


The Replica Trick, Wormholes, Island formula, and Quantum Extremal Surfaces

Stephane H. Maes¹ 

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Abstract:

The multi-fold theory factually encounters the AdS/CFT correspondence conjecture: the AdS(5) space is tangent dual to the multi-fold spacetime. On the other hand, while the derivation of the conventional conjecture involved branes in AdS(5) (+ ...), which amounts to physical dual tangency, and key derivations like the Ryu–Takayanagi conjecture, are based on the same model, the conventional AdS/CFT correspondence conjecture can also be understood as a mathematical duality, where the CFT spacetime is not necessarily physically tangent to AdS(5) (+...). It can be justified as a particular case of the holographic principle. Therefore, the paper will revisit, and derive the holographic principle, in a multi-fold universe. As General relativity (GR) encounters multi-folds at Planck scales, the proof applies to GR-based universes. We also debunk the use of Wheeler’s bag of gold as a counter example to the holographic principle.

Trying to resolve the black hole information paradox, different teams have been able to recover the black hole Page curve, by relying on the replica trick with generalized semi-classical gravitational path integrals, in asymptotic AdS, with arbitrary topologies, including spacetime (Euclidian) wormholes in between replicas, to justifies paths in between the replicas. The approach also relies on the island formula for the von Neuman entropy as fine-grained entropy. One knows that the Page curve had to be recovered one way or another as with the AdS/CFT correspondence conjecture, the unitarity of CFTs implies unitarity of gravity. The results, have also been extended to de Sitter asymptotic spacetime. The quantum extremal surface that appears in the process, bounding the island, leads to proposed physical interpretations of a black hole interior, that, in our opinion lead to more confusions than answers. In the multi-fold theory, we have already encountered an equivalent surface, with a much cleaner microscopic interpretation. For this paper, all what matters is that the microscopic interpretation validates the approach of replica trick, wormholes and island. But on the way, the paper discusses the differences between the multi-fold model and the conventional interpretation.

A few recent papers reused the approach to study two entangled gravitating universes, with one possibly without gravity. Admittedly, speaking of different universes is a bit of an oxymoron, as a universe embodies everything that is physical. Also, what is between universes, whatever that means, seems even more a red herring, unless if they were to share, or be a (global) embedding or dual tangent space that is physical. Phrased this way, the holographic AdS/CFT correspondence conjecture would be a particular case. The papers also rely on the replica trick, and its wormholes, adding some replica and wormholes swapping entangled universe parts. They recovers sensible, and unitary behaviors as well as the ER = EPR conjecture. This paper provides multi-fold inspired interpretations of the quantum extremal surfaces appearing among the disjoint universes.

Interpreting the replica trick, and its wormholes, in both the cases of black holes and entangled disjoint universes, one can see that the role played by the wormholes between different replicas amounts to the multi-fold mechanisms.. This is further reinforced by considerations on global symmetry in the presence of gravity or wormholes. The islands associated to the different entangled universe use cases also imply that, in universes with

¹ shmaes.physics@gmail.com

gravity, entanglement implies gravity effects, which is also known as the E/G conjecture, factual in multi-fold theory. With the (disjoint) AdS/CFT conjecture, this means that it, and therefore the M-theory encounters multi-folds. The multi-fold theory could have predicted such an outcome from the link between the Hilbert Einstein action and superstring action, and the fact that we already had GR encounter multi-folds at Planck scales.

1. Introduction

In a multi-fold universe [1,8-10,22,247], gravity emerges from Entanglement through the multi-fold mechanisms. As a result, gravity-like effects appear in between entangled particles, whether they be real or virtual. Long range, massless gravity results from entanglement of massless virtual particles. Entanglement of massive virtual particles leads to massive gravity contributions at very small scales. Multi-folds mechanisms also result into a spacetime that is discrete, with a random walk fractal structure and non-commutative geometry that is Lorentz invariant and where spacetime nodes and particles can be modeled with microscopic black holes. All these recover General Relativity (GR) at large scales, and semi-classical model remain valid till smaller scale than usually expected. Gravity can therefore be added to the Standard Model (SM) resulting into what we defined as SM_G : the SM with gravity effects non-negligible at its scales. This can contribute to resolving several open issues with the Standard Model and the cosmological standard model [210] without new Physics other than gravity [1,4-27,34,45,51-54,56,59-61,63,66-73,83,89,96-99,109-115,133,137-139,166,172,184,186-221,224-259].

Note added on November 23, 2023: References in italic were added on November 23, 2023.

Among the multi-fold SM_G discoveries, the apparition of an-always in-flight, and hence non-interacting, right-handed neutrinos, coupled to the Higgs boson is quite notable. It is supposedly always around right-handed neutrinos, due to chirality flips by gravity of the massless Weyl fermions, induced by 7D space time matter and scattering models, and hidden behind the Higgs boson or field at the entry points and exit points of the multi-folds. Massless Higgs bosons modeled as minimal microscopic black holes mark concretized spacetime location. They can condensate into Dirac Kerr-Newman soliton Qballs to produce massive and charged particles, thereby providing a microscopic explanation for a Higgs driven inflation, the multi-fold gravity electroweak symmetry breaking, the Higgs mechanism, the mass acquisition and the chirality of fermions and spacetime; all resulting from the multi-fold gravity electroweak symmetry breaking. Above the energy of the multi-fold gravity electroweak symmetry breaking, massless particles are patterns of the random walks of the massless Higgs bosons. The multi-fold theory has also concrete implications on New Physics like supersymmetry, superstrings, M-theory and Loop Quantum Gravity (LQG) [1,8-21].

The multi-fold paper [1] proposes contributions to several open problems in physics, like the reconciliation of General Relativity (GR) with Quantum Physics, explaining the origin of gravity proposed as emerging from quantum (EPR- Einstein Podolsky Rosen) entanglement between particles, detailing contributions to dark matter and dark energy, and explaining other Standard Model mysteries without requiring New Physics beyond the Standard Model other than the addition of gravity to the Standard Model Lagrangian. All this is achieved in a multi-fold universe that may well model our real universe, which remains to be validated.

Multi-folds are encountered in GR at Planck scales [5,6] and in Quantum Mechanics² (QM) if different suitable quantum reference frames (QRFs) are to be equivalent relatively to entangled, coherent or correlated systems [7]. This shows that GR and QM are different facets of something that they cannot well model: multi-folds.

² Standing in for Quantum Physics in general.

The present paper starts with a discussion of the AdS/CFT correspondence conjecture, conventional [30,32,33], and multi-fold [1,18,20,72], and derived conjectures like the Ryu–Takayanagi conjecture [37,38,74], and its multi-fold version [73,97], the ER=EPR conjecture [42], and the associated GR=QM conjecture [49]. Then, we discuss considerations on inter-, intra- and mixed representations [77], and physical vs. mathematical dualities. We justify the breadth of possible variations with the holographic principle [78]. We review its construction, and justify its covariant entropy bound formulation [78], in multi-fold and conventional universes. It is something that, to our knowledge so far, has never been achieved generically, and non-perturbatively.

The paper revisits the black holes and quantum extremal surfaces in conventional [79-88] in multi-fold universes [89]. In conventional theories, with the replica trick and associated wormholes, the black hole information paradox seems finally addressed through the island formula (See for example [79-88]), that recovers the Page curve³, needed for such a resolution, and the unitarity expected from the AdS/CFT correspondence conjecture⁴. Doing so, we explain, in our own way, the fine-grained and coarse-grained formulas, and reinterpret the microscopic effects, in conventional physics, based on past multi-fold results [89]. Indeed, our analysis, based on multi-fold theory, leads us to argue that, while the conventional island formula is correct, the conventional interpretation of the resulting quantum extremal function may not be, at least for the interpretations we have encountered, and in multi-fold universes, including in popular science articles like [92]. We provide what is, in our view, a better interpretation, and, in our view, it makes some much more sense. Doing so, we also validate the replica trick, at least in our context. Besides providing a much better microscopic interpretation, our approach helps understand the replica spacetime wormholes as implementing multi-fold mechanisms between entangled regions, which is a potential option [6,14,61].

Note added on November 23, 2023: Note that our perspective on the interpretation of the quantum extremal surface is driven by black holes in multi-fold universes, i.e., with a positive cosmological constant [1,188,259]. If the cosmological constant is strictly negative as in asymptotic AdS, it is possible that the original interpretation of the quantum extremal surface remain valid, as discussed in [259]. So everything is to be taken with a grain of salt. In asymptotic dS universes, we stand by our view that what we proposes makes more sense, and papers like [90] are more problematic. In general [259] takes the view that all these paths are plausible but with different probabilities. And so [90] may happen, but in our view, not that likely etc.

Then we discuss a few more recent papers proposing to apply the same approach to entangled disjoint gravitating universes, including the case where one of the universes is without gravity versus the cases, where both are with gravity [93-95]. It includes discussing the challenges of the meaning of “multiple disjoint universes”, and “what’s around or in between” these universes, which may or may not include gravity, GR, or Physics. Fortunately, we find some comfort in accepting it in the context of the holographic principle. Reviewing the results and interpretations of the involved wormholes, resulting into islands, the papers introduce path integrals with the usual replica trick, with universe replicas and wormholes between all possible parts, including between and among entangled universes (parts), and for all possible universe topologies, weighted by their probability/plausibility. See [1,222,259] for a discussion of such a view extensible to universe topologies.

In the context of these papers, we confirm that the involved spacetime and swap wormholes at hint implementing multi-fold-like mechanisms. More interesting, we also show that, while sketching our microscopic interpretation for the encountered islands, and their quantum extremal surfaces, the multi-folds from the replica trick generate gravity effects, as in [1,45], and therefore the E/G conjecture [70], factual in multi-fold universe [97]. The approach can also apply to the disjoint AdS/CFT correspondence conjecture with universes of different dimensions, in this case D and D-1. For example, in one case the quantum extremal surface will indeed correspond to the Ryu–

³ The results are typically for an asymptotic AdS spacetime. But there are examples of extensions to asymptotic de Sitter spacetime (dS). See for example, [90].

⁴ If it were not recovered, the conjecture would in fact be proven inconsistent.

Takayanagi conjecture. This last step allows us to confirm that M-theory encounters multi-folds, as the AdS/CFT correspondence conjecture is widely considered to be a window into the M-theory.

2. The AdS/CFT Correspondence

For sections 2.1 and 2.2, we mostly reuse the text from the corresponding sections in [14]. We added some new consideration in relation to [61].

2.1 AdS/CFT correspondence Conjecture

Often labelled as the biggest discovery in the field of the last 25 years, the AdS/CFT conjecture establishes a concrete duality between supersymmetric gauge CFT in $D-1$ dimensions and gravity in D dimensions [30,32]. It implements the holographic principle assumed from the Bekenstein-Hawking area formula for black hole entropy [28,31].

Reasoning-based derivation can be found in [57,33]. [33] explains both Maldacena's derivation, and a hindsight reasoning. Let us repeat the latter, but this time at the light of our work on multi-fold universes, which also recovers a factual (multi-fold spacetime)/AdS(5) correspondence [1,13,18,20,8,34], and a holographic principles with area laws (e.g. [1,8,17]).

We refer to [33], section 3.2, for the steps of the proof; just adding relevant comments:

- $D+1=5$ per the holographic principle. The duality or holographic principle is between a bulk AdS(5) universe (+ compactified dimensions in S^5 per supersymmetry: 10D is needed for conformance of superstrings [14]), where we know that superstrings can exist (per [35] and sections 2 and 3 in [14], they can't be in a positively curved (or rather, with a positive cosmological constant) universe, nor really even in a flat spacetime (no conformant solution) per an older result [36], although there could be ways around. This really does not matter too much here), where quantum gravity exists, and a particular 4D supersymmetric ($N=4$, for maximally supersymmetric) CFT (The choice of CFT is justified in next bullet) without gravity. So the graviton lives in AdS(5)(+...), something recovered in multi-fold universes [1,34], where the graviton appears as multi-folds. *Note added on November 23, 2023: See [14,15,259] for more details and references about the problems in a space with a strictly positive cosmological constant.*
- $D=4$ is a spacetime boundary to AdS(5)(+...). It is consistent, as there are different ways that we know for sure that spacetime is 4D [1, 13,216,239] (see also [51-54,71]). The justification for AdS(5) (+...) comes from proposing a 5th large dimension that can only be added to provide a negative curvature (negative cosmological constant), as mentioned above (de Sitter dS(5)(+...) would therefore not work as no superstring could live in it. See [14,15,259] and references therein). The criteria of having a stable energy behavior over large-scale ranges implies that scale can be used as 5th dimension. Such stability implies that CFT reigns on the $D=4$ spacetime. Based on the previous sections we would rather state that, if (Yang-Mills like) fields are to appear around superstrings they will be CFT-like (remember sections 2 and 3 in [14]): super Yang Mills in this case. Note that in a multi-fold universe, the 5th dimension rather results from the scale of the distance between entangled systems that generate gravity or gravity like effects [1].
- In addition, because the fields model matter and interactions with and among matter, and because they are expected – in the context of the derivation of the AdS/CFT correspondence – at the difference of a

multi-fold point of view [13-18,20], to be related to superstrings, the fields will be also supersymmetric. Therefore, in 4D spacetime, we have supersymmetric CFTs. Also, in order to bring comparable degrees of freedoms between CFTs (few) and superstrings, we need take the largest possible symmetry (in supersymmetry: we pick $\lim_{N \rightarrow +\infty} SU(N)$). Of course, it does not really matter, as we already know that supersymmetry is unphysical [13-18,20,25,259].

- Per section 3 in [14], superstrings on world sheets in AdS(5) (+...) in contact to the boundary D=4 spacetime, generate super Yang Mills CFT on D=4 spacetime, or following more properly [29], super Yang Mills CFTs (N=4) on D=4 result into superstrings in AdS(5) (+...). The superstrings further away from the boundary have little or no effect in terms of Yang Mills CFT generation in the D=4 spacetime. $N \sim$ different layers of spacetime or 3D-branes.
- The rest can be derived following the reasoning of [33] – section 3.1.
 - When coupling is weak ($g_s N \ll 1$), D=5 spacetime is not curved. Closed strings (gravitons) are in the bulk, away from surface (as no gravity is present). We have just 4D Yang Mills (supersymmetric) on 4D spacetime (flat). Gravity is present in the bulk, and decoupled from Yang Mills.
 - When $g_s N \gg 1$, many 3D-branes are added in the limit of N, and it becomes like a black hole, i.e. a 3D black brane in superstring theory. Gravity is responsible for the behavior near the surface. Further away, it is beyond a black hole horizon: only gravity really matters, described by closed strings (gravitons) in AdS(5) (+...).
 - Because Yang Mills is renormalizable, the solution on the 3D-brane for $g_s N \ll 1$ is also known for $g_s N \gg 1$. We have encountered the same solution for $g_s N \gg 1$: Yang Mills CFT (i.e. supersymmetric N=4) on 3D (+1) spacetime and gravity (i.e. modeled by superstrings) in AdS(5)(+..) describes the same Physics (for $g_s N \gg 1$). That is the correspondence.
 - The conjecture is that this works for all $g_s N$.

This correspondence can be seen first and foremost as a mathematical conjecture between two frameworks. It has been proved to work in other context of physics like material science [35], and in mathematics. But it is an unproven conjecture. And it results mainly from a) the superstring model elaborated in sections 2 and 3 of [14] (i.e. the string action, conformant invariance and gauge invariances) b) Black hole behaviors with respect to its horizon. Nothing more.

The conjecture models a universe (D=4, supersymmetric N=4, with CFT and no gravity) that is not exactly our real universe, although, it may be a closer match when energies are high. Remember that we said that all fields including gravity should look like CFTs⁵ [14,72,111].

This AdS/CFT correspondence conjecture is the closest link to multi-fold universes, where we have a factual holography due to the mappings and the multi-fold mechanisms, between entangled particles (i.e. fields) in 4D spacetime and AdS(5) [1]. Entanglement seems to be the biggest difference between statements (besides the factual vs. conjecture considerations), but it is recovered in the ER=EPR conjecture discussed in the next section [1,6,14,18,20].

Now, the relationship to entanglement from the multi-fold universe, is also present in the AdS/CFT correspondence conjecture: the entanglement entropy of the CFT fields can be expressed as proportional to an

⁵ Do not hold your breath. [13] killed the supersymmetry option for our universe as incompatible with SM/SM_G, because quantum gravity seems asymptotically safe. [16] confirms that view. *Note added on November 23, 2023: More confirmation result from [259].*

minimal surface area in AdS(5). It is known as the Ryu–Takayanagi equation [37], and it relates to the Hawking-Bekenstein formula. Again, it may appear surprising, but, in fact, it is a circular result because black holes were also the source of the derivation of the AdS/CFT correspondence conjecture: of course the area law is therefore baked in it. In fact, Ted Jacobson [38], ignoring the AdS/CFT correspondence conjecture, recovered variations of the formula in the 4D spacetime both for QFTs and CFTs, and showed that the framework implies recovering, linearized, but [39] can then be adapted to make it full, GR in 4-D spacetime. Entanglement, entropy of entanglement and entropy of Black holes, GR and AdS/CFT are directly related [37,38]. All this shows a duality between GR and its linearization in 4D spacetime vs. gravitons in AdS(5) and the ability to use one (gravitons in spacetime) or the other (gravitons, or multi-folds, in AdS(5)) to model gravity. It was also our contention in [1,34] for multi-fold universes and the reason why, despite the outcome of [13], we still consider that gravitons in AdS(5) and the holographic correspondence, as well as the principle that gravity = entanglement and entanglement = gravity, survive even if superstrings are not physical: the AdS(5) graviton effects seem physical, at least for a multi-fold universe. *Note added on November 23, 2023: Again as explained multiple time, gravitons are not physical. They only give hints of multi-folds [12-18,20,34,212].*

Going further, [39] showed that the AdS/CFT correspondence conjecture, its dictionary and entanglement (entropy) implies that GR governs AdS(5) (+...). In other words:

$$\begin{aligned} \text{GR in AdS(5)} \Rightarrow \text{superstrings in 10D} \Rightarrow \text{AdS/CFT correspondence conjecture} \Rightarrow \text{CFT (supersymmetric etc.)} \\ + \text{entanglement} \Rightarrow \text{Ryu–Takayanagi equation} \Rightarrow \text{(full) GR in AdS(5)} \end{aligned} \quad (1)$$

(1) allows to invert the arrows as a mathematical proof. It can be considered as a strong example of a consistent approach.

About [39], we may ask if it means that GR reigns in AdS(5) implies superstrings in AdS(5) (+...). [29] only studied AdS(D) via the AdS/CFT correspondence conjecture. To that effect, let us consider two other papers [40,41] that infer that the outcome seems to be that indeed AdS(5) (+...) is populated with extended objects (i.e. strings or D-branes), detected by their energy spectrum that provides a characteristic Hagedorn phase, at very high energy (i.e. exponential growth with energy due to each element of the strings / D-branes contributing significantly), that differs from the behavior of CFT or QFT (or conventional black holes): AdS(5) appear stringy. The derivation chain from above therefore becomes:

$$\begin{aligned} \text{GR in AdS(5)} \Rightarrow \text{Super strings in 10D} \Rightarrow \text{AdS/CFT correspondence conjecture} \Rightarrow \text{CFT (supersymmetric etc.)} \\ + \text{entanglement} \Rightarrow \text{Ryu–Takayanagi equation} \Rightarrow \text{(full) GR in AdS(5) \& superstrings in AdS(5) + (...)} \end{aligned} \quad (2)$$

In a multi-fold universe, we also recover a version of Ryu–Takayanagi equations, which explains the minimum surface area as being as far as the multi-fold reach in AdS(5)[28], but GR may or may not reign in AdS(5), and so superstrings may or may not be physical... Also, multi-fold dynamics may or may not be governed by GR/Hilbert Einstein action or variations [1,18,20,27,8].

Is there a problem for multi-fold universes because of [13] vs. [40,41]? The answer is no: the result of [40,41] assumes the full dictionary and properties of AdS/CFT correspondence conjecture. (1) was satisfied, and (1) implies superstrings in AdS(5) and therefore also in AdS(5) (+...) = (+ S⁵). The outcome was circularly baked in.

The multi-fold universe changes the AdS/CFT correspondence into a spacetime of Multi-fold Universe with matter gravity & QFT / AdS(5) mapping, and, as a result, (1) and (2) evolve as:

4D spacetime with gravity



Entanglement => AdS(5)/spacetime mapping and multi-fold mechanisms (=> Ryu–Takayanagi equation)



multi-folds in AdS(5) => maybe or maybe no GR in AdS(5) to govern multi-folds. (1')

GR in AdS(5) => Maybe or Maybe no superstrings in AdS(5) (+...) (2')

(1') and (2') are now fully decoupled and (2') can be a valid mathematical result. [13] only says that no matter what, GR in AdS(5), if it exists, has no impact on the 4D spacetime with gravity.

The 2D regime and model of [16] may allow the relationship to work between AdS(3) & 2D spacetime. Amazingly, AdS(3)/CFT2 correspondence is a mathematically proven theorem, not just a conjecture [55]. In our view this is another hint that this may be the only case that is physical as hinted in [16,259].

To conclude this discussion, we will also point out, as mentioned in [18], that GR is unstable with matter in AdS(5) [58]. In our view, it implies that superstrings other than closed strings, aka massless gravitons, probably cannot physically live in AdS(5) (even with (+...)); therefore destroying the conventional derivations of AdS/CFT correspondence conjectures as above; while the multi-fold approach still works. However, (1') and (2') may remain valid with GR reigning in AdS(5) (it is of course valid without GR) if, as expected, paths in the multi-folds are not resulting to any energy momentum stress tensor leaks into AdS(5)[1,252].

2.2 ER = EPR Conjecture (and more)

ER=EPR was proposed by Maldacena and Susskind (fathers of the holographic principle, and its AdS/CFT version), as their way to use entangled black hole analogies to handle the hints of a link between entanglement and gravity [42]; which [1] explains for multi-fold universes where we discover that entanglement generates gravity (like effects and gravity is due to entanglement of virtual particles, as in the E/G conjecture [10,80].

The original reasoning behind ER=EPR goes as follows:

- Consider two black holes (in AdS) with entangled horizon. We are continuing to use black holes analogies to AdS/CFT correspondence conjecture to model CFTs or Gravity.
 - They start from one black hole that radiates per Hawking's theory
 - The second black hole is composed of all the particles produced by Hawking radiation: if all these particles are brought together they will collapse into another black hole entangled with the first one acting as source.
- Link their regions behind the horizon to form a ER (Einstein Rosen) bridge, i.e. a wormhole which is non-traversable.
 - Doing so, one can resolve the Black Hole information paradoxes (AMPS / firewall, complementarity) relying on the entanglement between the inside and the outside radiations.
 - It correctly "emulates" the EPR paradox with Alice and Bob's experiments⁶.
 -
- The result is conjectured to be generalizable to any entanglement.

⁶ This emulation is probably the inspiration for the GR=QM conjecture mentioned later in this section [49].

- It is quite, a jump but it seems a good model:
 - [43] proposes an example between pairs of particles, using the AdS/CFT correspondence conjecture, and [44] shows that entropy of ER bridges follows the entropy (or information) laws and inequalities.
 - As discussed next, [1,126,127] proposes a stronger model for it with many more consequences, including ultimately the demise of superstrings as TOE, or even as correct gravity model, beyond approximations of gravitons (but preserving the idea of AdS/CFT correspondence).

Multi-folds mechanisms were proposed in [1], without knowledge of ER=EPR and therefore without using its model. In hindsight:

- Multi-folds are equivalent to many aspects of ER=EPR, but without resorting to strings or holographic postulates to derive the model. Instead the goal was only to address the ER paradox and locality vs. non-locality.
 - Many would now argue on the string side that the AdS/CFT correspondence is often used without any reference to strings anymore.
- Multi-fold mechanisms result into showing that:
 - Entanglement is gravity: entangled systems are attracted by gravity like effects [45]
 - Gravity is entanglement: by relying on virtual pair productions, and therefore entangled, around sources, gravity appears, and GR can be recovered at the right scales [1,6].
 - Massive virtual particles add massive gravity effects [59] at very small scales that renders possible a standard model with non-negligible gravity at the SM scales, aka SM_G [1,8].
 - Multi-fold universes, and / or SM_G explain many open issues with the standard model and standard cosmological model [1,4-27,34,45,51-54,56,59-61,63,66-73,83,89,96-99,109-115,133,137-139,166,172,184,186-221,224-259]. It also demonstrates that supersymmetry, higher dimensions and superstrings are not compatible with SM and unphysical (as are most conventional GUTs and TOEs) [1, 13,18,20,27]. *Note added on November 23, 2023: See also [259] for more conclusive proofs.*
- Multi-fold mechanisms add to ER=EPR (besides a completely different derivation and model), the ability to traverse them (so that path integrals include paths in the multi-folds) [61,252]. This is what is responsible for the appearance in spacetime of gravity like attractive effective potentials (or positive effective curvatures).
 - Wormhole in ER=EPR proposal are not traversable
 - Wormholes / blackholes in AdS may be traversable but only when involving unphysical considerations like exotic matter with negative mass/energy, and/or, with unreasonably large duration.
 - [6,10,23,61,111,252] discusses additional possible contributions facilitating wormhole traversability like Casimir effects or wormhole rotations, and if implementing multi-fold behaviors, or in asymptotic dS (instead of asymptotic AdS), the resulting stretching of the wormholes.

- Recent work [46] suggests that in the presence of ER=EPR, reasonable traversability may be possible without exotic matter[13]. [47,48] provide a possible way to ensure that traversability of wormholes in GR governed AdS(5) can be achieved with couplings of their Left and Right boundaries; which is exactly what happens when they are entangled... If any of these were the case:
 - It would validate the multi-fold mechanisms (when wormholes are in AdS for [47]).
 - It should derive gravity as we did in [1].
- [56] shows examples of traversable wormholes associated to entangled massive fermions which could also be a way to realize multi-folds (It also work with massless fermions and with entangled bosons to match multi-folds [23,61,252]).
 - We can adopted this approach for multi-fold universes where multi-fold would be implemented by traversable wormholes [1,23,24,26,60,61]. [61] relies on [62]. The solution of [62] works with charged fermion, but also with uncharged fermions, provided that there is matter at the throat, which is the case with neutrinos and Higgs bosons at the entrance⁷. Massless fermion also work, especially when we are at energies above the multi-fold gravity electroweak symmetry breaking, where everything is massless [61,67]. The wormholes can be stabilized and rendered traversable with pairs of right-handed neutrinos and left-handed anti-neutrinos behind Higgs bosons at the entrance of the wormholes [61].
- Interestingly, the absence of multi-fold mechanisms in hierarchical entanglements without local entanglement initiation [1] is related, or somehow equivalent to the entanglement cases where classical ER bridges appear in [44,252].
- In the multi-fold model, nothing is limited to CFTs (without gravity) in spacetime, and superstrings are not involved, just possibly living in the tangent dual space.

Our statement that entanglement is gravity, and gravity is entanglement [1,8,45] is stronger than ER=EPR, factual in multi-fold universe, and a proposal for experimental verifications in our real universe [1,45]. In its quantum computing with Qubit variations, it encompasses, and it is stronger than, the GR = QM conjecture [49], that we also discovered post our proposal. GR = QM is about Qubits proposed wormhole-based teleportation in lab experiments resulting from the connection of entangled Qubits via wormholes, and this way derive properties of bulk gravity (in AdS(5) (+...)). Of course traversability is not resolved, nor the resulting gravity effects in spacetime.

⁷ In [23], we discussed how Casimir effects can also contribute [64] to the traversability and stability of the wormhole. Note also that rotation of the wormhole could similarly help [65]. Also as discussed in [23], above gravity electroweak symmetry breaking energy levels [66], all particles are massless [67], and traversability by massless particles is not an issue [23,62]. The Casimir effects is assuming no contradiction with the tenancy model for multi-folds [1,9,66,68,69], which means that vacuum also exists in the folds, something that could sense once we know that it is also formed of concretized points marked by Higgs bosons, but it would also imply quantum fluctuations in the multi-folds, and particle / anti particle creation and annihilation something we preferred to avoid in [23]. Interestingly, rotations would even work for spin 0 cases, if we were to consider that, as in [1], the spin 0 case is the result of rotations in opposite directions. It also has the advantage to not require questioning the tenancy model of the multi-folds [1,9,66,68,69]. Revisiting and expanding the spin model and its implications, not just for traversability, is for future work. And so, vacuum fluctuations and their consequences is a option that for now we will prefer to avoid, but can keep in mind if traversability is otherwise a problem. After all, it does not seem that Casimir forces are required, to reach traversability, they would just be a plus. However, [23] rather precludes them. *Note added on November 23, 2023: See also [23,252].*

The need to invoke CFTs, requires a lab beyond critical point, which may not be achievable within the promised timeframe either. But yes, everything proposed in [49] and more is also plausible with multi-folds [1] where traversability problems and critical point/CFT concerns do not apply.

Note that the wormhole on a quantum computing chip controversy discussed in [213], also relates to this conjecture. In conventional physics, no traversable wormhole is created [114]. In a multi-fold universe, entanglement across the chip activates multi-folds. If they are implemented with wormholes [61,252], then it turns out that wormholes would actually have indeed been created and traversed [213].

In these different examples, the non-traversability in ER=EPR has probably been the hurdle that has so far prevent the String community to uncover that entanglement is gravity and gravity is entanglement...

In hindsight, we can see that the string community has grasped for a non-perturbative theory, and modeled it essentially only through (some of) its dualities. Doing so, it only got a blurry picture of Physics, using mathematical and unphysical concepts that have some relation to Physics. The circumvoluted way to work with CFTs and AdS and role of black holes in AdS vs. entanglement in CFT, led them to hints of the multi-fold mechanisms, but not the full pictures of [1,252]..

We do not know how much of AdS(5) is governed by an GR/Hilbert Einstein action and if multi-fold are therefore wormholes, or can have unrelated dynamics. Sure, AdS(5) is a solution of GR, so we can mathematically model it via GR and end up with superstrings and the circular reasonings of (2). Yet, the reasoning in (2) does not model the real universe per [13]. Having wormholes, as multi-folds, has no impact if they can't support our proposed mechanisms. *Note added on November 23, 2023: See [252] for how wormholes can implement multi-folds and all their behaviors and properties.* In such case, physically, we would still follow (1') and (2') and not imply that superstrings are physical (other than the fact that they can model gravitons in AdS(5)).

2.3 The factual Multi-fold AdS/CFT correspondence conjecture

As mentioned above, [1] derived the existence of an AdS(5) space tangent dual to every point of the multi-fold spacetime, and where the multi-folds live. More details are also discussed in [1,18,20,72]. The conjecture is therefore considered as factual in multi-fold theory.

A factual multi-fold version of the Ryu–Takayanagi conjecture is established in [73].

2.4 The holographic principle

The holographic principle was initially introduced before the AdS/CFT correspondence conjecture was proposed [75,76]. In [75], it results from arguments for dimensionality reduction from 4D spacetime to 3D spacetime⁸, or a surface evolution that can now be even considered to be a faraway plane: a viewing screen.

⁸ Note in [1,16,98,99], we argue and rely on a reduction to 2D not 3D. But it is a different playground. The 2D dimension reduction is happening at very high energy, or very small scales. The 3D dimension reduction takes place at all energies above the energy levels of the (multi-fold) gravity electroweak symmetry breaking, where all particles are massless: the gravity processes lose one more degree of freedom [98].

[100,101] discovered that string theory in the light front gauge has the form of a two plus one dimensional theory with no explicit mention of a longitudinal direction.

[76] combines the results into a proposal that the world is, therefore, a (time evolving) hologram, with a projection defined by light rays orthogonal to the hologram screen, intersecting in the 3D space to define the physical location of particles. The focusing theorem of GR [102,103], ensures that this result into a consistent focused model, where the rays converged in well-defined locations.

The holographic principle can also be considered as inspired from the black hole entropy. Indeed, it is often positioned as assumed from the Bekenstein-Hawking area formula for black hole entropy [17,104,105]. Yet, the holographic principle is still considered as a conjecture [106], with the Wheeler's bag of gold as possible counter examples [107,108]⁹. The AdS/CFT correspondence conjecture is probably its best particular case, and it has attracted most of the attention and efforts.

[78], and references including other reviews therein, provide an interesting, albeit dated, review of the holographic principle. Yet to our knowledge not much has changed other than in terms of the progresses with the AdS/CFT correspondence. Unfortunately, the AdS/CFT correspondence applies only to a narrow class of spacetimes of limited physical relevance. By contrast, the holographic principle claims a far greater level of generality—a level at which it still lacks a concrete implementation.

In trying to detail the holographic principle, [78], provides a series of bounds of the entropy that can exist within spacetime surface: the spherical entropy bound, the spacelike entropy bound and the covariant entropy bound. It also claims the absence of a derivation of the latter, albeit the conjecture (again!) that it is satisfied. Since, a weak gravity proof has been published as [181-183]. Our treatment of the bag of gold alleged counter example in footnote 9, makes this statement meaningful; otherwise [78] and [106], would have been incompatible. In the next subsection we provide a derivation of the bound in a multi-fold universe.

2.5 The covariant entropy bound

⁹ [108] is easier to understand. Wheeler's bag of gold is obtained by stitching together a ER bridge described by a Schwarzschild metric, and a FLRW metric that describes a homogeneous, isotropic, expanding solution to GR. In our view, it is not a counter example: ER is not a black hole, but a wormhole so the derivation of the Bekenstein-Hawking area formula wouldn't apply. If it were a black hole then only one half would count and we would recover the formula. Similarly the covariant entropy bound discussed after, for a region around the throat would have to include contributions from both side. As the covariant entropy bound is respected, let us assume for now, on both sides of the wormholes, it would still hold. This is to our knowledge the first debunking of this alleged counter example. We haven't look at the other ones cited in [108], but we feel confident that all counter examples can probably be similarly disposed. If it were not the case, we would love to hear, which one to investigate. Note that this does not contradict [186]: there are no shields to gravity in multi-fold universes, however, the boundary of a universe will by definition block gravity effects.

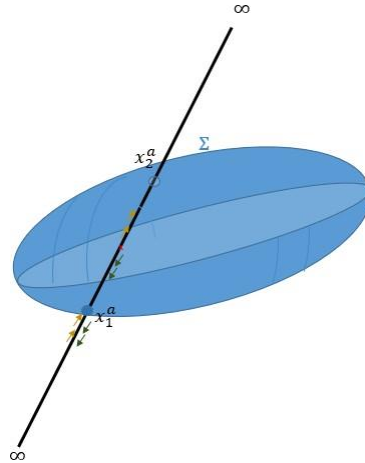


Figure 1: Static distribution means that each particle within the surface Σ contributed an infinite number of virtual particles and their associated multi-folds. If a contributing point is moved between $[x_1^a, x_2^a]$, the gain or loss felt by x_1^a is compensated by loss or gains coming from the change on the x_2^a side. As a result points can be moved to a more internal Σ' surface and not change the effects felt on Σ . (Source: Figure 11 in [1]).

In a multi-fold universe, section 7.4 of our original paper [1], shows that, in a static situation, or in a stabilized situation, where the system within the surface has been static for a long time, the multi-folds follow the Gauss law. Here static or stabilized means that the configurations has been in place for such a long time that we have an infinite, or long enough, distribution of entangled virtual particles emitted by each particle within, or at a time like distance of, spatial surface to ensure that the surface can be changed at will (at the difference of dynamic cases as discussed in [1,45,230,252]). If not enough time passed, then one must “correct the missing effects” versus figure 1. It is not exactly the Gauss law anymore, but when integrating over the surface (same time slice) defined by Σ , and its light sheet cones (See figure 3 in [78]), compensating effects that were done only with different space point within the same time slice, are now compensated by points in different successive time slices. The sum of all the effects is again essentially the same, independent of the change that took place. That is our version of holography.

In such case, to compensate in all time slices amounts to considering the two light sheet spacetime cones built on the initial spatial surface Σ , with each future and past cone converging per the focusing theorem already mentioned [102,103]. What is missing as multi-fold contributions for a given point on Σ , can come from the past or the future cone: one click in time, discrete in multi-fold theory [1,109], can provide the missing contributions from the x_2^a side.

This is why the covariant entropy bound works with the spacetime surface defined by figure 3 in [78], and the spacelike entropy bound (See figure 1 in [78] – it matches Σ in our figure 1) doesn't.

You will note that, so far, this proof entirely based on multi-fold properties:

- 1) the multi-fold mechanisms and their relationship to gravity emergence
- 2) the discreteness, yet Lorentz invariance of spacetime (discreteness is not essential but it allows to have discrete changes only to consider. Lorentz invariance is key to ensure that we are not losing the notion of covariance).

We can't reproduce directly these steps so far for a conventional universe and Physics. Indeed, we do not have the multi-fold mechanisms to compute the effects of gravity. It would amount to accepting the holographic principle then. Or alternatively, we could invoke [6], that indicates that GR-based universes are multi-fold. Also, we note that [1,6] recover GR through respectively the reconstruction steps or the top-down-up-and-upper derivation. It proves that the result holds for GR and universes with GR-based quantum gravity.

The result and proof so far is a priori only for gravity at best sourced by pure energy. What about matter: fermions and Yang Mills fields? There are two ways to discuss this:

- i) As argued by [75,76]: the entropy of spacetime represents the contribution of all the SM fields (fermions and Yang Mills), not just of the gravity metric. Indeed if we expect that the entropy of black hole with matter (inside and outside) to be finite then it means that the Bekenstein (generalized) formula also includes these fields otherwise, the total entropy would be infinite, as there is an infinite amount of matter than can be thrown into a black hole, and then evaporate from it. The result is consistent with our multi-fold proof of the area law for entropy black hole provided in [1,89]: it is entirely and solely based on the quantum fields or particles, fermions and bosons, not gravity. This argument is strong enough to convince ourselves. It is confirmed for black holes for examples as in [185].
- ii) Or we can follow our models from [17,110,111], implying that Yang Mills field are duals to gravity fields, and, therefore, will respect a similar holographic principle and support an equivalent area law in the presence of the associated gravity. [23] then showed that the SM (or rather SM_G , i.e., the Standard Model with gravity effects non-negligible at its scales [1,8-10]), results from multi-fold space time matter induction and scattering [72,27,113]. That is GR in 5D (or 7D), hence also following an holographic principle. In fact the transfer of symmetries between the embedding space, and SU(2) axial symmetry of the multi-fold mechanism implies the SU(2) symmetry for the SM, also linking the holographic principle to it: the behavior of figure 1 for Σ and its light sheet cones, extends to the SM, at least when considering the SM_G . Reasoning with the black holes then leads to the same bound for the proportionality constants.

The latter path leads to the same conclusion as t' Hooft: the black entropy accounts for all the fields, not really just the spacetime metrics. We already knew that from multi-fold spacetime reconstruction per [1,89] (spacetime are minimum Schwarzschild black holes and particles are excited versions as Schwarzschild, or (extremal) Reissner-Nordström or Kerr-Newman black holes - which also shows that vacuum also consists of spacetime excitations), and our proofs of the area law for black holes [1] (entropy measure entanglement across the horizon due to horizon fluctuations (or tunnelling [184]) between inside and outside particles, aka fields or micro hairs).

With this we have proved that a bound in Equation (5.1) from [78], holds in a multi-fold universe. Under the condition that one accepts the case that we have built [1,8-10], especially [6], that our real universe seems to behave like a multi-fold universe, we have also proven that that the Equation (5.1) from [78] holds in our real universe. Doing so we have essentially proven the holographic principle. Indeed for example, now take Σ to be an infinitely far away screen, and use our result of repeat [75,76] with our entropy results.

To find the exact value of the bound, i.e. equation (5.1) in [78], one now just need to remember that the maximum entropy is associated to a Schwarzschild blackhole (add more matter and a black hole forms). Therefore it is in each time slices $A(t)/4$. Integrating over the time sheets provides (5.1) in [78].

For the real universes, we can treat the covariant entropy bound as a conjecture as in [78], or accept that our real universe is multi-fold per [6], and the reasoning above, or accept that in weak gravity, we have the proof of [181-183].

Alternatively, we can use a different proof altogether for multi-fold or non-multi-fold Universes (**). We can follow our derivation, from section 4 in [73], that entanglement entropy of a stable system across a spacetime surface, is proportional to the area of that surface. Indeed, when the entanglement correlation is larger than the discrete spacetime (or a lattice model), but short compared to a characteristic length of the surface, equation (1) in [73] provides that relationship. It addresses strong and weak interactions, and fermions in recent interaction. When the correlation distance is way larger than the characteristics of the system, it is also dominantly proportional to the area of that surface, which covers electromagnetism, gravity and fermions in older interactions. Handling time evolution, instead of static systems, then adds the need to use the same surface as figure 3 in [78], for the same reasons. This way, we have demonstrated that the entanglement entropy will be proportional to area of the light

sheet cones built on the surface. Based on section 3.1, we are good enough to assume again that a covariant entropy bound exists, and that we have therefore a valid holographic principle.

These approaches, and the different options, are, to our knowledge, the first generic and non-perturbative proof presented of the validity of the covariant entropy bound, including in the presence of free and interacting fields. We later encountered [181-183], which present perturbative proofs when gravity is weak, for free and interacting fields. with quite different approaches. In any case, our proof is required for multi-fold universes, and any analysis we would want to do in such a universe. Applicability to real universe relies on [6], or just on using option (**) that can also be argued in the real universe.

It is probably also worth noting that the Hilbert Einstein action extremize surfaces, i.e. makes them invariant, see discussion in [1], coupled with the double copy duality for gravity, see [110], is also a way to anticipate the holographic principle: covariant infinitesimal surfaces are invariant, and extremized, both for gravity, and coexisting fields, and fully characterize the effect of the volume content. Combine the infinitesimal covariant surfaces to form a larger covariant surface and the property will persist (they can within the volume of the larger surface). The proof was straightforward, yet, before this, it took years to partially achieve it for weak gravity. The result holds in QFT as the extremized action builds on the same actions.

Finally, note that such bounds are not a function of the species of fields that can be encountered: their contributions are all included in the gravity entropy as argued in i). But even with ii) we obtain the same conclusion: as SM (in fact SM_G) symmetries [23], and number of flavors per family [1,114], result from gravity and multi-fold, that number is fixed, not subject to other considerations [231]: we know that we have a desert of new fundamental particle above the gravity electroweak energy scale [115], no concerns that some would eventually pop at higher energies¹⁰.

2.6 Duality, correspondence, inter- or intra-theoretical relations of the AdS/CFT duality

The AdS/CFT correspondence conjecture really exists in multiple different forms¹¹:

- 0) the original derivation for a 4D brane in the 10 D superstring spacetime which is essentially $AdS(5) \times S^5$. See [30,33]
- a) The case where the CFT spacetime is the boundary of the $AdS(5)$ space, but without gravity.
- b) The case where the CFT spacetime is the boundary of the $AdS(5)$ space, but with gravity. This is not conventional AdS/CFT duality but was encountered i) in [72,116], and ii) it is of course the case of the multi-fold AdS/CFT correspondence [1,18,20,72].
- c) The case where holography is done without holography as discussed in [77] and reference therein. These include the Jacobson derivation of an in-spacetime equivalent to Ryu–Takayanagi formula [38], that recovers GR, and Verlinde’s approach¹² of gravity emergence [117].

[77] classifies a) as inter-representation dualities and c) as intra-representation. It misses a mixed case as for example the multi-fold approach that mixes b) and c), as the theory addresses both the multi-fold 4D spacetime and the dual tangent $AdS(5)$, in one shot.

¹⁰ In fact, the conjecture, apparently satisfied so far, of the covariant entropy bound reinforces, or even proves also our thesis for a desert of new fundamental particle above the gravity electroweak energy scale, as claimed in [114].

¹¹ Here, we will not discuss double copy extensions as in [111].

¹² [117] is the paper that pushed us to formulate and publish the original paper on the multi-fold theory [1].

[77] also argues that the holographic principle is more generic, as it can cover dualities between many different types of universes. In fact once the mapping or projection rules are understood, they can become purely mathematical, i.e. between physically disjoint universes, that can be embedded in a larger space with or without (known) Physics in it. Such a consideration will be relevant to make sense of the papers on disjoint entangled gravitating universes that we want to discuss. Indeed one can go from one space to its dual holographic representation, take the representation elsewhere and go from that representation to the second universe.

We also believe that the holographic principle explains why so many different types of AdS/CFT dualities can be encountered, all making sense in the right context.

3. Reinterpreting Quantum Extremal Surfaces and the Page Curve of a Black Hole

As the papers [93-95] rely on the concepts and tools like the replica trick in QFT with multiple copies of the system's¹³ spacetime as Euclidian semi-classical gravity path integrals [85,88], with wormholes between replicas, and the resulting island formula for fine grained entropy [88], it is important to also ensure first that we confirm that we agree with such a methodology.

Why? Who are we to raise such a concern? Well firstly, while some are enthusiastic with the results, especially on the basis that it recovers the Page Curve for black holes [91], and as a result seems to address the black hole information paradox, even the popular science articles [92], that seem to idolize the approach, admit that there are many new unproven concepts, and that the jury is still out on the validity of the different steps, including the approximations of the path integrals, and disregard of what are considered as negligible contributions in certain regimes. Secondly, in [89], we showed how, in a multi-fold universe, we can also recover the Page curve for a black hole. And therefore, solve the black hole paradox, without any use of the replica trick and its components¹⁴. Therefore, it seems appropriate to understand what the implications of our paper on multi-fold black holes are. Isn't it contradicting (the interpretation of) the island formula?

[85,88,119], and references therein, provide an excellent discussion of the replica trick and island formula.

3.1. Coarse and fine-grained entropy of black hole

The replica trick approach, to address the black hole entropy and information paradox, starts with a generalized entropy formula that is the sum of the (thermodynamic) entropy of the black hole plus the entropy of its entanglement with the rest of the universe, i.e. the von Neuman entropy [88], and of the entropy of the outside, i.e. the rest of the universe, which consists of all the black hole radiation.

The thermodynamic entropy is the coarse-grained entropy of the black hole (or any other system). Indeed, The von-Neumann entropy is the fine-grained entropy. Proofs can be found in [120-122,127], all with their

¹³ E.g., black hole, or universe in later sections.

¹⁴ Of course if wormholes and replicas amount to multi-fold mechanisms, we may have to revise such a statement. A big part of the rest of this paper is devoted to answering such questions.

controversies that honestly do not seem to hold (See for example [123-126]). Yet, all these proofs and definition glance over the notions of fine-grained or coarse grained entropy, used in [88], and well-illustrated in [128].

To agree on the definitions to use in the case of the island formula, we prefer to revert to the functional definition and properties provided in [129,130]. Indeed, we know that the thermodynamic entropy must count all the possible equivalent microstate configuration giving the same macroscopic effects. It is therefore the maximizing functional: any non-maximizing case could become a future evolution and be associated with decreased entropy, what we simply can't have. The only way to prevent that and to fit the second law of thermodynamics is to ensure that we select the maximum. On the other hand, per the Jensen's inequality [131], the von Neumann entropy is smaller than any other functional of the same form, it must therefore be the minimizing functional, for whatever criteria is being considered. On that basis, we agree we have motivated the island formula used for example in [88]. At this stage, with these understanding, we are now confident that [88], and reference therein, perform the correct computations.

3.2. The replica trick, and its wormholes, for black holes

The idea behind the replica trick for black holes is to apply the principle of path integrals to black holes, well modeled in classical or semi classical gravity, and QFT fields (or particles, or strings for that matter): all paths must be considered, suitably weighted by the contribution of their associated action [88,135]. Physically explaining how the fields, in their different configurations, can encounter these different spacetime configurations, i.e. different types and state of black holes, requires modeling them as different replicas with wormholes connecting them [88,134,135,136].

A path for the path integral is now one of the possible evolution with a certain combination of all the possible fields, i.e. matter particles, and replica configurations (where one of the latter is one of all the possible configurations of spacetime and the black hole). Without being able to say much more in terms of the probability associated to different spacetime replicas, they are assumed to be all equiprobable, which is reasonable. It is in principle identical to how we weighted the multi-fold on their impact as attractive effective potential in spacetime, i.e. gravity [1].

For ease of computation, Wick rotations are applied and the work is done in an Euclidean space.

Entropies can now be computed via the Rényi entropy algorithm: compute entropy across n replicas (i.e., n th order) as the Rényi entropy, continue it analytically towards 1, and the entropy is obtained when n converges to 1 copy [85,88,119,134,135,136]. A very astute approach.

3.3. The island formula for Black holes, and resulting quantum extremal surface

As we need to consider all possible effects, we want to stick to the most rigorous measure we have: the fine-grained entropy. As already mentioned the replica trick allows the entropy to be obtained by computing the entropy estimates for n replicas, and then via the Rényi algorithm, i.e., the limit for $n \rightarrow 1$ of the Rényi entropy [85,88,119,134,135], and trying to minimize the result over all possible spacetime (around, and, in particular, inside the black hole) surfaces. The set of possible surfaces includes the black hole horizon and another (or many other) surface(s) within the black hole.

Doing so (See [85,88,119,135,136], references therein, and many other related papers; it's been a hot topic recently as well as references in [89]), a new surface appears near the horizon. This surface extremizes the entropy. It is called the quantum extremal surface. While the main part of the computation involves semi-classical path integrals, the term quantum is supposedly introduced because it results from the behavior in presence of fields, i.e. QFT, and the use of path integrals. The interpretation is that this surface creates a new spacetime regions, separated from the rest of the black hole interior. What enters the quantum extremal surface is separated from the black hole interior, and no more contributing to the black hole mass, entanglement or radiation. This surface is internal to the horizon, and closely follows it on the inside, and contracting, as the black hole contracts.

The outcome of the model is a Page curve, as needed [79-81, 82, 84-88,90-92,135,136].

The explanation provided in [92] is particularly helpful. As illustrated in figure "the Great Black Hole Information Escape" of [92], one sees that radiations now would occur through the black hole horizon, and through the quantum extremal surface. What is within the quantum extremal surface, or radiated through the quantum extremal surface, is technically no more part of the black hole, and therefore no more a source of radiation through the horizon, or of the black hole entanglement with the outside of the black hole.

Note that while the picture and interpretation of [92], may be for the masses, a bit like the Hawking radiation explanation is itself (see [137-139]), it seems the orthodox view (e.g. see the "state purification" [140], and the convoluted mumbo jumbo¹⁵ of [88]), just amount to the same: they attribute the entropy evolution à la Page curve to entanglement across the Quantum Extremal Surface. More on this later.

As a microscopic interpretation, the quantum extremal surface appears as if it could be the boundary of something like a baby universe [132], and reminiscent of Wheeler's bag of gold [107,108], as also shown in [88]. What is within the bag would, though a black hole would not interact any more with the exterior. Based on our discussion in footnote 9, our interpretation with a wormholes and an expanding universe, this microscopic interpretation is plausible, yet very unsatisfactory, Indeed:

- If nothing else, while information and unitarity are preserved, baby universes may imply excessive loss of energy to our taste, albeit ideas like total collision near the singularity may fit this picture [133].
- The associated non-conservation of charges (Electromagnetic and others) is in our view quite a problem. It is related to a discussion we will have in section 3.5, where it is argued that the existence of wormholes break all global symmetry. The situation here is even worse: it is not just about non-local conservation, but when the part inside the quantum extremal surface disconnects from the black hole exterior (i.e., the throat is strangled), the charge is not just locally lost but it is lost for the whole universe: to our knowledge, there are no proposals if say it was assumed that the baby universe would materialize as a charge in the universe, a bit like some remnant idea. It is just not likely, if a better interpretation exists as in [89], and section 3.4.
 - Note that this argument may be partially countered for the electromagnetic charge (but not lepton or baryon numbers for example) with the view that charged black holes, especially close to extremality, emit mostly charged particles, driven by the Schwinger effect [146-151], instead of neutral. This way, one may hope that the charge imbalance is reduced, yet if radiation also takes place through the quantum extremal surface, the Schwinger effects should also occur, until it creates a field in the back balancing it and privileging neutral radiation. A mix of effect partially reducing each other occur, but in general, the effects remains.
- (*) Linking wormholes and blackholes is highly speculative as mentioned in footnote 9. In particular, while we understand the behavior of the entropies, and proposed behavior of the entanglement and evaporation, it is microscopically unclear how and when the black hole singularity and interior morphs

¹⁵ By this we complain that the explanation is way too convoluted to describe what happens versus the simple explanation we give in [89].

from a black hole behavior to a wormhole. The replica tricks says that, after the scrambling time [159-161] (and also see later), the quantum extremal surface forms. But that means around that time quasi-instantaneous disappearance of the singularity, exchanged by magic with the bag of gold. And the bag of gold would be quasi immediately able to emulate the black hole for all purpose of the outside world: the apparent mass would remain the same, except for whatever has just been radiated (so far). How can this be? Besides asking if the singularity is present, or has disappeared in the bag of gold, it is unclear how the mass/energy contribution from within would maintain an external black hole behavior to the external world: the matter/mass/energy between the newly formed quantum extremal surface and the black hole horizon, may be too small to emulate the black hole. Yes, after a while, mass going with the quantum extremal surface can match the radiated mass but that would mean that the black hole mass reduces roughly two times faster than it radiates towards the outside world. Alternatively, the bag of gold cannot contain a baby universe that expands to keep mass (through gravitons/multi-folds/...) to be communicated back towards the entrance (blackhole or wormhole), so that its mass content remains available to the black hole mass. In that latter case half of the black hole mass would never disappear from the point of view of the outside of the black hole. Both cases are clearly non-sense and not what the Page curve tells us. In our view this on this own is sufficient to invalidate the interpretation (In a universe with a positive cosmological constant – we are not analyzing here what happens in AdS, and AdS black holes. The latter are different beast, as they do not match black holes in multi-fold universes, we lack the microscopical interpretations we used in [89]. *Note added on November 23, 2023: this will be used in [259] to consider the scenario as part of proving that our real universe cosmological constant is strictly positive.*).

- One could also imagine that the bag of gold itself has at some point too much mass density, and forms itself as a black hole. What happens then? Have such options actually be added to the topologies considered for the path integrals, or are there arguments to keep their effect negligible¹⁶?
- What happens at the end when all is evaporated and the quantum extremal surface detaches? Where does it go? Does it carry missing charges? In the electromagnetic case do we have a polarization of spacetime/vacuum near the end then a capacitor effect before it all disappears? Is it a different kind of remnant? Could it be a way to form another disjoint universe as envisaged in section 4¹⁷, or is it out of sight causality disjoint and lost forever?
 - In such a model, baby universe may be charged, a discrepancy also in case we assume that this is what happened with the big bang in our universe.

It seems that internal transition is left without a microscopic model.

So, on one hand we have a clear success that seems to corroborate the approach recovering the Page's prediction using a, by now, relatively uncontroversial model of path integrals, but with hard to physically interpret replicas and spacetime wormholes. On the other hand, the resulting island model accepted by all those convinced, leads to what might otherwise be a puzzling picture: the black hole interior would develop such a disconnected internal island, à la Wheeler's bag of gold, maybe, it is not that certain after all.

¹⁶ After all, [89] could be understood as having a similar internal behavior: multi-fold can't propagate outside the quantum extremal surface. So in our view these contributions may warrant considerations. With say at least half of the black hole mass at Page time being radiated into the bag of gold, this could happen, as we say nothing about the geometry of the bag beyond its wormhole throat. After all, a FLRW geometry, was just an example in [108].

¹⁷ The answer is no, as per its genesis, we know that it, and its content, can't be entangled with the black hole exterior anymore.

In our view, with the hindsight of [89], and considering in particular (*), we argue that the microscopic/physical¹⁸ interpretation seems potentially wrong. It is our claim. The next subsection will refer to what we believe to be the correct interpretation.

Note that while many of the proponents of the conventional quantum extremal surface preferred to work in AdS spacetime, the same analysis was also produced in dS (and flat) spacetime. See for example [90,95,141-146,152-158]. It is in such a case that [89] analysis was produced and where we have a more compelling story than the conventional interpretation discussed so far.

Also in much of the work, the spacetime is 2D, and the gravity theory used is a Jackiw-Teitelboim (JT) gravity, i.e. two-dimensional dilaton theory of gravity [162-165]. The usage of 2D¹⁹, AdS and JT are really²⁰ for the ease of the already complicated calculations with many approximations and unknown. In a multi-fold universe, all this makes a lot of sense: the Higgs boson is the dilaton resulting from the kinematics and dynamics of the multi-fold [23,67,187,227,239,252]. It is also the particle driving the 2D random walks at very small scales, where gravity is also 2D and dilaton based.

3.4. Multi-fold analysis of Black Holes and The Quantum Extremal Surface

In [89], we also encountered a Page curve with a different analysis, and with a detailed microscopic explanation of the effects. We also encountered conceptually a region that we related to the quantum extremal surface, and called it so, because we felt it was the same. However, at a major difference with the approach of section 3.3, it is not like an horizon defining an island to which the interior also radiates, and contributes particles that are then lost to the black hole. There are no Wheeler's bag of gold or related ideas. Instead, in a multi-fold universe, the quantum extremal surface reflects an interior surface, almost like an internal second horizon, such that radiations coming from in between that surface and the black hole horizon amount to mostly only radiating the particles that are so far entangled with previous radiations (see it as encompassing the set of particles that fell in the black hole versus the ones that escaped and are entangled to them), and therefore, it reduces the entanglement of the black hole interior with the outside.

Note that the multi-fold quantum extremal surface [89] essentially matches the quantum extremal surface obtained by the conventional island formula, as discussed above, even if the microscopic interpretations differ. Indeed, at any given time, we can maximize the radiation through the horizon and quantum extremal surface, and among all the possible solutions, the multi-fold version of the quantum extremal surface is also the one that, at that moment, minimizes the remaining entanglement entropy, as no particle within that surface (and hence towards the center of the black hole) can further contribute, at that moment, to changing (increasing or decreasing) the entanglement, radiation, or area of the black hole. Anything above it, up to and including the horizon can change the entropy. This is a very important additional consideration. One can't say that the concepts, or defined surfaces are different: it is exactly the solution in a multi-fold universe to the same equation, i.e., the island formula, as the quantum extremal surface.

Furthermore, the idea of awaiting, at the beginning before any , a scrambling time $\sim r_{\text{Sch}} \log S_{\text{BH}}$, with r_{Sch} being the Schwarzschild radius of the black hole, and S_{BH} , its entropy (uniquely defined in our view as before any Hawking

¹⁸ As based on semi classical path integral and gravity, it is rather a macroscopic physical interpretation, at the difference of say [89].

¹⁹ Of course, it does not hurt to also know that gravity and spacetime are essential 2D [1,16,98,99]: we know that the analysis will have the correct simplifications, and model to still capture the relevant essential effects at higher dimensions, even if it may miss other effects of course.

²⁰ See for example [50] for analyses with GR.

radiation takes place), to see the quantum extremal surface appear [88, 159-161]. Again this is exactly aligned with [89]: the surface appears as soon that one Hawking radiation takes place, and the stream becomes regular. This is essentially the proposal of [159-161] with their definition of scrambling time, i.e., when a small document thrown into the black hole can start to be retrieved in the Hawking radiation.

In [89], as radiation takes place, increasing or decreasing entanglement, the mass of the black hole decreases and that reduces the black hole area, as well as the surface associated to the multi-fold quantum extremal surface, which stays close below the horizon, and in fact getting closer to it, in agreement with for example figure 11 in [88]. As the latter shrinks, new candidates, to be radiated while decreasing entanglement, become available.

So, we have established that multi-fold black holes encounter the same (kind of) quantum extremal surface as conventional black holes encounter with the replica trick. However, while both the models of section 3.3., and [89], lead to the same results, the island approach misses with its microscopic explanation. At least based on the problems we have identified, and now applying Occam's razor principle, and within an asymptotic dS / positive cosmological constant as in a multi-fold universe [1,188,259].

Our proposal is that, in an asymptotic dS universes, instead of radiation towards the island(s), even in the case of the replica trick, with its wormholes and islands approaches, all the radiations continue the same only through the black hole horizon towards the outside of the black hole. In other words, no Wheeler's bag of gold or equivalent region appears to receive additional incoming radiations, and later separate from the main universe. But the portion of black hole radiation at a given time is only coming from the region between the horizon and the quantum extremal surface, even if coming from the horizon, with an \hbar uncertainty region as discussed in [1,89]²¹, when it the particles in that region are eventually reached by the black hole horizon fluctuations²². This does reduce the entanglement with the outside.

As the horizon shrinks, so does the quantum extremal surface. This way, entanglement across the quantum extremal surface towards the island, as in section 3.3, is really not what happens. What is inside the quantum extremal surface is just not entangled with the radiation outside the black hole. That's it. And what is inside the quantum extremal surface, will have to wait to be eventually reached by the horizon to play a role: being radiated but with no entanglement with what has already been radiated.

As this explanation make way more sense, with Occam's razor, we can only infer that the orthodox conventional interpretation of the quantum extremal surface is just misled or misleading. Indeed, while derived in a multi-fold universe, this interpretation, in such a universe, is microscopically tight. With [6], we can expect that it applies to any GR based universe and black hole, including in 2D, JT setup. Assuming, that GR reigns in AdS would also lead to the same considerations [72], but there is more leeway for the conventional explanation as [89] is for (asymptotic) dS, not (asymptotic AdS) [259].

With the multi-fold microscopic interpretation, we have no issue with understanding what happens in the island, and with the singularity region (even if the singularity does not exist in a multi-fold black hole because of the combination of discrete spacetime, non-commutativity, multi-fold dark energy, and possible torsion effects [1], or total collision as in [133]). Indeed, that region exists, unaffected, behind the quantum extremal surface. It reappears eventually, as part of the universe, and with effective potentials or curvature matching the outside spacetime, at the end of life of the black hole: no more mindboggling remnants, baby universes, or detaching bags of gold, etc.

Of course, the differences fundamentally come from what we had already identified as key very early on, in [1,166]. QFT, and related approaches [1,18], with their obsessed rejection of particles, miss the boat. The model

²¹ This model can also now be seen as confirmed with Hawking radiation viewed as tunneling. It is equivalent to the horizon \hbar horizon fluctuations due to quantum uncertainties presented in [1,89].

²² And, as the horizon contracts due to loss of mass and energy in previous radiations.

we propose works the same, but preserves particles inside the black hole. All the concerns expressed at the end of section 3.3 disappear. As so often encountered so far, continuous QFT models are approximations of what happens, instead of the supreme²³ model as too often considered [1].

We would recommend that the community engaged in addressing the black hole information paradox via the replica trick, wormholes and island formula, should consider our proposal for physical / microscopic interpretation²⁴. Especially as for the rest, this seems to require no differences at all in the mathematics and algorithm used, as in [88].

3.5 Problems with replica spacetime wormhole and global symmetries / charge problems, or multi-folds to the rescue?

Have all concerns with the replica trick, wormholes and island formula from section 3.3 disappeared? We think so with section 3.4, i.e., [89]: for us, we know that we can trust the results, but we also learned that we must be careful with the physical interpretation. As discussed, it can still go wrong. We must keep it in mind when applying the approach to disjoint universe, the original intention of the present paper.

We know that, even if we tried to appease some of the concerns, it relies on toy models (often 2D, JT gravity, AdS universe or black holes). All the numerous papers, mentioned above and references therein, also rely on strong approximations, simply because one does not know how to do better, and compute the gravity path integrals, with dominant contributions identified and computed or rather often also approximated. Even all that remains very complicated, and “unproven”, or controversial²⁵.

The replica spacetime wormholes are inspired from [134,167], and the island to their notion of baby universe [167]. However, [168] argues that replica spacetime wormholes lead to violations of global symmetries and associated charges conservation. Indeed, as shown in [168], the spacetime replica wormhole introduce mixing between pure states associated to a given quantum charge. Charges associated to Global symmetries are not conserved.

Frankly, this is not entirely new; it was already known from past considerations that wormholes and global (continuous) symmetries do not coexist [169-171]^{26, 27}. In fact, it is easy to see the issues with wormholes and charges. We already encountered it with Wheeler’s bag of gold as plausible, we eventually rejected it from our angle, physical interpretation of the (conventional) quantum extremal surface, which does not mean that it cannot appear in spacetime; it can as argued by Wheeler [107]. In any case, wormholes remain problematic for global

²³ Granted that, since superstrings have been positioned by their supporters as the only promising approach, supplanting QFT and other approaches like for example Loop Quantum Gravity (LQG). Yes we are rolling our eyes. We do not subscribe with that view. Again, superstrings are a tool, among many, and not necessarily physical considering for example [1,6,8-19].

²⁴ To that effect, we expect to republish these sections in the future in a paper dedicated to the subject to increase the visibility of this contribution.

²⁵ Of course, we are under no illusion. Our interpretation will also be seen as controversial, if only because it relies so much on understanding and tracking particles.

²⁶ Note that [169], studies global symmetry violation by gravity in the context of axions. We have conclude elsewhere that there are little reasons to expect the existence axions, simply because we do not need them [1,10,60,69,172,247]. However the conclusions in terms of global symmetry violation remain relevant.

²⁷ With the results of [89], in a multi-fold universe, the blackhole arguments of [172] seem moot. Wormholes remain a problem though. If we do not accept our proposed physical interpretation of the quantum extremal surface in a conventional spacetime, then the argument holds of course.

symmetries. Indeed, consider a (traversable) wormhole, real, or virtual, if that exists, between x^μ and x^ν , two different space locations, on a same time slice. A charge at x^μ can appear (quasi) instantaneously at x^ν , or a current can flow through the wormhole, without any current in the main manifold.. Quantum wise, the quantum wavefunction, or field, can now leak from x^μ to x^ν , without any current on the “main” spacetime manifold. Furthermore, with wormholes, one can build Wheeler’s bag of gold, without needing the black holes, as for example suggested in [108]. It remains an option, even if the wormhole throat is choked, the charge is lost, as in the black hole case, and even if the model is not suitable as physical explanation for black hole islands, as we argued in section 3.3. This way the charges is neither locally nor globally guaranteed to be conserved in the presence of gravity admitting (traversable) wormholes. That would affect for example charges like the electromagnetic charge, or the lepton and baryon numbers. We also saw that it affects energy conservation, but we also already know that it is a potential characteristics for GR / spacetime gravitational contribution, and say in cosmology for an expanding universe (*Note added on November 23, 2023: this is strictly not correct, energy is conserved as discussed in [242]*). The conundrum of [168] related to finite and infinite actions comes from the setup considered. Globally, it is not the wormhole that carries the charge but the bag of gold. That would allow for charge carrying wormhole²⁸ + say a FLRW baby universe. That would lead to finite Euclidian actions.

Separately, in [1], we saw that (many) global symmetry are broken by multi-folds effects, gravity and entanglement, at least in a multi-fold universe. So a priori the incompatibility between global (continuous) symmetries (discrete may still exist if modulo symmetries were in the black hole evaporation [171], not too likely though, as we did not encounter such behavior in the section 3 so far), is not shocking. It certainly look like it is the case, and it is not just due to the curvature spacetime, as sometimes claimed: it affects the internal symmetries, but of course with [23,231], we know that these are intimately connected to spacetime too.

However, we can claim that global symmetries are not locally affected. Globally, wormholes violate regional charge conservation, and bags of gold violate it globally.

In terms of the replica trick, it may have again been misinterpreted. Consider wormholes implementing multi-fold mechanisms [1,23,24,26,60,61]. When fields are exchanged as in (3.3) and (3.4) in [168], it results in precise entanglement of the exit points (or particles). So the fields are already entangled. This can creates singlet states [173], violating the assumptions for the computations provided in [168], where no singlet state is assumed. In other words, the explicit concerns raised about wormholes, as presented in [168], can be alleviated if understood as implementing the multi-fold mechanisms. Indeed multi-folds do not transfer charge for the sake of it. The transfer takes place on the main manifold. Only (ϵ fraction for any measured quantity or charge) paths are allowed in the multi-folds, and they disappear as the multi-fold are deactivated, resulting into conservation of charge [1]. In other words, if we are talking of multi-folds, or their implementation by wormholes, there are no issues. Furthermore, non-traversable wormholes should not matter. More generic and traversable wormholes²⁹, e.g., per GR, remain potentially affected by the questions raised in [169,170], as well as [171], the latter being subject to footnote 27. It is unclear if traversable wormholes physically exist in GR-based universes, without the mechanism ensuring traversability. That is to say that wormholes, real or virtual, in general would not break global symmetry, because non-traversable, unless in particular cases with entangled massive fermions in their throat. In such case, symmetry breaking is local (around the entry and exit location) and accidental in spacetime. The symmetry remains at least approximately global, as already predicted also by [171].

To the unconvinced, please also remember that this is used to compute Rényi entropies. If the parts are not entangled, there is no entanglement entropy: all the conditions are aligned to support multi-folds.

²⁸ It can also be just 1 at a time.

²⁹ Even if multi-fold were not implemented by traversable wormholes say as in [1,23,24,26,60,61,252] and footnote 7.

With this, reverting back to section 3.4, we hope that the reader realizes that the multi-fold theory habit to track particles, not (just) fields, get us a out of all these charge troubles. Indeed it is not only that entanglement is released as evaporation is released, but also the charges that were captured with particles that had collapsed into the black hole. Black holes have micro hairs after all. We knew that already since [1,174]. That's how we could say that conservation laws are not violated a priori by black hole and their life cycle.

We think that [171], is the strongest argument to justify that global symmetry must be broken by gravity in general, as they admit wormhole solutions, even if they may never be encountered, or if we expect that wormholes implement the multi-fold mechanisms [1,23,24,26,60,61]. Yet we can't pronounce ourselves at this stage. Remember that, in [1,83], baryon and lepton numbers are considered salvaged by chirality flips due to gravity. In our view, even if the wormhole impact on global symmetry were to be valid, this effect seems to compete with any global symmetry breaking, granted that we do not know if these symmetries are continuous or discrete. So, we assume that the conservation of L and of B , as in [1,83], is not invalidated by [168-171]. This analysis may be re-visited, and published, in future works.

3.6 Lessons learned for the Island formula and replica trick

The replica trick with its replica wormholes, and the island formula approach, works, recovers the Page curve and resolves the information paradox; even if the physical interpretation is disputed. The lessons of the multi-fold microscopic information is that the island is not a different spacetime region, à la Wheeler's bag of gold [88,107,108], that would later separate from the black hole interior and the outside universe, and where the black hole also radiate, besides what it radiates towards the black hole exterior. Instead it is, at a given time, the lower limit of the region between it and the horizon, from where radiation will result into releasing the particle entangled with the outside and therefore reduce the overall entanglement of the black hole. The physical or microscopic interpretation of the quantum extremal surface has changed.

That is a different, and, in our view, a clearer, and more logical microscopic interpretation of the quantum extremum function, and the black hole island.

Yet, it also validates the approach; it was just problematically interpreted. The conventional interpretation may have been incorrect, but it turns out that there is a cleaner microscopic interpretation, provided by the multi-fold approach that works as well and relies on the same concepts of island delimited by a quantum extremal surface. Mathematics remain the same, as do the results. So, overall, the methodology works.

Adding the understanding that spacetime wormholes implies entanglement between replica content, or replicas, resolve the charge and global symmetry concerns.

On that basis, for now, we accept that the approach is sane and consistent. Now, we are ready to see where it will lead us in other use cases.

Note added on March 12, 2023: See also [187] for a discussion of black hole evolution via evaporation, as discussed here, or split into smaller black holes, which are key to our introduction of the Ultimate Unification (UU) [1,25].

Note added on November 23, 2023: See [259] for a discussion on how wormholes considerations allow us to prove that the cosmological constant of our universe is strictly positive.

4. Disjoint (gravitating) entangled universe

4.1 Modeling challenges

[93-95] extend the use of replicas and wormholes in a model of entangled (with entangled fields) disjoint (gravitating) universes, including the case where one of them does not contain gravity, detailed in [93]. It also encounters the ER = EPR conjecture, in the toy model that it considers.

Entanglement between the universes is achieved with a double thermofield-like state, i.e. a field whose distribution is shared by both universe and characterized by a temperature (same, or different in both universes). This choice comes from working with Euclidian time and Euclidian Path integrals. The fields are assumed CFTs, they are the same for ease of the computations but could differ. How such a setup is actually physically achieved is left silent though.

In general, all the entanglement entropy estimates in [93-95] are computed, following the same approach as in sections 3.2 and 3.3. Doing so the papers show that ensuring unitarity of the model, more replicas need to be considered, involving swap wormholes, inspired from [88,132] and reference therein, as discussed in section 3. If say two fields are defined in say two half of each universes, the replicas must include all the configurations where say one half of one universe is glued to the other half of the other universe, and conversely. The associated wormholes are the “swap” wormholes. Although maybe hard to imagine, [94] gives good generic illustrations (generic without swap wormholes, and then specific figures where swaps take place).

[93,94] also model the case where one universe does not include gravity (turned off).

The following results were encountered by [93-95]:

- In an AdS spacetime with JT gravity, hence 2D, in the case involving a universe without gravity, the universe with gravity sees an island appearing inside its bulk, which minimizes the entanglement entropy (fine grained formula). It ensures preservation of the covariant entropy bound: if we tune the fields setup so that entanglement increases, the island increases in size until filling essentially completely the gravitating universe, except for a tiny region. It seems to directly relate to the Ryu–Takayanagi conjecture [38].
- When between two gravitating universes, [94], finds that the island rather appears in the (common) wormhole appearing between the different replicas, and their various topologies to implement the swap wormholes. All this amounts to implement the ER = EPR conjecture: wormholes between and among the entangled (part of) the universes. A simplified case of ER=EPR between the universes is explicitly computed. As explained earlier, multi-folds fundamentally match ER = EPR, without being tied to the wormhole physics, if needed.
- With one gravitating and another one without gravity dS universes, with cosmological horizon, [95] encounters equivalent results, but the island becomes very difficult to visualize or understand (a Cauchy slice minus a point). Other dS universes lead to more complex sets of islands. We already had problems with the interpretation of the replica trick for black holes in dS; so the complexity may be another signs that some considerations, in terms of the interpretation in dS, may have to be revised.

In all cases, the results obtained this way are unitary.

4.2 Challenges with the model

There are strange considerations with this model.

First of all, nothing really explains how entanglement is initially achieved. If one assumes that entanglement requires proximity at some point, one may wonder how the models of [93-95] can work, or otherwise occur. Unless if it was a case of a baby universe splitting apart from its parent universe, with entanglement between the two.

Secondly, the notion of two entangled universes is ambiguous, as a universe is supposed to include everything... However, it seems that any disjoint region in a universe would also fit the models that are practically used. In our view, this way, we may in fact also be able to consider regions sharing a boundary, if they were of different dimensions (e.g. a D-1 boundary to a D bulk as in the AdS/CFT duality).

Finally, if we are truly speaking of different universes, then what exists in between? [72] discusses embeddings. But as repeatedly discussed in our multi-fold work so far, we always consider that embedding or dual tangent space may or may not be (fully) ruled by GR, or even Physics in general. For multi-fold universes, the embedding is generated by the multi-fold mechanisms only as a local ϵ neighborhood [1,23,27,72,113,252]. It is one of the reasons why we maintain that multi-folds are more generic than traversable wormholes that may just be ways to implement multi-fold when allowed by Physics that would govern them. All this leads to next question: how can entanglement exist between such universes, and how can wormholes connect entangled regions across disjoint universe if not GR or no Physics exists in between? [93-95] are silent about it. The split baby universe comment above is probably the only way forward, at best.

Also, we know, and it is certainly the case in the multi-fold theory, that entanglement models require the past history of local interaction, possibly via real intermediaries (and we don't mean force carriers). So what does it mean for entangled disjoint universes? [93-95] are also silent about it. Again, the split baby universe comment above is probably the only way forward, at best.

So, even as a pure thought experiment or use cases, it is rife with pitfalls, ambiguities and possible inconsistencies.

4.3 New Interpretation to the Quantum Extremal Surface in the entangled disjoint universe use cases?

Just as in section 3, where we encountered islands and quantum extremal surfaces, and provided new microscopic interpretations, it may be worth revisiting the different islands encountered in section 4.1, and see what possible microscopic interpretations could be provided to explain them.

As entanglement between increases, in the case between disjoint AdS universes, one without gravity and one with gravity, the island grows, until it occupies almost the whole AdS with gravity. We claim that it is a manifestation of the E/G conjecture³⁰ [10,80], derived from [1]: gravity results from entanglement and entanglement produces gravity. So, as entanglement increases, in the universe with gravity, it bounds more and more strongly the matter

³⁰ The E/G conjecture is factual in multi-fold universe, and per [6], in GR based universes like our real universe.

behind the CFTs of that field³¹. That is the origin of the quantum extremal surface that separates that region from the rest.

In the case between entangled disjoint dS universes, with a cosmological horizon, one without gravity and one with gravity, the island is a Cauchy surface in the universe with gravity, between the cosmological horizon, and the wormhole. It could be seen as a quasi-black hole horizon resulting from the attraction between all the entangled fields that are now attracted together like in a black hole collapse as a result of the entanglement. The missing “pixel” in the Cauchy surface is because as long that no black hole forms, particles can pass through. Again entanglement results into gravity.

In the case of two entangled disjoint gravitating AdS universes, the different replicas and wormholes linking all the different replicas feel gravity effects due to entanglement. It is now spread everywhere “equally”, and the dominant effects end up adding up in the wormhole structure³². The extremal surface adds a black hole contribution that limits (minimize) the fine grained entropy.

If we were to envisage two entangled disjoint universe with gravity, one dS and one AdS, we expect a combination of the first and third case respectively in the AdS and dS space. As already mentioned, although harder to model, even these could involve universe of different dimensions.

The bottom line is again a confirmation of the multi-fold mechanisms [1] and the E/G conjecture: entanglement creates gravity because of the multi-folds, in this case implemented via replica spacetime and swap wormholes.

4.4 Encountering Multi-folds

As already mentioned, [93-95] do not need to require gravity or Physics in the embedding between the disjoint universes. This is reminiscent of the AdS/CFT correspondence conjecture as discussed in section 2.7.

[93-95] do not explicitly consider that the entangled spacetime would be of different dimensions. However it is not forbidden.

It is clear that disjoint 4D and 5D universes in a 10D superstrings or 11D supergravity, or M-theory space, would not have challenges to be modeled this way: Physics and GR govern the embedding space, and therefore, entanglement can exist across it, as well as wormholes, whatever they may look like. One just need a story on how the entanglement took place: past contact or interaction or some leakage, or detached baby universes. Such a leakage exists for multi-fold [1], where multi-fold create an embedding space where it could encounter another (albeit of course it does not happen as it is created inside out). In multi-fold, and in conventional physics one knows also that it could happen per [1,14,72]: Yang Mills locally leaks from its universe, into an embedding space. However, beyond the AdS/CFT correspondence conjecture use case, i.e., M-theory, and superstrings/supergravity [223,260], in conventional physics there are no other theories for such a larger embedding space other than Kaluza Klein [72, 175-177] (when seen as in [72]), and space time matter induction and scattering [27,113,178-180], and their embedding aspects as in [72]. The formers are covered by the AdS/CFT correspondence conjecture. The latter two are similar in principles, as shown in [72], and in our view covered by the 7D multi-fold embedding Ricci flat

³¹ Without gravity (e.g. with a zero coupling constant to the multi-fold path integrals), such an effect cannot occur. Between the universes, we do not know, nor care, if Physics or gravity reigns. So, any such possible effects, outside the two universes are not observable, and any discussion is in advance moot, till what’s in between is defined.

³² This did not happen in any of the other cases as gravity was always turned off on one side. Therefore, dominant effects had to be beyond the wormhole throat, i.e. different for each replica, and inside the universe with gravity.

space. However, the constrained KK case, i.e., the conventional KK with dimension compactification, may be harder to explain, as it would require the disjoint entangled universes to share these compactified dimensions, and / or, probably to have gotten entangled before compactification. It is a bit hard to justify with a straight face. Again the multi-fold setup of unconstrained KK seems better [27,72,113].

Yet there is another avenue: the holographic principle. With it, entanglement might simply result from the transformation:

$$\text{Universe}_1 \Leftarrow \text{Holographic Mapping} \Rightarrow \text{Hologram Screen} \Leftarrow \text{Holographic Mapping} \Rightarrow \text{Universe}_2 \quad (3)$$

The two sides are entangled: one does not know if a field that we observe comes from universe₁ or universe₂ or if we observe in one or another universe.

In (3), if the universes dimensions differ, it will impact how replicas, and associated (swap) wormholes are defined. But the process is always the same as across sections 3 and 4. As discussed in section 3, we really implement a multi-fold mechanism between entangled regions, not between non-entangled regions, and all the replica spacetimes and swap wormholes implement multi-fold-like mechanisms between universes, and among entangled pieces of the universes (across and within). When it makes sense they generate gravity exemplify with disjoint universe by gravity behavior consistent with the E/G conjecture.

So the replica trick with its wormholes and islands encounters multi-folds, as wormholes within and among the universes. When no physics exists between the universes, one is tempted to claim that the wormholes can't be wormholes, but they really rather are multi-folds, that are not tied to the existence of known physics in between (e.g. GR and wormholes) to appear.

More interestingly, if we go towards two disjoint universes, one 4D dS (or flat) without gravity, and the other AdS(5) (+...), we recover the setup of the conventional AdS/CFT correspondence conjecture, considered as a non-perturbative window into the M-theory. Multi-fold appear between entangled regions within the CFT space, and within the AdS(5) space (within the Ryu–Takayanagi region, which would be the region within the quantum extremal surface in this model³³). Note that multi-folds also recover the Ryu–Takayanagi region, based on maximum reach, into AdS(5), of the multi-fold from a multi-fold spacetime region [5].

With this, we conclude that, even more fundamentally than with the ER = ERP conjecture, the AdS/CFT correspondence conjecture encounters multi-folds. Note that it was expected. Indeed, [6] showed that the Hilbert Einstein action encounters multi-fold at Planck scales. As the AdS/CFT correspondence conjecture is a window into M-theory which itself manifest itself through the different superstrings and super gravity theory, with these latter theories relying on actions that encompass the Hilbert Einstein Action [6], M-theory was bound to also encounter multi-folds. Especially as we also knew from [1,8-20] that multi-fold encounters AdS(5), and hints of superstrings living in it as well as a factual AdS/CFT correspondence [1,8-20,97]. It had to be the case. Therefore, we see this result therefore also as a strong sign of consistency across all the approaches followed.

5. Conclusions

In this paper, we have shared a few interesting new results.

While reviewing the conventional and multi-fold AdS/CFT, ER=ERP and GR=QM conjectures, and its different versions, aiming at linking it to disjoint universes, we reviewed its generalization to the holographic principle,

³³ That was absolutely needed for consistency of the whole analysis.

instead of the other way around, where AdS/CFT is seen as a particular use case. Doing so, we used the multi-fold theory to prove the principle in our way, and as a result provide, to our knowledge, a first generic, and non-perturbative, proof of the covariant entropy bound, even if weak gravity proofs have been encountered in the literature. With previous encounter of multi-folds in GR-based universes, we consider the proof also applicable to conventional physics. In the process, we refuted that Wheeler's bag of gold would be a counter example to the holographic principle; something that, to our knowledge, also had not been done so far.

Then, the paper analyzed the replica trick to model the evolution and interior of black holes, and resolved the black hole information paradox. Comparing with our multi-fold resolution of the paradox, we identified the similarities and differences in recovering the Page curve. We concluded that the replica trick works, but that the microscopic of the quantum extremal surface interpretation is in our view incorrect, compared and should instead adopt the multi-fold model: radiation is always going to the outside, of the black hole, not towards a new baby universe. Of course, this requires re-privileging a particle approach that conventional QFT abhors. This result is so important that we plan to re-publish into a standalone paper devoted to the subject. The analysis applies to a universe with a positive cosmological constant. In a universe with a strictly negative one, it is possible that the conventional interpretation holds its own also.

On the way, we also encountered how ensuring that wormholes really implements multi-fold mechanism between entangled field, allows us to avoid deciding on the issue of compatibility or not of gravity and Global symmetries. We admit that this topic raises interesting questions: could it be used to produce a proof, in a multi-fold universe that global symmetries can't exist, or rather that traversable wormholes can't exist, or shouldn't implement multi-fold mechanisms. All this is potentially for future work.

Based on this analysis, we accepted the strength and validity of the replica trick, and island formula. We were therefore pleased to encounter it in works which study the entanglement between disjoint universes. Following the reason for entangled disjoint universes, and confirmation of our interpretations of replica spacetime and swap wormholes as implementing multi-fold mechanisms, we extended the model to the AdS/CFT correspondence conjecture. As a result, it turns out that this conjecture, and therefore M-theory, encounters multi-folds. It was an expected result, but quite impressive anyway, and in our view a further sign of the consistency of the approaches followed so far. Also, we provide multi-fold inspired interpretations of the quantum extremal surfaces appearing among the disjoint universes.

We also confirmed that, when encountering multi-folds, implemented via replica spacetime and swap wormholes, they create gravitation effects, confirming the multi-fold theory and its E/G conjecture factual in multi-fold universe, and per [6], likely factual in GR based universes like our real universe.

Understanding how a factual AdS/CFT correspondence with gravity and CFT on both sides, would also be of great interest. It is for future work, and an invitation to the Physics community for collaboration on the subject.

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Note: Some versions of other papers may have referred to this paper instead of [187] when it comes to discussing aspects of UU. We have added an explicit note referring to [187], for any reader reaching this version instead.
