A DARK MATTER THEORY BY GRAVITATION FOR GALAXIES AND CLUSTERS –V3

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1. ABSTRACT

This paper develops carefully an original theory of dark matter, whose main hypothesis is that DM is generated by the own gravitational field. This work introduces the best version of the theory physical and mathematically, which has been developed and published since 2013.

The hypothesis of DM by gravitation has two main consequences: the first one is that the law of DM generation has to be the same, in the halo region, for all the galaxies and the second one is that the haloes are unlimited so the total DM goes up without limit. Both properties are crucial for the success of this theory.

The newness of this work regarding the previous paper, [13] Abarca,M. 2023, is the chapter 12, where it is studied the DM by gravitation theory at the disk and halo regions. The numerical method used in this chapter is a bit rough but using the rotation curve at disk and its baryonic mass profile has been possible to estimate roughly DM density at disk depending on the field.

Three main conclusions have been got in this chapter.

1º at the disk region, there is a clear functional dependence between DM density versus field E according a potential function $D_{DM} = A·E^B$, although the exponent of the power is not 5/3 because the baryonic mass density is not zero in this region.

2º at the halo region, the parameters A & B got with a quite rude method are similar to the ones got in chapter 11 with the analytical method. This similarity shows that this rude method may be accepted in order to study the function DM density versus a power of field as a first approximation to this law.

3º Even with a method so rough it is possible to estimate the Solar DM density. The result got with that method differs only 15% regarding the current value accepted by the scientific community.

To study the DM theory more in depth at the disk region it is needed to solve the Poisson equation using together the baryonic density and the DM density, but unfortunately it is required sophisticated software, however at the halo region the Poisson equation is easy to solve, as it was shown in [13] Abarca,M. 2023, chp.15. Fortunately it is the galactic halo the most important region of galaxies to study DM in Cosmology.

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2. INTRODUCTION

Since 2013 up to 2019 I have published several papers studying DM in galactic halos, especially in M31 and Milky Way although also I have published some papers studying DM in Coma cluster [3] Abarca,M.2019.

As reader knows M31 is the twin galaxy of Milky Way in the Local Group of galaxies. According [5] Sofue, Y. 2015. $M_{BARYONIC-M31} = 1{,}61\cdot10^{11}\ M_{SUN}$ and according [6] Sofue, Y. 2020. $M_{BARYONIC-MILKY\ WAY} = 1{,}7\cdot10^{11}\ M_{SUN}$. Where Msun represents the mass of Sun. Msun = 1.99E+30 kg. Hereafter Msun represents such value.

The DM by gravitation theory was introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* . It considers that DM is generated by the own gravitational field. In order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. i.e. radius bigger than 30 kpc for MW and 40 kpc for M31, as it will be shown in chapter 6.

This hypothesis has two main consequences: the first one is that the law of dark matter generation, in the halo region, has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit.

In the chapter 9, is demonstrated mathematically that total mass by Direct mass formula goes up proportionally to the root square of distance, so this property may explain how the ratio of dark matter versus baryonic matter at cluster scale is bigger than such ratio at galactic scale.

By other side the growing of the total mass is so slow that Dark energy phenomenon may counter balance the DM when it is considered radius measured in mega parsecs. Precisely, this fact may explain the size of galactic clusters. This issue can be studied in [13] Abarca, M.2023. Chapter 16.

The first consequence before mentioned, dark matter generated by a Universal law, has been studied by all my papers, especially inside M31 and Milky Way thanks the remarkable data of rotation curves published in papers [5] Sofue, Y.2015 and [6] Sofue, Y.2020.

In fact I could develop rigorously the theory because the rotation curve of M31 at halo region decreased with a power regression fitted curve whose exponent is -1/4. However with data published for Milky Way at the same paper (2015) it was not possible to fit rigorously the rotation curve with such exponent.

Fortunately, in a new paper [6] Sofue,Y.2020, the author gives a new rotation curve data for Milky Way at halo region whose fitted curve has an exponent -1/4. Such result is good news for DM by gravitation theory, because the theory states a universal law of DM generation in the halo region of galaxies or clusters.

In this paper it is firstly developed all the theory carefully with M31 rotation curve data up to chapter 10 and the chapter 11 is dedicated to apply the theory to Milky Way with magnificent results.

The newness of this paper is the chapter 12,12 pages, where is studied the DM by gravitation theory inside the disk, and halo and three main conclusions have been got:

1º At the disk region, there is a clear functional dependence between DM density versus field E according a potential function $D_{DM} = A*E^{B}$, although the exponent of the power is not 5/3 because the baryonic mass density is not zero in this region.

2º At the halo region, the parameters A & B got with a quite rude method are similar to the ones got in chapter 11 with the analytical method. This similarity shows that this rude method may be accepted in order to study the function DM density versus a power of field as a first approximation to this law.

3º Even with a method so rough it is possible to estimate the Solar DM density. The result got with that method differs only 15% regarding the current value accepted by the scientific community.

As I have mentioned before, this theory has been developed assuming the hypothesis that DM is a quantum gravitational effect. However, it is possible to remain into the Newtonian framework to develop the theory. In my opinion there are two factors to manage the DM conundrum with a quite simple theory.
The first one, that it is developed into the halo region, where baryonic matter is negligible. The second one, that the mechanics movements of celestial bodies are very slow regarding velocity of light, which is supposed to be the speed of gravitational bosons. It is known that community of physics is researching a quantum gravitation theory since many years ago, but does not exist yet.

Use a more simple theory instead the general theory is a typical procedure in physics.
For example the Kirchhoff ´s laws are the consequence of Maxwell theory for direct current and remain valid for alternating current, introducing complex impedances, on condition that signals must have low frequency. However these laws do not work for electromagnetic microwaves because of its high frequency.

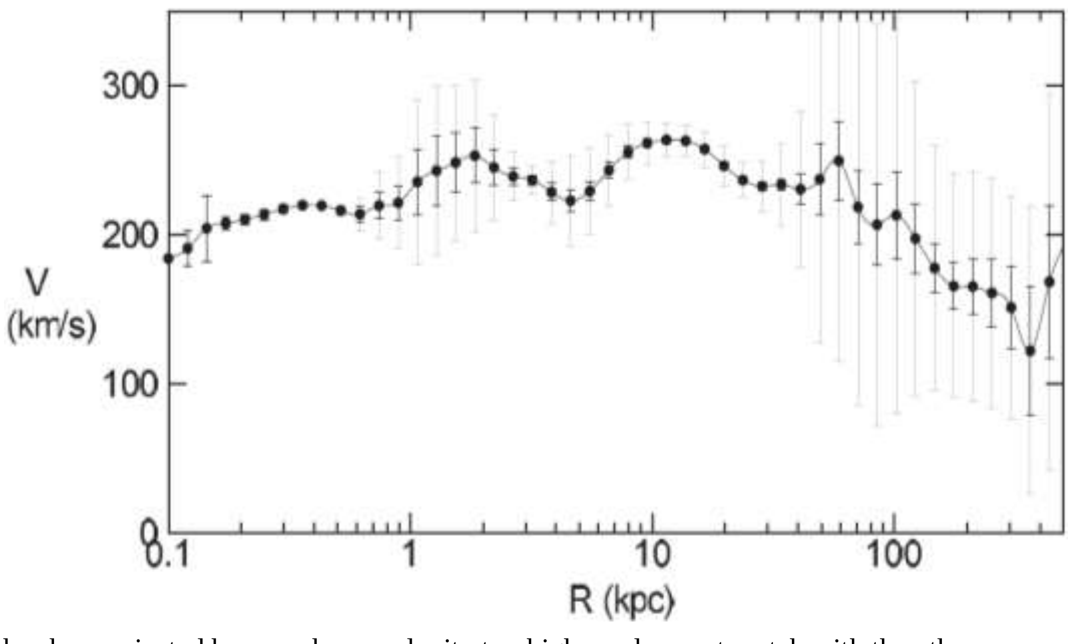
Thanks the possibility to study the gravitational effect of DM pure, in halo regions of M31 and MW, it have been possible to develop a theory mathematically simple. When baryonic mass is mixed with dark matter as it happens inside the galactic disc the mathematical treatment is by far more complex.

Taking into account that the only ones giant galaxies quite close to be able to study with accuracy the rotation curve at halo region are Milky Way and M31, the coincidence of the same exponent to the fitted function for the rotation curves for both galaxies is crucial in order to state that dark matter is generated according an Universal law.

3. OBSERVATIONAL DATA FOR M31 GALAXY FROM SOFUE. 2015 DATA

Graphic come from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. Although Sofue rotation curve ranges from 0,1 kpc up to 352 kpc the range of dominion considered for this work is only the halo region where ratio baryonic matter is negligible. In chapter 6, will be shown that this happens for radius bigger than 40 kpc, despite the fact that disc radius for M31 is accepted to be 35 kpc.

kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8



The measure at 352 kpc has been rejected because has a velocity too high, so does not match with the other measures. May be an celestial object captivated by the gravitational field of M31 afterwards to M31 formation and it is right to think that it is not in dynamical equilibrium with M31.So it is a good criteria to consider the Virial mass associated to M31 as the dynamical mass up to 302,9 kpc.

3.1 POWER REGRESSION TO ROTATION CURVE

The measures of rotation curve have a very good fitted curve by power regression.

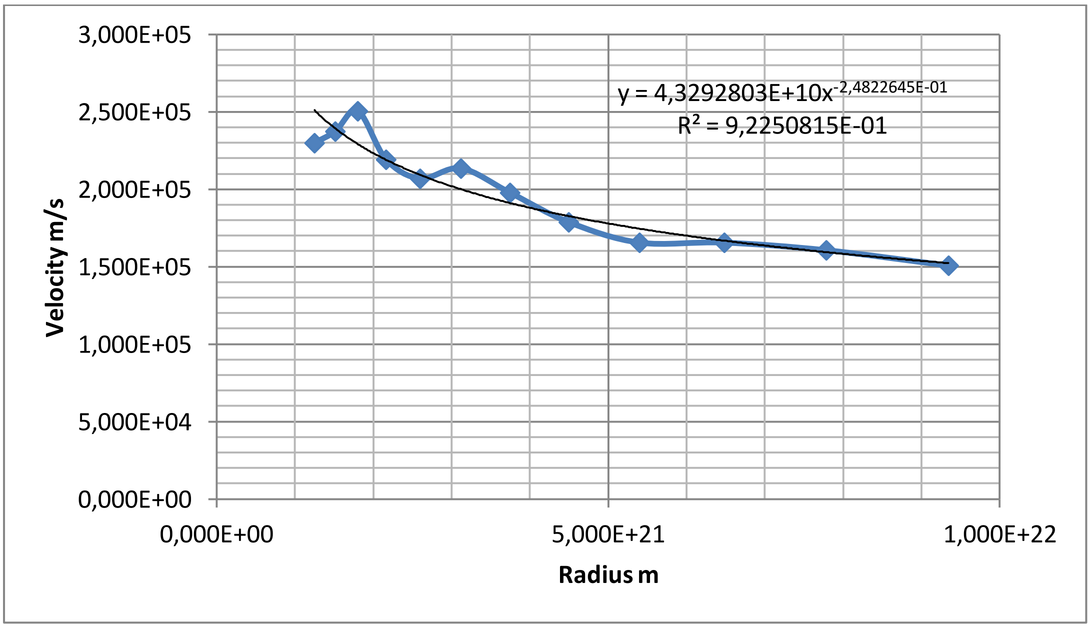
Power regression for M31 rot. curve	
V=a*r^b	
a	$4{,}32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Data fitted are in grey columns below. In fifth column is shown results of fitted velocity and sixth column shows relative difference between measures and fitted results.

Below is shown a graphic with measures data and power regression function.



Correlation coefficient equal to 0,96 which is a superb result especially when dominion measures is up to 303 kpc. There is not any other galaxy to measure a rotation curve so magnificent. According theory of DM generated by field, galaxy haloes are unlimited although up to a half of distance i.e. 375 kpc toward Milky Way direction is dominated by M31 field whereas the other half distance is dominated by Milky Way.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a{\cdot}r^b$. As $M_{DYNAMIC}(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$. Hereafter this formula will be called Direct Mass

$M_{DIRECT}(<r) = \frac{a^2 \cdot r^{2b+1}}{G}$ because it has been got rightly from rotation curve.

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $dm = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G}$ and $dV = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G}$ results $D_{DM}(r) = L{\cdot}r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L{\cdot}r^{2b-2}$ kg/m^3

It is important to highlight that at this moment this formula is only a statistical approximation of DM density able to explain the rotation curve, without any physic meaning.

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E BY VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, Virial gravitational field, E, got through this formula will be called E.

The key to state the Virial theorem is the dynamical equilibrium. It is supposed that celestial bodies are quite close to dynamical equilibrium, because the most of celestial bodies belong to a specific galactic system from its formation times.

By substitution of $v = a \cdot r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E = a^2 \cdot r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A \cdot E^B$ .Where A & B are parameters to be calculated. This hypothesis has been widely studied by the author in previous papers. [1] Abarca,M. [2] Abarca,M. [8] Abarca,M. y [10] Abarca,M. This hypothesis fulfils the physic meaning of D.M. Density formula in the halo region because it is supposed that such D.M. density is generated as a consequence of gravitational field propagation in the framework of a quantum gravitation theory.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	3,6559956 ·10^{-6}
B	1,6682469

According parameters a & b got in previous chapter, A & B parameters are written beside.

Conversely $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi G A (B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1$ *and* $B \neq 3/2$.

As conclusion, in this chapter has been demonstrated that a power law for velocity

$v = a \cdot r^b$ *is mathematically equivalent to a power law for DM density depending on E.* $D_{DM} = A \cdot E^B$ *if it is considered as physic hypothesis that D.M. is generated by the gravitational field.*

5.3 THE IMPORTANCE OF $D_{DM} = A.E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ and $E = a^2 \cdot r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A \cdot E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be similar for different galaxies on condition that the galaxies are similar. In further chapters will be got that power B is exactly the same for M31 and Milky Way although parameter A will be a bit different.

However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters. As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, at galaxy scale or cluster of galaxies.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy. [5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

To convert superficial baryonic density to volume density it is right to get the formula $D_{BARYONIC}^{VOLUME} = \frac{\Sigma(r)}{2r}$ so $D_{BARYONIC}^{VOLUME}(40kpc) = 3.1 \cdot 10^{-25}\ kg/m^3$.

The formula of Direct Dark matter density is got afterwards, in page 17. $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ being L = 1,33E+30. For example D_{DM} (40 kpc) = 2,5E -23 kg/m^3 . So the ratio of both volume density is 0,0124 .In conclusion it is right to consider negligible the baryonic density at 40 kpc, therefore it is possible to estate that halo dominion begins at 40 kpc in M31.

7. A DIFFERENTIAL EQUATION FOR THE GRAVITATIONAL FIELD IN THE HALO

7.1 INTRODUCTION

This formula $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \frac{G \cdot M(<r)}{r^2} = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A \cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION IN THE GALACTIC HALO

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r^3}$ As $\varphi_{DM}(r) = A\cdot E^B(r)$ it is right to get $E'(r) = 4\pi\cdot G\cdot A\cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K\cdot E^B(r) - 2\frac{E(r)}{r}$ being $K = 4\pi\cdot G\cdot A$ Calling y to E, the differential equation is written in this simple way $y` = K\cdot y^B - \frac{2\cdot y}{r}$ Bernoulli family equations $y` = K\cdot y^B - \frac{2\cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$. This is $\frac{u`}{1-B} + \frac{2u}{r} = K$

The homogenous equation is $\frac{u`}{1-B} + \frac{2u}{r} = 0$ whose general solution is $u = C\cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u = z*r then it is got that $z = \frac{K\cdot(1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C\cdot r^{2B-2} + z\cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{K(1-B)\cdot r}{3-2B}\right)^{\frac{1}{1-B}}$ $with\ B \neq 1\ and\ B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B-2$ $\beta = \frac{1}{1-B}$ and parameter $D = \left(\frac{K(1-B)}{3-2B}\right)$ then $E(r) = (Cr^{\alpha} + Dr)^{\beta}$ .Notice that $\alpha.\beta = -2$ for any value of B, that means that the field has a term which depend on radius as the Newtonian field i.e. $E \propto r^{-2}$, namely the term associated to initial parameter C.

Calculus of parameter C through initial conditions Ro and Eo

Suppose R_0 *and* E_0 are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{1/\beta} - D\cdot R_0}{R_0^{\alpha}}$

The values R_0 and E_0 may be anyone belonging to radius dominion, not only the border of halo between disk region and halo region. In the epigraph 9.6 is calculated parameter C using different values of R_0 and E_0.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E BY BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ Being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1, 6682469

In this case relative difference between B = 1, 6682469 and 10/7 is 16, 7 %. A 17% of error in cosmology could be acceptable. However by the end of the chapter it will be found the right value for B.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G, h and c =2.99792458$\cdot 10^8$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which cannot be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1$ *and* $B \neq 3/2$.

As A is a positive quantity then $2b+1>0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ therefore $B \in (-\infty,1)\cup(3/2,\infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph, namely for $B \in (3/2,\infty)$.

The main consequence this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only one pi monomial is wrong because B=10/7 < 3/2. Therefore formula $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$, being $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$, got by two pi monomials it is the right physic formula of D.M. density depending on gravitational field E.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As the hypothesis of the theory estates that DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3{,}6559956 \cdot 10^{-6}$
B	1,6682469

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A \cdot E^B$ will be 5/3, which is the best approximation to B= 1.6682469.

Finally $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs less than 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a & b parameters and A&B parameters. Now considering B= 5/3

as $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$. It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $D_{DM} = A \cdot E^{\frac{5}{3}} = \dfrac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$. In chapter 11 will be studied the rotation curve of Milky Way according the data published by Sofue in 2020, and it will be shown that in the halo region the rotation curve decreases with the same exponent b = -1/4. This fact is crucial for DM by gravitation theory because both giant galaxies are the only ones with rotation curves measures in halo region.

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Regression for M31 dominion 40-303 kpc	
$V=a*r^b$	
a	$4{,}32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Due to Buckingham theorem it is needed that b= -1/4. Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a &b that fit perfectly to experimental measures of rotation curve in M31 halo.

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula $V= a*r^{-0.25}$ on condition that $\sum_e (v-v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

New parameters a &b and A&B	
B	5/3
$b = \dfrac{B-2}{2B-2}$	b = -1/4
a new	$4{,}727513*10^{10}$
A new	$3.488152*10^{-6}$

Where r_e represents each radius measure and V_e represents its velocity associated. See table page 5 data in columns in grey.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \cdot 10^{10} \frac{m^{5/4}}{s}$$

9.2 FORMULAS OF DIRECT D.M. FOR DENSITY FOR MASS AND FIELD

With these new parameters recalculated it is going to get the direct formulas got at the beginning of paper.

Function of Density DM depending on radius.

$$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}} \text{ being } L = \frac{a^2 \cdot (2b+1)}{4\pi G} = \frac{a^2}{8 \cdot \pi \cdot G} = 1{,}3326*10^{30} \frac{Kg}{m^{1/2}}$$

Function of E depending on radius $E = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}}$ being $a^2 = 2{,}235*10^{21} \dfrac{m^{5/2}}{s^2}$

Mass enclosed by a sphere of radius r, known as dynamical mass because it is calculated with velocity.

$M_{DYN}(<r) = \frac{v^2 \cdot R}{G}$ When velocity is replaced by its fitted function it is got $M_{DIRECT}(<r) = \frac{a^2 \cdot r^{2b+1}}{G} = \frac{a^2 \cdot \sqrt{r}}{G}$

9.3 BERNOULLI SOLUTION FOR E AND DENSITY IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to get the formulas because some parameters and algebraic expression, now are simple fractions. Namely:

$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ being $\alpha = 2B - 2 = \frac{4}{3}$ and $\beta = \frac{1}{1-B} = \frac{-3}{2}$ As it was pointed in paragraph 7.2 $\alpha.\beta = -2$ for any value for B, so the field E will have a term equal to $C.r^{-2}$i.e. will be the Newtonian term.

By other side, the initial condition $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$ becomes $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ and $D = \left(\frac{4 \cdot \pi \cdot G \cdot A(1-B)}{3-2B}\right) = 8 \cdot \pi \cdot G \cdot A$ As $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$ (see chap. 9 at the beginning) then $D = a^{\frac{-4}{3}} = 5{,}85 \cdot 10^{-15}$

Therefore $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being C the initial condition of differential equation solution for E and $D = a^{\frac{-4}{3}}$ is a parameter closely related to the global rotation curve at halo region, being parameter **a** =4,7275*10^{10}

AND THE BERNOULLI SOLUTION FOR DENSITY IN HALO REGION

$$D_{BERNI} = A \cdot E^{\frac{5}{3}} = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}} = \frac{D}{8\pi G}\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}}$$

9.4 DARK MATTER AT A SPHERICAL CORONA BY BERNOULLI SOLUTION IN HALO REGION

Formula below express the dark matter contained inside a spherical corona defined by R_1 and R_2 belonging at halo.

$$M_{BERNI} = \int_{R1}^{R2} 4\pi \cdot r^2 \cdot D_{BERNI} dr = \int_{R1}^{R2} 4\pi \cdot \mathrm{r}^2 A E^{B} dr = 4\pi A \int_{R1}^{R2} \mathrm{r}^2 \left[C \cdot r^{4/3} + D \cdot r\right]^{\frac{-5}{2}} \cdot dr$$

The indefinite integral $I = 4\pi A \cdot \int \frac{r^2}{\left(C \cdot r^{4/3} + D \cdot r\right)^{2.5}} = \frac{8\pi A\sqrt{r}}{D \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$

As $\frac{8\pi A}{D} = \frac{1}{G}$. Calling $M_{BER}(r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ and **by** the Barrow`s rule, it is got

$M_{BERNI}\Big|_{R1}^{R2}$ = M(R2)-M(R1) that provided the DM contained inside the spherical corona defined by R2 and R1.

9.5 NEWTON`S THEOREM WITH BERNOULLI MASS FORMULA

The name for this theorem has been chosen because the relation between field E and total mass M(<r) is the same that in Newton's theory.

From Bernoulli field $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}} = \dfrac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D\right)^{3/2}}$

Bernoulli mass formula $M_{BERNI}(r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ so $G \cdot M_{BER}(r) = \dfrac{\sqrt{r}}{\left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ and

$$\frac{G \cdot M_{BER}(r)}{r^2} = \frac{\sqrt{r}}{r^2 \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D\right)^{3/2}} = E(r)$$

Therefore $E(r) = \dfrac{G \cdot M_{BER}(r)}{r^2}$ this is the intensity of field in Newton´s theory. This identity shows how the ***DM by gravitation theory,*** adding an extra of mass depending on radius, being the halo region unlimited, is able to explain the DM measures in galaxies and cluster in the Newtonian framework.

COROLLARY

According the Newtonian framework, the function of mass included in the formula of field E, means the total mass included inside the sphere with radius r. Therefore M_{BER} (R) must be renamed as $M_{BER}(< r) = M_{TOTAL}(< r)$ where r ranges in the halo region. In conclusion, the total mass enclosed by radius r is given by the formula:

$M_{TOTAL}(< r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ where r belong to halo region. i.e. r > 40 kpc for M31 galaxy.

9.6 CALCULUS OF PARAMETER C

$$C_O = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

This parameter is calculated by a data belonging to rotation curve in halo region, (R_0, V_0). E_0 is the gravitational field at R_O radius. Considering that data measures are in dynamical equilibrium, it is possible to estimate $E_O = V_0^2 \;/ R_0$ In the following epigraph it will see studied more in deep the hypothesis about the dynamical equilibrium of data.

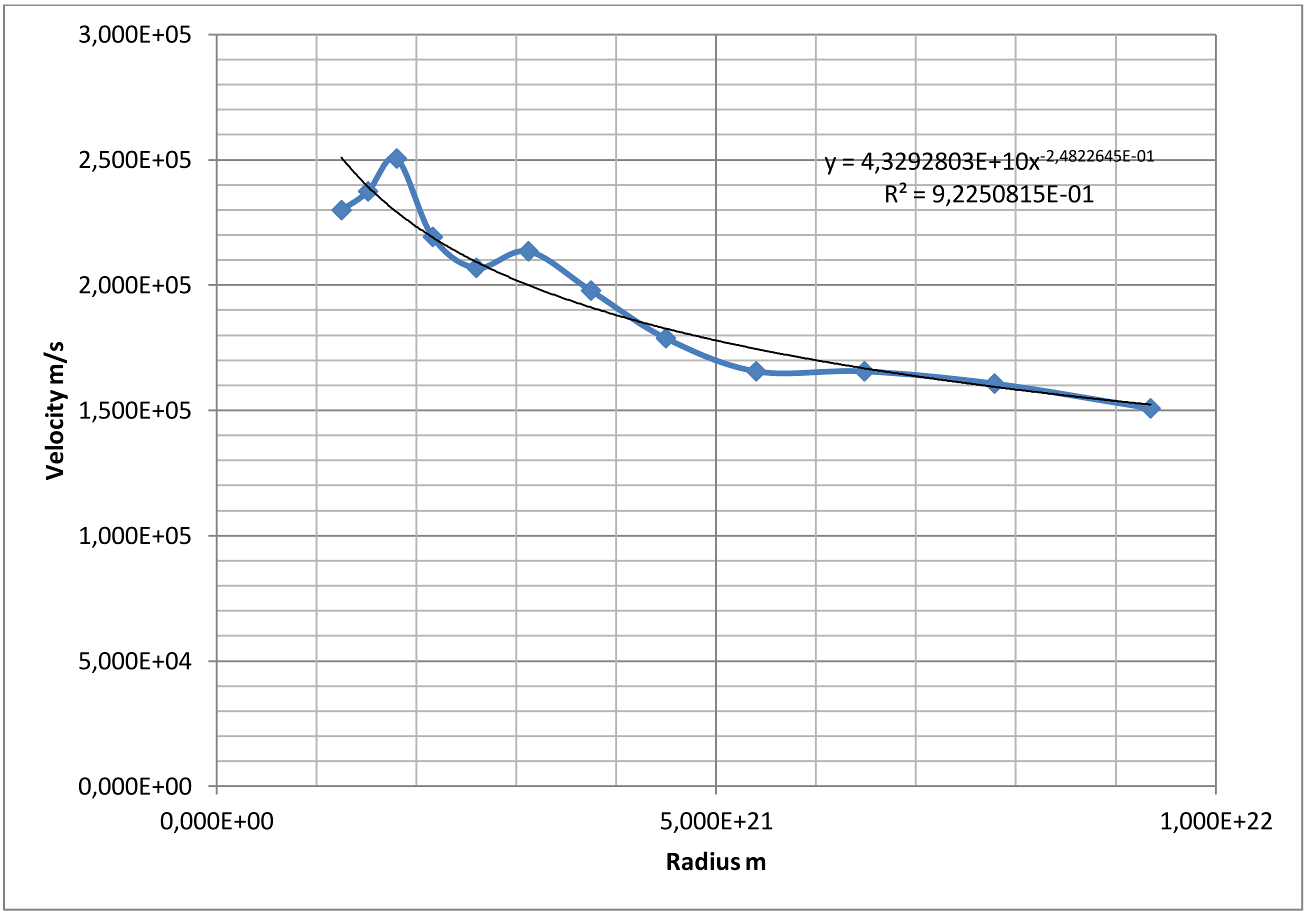
It is only an approximation because the celestial bodies are not in perfect dynamical equilibrium, but data selected in the halo region are quite close to the dynamical equilibrium. In addition experimental measures have errors. By these two reasons, it will be calculated Co for every data in order to calculate Bernoulli mass formula for all of them.

Points	Radius	Radius	Velocity	Field Eo	Eo^(-2/3)	Eo^(-2/3)-D*R_0	Parameters C_0
	kpc	m	m/s	m/s^2			
1	40,5	1,250E+21	2,299E+05	4,23E-11	8239256,95	9,27E+05	6,8830E-23
2	49,1	1,515E+21	2,374E+05	3,72E-11	8973615,749	1,11E+05	6,3753E-24
3	58,4	1,802E+21	2,505E+05	3,48E-11	9377625,482	-1,16E+06	-5,3082E-23

4	70,1	2,163E+21	2,192E+05	2,22E-11	12654654,13	1,20E+03	4,2804E-26
5	84,2	2,598E+21	2,069E+05	1,65E-11	15443489,73	2,45E+05	6,8683E-24
6	101,1	3,120E+21	2,135E+05	1,46E-11	16732951,68	-1,52E+06	-3,3317E-23
7	121,4	3,746E+21	1,978E+05	1,04E-11	20928758,45	-9,85E+05	-1,6934E-23
8	145,7	4,496E+21	1,788E+05	7,11E-12	27043372,43	7,42E+05	9,9988E-24
9	175	5,400E+21	1,656E+05	5,08E-12	33846627,1	2,26E+06	2,3822E-23
10	210,1	6,483E+21	1,656E+05	4,23E-12	38232870,38	3,08E+05	2,5446E-24
11	252,3	7,785E+21	1,607E+05	3,32E-12	44959131,06	-5,83E+05	-3,7771E-24
12	302,9	9,347E+21	1,508E+05	2,43E-12	55281485,72	6,02E+05	3,0572E-24

Values in yellow are negatives because these points are above the fitted curve . See graph below. In addition, the more close to curve the point is, the smaller, in absolute value, the parameter C is. The cyan value is the smaller.

Afterwards will be shown that this fact explain that data measures are close to dynamical equilibrium. For example points 1 and 3 are quite far away from dynamical equilibrium despite to be placed in halo region.



The close oscillation around the fitted curve suggests strongly that this curve might be the ideal curve of perfect ideal dynamical equilibrium for the celestial bodies belonging to M31 galaxy.

In the following epigraph it will be demonstrated that if it is considered as initial condition a point belonging to the ideal fitted curve at halo region, this point will have parameter C = 0. In addition in the epigraph 9.8 it will be shown that Bernoulli mass formula becomes direct mass if parameter C = 0.

9. 7 PARAMETER C EQUAL ZERO THEOREMS

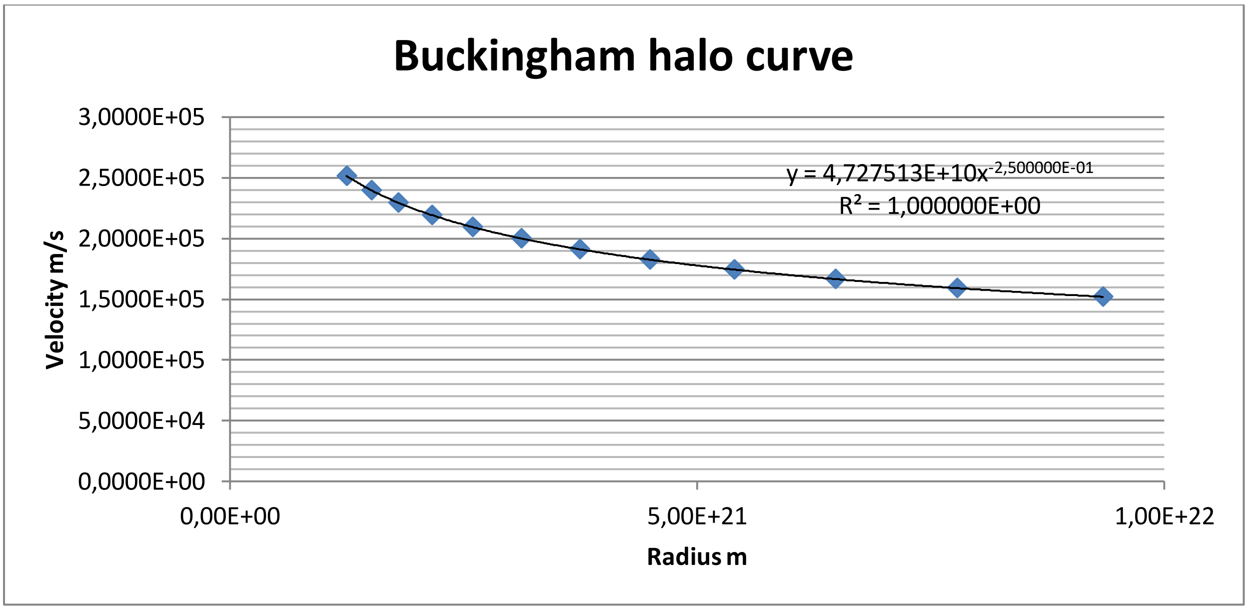
Definition. Hereafter, it will be named Buckingham halo curve to the points **(r, v)** **r** belonging to the halo region and the velocity $v = a \cdot r^{-1/4}$ being **a** the parameter associated to galactic halo. It is an ideal curve because its points are in perfect dynamical equilibrium.
As the exponent -1/4 was got by the Buckingham theorem, it has been select such name for that curve.

Direct Theorem: If it is supposed that a point belonging to Buckingham halo curve is in dynamical equilibrium and if it is selected such point as initial point to calculate C, then such parameter is zero.
Proof : Suppose a point (Ro, Vo) belonging to Buckingham halo curve, then $V_0 = a \cdot R_0^{-1/4}$ As dynamical equilibrium leads to $E_0 = \frac{GM(<r)}{r^2} = \frac{V_0^2}{r}$ then $E_0 = a^2 \cdot R_0^{-3/2}$ and $E_0^{\frac{-2}{3}} = a^{-4/3} \cdot R_0 = D \cdot R_0$ because $D = a^{\frac{-4}{3}}$ when B=5/3 as it was shown at epigraph 9.3. Therefore C= 0 because its numerator is zero. $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$

It is important to highlight that such points are in perfect dynamical equilibrium, whereas the data measures are **close** to dynamical equilibrium. In addition, the real gravitational field never has a perfect spherical symmetry. See in table below the relative difference between data and Buckingham points.

Reverse Theorem
If it is selected a point (Ro,Vo) which is supposed to be in dynamical equilibrium and its parameter C=0 then such point belong to Buckingham halo curve. Suppose that $V_0 = a \cdot R_0^b$ being exponent b unknown.

Proof: If C = 0 then $E_0^{-2/3} = D \cdot R_0$ and as there is dynamical equilibrium $E_0 = \frac{V_0^2}{r}$ then $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ and $E_0^{-2/3} = a^{-4/3} \cdot R_0^{\frac{2-4b}{3}}$ so $a^{-4/3} \cdot R_0^{\frac{2-4b}{3}} = D \cdot R_0$ or $D \cdot R_0^{\frac{2-4b}{3}} = D \cdot R_0$ which leads to $R_0 = R_0^{\frac{2-4b}{3}}$ so b= - 1/4



Radius kpc	Radius m	Buckingham Velocity m/s	Data velocity m/s	Relative Diff. %
40,5	1,25E+21	2,5144E+05	2,2990E+05	8,57E+00
49,1	1,52E+21	2,3962E+05	2,3740E+05	9,27E-01
58,4	1,80E+21	2,2945E+05	2,5050E+05	-9,17E+00
70,1	2,16E+21	2,1921E+05	2,1920E+05	5,58E-03
84,2	2,60E+21	2,0939E+05	2,0690E+05	1,19E+00
101,1	3,12E+21	2,0004E+05	2,1350E+05	-6,73E+00
121,4	3,75E+21	1,9109E+05	1,9780E+05	-3,51E+00
145,7	4,50E+21	1,8257E+05	1,7880E+05	2,06E+00
175	5,40E+21	1,7440E+05	1,6560E+05	5,04E+00
210,1	6,48E+21	1,6660E+05	1,6560E+05	6,03E-01
252,3	7,79E+21	1,5915E+05	1,6070E+05	-9,72E-01
302,9	9,35E+21	1,5204E+05	1,5080E+05	8,18E-01

The third column got with the formula $V_0 = a \cdot R_0^{-1/4}$ is called Buckingham velocity because such formula is calculated by the Buckingham theorem. See the graph above.

Parameter $a_{M31} = 4{,}727513E{+}10\ m^{5/4}\ /s$

The fourth column are the measures of velocity.

Data at radius 302,9 kpc has the lowest relative difference. Although data radius 252, 3 kpc has almost the same relative difference but negative.

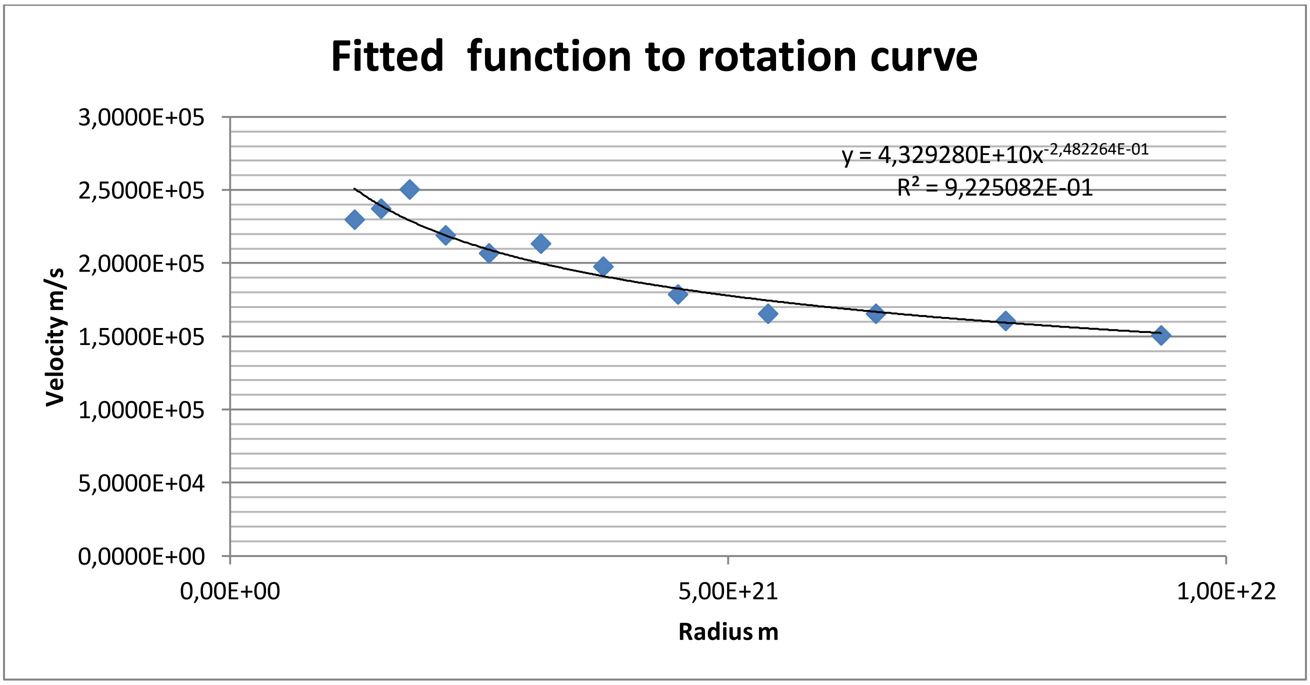
Final comments

Data measures do not belong to Buckingham halo curve by two reasons:

The first one it is simple: measures have experimental errors. The second one is more subtle, the celestial bodies are not in perfect dynamical equilibrium. It is right to think that celestial bodies which belong to M31 gravitational system from its formation times, more than ten billions years ago, will be closer to dynamical equilibrium regarding other ones that were captivated by the gravitational field of M31 afterwards.

Watching the graph, it is clear that point 1 and point 3 at 40,5 kpc and 58,4 kpc are the points more distant regarding Buckingham halo curve. This important difference regarding dynamical equilibrium curve may be explained by the asymmetries of gravitational field during the history of dynamic evolution.

Anyway it is undeniable that in general data are very close to dynamical equilibrium, just as Sofue data measures are magnificent.



As it is shown in the graph the exponent of fitted function differs 18 thousands regarding -0.25

9.8 BERNOULLI FORMULAS BECOME DIRECT FORMULAS WHEN PARAMETER C = 0

Thanks demonstration made above, it is clear why data measures close to Buckingham halo curve give values for C very close to zero. The more close point measure to Buckingham halo curve is, the more close to zero parameter C is. Now parameter C will be zero, because at C = 0 are got the formulas with initial point belonging to Buckingham halo curve. i. e. a initial point which is in perfect dynamical equilibrium.

FOR FIELD E

When in formula $E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}}$ C= 0 then it is got $E=a^2\cdot r^{\frac{-3}{2}}$ because $D=a^{\frac{-4}{3}}$, being a^2 = 2.235*10[21]

which is precisely direct formula for E. Which is the field calculated by perfect dynamical equilibrium.

FOR D.M. DENSITY

As D_{DM}= A*E$^{5/3}$ Using field got by Bernoulli solution it is right to get

$D_{DM}(r)=A\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-5}{2}}$ Being $A=\dfrac{D}{8\pi G}$ and $D=a^{\frac{-4}{3}}$ if **C** = 0 then formula becomes

$D_{DM}(r)=A\cdot D^{\frac{-5}{2}}\cdot r^{\frac{-5}{2}}=L\cdot r^{\frac{-5}{2}}$ being $L=\dfrac{a^2}{8\cdot\pi\cdot G}$ = 1.3326*10^{30} which is direct DM density formula.

FOR DIRECT MASS FORMULA

If C=0 then $M_{BERNI}(<r)=\dfrac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$ becomes $M_{DIRECT}(<r)=\dfrac{a^2\cdot\sqrt{r}}{G}$ being $\dfrac{a^2}{G}=3.349\cdot10^{31}$

Final comment

As it is clear Direct formulas are a particular case of Bernoulli formulas when parameter C =0.

At the beginning of this paper, the direct formulas were only right up to 300 kpc, and they were statistical formulas to calculate the extra additional amount of matter needed to explain the rotation curves. However, thanks to findings got in this chapter, Direct formulas are the Bernoulli formulas when it is considered as initial point a theoretical point of rotation curve in halo region, which are in perfect dynamical equilibrium, therefore according DM theory by gravitation its dominion is unbounded. Also direct formulas depend on parameter **a** solely, instead two parameters C and D associated to Bernoulli formulas, which is a magnificent simplification of the theory.

In the last chapter will be shown how the dark energy is able to counter balance dark matter at cluster scale, so the total mass of dark matter generated by a galaxy or cluster of galaxies do not diverge at infinitum.

Conclusion Hereafter it will be use Direct formulas with unbounded dominion instead Bernoulli formulas.

10. MASSES IN M31

In this chapter, It will be calculated and compared three different types of masses related to M31.

10.1 DYNAMICAL MASS VERSUS DIRECT MASS

As it is known, dynamical mass represents the total mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius, so it is right to consider dynamical mass as the total mass, baryonic and DM mass, enclosed at radius R. Ranging radius in the interval of radius measured.

The formula of dynamical mass is $M_{DYN}(<r) = \frac{V^2 \cdot r}{G}$. and $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3{,}35.10^{31}$

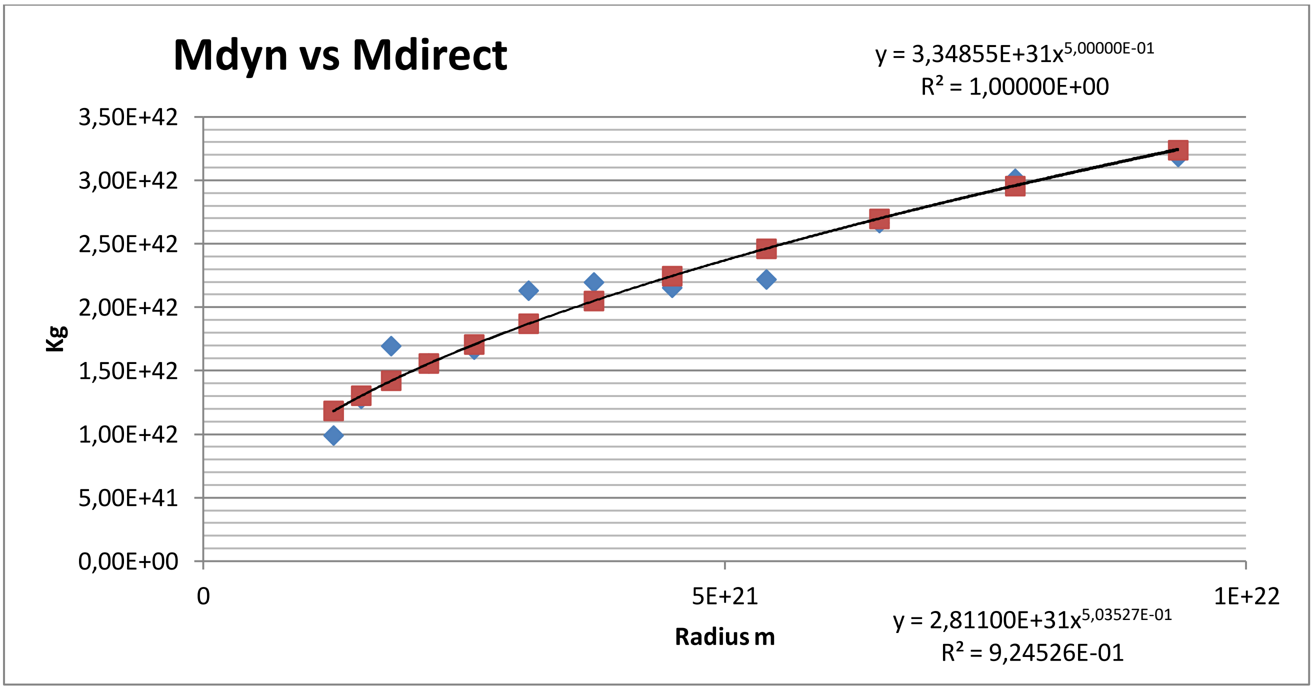
			Dyn Mass	Direct mass	Rel diff
kpc	m	m/s	Msun	Msun	%
40,5	1,250E+21	2,299E+05	4,974E+11	5,95E+11	1,639E+01
49,1	1,515E+21	2,374E+05	6,429E+11	6,55E+11	1,849E+00
58,4	1,802E+21	2,505E+05	8,514E+11	7,14E+11	-1,919E+01
70,1	2,163E+21	2,192E+05	7,825E+11	7,83E+11	1,419E-02
84,2	2,598E+21	2,069E+05	8,373E+11	8,58E+11	2,373E+00
101,1	3,120E+21	2,135E+05	1,071E+12	9,40E+11	-1,392E+01
121,4	3,746E+21	1,978E+05	1,103E+12	1,03E+12	-7,143E+00
145,7	4,496E+21	1,788E+05	1,082E+12	1,13E+12	4,087E+00
175	5,400E+21	1,656E+05	1,115E+12	1,24E+12	9,833E+00
210,1	6,483E+21	1,656E+05	1,339E+12	1,35E+12	1,204E+00
252,3	7,785E+21	1,607E+05	1,514E+12	1,48E+12	-1,951E+00
302,9	9,347E+21	1,508E+05	1,600E+12	1,63E+12	1,629E+00

In the fifth column is tabulated the direct masses in order to be compared with dynamical masses.

Below in the graph are plotted both functions, blue points are dynamical masses and brown point are direct masses.

The first and third points have the maximum difference regarding fitted curve whereas relative differences decreased as radius increased. Namely, relative differences are below 10% for radius bigger than 120 kpc and are below 2 % for radius bigger than 210 kpc. So direct mass is a very good approximation for total mass (baryonic and DM) enclosed at radius R. Ranging radius in the interval of radius measured.

As it was shown in chapter 3, the data set was selected up to 303 kpc because the following data was too different to value associated to Buckingham halo curve, which means that such data is not in dynamical equilibrium.
In conclusion 303 kpc may be considerate as the Virial radius of M31.



10.2 BERNOULLI MASS VERSUS DIRECT MASS

In this epigraph will be shown that relative difference between both kinds of formulas is negligible.

Below are both function formulas.

$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $a^2 / G = 3{,}35.10^{31}$, as was pointed in previous paragraph, will be used to approximate total mass at radius R.

In the corollary of Newton´s theorem, epigraph 9.5, was demonstrated that the Bernoulli mass is the total mass enclosed by a sphere with radius r. Now it will be shown that relative differences between Bernoulli and direct mass are quite small, even for extended haloes.

In the paper [20] Abarca,M.2023, epigraph 9.6 it is developed a method to calculate the optimal parameter C, taking into account that every data have errors not only as a result of measures but celestial bodies are not in perfect dynamical equilibrium, so such value for parameter C is the average associated to different data inside the M31 halo.

M31 PARAMETERS C & D	This selected value C_{M31} belong to the initial point radius equal to 252,3 kpc. See epigraph 9.6
$C_{M31} = -3{,}777*10^{-24}$	
$D_{M31} = a^{\frac{-4}{3}} = 5{,}85*10^{-15}$	

$$M(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

Below are tabulated both function and its relative difference. It is remarkable that even at 3 Mpc its difference is only 4,35 %, despite the fact that its dominion has been extended 10 times.

		Direct mass C=0	Bernoulli mass $C \neq 0$	Rel diff
kpc	m	Msun	Msun	%
40,5	1,250E+21	5,949E+11	6,011E+11	1,04E+00
60	1,851E+21	7,240E+11	7,327E+11	1,19E+00
80	2,469E+21	8,361E+11	8,471E+11	1,31E+00
100	3,086E+21	9,347E+11	9,481E+11	1,41E+00
200	6,171E+21	1,322E+12	1,346E+12	1,77E+00
385	1,188E+22	1,834E+12	1,875E+12	2,20E+00
500	1,543E+22	2,090E+12	2,142E+12	2,40E+00
770	2,376E+22	2,594E+12	2,668E+12	2,77E+00
1000	3,086E+22	2,956E+12	3,048E+12	3,02E+00
1500	4,629E+22	3,620E+12	3,750E+12	3,46E+00
2000	6,171E+22	4,180E+12	4,346E+12	3,80E+00
3000	9,257E+22	5,120E+12	5,353E+12	4,35E+00

Bernoulli mass have been got with the DM by gravitation theory so its dominion is the unlimited haloes.

It is obvious that direct mass is easier to calculate than Bernoulli mass because it has only one parameter. Hereafter it will be used Direct mass instead Bernoulli mass.

11. DARK MATTER BY GRAVITATION THEORY IN THE HALO OF MILKY WAY

In the new rotation curve published by [6] Sofue.2020, the radius of data range from 0,1 kpc up to 95,5 kpc whereas in the previous rotation curve [5] Sofue.2015 the radius range up to 300 kpc. Afterwards will be discussed the importance to reduce the dominion up to 95 kpc. However firstly it is needed to calculate the lowest radius for the halo region, where the baryonic density is negligible versus DM density.

11.1 AN ESTIMATION FOR THE HALO RADIUS

As it is known, Bernoulli solution is only right into the halo region, where the baryonic mass is negligible. To calculate baryonic volume density has been used model provided by Sofue for baryonic disc.

This Table comes from Sofue [6],see table 3, page 12. Parameters for baryonic matter at disc in Milky Way.

Component	Parameter	Fitted Value	χ^2
Expo. disk	a_d	4.38 ± 0.35 kpc	
	Σ_0	$(1.28 \pm 0.09) \times 10^3 M_\odot \mathrm{pc}^{-2}$	

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disc region. To convert superficial baryonic density to volume density it is right to get the formula $D_{BARYONIC}^{VOLUME} = \dfrac{\Sigma(r)}{2r}$ so $D_{BARYONIC}^{VOLUME}(30{,}5\,kpc) = 1.34 \cdot 10^{-24}\, kg/m^3$.

The formula of Direct Dark matter density was got in page 17. $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ being $L_{MILKY\text{-}WAY}$ = 9,1E+29, according with parameter **a** got in epigraph 11.3. For example D_{DM} (30,5 kpc) = 3,35E-23 kg/m^3 . So the ratio of both volume density at 30,5 kpc is 0,04 .In conclusion it is right to consider negligible the baryonic density for radius bigger than 30,5 kpc, therefore it is possible to estate that halo dominion begins at 30,5 kpc for Milky Way. According with DM by gravitation theory, the halo region is unbounded as it is the gravitational field.

11. 2 ROTATION CURVE OF MILKY WAY BY SOFUE 2020 DATA

kpc	km/s	radius m	vel m/s
30,448	229,6	9,40E+20	229600
33,493	222,5	1,03E+21	222500
36,842	215	1,14E+21	215000
40,527	207,1	1,25E+21	207100
44,579	200,3	1,38E+21	200300
49,037	194,7	1,51E+21	194700
53,941	189,8	1,66E+21	189800
59,335	186,2	1,83E+21	186200
65,268	184,7	2,01E+21	184700
71,795	183,9	2,22E+21	183900
78,975	181,4	2,44E+21	181400
86,872	175,5	2,68E+21	175500
95,56	167,7	2,95E+21	167700

This table of rotation curve of Milky Way comes from [6] Sofue.2020, and there have been selected data with radius bigger than 30 kpc.

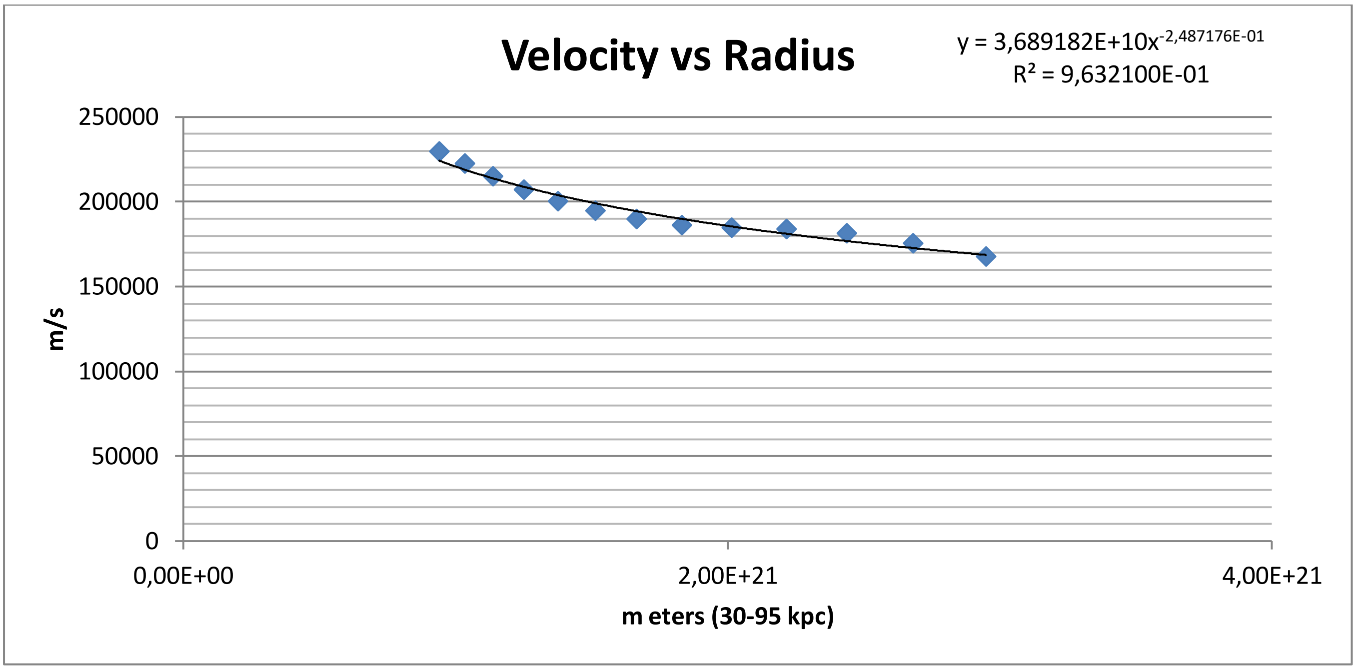
This new set of Sofue data is very important for the theory of DM by gravitation theory because gives a rotation curve at halo region with a power for radius very close to -1/4 which is the same for M31. This fact backs strongly the hypothesis of this theory.

In the previous paper [5] Sofue, Y.2015, the author gave an extended dominion up to 300 kpc, However data with radius bigger than 100 kpc have too high velocity and fitted power function did not fit properly with exponent -1/4.

The logical explanation about the "bad" behaviour of these data is to consider that such celestial bodies are not in dynamical equilibrium. Perhaps they came from the outskirts of MW and were captivated by MW gravitational field afterwards so it is right to consider that these data are far away to dynamical equilibrium, whereas celestial bodies below 100 kpc of radius are properly in dynamical equilibrium with Milky Way.

Anyway, the important data are those closer, because it is right to think that celestial objects with lower radius belong to MW from times of MW formation so these objects may have a better dynamic equilibrium.

11.3 FITTED FUNCTION VELOCITY VERSUS RADIUS AT HALO REGION



According the statistical procedure $v=a*R^b$ Being a = 3.68918E+10 and b = - 0.248717

$$a_{OPTIMAL} = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 3.90787373\,10^{10}$$

Using Buckingham theorem it has been stated b= - 1/4 so it is needed recalculated parameter **a** through the formula as it was made with M31 rotation curve, using the formula for **a** optimal, where Ve is the experimental velocity and r_e is its associated radius.

New parameters a&b - A&B for Milky Way	
B	5/3
$b = \dfrac{B-2}{2B-2}$	b = -1/4
a optimal	$3{,}90787373*10^{10}$
$A = \dfrac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A $4{,}496262*10^{-6}$

A good approximation for parameter $\mathbf{a}_{\mathbf{M\text{-}W}}$ = 3,9E+10

Which is lightly bigger compared with the which one associated to b= - 0.248717

Parameter **a** is similar for similar galaxies, for example $\mathbf{a}_{\mathbf{M31}}$ = 4,7275E+10

Dark matter by gravitation theory stated that B has to be the same for all galaxies. However parameter **a** depend on each galaxy because it depend on baryonic matter enclosed by the galaxy.

Table beside shows the most important parameters for M-W for the halo region.

11.4 MASSES SSOCIATED TO MILKY WAY UP TO 3 MPC

Radius	Radius	Masses
kpc	m	Msun
30,448	9,3953E+20	3,524E+11
33,493	1,033E+21	3,696E+11
36,842	1,137E+21	3,877E+11

40,527	1,251E+21	4,066E+11
44,579	1,376E+21	4,264E+11
49,037	1,513E+21	4,473E+11
53,941	1,664E+21	4,691E+11
59,335	1,831E+21	4,920E+11
65,268	2,014E+21	5,160E+11
71,795	2,215E+21	5,412E+11
78,975	2,437E+21	5,676E+11
86,872	2,681E+21	5,953E+11
95,56	2,949E+21	6,244E+11
770	2,38E+22	1,772E+12
1000	3,09E+22	2,020E+12
2000	6,17E+22	2,856E+12
3000	9,26E+22	3,498E+12

As it has been demonstrated previously, the direct mass is the Bernoulli mass when parameter C is zero.

$$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$$ being $a^2/G = 2{,}2885.10^{31}$

It is remarkable data of mass at 95,5 kpc equal to 6.2E+11Msun, such quantity may be considerate the virial mass because that radius is the biggest value where data are close to Buckingham halo curve, which represents data in dynamical equilibrium.

11.5 COMPARING DIRECT MASS WITH RESULTS FROM GAIA DR2 PUBLISHED IN JCAP 2020

In this section will be compared result got by direct mass formula with result published in the prestigious Journal of Cosmology and Astroparticle Physics by [15] E.V. Karukes et al. 2020 in the paper *A robust estimate of the Milky Way mass from rotation curve data.*

These results come from GAIA DR2 and others remarkable sources.

In table below is made the comparison only with the four radiuses bigger than 30 kpc, as DM by gravitation theory only works in the halo region.

In the last column is shown the relative difference between both kind of mass, being quite small indeed.

Radius kpc	Radius m	Direct Mass Msun	Karukes et al. Msun	Relative difference %
45,79	1,4129E+21	4,305E+11	4,27E+11	8,23E-01
74	2,2834E+21	5,473E+11	5,68E+11	-3,78E+00
119,57	3,6896E+21	6,957E+11	7,26E+11	-4,35E+00
193,24	5,9628E+21	8,845E+11	8,95E+11	-1,19E+00

It is awesome how a simple theory which associates only one parameter to galactic halo is able to give results so close with results got by GAIA DR2 which have been got with the highest current technology and processed through sophisticated software.

These tables comes from **[**15**]** E.V. Karukes et al. In page 25

Total mass in MW at some radius E.V. Karukes et al. 2020 in page 25	
Radius kpc	Total mass x 10^{11} Msun
45.79	$4.27^{+0.22\,(0.43)}_{-0.19\,(0.37)}$
74.0	$5.68^{+0.40\,(0.83)}_{-0.37\,(0.65)}$
119.57	$7.26^{+0.66\,(1.40)}_{-0.58\,(1.03)}$
193.24	$8.95^{+0.98\,(2.07)}_{-0.84\,(1.48)}$

r [kpc]	$M_{\rm tot}$ [10^{11} M$_\odot$]	$M_{\rm DM}$ [10^{11} M$_\odot$]
2.57	$0.23^{+0.006\,(0.011)}_{-0.005\,(0.011)}$	$0.10^{+0.03\,(0.05)}_{-0.02\,(0.05)}$
4.15	$0.45^{+0.008\,(0.014)}_{-0.007\,(0.012)}$	$0.22^{+0.04\,(0.07)}_{-0.03\,(0.08)}$
6.71	$0.80^{+0.01\,(0.02)}_{-0.01\,(0.03)}$	$0.45^{+0.05\,(0.09)}_{-0.05\,(0.11)}$
10.85	$1.33^{+0.02\,(0.04)}_{-0.03\,(0.05)}$	$0.87^{+0.06\,(0.12)}_{-0.06\,(0.12)}$
17.53	$2.07^{+0.04\,(0.09)}_{-0.04\,(0.08)}$	$1.51^{+0.07\,(0.14)}_{-0.07\,(0.13)}$
28.33	$3.04^{+0.10\,(0.19)}_{-0.08\,(0.17)}$	$2.46^{+0.08\,(0.18)}_{-0.12\,(0.21)}$
45.79	$4.27^{+0.22\,(0.43)}_{-0.19\,(0.37)}$	$3.63^{+0.23\,(0.42)}_{-0.18\,(0.36)}$
74.0	$5.68^{+0.40\,(0.83)}_{-0.37\,(0.65)}$	$5.06^{+0.38\,(0.79)}_{-0.38\,(0.67)}$
119.57	$7.26^{+0.66\,(1.40)}_{-0.58\,(1.03)}$	$6.59^{+0.69\,(1.43)}_{-0.55\,(0.97)}$
193.24	$8.95^{+0.98\,(2.07)}_{-0.84\,(1.48)}$	$8.26^{+1.21\,(2.09)}_{-0.92\,(1.43)}$

It is important to notice that results by the direct mass formula at 216 kpc is calculated through Dark Matter by gravitation theory using a formula which was got with a data set whose dominion ranges between 30 kpc and 100 kpc and there is a perfect concordance if it is considered the interval of errors.

11.6 RESULTS GOT BY JEFF SHEN ET AL.ApJ.2022 VERSUS DIRECT MASS AT MW HALO

In this epigraph will be compared result published in The astrophysical journal 2022. See [16] Jeff Shen, with results calculated by the Direct mass formula in Milky Way.

Below is placed rightly the abstract of the paper where it is possible to see two masses results at different

Abstract

The mass of the Milky Way is a critical quantity that, despite decades of research, remains uncertain within a factor of two. Until recently, most studies have used dynamical tracers in the inner regions of the halo, relying on extrapolations to estimate the mass of the Milky Way. In this paper, we extend the hierarchical Bayesian model applied in Eadie & Juri to study the mass distribution of the Milky Way halo; the new model allows for the use of all available 6D phase-space measurements. We use kinematic data of halo stars out to 142 kpc, obtained from the H3 survey and Gaia EDR3, to infer the mass of the Galaxy. Inference is carried out with the No-U-Turn sampler, a fast and scalable extension of Hamiltonian Monte Carlo. We report a median mass enclosed within 100 kpc of $M(<100\,\text{kpc}) = 0.69^{+0.05}_{-0.04} \times 10^{12}\,M_\odot$ (68% Bayesian credible interval), or a virial mass of $M_{200} = M(<216.2^{+7.5}_{-7.5}\,\text{kpc}) = 1.08^{+0.12}_{-0.11} \times 10^{12}\,M_\odot$, in good agreement with other recent estimates. We analyze our results using posterior predictive checks and find limitations in the model's ability to describe the data. In particular, we find sensitivity with respect to substructure in the halo, which limits the precision of our mass estimates to ∼15%.

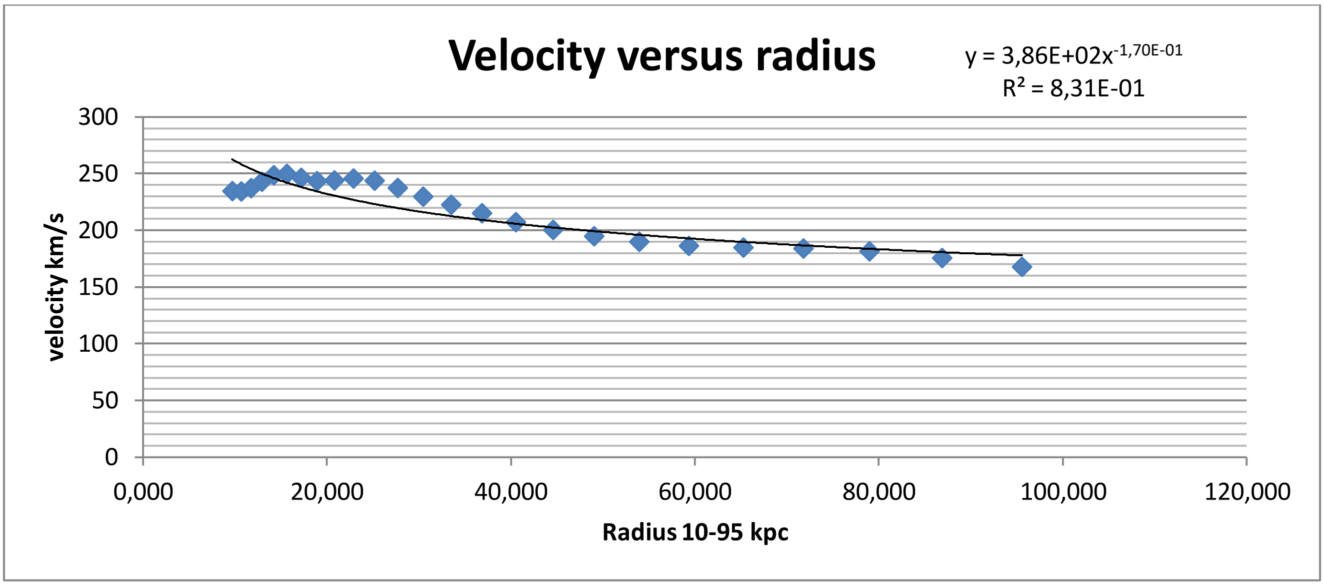
radiuses.Now it will be calculated the total mass at the same radius with direct mass formula $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $\frac{a^2}{G} = 2.2885 \cdot 10^{31}$, (units in I.S.). Msun =1.99E+30 kg. In the last column is shown the relative difference between both results.

Radius kpc	Total mass x 10^{12} Msun Direct formula	Data total mass x 10^{12} Msun Jeff Shen et al.	Relative difference %
100	0.64	0.69 ± 0.04	7
216 ± 7.5			
216+7.5	0.96	1.08 ± 0.11	11

As it is shown the relative difference is small especially at 100 kpc. In adittion both results match if it is considered the range of errors of measures.

It is important to notice that results by the direct mass formula at 216 kpc is calculated through Dark Matter by gravitation theory using a formula which was got with a data set whose dominion ranges between 30 kpc and 100 kpc and there is a perfect concordance if it is considered the interval of errors.

12. DARK MATTER BY GRAVITATION THEORY AT DISK AND HALO IN MILKY WAY



Data Sofue,Y.[6]	
Radius	Velocity
kpc	Km/s
9,702	234,5
10,672	234,2
11,739	237,1
12,913	242,8
14,204	248,5
15,625	249,7
17,187	246,2
18,906	243,3
20,797	243,9
22,876	245,6
25,164	243,7
27,680	237,3
30,448	229,6
33,493	222,5
36,842	215
40,527	207,1
44,579	200,3
49,037	194,7
53,941	189,8
59,335	186,2
65,268	184,7
71,795	183,9
78,975	181,4
86,872	175,5
95,560	167,7

In this chapter will be studied the dark matter considering as dominion from 10 kpc up to 95 kpc. In this radius interval the baryonic matter is not negligible, so it will be used a numerical method a bit rough but with the ability to calculate the DM density inside the disk region.

In the graph above are represented data set of this table.

12.1 DISK PARAMETERS IN MILKY WAY

As it is known the profile for the SMD (Surface density mass) at the disk follows an exponential function and in the clipped text taken from Sofue,Y.2020 [6], are shown these parameters:

3.2. Disk

The galactic disk is approximated by an exponential disk, whose surface mass density is expressed as

$$\Sigma_d(R) = \Sigma_0 \exp(-R/a_d), \quad (9)$$

where Σ_0 is the central value and a_d is the scale radius. The total mass of the exponential disk is given by

$$M_d = \int_0^\infty 2\pi r \Sigma_d dr = 2\pi \Sigma_0 a_d^2. \quad (10)$$

Component	Parameter	Fitted Value
Expo. disk	a_d	4.38 ± 0.35 kpc
	Σ_0	$(1.28 \pm 0.09) \times 10^3 M_\odot \mathrm{pc}^{-2}$

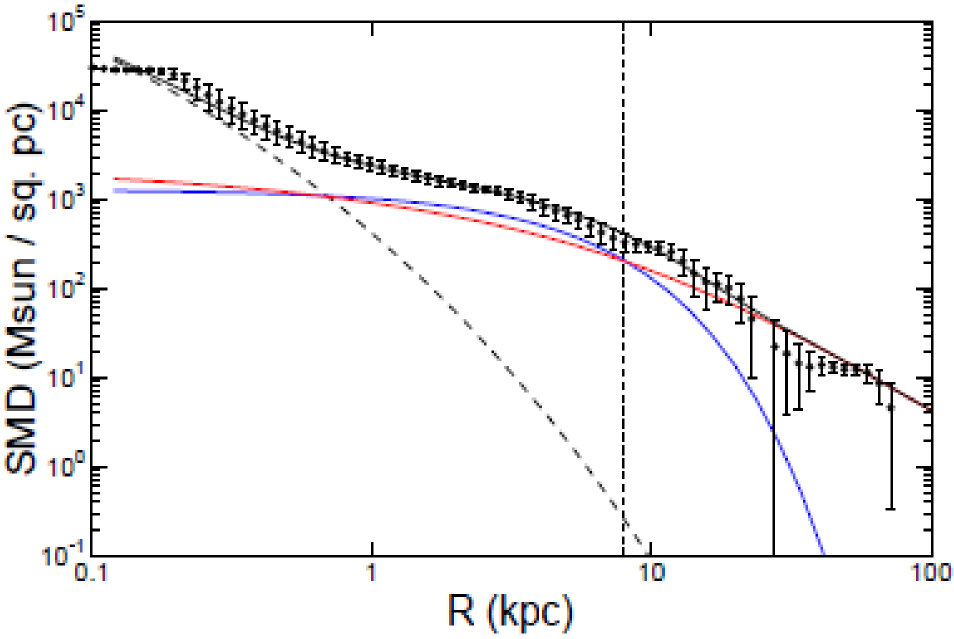
It is right to get the value for Mdisk = $1.54 \cdot 10^{11}$ Msun

Notice that error for $\mathbf{a_d}$ is about 8% and error for $\mathbf{\Sigma_0}$ is about 7% , which have a non negligible effect in the accuracy of calculus in the whole chapter.
Namely $\Delta M = 3.5 * 10^{10} M_{SUN}$

Thanks this function for baryonic surface density mass, it is possible to calculate the baryonic mass contained inside a corona defined by two radiuses. Namely from 10 kpc up to certain radius r. It is not hard to find a formula for this mass:

$\Delta M_{10}^{r} = \boldsymbol{J} - \boldsymbol{K}(4.38 * r + 19.1844) * \exp\left(-\frac{r}{4.38}\right)$ Units Msun. Corona baryonic mass in disk from 10 kpc up to r. Where **r** is the upper radius in kpc. **J**= 5.165149801E+10 Msun and **K**= 8.0424772193E+9 Msun/ kpc^2.

Notice that the unit of the parenthesis (4.38*r+ 19.1844) is kpc^2, because at the disk there is a surface density of baryonic mass, SMD.



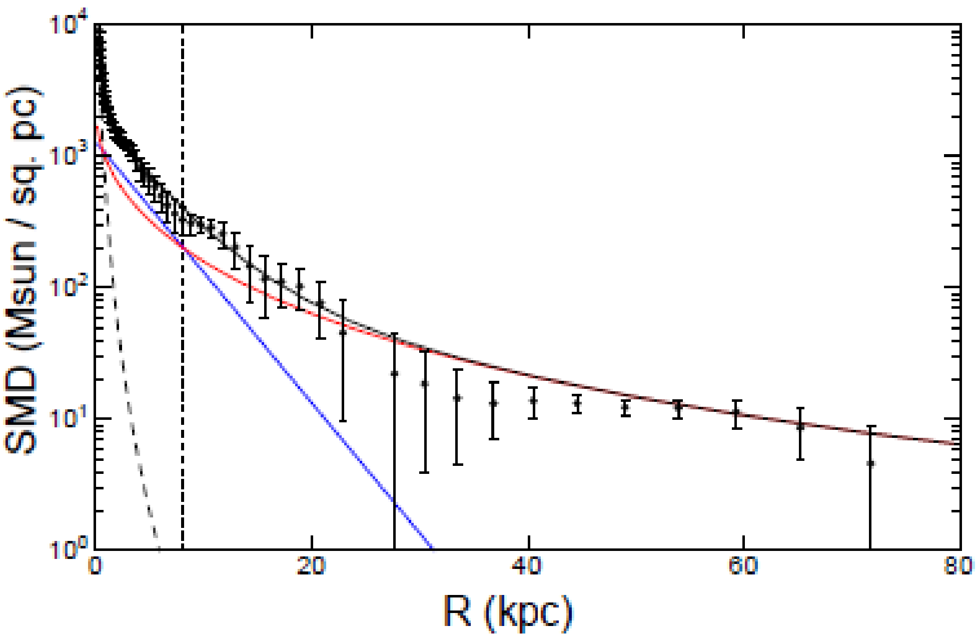
The reason to begin at 10 kpc is why at 10 kpc the baryonic mass associated to bulge is totally negligible, so for bigger radius to 10 kpc the baryonic matter to consider will be calculated exclusively by the previous formula.

In the left graph Sofue,Y. 2020 [6] The black solid line is the $\aleph^2$ fit curve associated to data.

The dashed line represents the SMD associated to bulge, which is negligible at 10 kpc, and this is the reason why the baryonic mass at disk is calculated from 10 kpc.

The blue line is the Surface Mass Density at disk, SMD disk, which become negligible at 30 kpc.
The above graph is a semi-logarithmic plot and the graph below is logarithmic but both represents the same magnitudes.

In the graph below it is shown that at 30 kpc density of DM (red line) is more than 30 times bigger than baryonic mass (blue line), however in this epigraph the baryonic mass will be calculated up to 95 kpc.



The red line is associated to DM by a NFW profile,but is not used in this paper.

It is crucial for this work the fact that at 10 kpc the baryonic mass associted to bulge is negligible because for bigger radiuses the baryonic mass is calculated only by the disk mass formula.

12.2 A NUMERICAL METHOD TO GET THE DENSITY D.M. AT THE DOMINION DATA SET

Taking in consideration the Sofue data set [6], it is a good approximation to consider that velocity at 10 kpc is 234.4 km/s. So considering the formula for dynamical mass = v^2 *R/G, it is got that M_{DYN}(<10 Kpc) = 1.2767E+11 Msun. This mass includes baryonic mass and dark matter under 10 kpc so it is considerate the total mass under 10 kpc . So M_{TOTAL}(<10kpc) = 1.28E+11 Msun. This value will be added to ΔM_{10}^{r} (Corona baryonic mass. See column [3]) to get the the column [4] in the table below. In other words, in column [4] is considered the total mass up to 10 kpc plus the baryonic mass enclosed inside the corona defined by 10 kpc and the radius **r.** Obviously in the column [4] , it is not included the DM enclosed in that corona region.

Thanks velocity values is possible to calculate M_{TOTAL}(< R). See column [5] , and subtracting [5]-[4] it is possible to get the DM contained from 10 kpc up to indicated radius. See column [6]. Using different consecutives radiuses is possible to get the DM contained inside the spherical corona defined by such radiuses. See columns [7]. In the column [8] is calculated the volume of the spherical corona (V.S.C.) defined by two consecutives radiuses.

Velocity [1]	Radius [2]	Corona Baryonic Mass from 10 kpc up to R [3]	M_{TOTAL}(<10kpc)+[3] [4]	M_{TOTAL}(<R) or dynamical mass [5]	Dark matter from 10 kpc up to R [6]	Corona DM between consecutives radiuses [7]	V.S.C between consecutives radiuses[8]
km/s	kpc	Msun	Msun	Msun	Msun	Msun	Kpc^3
2,344E+02	1,00E+01	0,00E+00	1,28E+11	1,2767E+11	0	0	0
2,342E+02	1,07E+01	5,28E+09	1,33E+11	1,360E+11	3,051E+09	3,05E+09	9,02E+02
2,371E+02	1,17E+01	1,27E+10	1,40E+11	1,533E+11	1,293E+10	9,87E+09	1,68E+03
2,428E+02	1,29E+01	1,97E+10	1,47E+11	1,769E+11	2,948E+10	1,66E+10	2,24E+03
2,485E+02	1,42E+01	2,61E+10	1,54E+11	2,038E+11	5,003E+10	2,05E+10	2,98E+03
2,497E+02	1,56E+01	3,18E+10	1,59E+11	2,263E+11	6,692E+10	1,69E+10	3,98E+03
2,462E+02	1,72E+01	3,66E+10	1,64E+11	2,420E+11	7,773E+10	1,08E+10	5,29E+03
2,433E+02	1,89E+01	4,07E+10	1,68E+11	2,600E+11	9,164E+10	1,39E+10	7,04E+03
2,439E+02	2,08E+01	4,40E+10	1,72E+11	2,874E+11	1,158E+11	2,42E+10	9,37E+03
2,456E+02	2,29E+01	4,65E+10	1,74E+11	3,206E+11	1,464E+11	3,06E+10	1,25E+04
2,437E+02	2,52E+01	4,83E+10	1,76E+11	3,472E+11	1,712E+11	2,48E+10	1,66E+04
2,373E+02	2,77E+01	4,96E+10	1,77E+11	3,621E+11	1,848E+11	1,36E+10	2,21E+04
2,296E+02	3,04E+01	5,05E+10	1,78E+11	3,729E+11	1,948E+11	9,92E+09	2,94E+04
2,225E+02	3,35E+01	5,10E+10	1,79E+11	3,852E+11	2,066E+11	1,18E+10	3,91E+04
2,150E+02	3,68E+01	5,13E+10	1,79E+11	3,957E+11	2,167E+11	1,01E+10	5,21E+04

2,071E+02	4,05E+01	5,15E+10	1,79E+11	4,038E+11	2,247E+11	8,01E+09	6,94E+04
2,003E+02	4,46E+01	5,16E+10	1,79E+11	4,155E+11	2,363E+11	1,16E+10	9,23E+04
1,947E+02	4,90E+01	5,16E+10	1,79E+11	4,319E+11	2,526E+11	1,63E+10	1,23E+05
1,898E+02	5,39E+01	5,16E+10	1,79E+11	4,515E+11	2,722E+11	1,96E+10	1,63E+05
1,862E+02	5,93E+01	5,16E+10	1,79E+11	4,779E+11	2,986E+11	2,65E+10	2,18E+05
1,847E+02	6,53E+01	5,17E+10	1,79E+11	5,173E+11	3,380E+11	3,94E+10	2,90E+05
1,839E+02	7,18E+01	5,17E+10	1,79E+11	5,641E+11	3,848E+11	4,68E+10	3,86E+05
1,814E+02	7,90E+01	5,17E+10	1,79E+11	6,038E+11	4,245E+11	3,97E+10	5,13E+05
1,755E+02	8,69E+01	5,17E+10	1,79E+11	6,216E+11	4,423E+11	1,79E+10	6,83E+05
1,677E+02	9,56E+01	5,17E+10	1,79E+11	6,244E+11	4,451E+11	2,74E+09	9,09E+05

Below is detailed the meaning of different columns.

[1] Rotation velocity km/s

[2] Radius kpc

[3]Barionic mass from 10 kpc up to the indicated radius. It is used the formula for disk mass:

$\Delta M_{10}^{r} = J - K(4.38 * r + 19.1844) * \exp\left(-\frac{r}{4.38}\right)$, the unit mass is Msun.

Where J=5,16514980E+10 Msun and K= 8,04247719E+09 Msun/kpc^2

[4] M_{TOTAL}(< 10 kpc) added to column [3] where M_{TOTAL}(<10kpc) = 1.2767E+11 Msun.

[5]Dynamical or total mass up to indicated radius. M_{TOTAL}(<r)

[6]Dark matter contained from 10 kpc up to indicated radius. i.e. [5] – [4]

[7]Dark matter contained between the current radius and its previous radius. Hereafter named Corona DM.

[8]Volume spherical corona. i.e. The difference of volume between two spheres with consecutives radiuses.

[9]Mean Dark matter density in the volume spherical corona. i.e. [9] =[7] /[8] Unit used Msun/kpc^3

[10] The same that [9] using units kg/m^3

Radius [2]	Corona DM[7]	V.S.C. [8]	D.M. density[9]	D.M. Density[10]
kpc	Msun	Kpc^3	Msun/kpc^3	kg/m^3

1,00E+01	0	0	X	X
1,07E+01	3,05E+09	9,02E+02	3,38E+06	2,2889E-22
1,17E+01	9,87E+09	1,68E+03	5,86E+06	3,9679E-22
1,29E+01	1,66E+10	2,24E+03	7,38E+06	4,9970E-22
1,42E+01	2,05E+10	2,98E+03	6,88E+06	4,6603E-22
1,56E+01	1,69E+10	3,98E+03	4,25E+06	2,8762E-22
1,72E+01	1,08E+10	5,29E+03	2,05E+06	1,3850E-22
1,89E+01	1,39E+10	7,04E+03	1,98E+06	1,3373E-22
2,08E+01	2,42E+10	9,37E+03	2,58E+06	1,7451E-22
2,29E+01	3,06E+10	1,25E+04	2,46E+06	1,6642E-22
2,52E+01	2,48E+10	1,66E+04	1,49E+06	1,0106E-22
2,77E+01	1,36E+10	2,21E+04	6,17E+05	4,1756E-23
3,04E+01	9,92E+09	2,94E+04	3,37E+05	2,2844E-23
3,35E+01	1,18E+10	3,91E+04	3,01E+05	2,0374E-23
3,68E+01	1,01E+10	5,21E+04	1,94E+05	1,3153E-23
4,05E+01	8,01E+09	6,94E+04	1,15E+05	7,8160E-24
4,46E+01	1,16E+10	9,23E+04	1,26E+05	8,5101E-24
4,90E+01	1,63E+10	1,23E+05	1,33E+05	8,9910E-24
5,39E+01	1,96E+10	1,63E+05	1,20E+05	8,1004E-24
5,93E+01	2,65E+10	2,18E+05	1,22E+05	8,2381E-24
6,53E+01	3,94E+10	2,90E+05	1,36E+05	9,1992E-24
7,18E+01	4,68E+10	3,86E+05	1,21E+05	8,2209E-24
7,90E+01	3,97E+10	5,13E+05	7,73E+04	5,2323E-24
8,69E+01	1,79E+10	6,83E+05	2,62E+04	1,7719E-24
9,56E+01	2,74E+09	9,09E+05	3,01E+03	2,0386E-25

12.3 A NUMERICAL METHOD TO GET THE FIELD E AT THE DOMINION DATA SET

Below is detailed the meaning of different columns.

[11]Mean of two consecutives radiuses. [12]Mean velocity estimated for the mean radiuses [11].

[13]Field E calculated using the mean values for radius and velocity. i.e. [11] and [12] by formula $E = v^2/ r$

[14] The power 5/3 for field E. i.e. $E^{5/3}$.

The reason to consider the mean value for consecutives radiuses or velocities is to get a more properly value for these magnitudes inside each interval. Reader can check that up to 30 kpc the radius interval is 1, 2 or 3 kpc whereas at bigger distances the amplitude grows, for example the last interval is bigger than 8 kpc. It is obvious that this method to study the DM density and field is very rough regarding the mathematical method used in previous chapters, but it is a method quite close to data set, in other words, this method allow us to study DM density using the raw data.

Radius [2]	Vel rot [1]	Mean radius[11]	Mean velocity[12]	Field E [13]	Power field E^(5/3)[14]	D.M. density[10]
kpc	km/s	kpc	Km/s	m/s^2	m^(5/3)/s^(10/3)	kg/m^3
1,00E+01	2,344E+02	X	X	X	x	x
1,07E+01	2,342E+02	1,03E+01	2,34E+02	1,72E-10	5,32602E-17	2,2889E-22
1,17E+01	2,371E+02	1,12E+01	2,36E+02	1,61E-10	4,74521E-17	3,9679E-22
1,29E+01	2,428E+02	1,23E+01	2,40E+02	1,51E-10	4,2998E-17	4,9970E-22
1,42E+01	2,485E+02	1,36E+01	2,46E+02	1,44E-10	3,96691E-17	4,6603E-22
1,56E+01	2,497E+02	1,49E+01	2,49E+02	1,35E-10	3,54525E-17	2,8762E-22
1,72E+01	2,462E+02	1,64E+01	2,48E+02	1,21E-10	2,97823E-17	1,3850E-22
1,89E+01	2,433E+02	1,80E+01	2,45E+02	1,08E-10	2,43316E-17	1,3373E-22
2,08E+01	2,439E+02	1,99E+01	2,44E+02	9,69E-11	2,04339E-17	1,7451E-22
2,29E+01	2,456E+02	2,18E+01	2,45E+02	8,89E-11	1,77087E-17	1,6642E-22
2,52E+01	2,437E+02	2,40E+01	2,45E+02	8,08E-11	1,50873E-17	1,0106E-22
2,77E+01	2,373E+02	2,64E+01	2,41E+02	7,09E-11	1,21578E-17	4,1756E-23
3,04E+01	2,296E+02	2,91E+01	2,33E+02	6,08E-11	9,39297E-18	2,2844E-23
3,35E+01	2,225E+02	3,20E+01	2,26E+02	5,18E-11	7,1975E-18	2,0374E-23
3,68E+01	2,150E+02	3,52E+01	2,19E+02	4,41E-11	5,50394E-18	1,3153E-23

4,05E+01	2,071E+02	3,87E+01	2,11E+02	3,73E-11	4,16683E-18	7,8160E-24
4,46E+01	2,003E+02	4,26E+01	2,04E+02	3,16E-11	3,15866E-18	8,5101E-24
4,90E+01	1,947E+02	4,68E+01	1,98E+02	2,70E-11	2,43093E-18	8,9910E-24
5,39E+01	1,898E+02	5,15E+01	1,92E+02	2,33E-11	1,89574E-18	8,1004E-24
5,93E+01	1,862E+02	5,66E+01	1,88E+02	2,02E-11	1,50117E-18	8,2381E-24
6,53E+01	1,847E+02	6,23E+01	1,85E+02	1,79E-11	1,2237E-18	9,1992E-24
7,18E+01	1,839E+02	6,85E+01	1,84E+02	1,61E-11	1,02255E-18	8,2209E-24
7,90E+01	1,814E+02	7,54E+01	1,83E+02	1,43E-11	8,4659E-19	5,2323E-24
8,69E+01	1,755E+02	8,29E+01	1,78E+02	1,24E-11	6,68357E-19	1,7719E-24
9,56E+01	1,677E+02	9,12E+01	1,72E+02	1,05E-11	5,00443E-19	2,0386E-25

12.4 DISCUSSION THE LAW D.M. DENSITY DEPENDING ON FIELD AT THE DISK AND HALO REGIONS

Below are represented the two yellow columns at different range of dominions, DM density versus $E^{5/3}$. This law was got in the halo region of M31 and Milky Way, now will be studied in the disk region, 10kpc up to 30 kpc, and jointly disk and halo i.e. 10 kpc up to 95 kpc.

In the paragraph 7.2 was shown that the law $D_{DM} = A \cdot E^{B}$ leads to a Bernoulli differential equation for field whose solution is $E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ being $\alpha = 2B - 2$, $\beta = \frac{1}{1-B}$ and $\alpha . \beta = -2$, for any value of B.

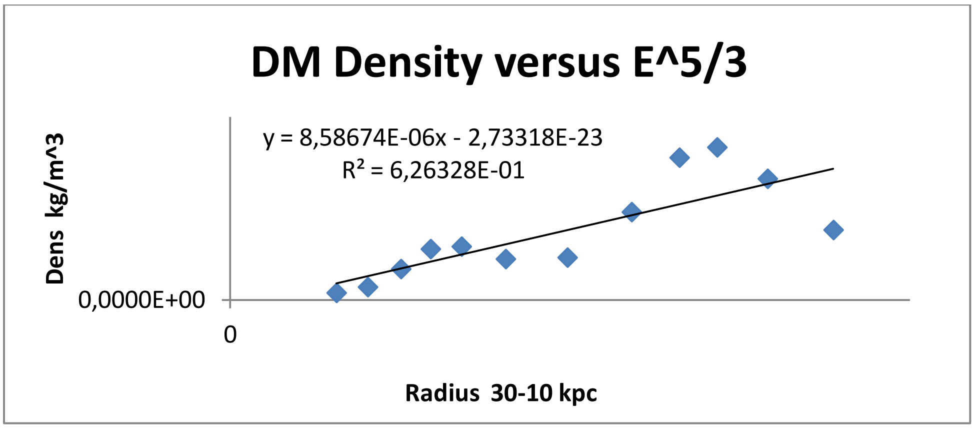
In the chapter 9 was mathematically demonstrated that the exponent b= -1/4 for the power of radius at velocity formula is equivalent to a B =5/3 for the exponent of field at the formula of dark matter density. However, at the disk is not possible to state such relation between the coefficients b and B because there is baryonic matter and $b \neq -1/4$. Anyway, firstly it will be studied the same functional dependence i.e $D_{DM} = A*E^{5/3}$, in order to compare with that function in the halo region and secondly it will be made a potential regression over field E and DM density i.e. $D_{DM} = A*E^{B}$

12.4.1 DISCUSSION OF DM DENSITY VERSUS E^(5/3) AT DISK AND HALO

In the graph below, the correlation is not so high, but r = 0.8 is acceptable. Although the independent coefficient, N hereafter, N= -2.7 E-23 is quite low, it is not negligible at the most part of dominion. Also the coefficient for field A=8.5 E-6 is almost twice that A= 4.5E-6 in the halo. These results might be explained not only by the rough mathematical method used in this epigraph but by the fact that in the disk region the velocity has no functional correlation with radius, see the graph below velocity versus radius (10 kpc-30 kpc.). Also errors related to disk parameters $\mathbf{a_d}$ and $\mathbf{\Sigma_0}$ have a non negligible importance about the calculus of baryonic mass and DM density at the disk region.

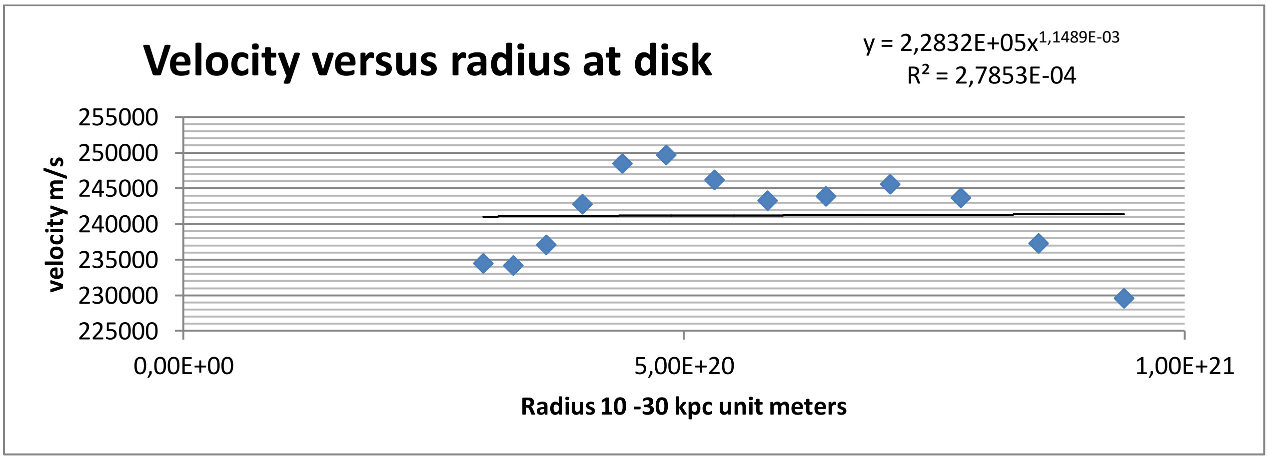
Below there are three graphs plotting DM density versus field $E^{5/3}$, which represent the above yellow columns (I.S.) although the X axe shows the legend radius 10-30 kpc. This legend is only to discriminate the different dominions in a simple way, although at the axe X is represented the value of $E^{5/3}$.

Notice that at the axe X are shown radius in decreasing order, because in the axe X is represented the field $E^{5/3}$ which is lower for bigger radius and greater for lower radius.



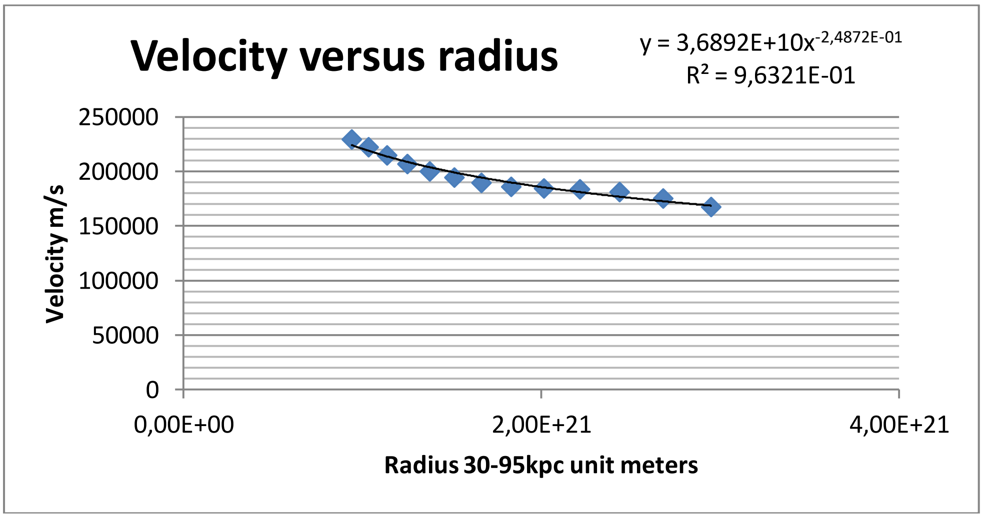
The above graph represents data set of yellow columns page 32, from 30 kpc up to 10 kpc, remember that the bigger the radius is, the lower the field will be.

However it is important to highlight that using a data set of radius and velocities, with correlation zero (see graph bellow) it is possible to get a formula for DM density depending on field with a correlation 0.8 with a coefficient "only" twice that the one got at halo region.



Notice that the correlation coefficient for a potential function is virtually zero.

Watching the above graph it is clear that correlation of potential regression is zero, whereas the same function at the halo region is bigger than 0.98.See graph below. So it is remarkable the fact that using a data set with no correlation law velocity versus radius, it is got a formula similar to the one at the halo region, DM density versus field $E^{5/3}$ with a correlation coefficient 0.8.

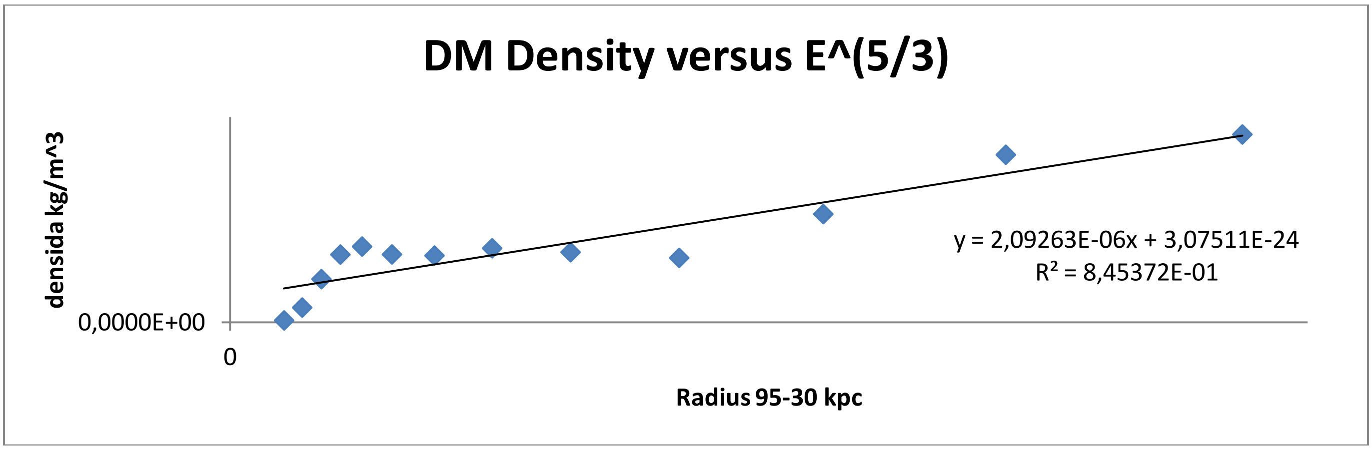


CHECKING THE FORMULA GOT BY THE SOLAR DM DENSITY

According Sofue,Y.2020 [6] Solar DM density $D_{SUN}^{DM} = 0.39\frac{Gev}{cm^3} = 6.42 * 10^{-22} kg/m^3$, Rsun =8.0 kpc and V_{ROT} = 238 Km/s.

Using these values it is got E_{SUN} =V^2/R = 2.295E-10 m/s^2 and using parameter A = 8.587E-6 (see graph page 33), it is possible to get rightly $D_{SUN}^{DM} = 7.4 * 10^{-22} kg/m^3$by the formula D_{DM} =A*$E^{5/3}$. The value got is only 15% bigger regarding the Sofue value, which is a magnificent calculus despite the fact that the dominion of the formula is for radius bigger than 10 kpc.

With the graph below, it will be compared the current mathematical method used in this epigraph with the rigorous method used in previous chapters, because the radius dominion is the same that used at chapter 11.

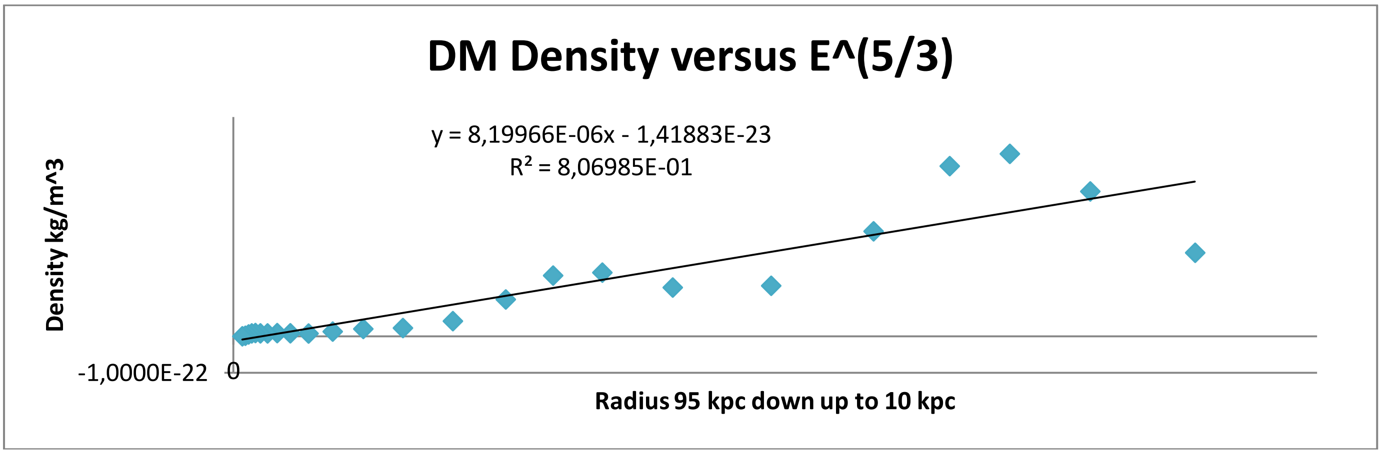


The above graph represents data set of yellow columns page 32, from 95 kpc up to 30 kpc.

The correlation is acceptable r= 0.9 and although the independent coefficient is almost zero (3E-24), it can be cheeked that is not negligible. In addition, the coefficient for field (A=2E-6) is below a half of A = 4,496*10^{-6} got in chapter 11. These results are expectable, because of the rough mathematical method used in this section.

Anyway, thanks this calculus it has been demonstrated that the hypothesis of DM by gravitation theory works perfectly in the disk region because the correlation coefficient is high.

Finally in the graph bellow is plotted DM density versus $E^{5/3}$ in the whole dominion 10 kpc up to 95 kpc. The coefficient for field is very similar to the one with dominion 10 -30 kpc, although the correlation is higher. Also it can be checked that the constant term N is not negligible, which prove that exponent 5/3 is not suitable.



The above graph represents data set of yellow columns page 32, from 95 kpc up to 10 kpc. remember that the bigger radius is , the lower the field will be.

In table below are summarized the coefficients got in this section.

Regression function	Density DM =A*E^5/3+N	N Constant term	Correlation coeff.
Dominion	A		r
10-30 kpc Disk region	8.6E-6	Non negligible -2.7E23	0.8
30-95 kpc Halo region	2.1E-6	Non negligible +3E-24	0.9
10-95 kpc Disk + Halo	8.2E-6	Non negligible -1.4E-23	0.9
Analytical method Halo region Epigraph 11.3 Dominion 30-95 kpc	4.496E-6	0	0.9814

In my opinion in this section it has been cleared four main characteristics.

1º Due to the rude method used, in the halo region the parameter A is only a half regarding the $A_{ANALYTICAL}$, in addition the constant term is not negligible.

2º At the disk region there is a clear functional dependence DM density versus field $E^{5/3}$ i.e. $D_{DM} = A*E^{5/3}+N$ as the correlation coefficient is 0.8, however the fact that the constant term N is not negligible proves that such functional dependence is not ruled properly by a power function with exponent 5/3, as it will be shown in the following epigraph.

3º The linear coefficient A at the disk region is four times bigger than the one at halo region got with the same method and the constant term is not negligible, this fact it is expectable because the baryonic density is not zero in this region.

4º The value of Solar DM density is only 15 % bigger than the one currently accepted. This fact suggest that the formula got at disk region with a so rude method might give reasonable values.

Summarising, as the constant term, N, is not negligible it is clear that DM density is not fitted properly by E^5/3

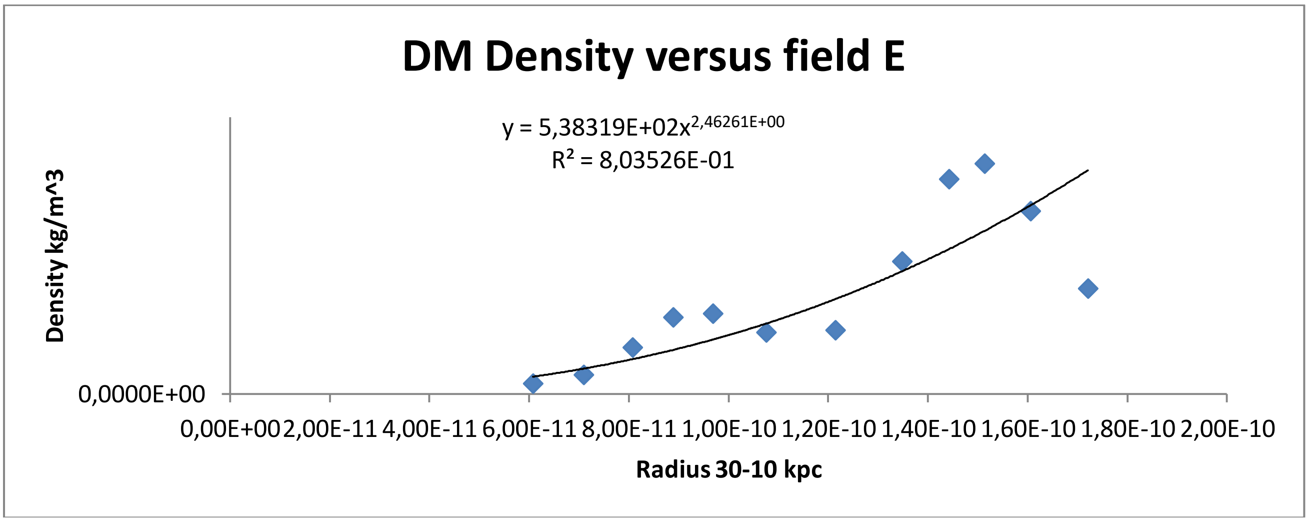
12.4.2 DISCUSSION OF DM DENSITY VERSUS E AT DISK AND HALO

In this section will be discussed DM density versus field E with a potential regression method, i.e. the green columns [13] field E and the yellow column [10] DM density (see page 30). i.e. will be calculated not only A but B. $D_{DM} = A*E^B$

Below there are three graphs plotting DM density versus field E. However the X axe shows the legend radius 30-10 kpc. This legend is only to discriminate the different dominions in a simple way, although at the axe X is represented the value of E.

In the graph below, with radius dominion 30- 10 kpc, the disk region, the correlation is quite high r = 0.9 . However power of E is B= 2.46 and A= 5.4 both are very different regarding the ones got in chapter 11.

Notice that at the axe X are shown radius in decreasing order, because in the axe X is represented the field E which is lower for bigger radius and greater for lower radius.



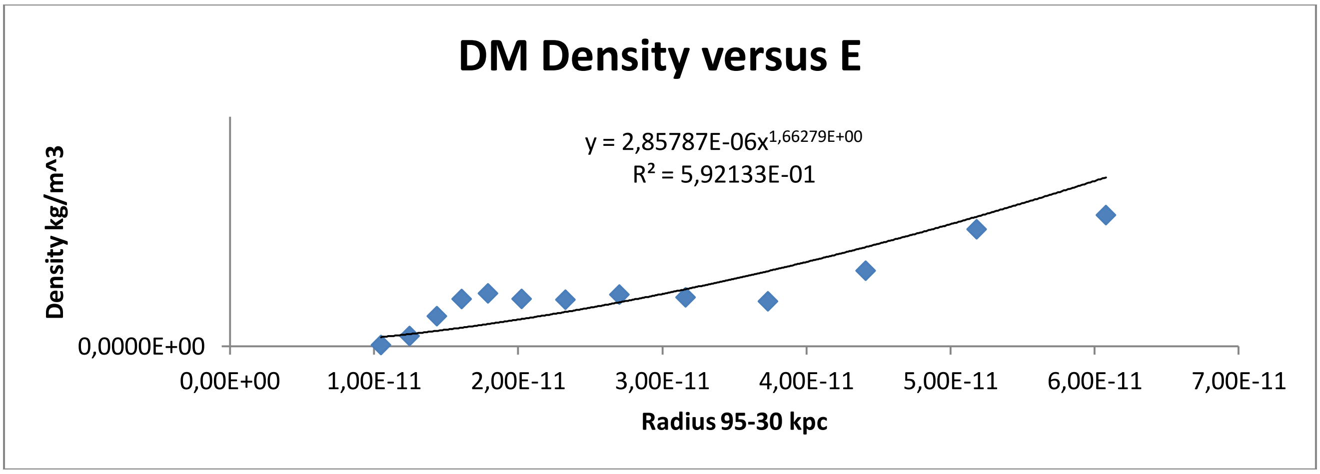
The above graph represents the green column[13] and yellow column [10] at page 32 from 30 kpc to 10 kpc.

CHECKING THE FORMULA GOT BY THE SOLAR DM DENSITY

According Sofue,Y.2020 [6] Solar DM density$D_{SUN}^{DM} = 0.39\frac{Gev}{cm^3} = 6.42 * 10^{-22}kg/m^3$, Rsun =8.0 kpc and V_{ROT} = 238 Km/s.

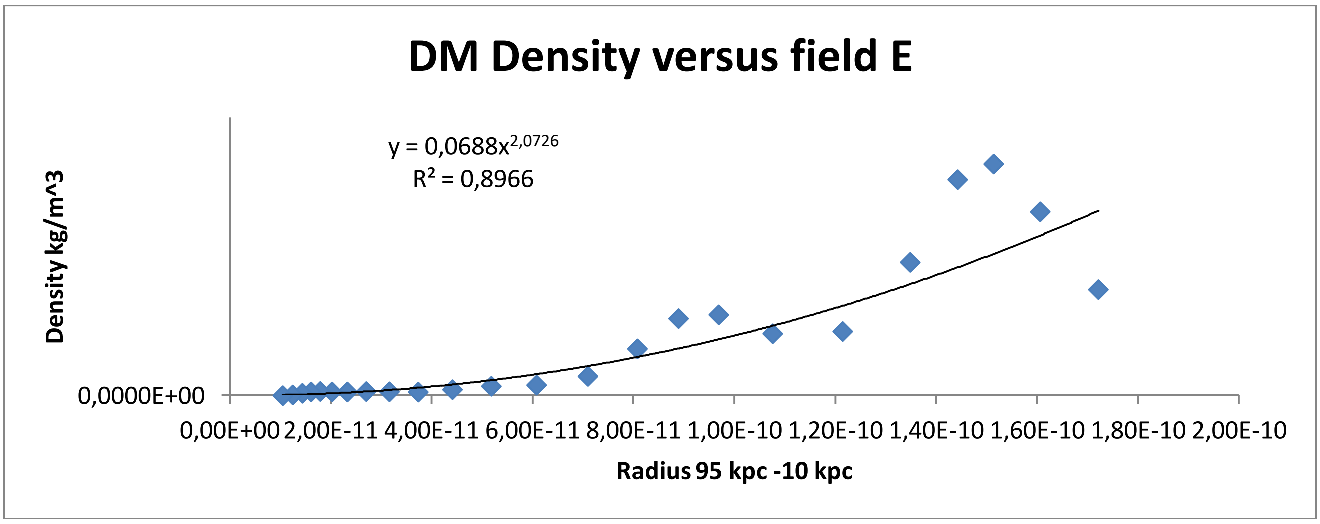
Using these values it is got $E_{SUN} = V^2/R$ = 2.295E-10 m/s^2 and using parameter A =5.38E+2 and B = 2,4626 (see the graph above), it is possible to get rightly $D_{SUN}^{DM} = 9.8 * 10^{-22}kg/m^3$by the formula $D_{DM} = A*E^B$. The value got is 52% bigger regarding the Sofue value, which is not a so good calculus as the one got in previous epigraph but remains being a reasonable calculus as the dominion for this formula begins at 10 kpc and the Solar radius is 8.0 kpc.

In the graph below, with radius dominion 30-95 kpc, the halo region, the correlation is not so high r = 0.77. However A= 2.86E-6 is very close to the one got at Epigraph 11.3,(A=4.5E-6), and B= 1.663 is almost 5/3. So despite the rough mathematical method used in this chapter the formula of DM density depending on field is very close to the one got at chapter 11.



The above graph represents the green column [13] and yellow column [10] at page 32 from 95 kpc to 30 kpc.

Finally, in the graph below, with radius dominion 10-95 kpc, the disk and halo region, the correlation is very high r = 0.95. However A= 0.0688 is very different to the one got at Epigraph 11.3, and B= 2.08 is quite different to 5/3.



The above graph represents the green column [13] and yellow column [10] at page 32 from 95 kpc to 10 kpc.

In table below are summarized the different coefficients got in this epigraph.

Regression function	Density DM =A*E^B		Correlation coeff.
Dominion	A	B	r
10-30 kpc Disk region	5.38	2.463	0.9
30-95 kpc Halo region	2.858E-6	1.663	0.77
10-95 kpc Disk + Halo	6..88E-2	2.073	0.95
Analytical method Halo region Epigraph 11.3 Dominion 30-95 kpc	4.496E-6	5/3	0.9814

In my opinion in this section it has been cleared four main characteristics.

1º Despite the rude method used, in the halo region the exponent B differs only 7 thousand from 5/3 and parameter A is a bit lower than $A_{ANALYTICAL}$ therefore this fact suggest that this "rude" method give reasonable results.

2º At the disk region there is a clear functional dependence DM density versus field E^B i.e. $D_{DM} = A*E^B$ because the correlation coefficient r = 0.9 despite that correlation coefficient velocity versus radius is almost zero, see graph pg33.

3º At the disk region, it is clear that A &B are quite different regarding the halo ones. It was expectable because the baryonic density is not zero.

4º The value of Solar DM density is "only" 52 % bigger than the one currently accepted, which is an acceptable value because the dominion of formula begins at 10 kpc, whereas the Solar radius is 8.0 kpc.

Summarising, despite to be used a so rude mathematical method it has been cleared that it exist a clear functional correlation between DM density and field according a power function. The exponent will be 5/3 in halo region where baryonic density is zero and not will be 5/3 in disk region because the baryonic density is not zero.

13. CONCLUSION

The newness of this work regarding the previous paper, [13] Abarca,M. 2023, is the chapter 12, where it is studied the DM by gravitation theory at the disk region. The numerical method used is a bit rough but using the rotation curve at disk and its baryonic mass profile it has been possible to calculate DM density at disk and his functional dependence on the field. The main conclusions in this chapter is that there is a clear functional dependence between field and DM density at the disk region, although the parameters A, B of the formula are different regarding the formula at the halo region because the baryonic density is not zero in this region, whereas is zero at the halo.

This paper develop the DM by gravitation theory using the rotation curve of M31 published by [5] Sofue, Y.2015 and a new of data for rotation curve in Milky Way published by [6] Sofue, Y.2020 . With these new data, the rotation curve at the halo region match perfectly with the one of M31. So it is possible to state that the rotation curve of MW and M31 at halo are governed by the ideal curve named Buckingham halo curve of velocity versus radius, which has the exponent -0.25.

This fact back strongly the main hypothesis of Dark gravitation theory i.e. Dark matter is generated according an unknown quantum gravitational mechanism, which depend on the gravitational field, so it is a Universal law.

Through the first ten chapters is developed the theory using M31 rotation curve. In the chapter 11 it is introduced the halo data set for Milky Way which comes from [6] Sofue, Y.2020.This chapters are identical to the previous paper [13] Abarca,M.2023

In the chapter 12 is studied the DM by gravitation theory inside the disk, and halo and three main conclusions have been got:

1º At the disk region, there is a clear functional dependence between DM density versus field E according a potential function $D_{DM} = A*E^{B}$, although the exponent of the power is not 5/3 because the baryonic mass density is not zero in this region.

2º At the halo region, the parameters A & B got with a quite rude method are similar to the ones got in chapter 11 with the analytical method. This similarity shows that this rude method may be accepted in order to study the function DM density versus a power of field as a first approximation to this law.

3º Even with a method so rough it is possible to estimate the Solar DM density. The result got with that method differs only 15% regarding the current value accepted by the scientific community.

To study the DM theory more in depth at the disk region it is needed to solve the Poisson equation using together the baryonic density and the DM density, but unfortunately it is required sophisticated software and skilful astrophysics teams.

Fortunately at the halo region the Poisson equation is easy to solve, as it was shown in [13] Abarca,M. 2023, chp.15

This theory introduces a powerful method to study DM in the halo and disk region of galaxies and conversely measures in galaxies and clusters offer the possibility to check the theory.

A natural way to develop more in depth the ***DM by gravitation theory*** would be to consider General Relativity. Namely, it is right to get the density of energy associated to DM, multiplying density of dark matter by c^2 i.e.

$D_{DM}^{ENERGY} = \dfrac{a^2 \cdot c^2}{8 \cdot \pi \cdot G} \cdot r^{-5/2}$. So this density of energy would be a new term to consider into the tensor of energy of Einstein's gravitational field equations.

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