elements as normalised latin squares with elements ordered so that the bit-wise 'exclusive or' (XOR) of binary representations of two element's numbering generate the numbering of their product. He uses "twist maps" to display the pattern of signs of products of unit elements. He designates $\mathbb{T}$ as the "ω_3 twisted Cayley-Dickson algebra for $\mathbb{A}_6$". Its twist map is:



C.0.1. An alternative construction of $\mathbb{T}$. A Cayley Dickson-type construction: $(a, b)(c, d) = (ac - b^*d, da^* + bc)$ is used by JW Bales to assemble the ω_2 algebra for $\mathbb{A}_5$. It contains an embedded S^β loop. It can be used to generate $\mathbb{T}$ from elements g and h in S^β using a procedure usually used to assemble Moufang loops from groups. A new element u, not in S^β, is defined. Then let $\mathbb{T} = S^\beta \cup (S^\beta u)$. Define the product in $\mathbb{T}$ as:
$(g, gu) \times (h, hu) = (g.h + gu.h + gu.h + gu.hu)$, where:
$g.h = (gh)$
$(gu)h = (gh^{-1})u$
$g(hu) = (hg)u$
$(gu)(hu) = h^{-1}g$

For its multiplication table arranged as a normalised latin square with elements ordered so that bit-wise 'exclusive or' (XOR) of binary representations of two element's numbering generate the numbering of their product, the twist map is:



The embedded loop of order 64 for this algebra is isotopic to the embedded loop T_L for the standard representation of $\mathbb{T}$. The isotopism is:

(2,25,21,19,26,45,39,12,22,43,14,31,32,8,20)(3,10,29,23,60,6,27,62,15,16,56,4,50,9,5)
(7,44,54,11,46,63,64,40,52,34,57,53,51,58,13)(18,41,37,35,42,61,55,28,38,59,30,47,48,24,36)

Applying the isotopism to $M_4(C)$ is equivalent to: $\mathbb{H}_L \otimes \mathbb{H}_R \otimes \mathbb{C} \to \mathbb{H}_R \otimes \mathbb{H}_L \otimes \mathbb{C}$ together with swapping the labels of some unit matrices with their negatives.

This indicates $[\sigma_\imath, \sigma_\jmath, \sigma_\kappa]$ and $[\lambda_o, \mu_o, \nu_o]$ generate the equivalent of left and right handed quaterionic subalgebras of $\mathbb{T}$.

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