A New Approach to Unification Part 2: Deducting particle physics

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In a series of 4 papers an approach to a unified physics is presented. In part 1 the foundation of such an approach is given. Here in part 2 it will be shown how particle physics follows. In part 3 gravitational physics will be derived. In part 4 open fundamental questions of actual physics are answered and the concept of a new cosmology is introduced.

Contents

Basic equations found in part 1.

To simplify quoting, here some results from part 1 of the series:

Our four dimensional (4d) world is a submanifold of the assigned spacetime, a six dimensional (6d) Minkowski space. In this it holds

- a 6d Klein-Gordon equation without mass term
$$\partial^\alpha \partial_\alpha \phi = 0 \text{ with } \alpha = 1, 2 \ldots 6 \text{ and the 6d wavefunction } \phi. \tag{1}$$
- a 6d Dirac equation without mass term
$$\gamma^\alpha \partial_\alpha \phi = 0 \text{ with (8x8)-matrices } \gamma^\alpha \text{ describing an SU(8) symmetry.} \tag{2}$$
ϕ is a spinor with 8 components.
- a Lagrangian $\mathcal{L}6_D$ based on the 6d Dirac equation
$$\mathcal{L}6_D = \Phi^+ \gamma^\alpha \left(\partial_\alpha - igA_\alpha\right) \Phi - \mathcal{L}_B \tag{3}$$
- a Lagrangian $\mathcal{L}6_{KG}$ based on the 6d Klein-Gordon equation
$$\mathcal{L}6_{KG} = \left(\left(\partial_\alpha - igA_\alpha\right) \Phi\right)^+ \left(\left(\partial^\alpha - igA_\alpha\right) \Phi\right) - \mathcal{L}6_B. \tag{4}$$
with $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$ (T means transposed), where the ϕ_i are solutions of the Dirac resp. Klein-Gordon equation. The A_α representing the boson fields are Hermitian 4x4-matrices described by the 15 generators $\hat{\lambda}_k$ of $SU(4)$ and an according number of Yang-Mills fields $W_{\alpha k}$ as $A_\alpha = \sum_{k=1}^{15} \hat{\lambda}_k W_{\alpha k}$. g is a coupling factor defining the strength of interaction.
The free boson field part $\mathcal{L}6_B$ is given by
$$\mathcal{L}6_B = -\frac{1}{4} \sum_{i=1}^{15} W_{\alpha\beta i} W_i^{\alpha\beta} \text{ with } W_{\alpha\beta i} = \partial_\alpha W_{\beta i} - \partial_\beta W_{\alpha i} + 2g \sum_{j,k=1}^{15} f_{ijk} W_\alpha^j W_\beta^k. \tag{5}$$

The f_{ijk} are the structure constants of $SU(4)$.

- With the 6d coordinates $x6^{\alpha} = x6^{\alpha}(T, u, v, x)$ (x stands for x^1, x^2, x^3, coordinates occurring also in 4d, and u,v for coordinates not accessible for a 4d observer) and the Jacobi determinant $J(T, u, v, x)$ the 6d action integral S can be written as

$$S = \int dx^3 dT \int dudv J(T, u, v, x)\mathcal{L}6(T, u, v, x).$$

The inner integral

$$\hat{\mathcal{L}4} = \int dudv J(T, u, v, x)\mathcal{L}6(T, u, v, x). \tag{6}$$

is called non-interpretable Lagrangian.

1 Introduction

The current state of particle physics is represented by the standard model. It deals with forces other than gravity and allows classifying the elementary particles and deriving the forces in a systematic way.[1]

Since Heisenberg introduced the first flavor quantum number, the isospin, and linked it to the SU(2) group, symmetries have become increasingly important in particle physics. To enable classification of the growing number of newly discovered particles the SU(2) group was expanded. At first it was sought that the multiplets of an SU(3) symmetry would be suitable to map the structure of the particle families.[2, 3] This made it necessary to find three interacting entities causing the symmetry. After years of debate, the view prevailed that most elementary particles are not really elementary, but are made up of a new type of particles called quarks, fermions with spin ½.

Subsequent accelerator experiments however showed, that there are particle families which cannot be arranged in the structure of the SU(3) multiplets. It needs the multiplets of SU(4) to get an enough powerful structure. A fourth quark was postulated and also found in a particle formed from this fourth quark and its antiquark.

In the meantime, 6 quarks are known and the standard model assumes that there are no more

Combining a quark and an antiquark the mesons and combining three quarks or three antiquarks the baryons are generated. All families can be arranged in the multiplets of SU(4).

If one smashes any elementary particles in a particle accelerator to get a quark, one always gets only new particles in which there are several quarks, never a quark alone.

For a comprehensive theory the question arises why just the SU(4) symmetry allows classifying elementary particles and why there are 6 quarks.

Demanding that the Lagrangians do not change under local transformations described by the U(1), SU(2) or SU(3) group, what is achieved by introducing suitable bosonic fields, the gauge theories of the standard model allows deducting the electromagnetic, weak and strong force.[4] Interaction is executed by the exchange of spin 1 gauge bosons. Each symmetry demands its specific bosons: one photon without rest mass for U(1) , three massive particles W^+, W^- and Z^0 for SU(2) and eight gluons for which it is not yet clear whether they are massless or have small masses for SU(3).

All attempts to formulate also a gauge theory of gravitation are not yet successful. Trying to formulate a quantum field theory of gravity demanding instead of the unitary symmetries a spacetime symmetry as e.g. given by the Poincaré group the problem of non-renormalizabilty occurs.[5]

It is not understandable that different symmetries are required for the three forces that usually act simultaneously in an atom. To solve this problem under the term GUT (Grand

Unified Theory) research is done on a more general symmetry of the Lagrangian that allows deducting all three forces. As promising candidates Lie groups as SU(5) or SO(10) were considered containing the required groups as subgroups. But up to now all approaches did fail.[6, 7, 8] Srednicki writes in [8]: "A great variety of grand unified models can be constructed, with and without supersymmetry. Which, if any, are relevant to the natural world is a question yet to be answered."

The task for UR is to answer the question why just these symmetries allow deriving the forces and how they can be unified? Does there exist further forces (except gravity)?

Introducing particle mass is one of the fundamental problems of physics. It was first found in quantum electrodynamics that can be understood as a gauge theory with U(1) symmetry. For some time it was a problem that some series describing the solution diverge. This happens e.g. if in the interaction a particle with mass is assumed. By a mathematical trick, the renormalization, this divergence can be overcome. However, the mass attributed to the particle does not follow from the theory but the experimentally found value must be inserted.[9]

The SU(2) and SU(3) symmetries of the gauge theories do not allow any particles with mass at all. It needs the Higgs mechanism destroying afterwards these symmetries to make it possible that some particles can have a mass.[10, 11] The mechanism allows particles to get mass but it cannot tell which particle has mass and how large it is.

Introducing this rather unmotivated additional field in a harmonic picture seems to be an alien element.

It was considered a great success and confirmation of the standard model of particle physics when 2012 in accelerator experiments a boson was found, which was attributed to the Higgs field.

In the perspective of UR the mass-problems are not a surprise. Mass is not an original feature of 6d physics. It is one of the entities filling the gap in information on the way from 6d to 4d physics.

The standard model does not explain dark matter and some actual observations with muonic hydrogen.

Looking at the way in which the standard model was developed, we can see that it is essentially a theory defining individual solutions for certain experimental findings without asking why the findings are as they are. It is the task of a unification theory to derive the ad hoc explanations made in the standard theory from a uniform point of view and to point out ways beyond the standard model.

In the following it will be shown that the UR approach is able to this.

As found in part 1 of the series the aim of UR in deducting 4d physics is finding the 4d Lagrangian equivalent to the non-interpretable Lagrangian. In general this means a nonlinear integral transform. Nonlinear because the 6d wave functions and Yang-Mills fields in $\hat{\mathcal{L}}4$ and the 4d wave functions and Yang-Mills fields in $\mathcal{L}4$ occur each at least quadratic.

The basic approach to find the 4d Lagrangian means solving the variational problem of the 6d Lagrangian, solving the found equation of motion, putting the hereby found wave-functions and Yang-Mills fields in the non-interpretable Lagrangian and adapting a 4d Lagrangian.

Regarding the (mathematical) difficulties occurring in solving the equations of nowadays physics the same or even larger problems can be expected for a more comprehensive theory like UR. The computational effort to implement the procedure for the whole Lagrangian so currently seems not to be feasible.

Often if the whole system (at the moment) is too complex to be examined in physics simpler subsystems are considered. The so gained knowledge allows already substantial conclusions. The same procedure is taken here. To get basic information at first simple versions of the 6d Lagrangian are considered. Additional terms are successively added to the Lagrangian and discussed. The examination is completed by symmetry aspects.

One of the strengths of UR compared to other unification approaches is that it allows deriving physically relevant results using approximations.

The nonlinear transform makes that the symmetry of the Euclidean 6d space has far reaching consequences for 4d physics. Depending on the 6d symmetry the character of 4d metric is different. In this part physics as a 4d observer it sees will be derived starting with translational symmetry in 6d.

2 Systematics of bosons

Starting point is the 6d Lagrangian built on the 6d Klein Gordon equation without a mass term. It will be investigated how this equation translates for a 4d observer in generating 4d physics.

2.1 Free particles and particle mass

The easiest situation is given assuming all Yang-Mills fields being zero and neglecting all effects caused by spin. With these simplifications equation (3) becomes

$$\mathcal{L}6_{KG} = \frac{\partial\Phi^*}{\partial T}\frac{\partial\Phi}{\partial T} - \frac{\partial\Phi^*}{\partial u}\frac{\partial\Phi}{\partial u} - \frac{\partial\Phi^*}{\partial v}\frac{\partial\Phi}{\partial v} - \sum_{i=1}^{3}\frac{\partial\Phi^*}{\partial x_i}\frac{\partial\Phi}{\partial x_i} \tag{7}$$

(Due to the proximity to the special theory of relativity a metric with positive dT^2 is chosen). Variation gives four equations of motion all independent of each other and all equal

$$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial u^2} - \frac{\partial^2\phi}{\partial v^2} - \frac{\partial^2\phi}{\partial x_1^2} - \frac{\partial^2\phi}{\partial x_2^2} - \frac{\partial^2\phi}{\partial x_3^2} = 0. \tag{8}$$

The solution of one is also valid for the others.

This Klein-Gordon equation is a 6d partial differential equation with constant coefficients. With a product ansatz $\phi = \psi(x, T)\, U(u)\, V(v)$ it can be separated in three equations

$$\frac{\partial^2\psi}{\partial T^2} - \sum_{i=1}^{3}\frac{\partial^2\psi}{\partial x_i^2} = k_1^2\psi, \quad \frac{\partial^2 U}{\partial u^2} = k_2^2 U \text{ and } \frac{\partial^2 V}{\partial v^2} = k_3^2 U.$$

k_1, k_2 and k_3 are constants with $k_1^2 = k_2^2 + k_3^2$.

The non-interpretable Lagrangian becomes $\hat{\mathcal{L}}4 = \int dudv\mathcal{L}6_{KG}(T, u, v, x)$ and is equal to the sum over four Lagrangians expressed in the single wavefunctions. Introducing ϕ in one term of $\hat{\mathcal{L}}4$ using the two last above equations integration over u and v can be carried out by partial integration. Since the wave function on the infinitely distant surface is zero, it follows

$$\hat{\mathcal{L}}4 = \frac{\partial\psi^\star}{\partial T}\frac{\partial\psi}{\partial T} - \sum_{i=1}^{3}\frac{\partial\psi^\star}{\partial x_i}\frac{\partial\psi}{\partial x_i} - \left(k_2^2 + k_3^2\right)\psi^\star\psi. \tag{9}$$

This is an equation of the form of a common Lagrangian of the 4d Klein-Gordon equation but expressed in 6d variables.

So the search for the correct 4d Lagrangian is rather easy. It just means interpreting the 6d variables as 4d variables to get the proper 4d Lagrangian $\mathcal{L}4$.

Defining time t by

$$cdt = dT \tag{10}$$

and identifying λ_C given by

$$1/\lambda_C^2 = k_2^2 + k_3^2 \tag{11}$$

as the reduced Compton wavelengths, the adapted 4d Lagrangian is given by

$$\mathcal{L}4 = \frac{1}{c^2}\frac{\partial\psi^\star}{\partial t}\frac{\partial\psi}{\partial t} - \sum_{i=1}^{3}\frac{\partial\psi^\star}{\partial x_i}\frac{\partial\psi}{\partial x_i} - \frac{1}{\lambda_C^2}\psi^\star\psi. \tag{12}$$

This shows that UR starting with a timeless 6d Euclidean space via the assigned spacetime for translational symmetry gives in 4d a Minkowski spacetime as is demanded by special relativity.

2.1.1 Dispersion relation and conservation of energy

Solving the equation of motion related to equation (12) we get $\psi = \exp i(\omega t - kx)$, a plane wave, where ω is the frequency and k the wave number. Implementing this solution in the Lagrangian gives the dispersion relation

$$\omega^2 = c^2k^2 + \frac{c^2}{\lambda_C^2}. \tag{13}$$

The identical result is achieved solving the 6d Klein-Gordon equation in the same way. So the 4d dispersion relation from the point of view of the assigned space is exact. This shows its fundamental significance.

We see that not mass is an original entity but frequency, wave number and Compton wavelengths. This differentiates the top down approach of UR from the classical approach based on Newton's basic assumption of mass.

Historically the 4d Klein-Gordon equation is interpreted as describing the behavior of a relativistic particle and its antiparticle with mass m_0.[12] The relationship between dispersion relation, following from the quantum physical equation, and the characteristics of a classical particle is established by the wave-particle dualism. With

$$m_0 = \frac{\hbar}{\lambda_C c}\ ,\ E = \hbar\omega \text{ and } p = \hbar k \tag{14}$$

(p is the momentum, E the energy of the particle) the dispersion relation for a particle reads $E^2 = p^2c^2 + m_0^2c^4$. This is the relativistic law of the conservation of energy.

It follows that m_0c^2 can be interpreted as the equivalent of the kinetic energy in the two not accessible dimensions.

Deducting 4d from 6d physics "particle" up to now however is a meaningless entity whose relevance still has to be found.

2.1.2 Bondary conditions

In addition to the differential equations there are boundary conditions defining the possible eigenvalues k_1, k_2 and k_3. Here only the boundaries of U and V are to be discussed. Knowing the boundary conditions not exactly to simplify the considerations it is assumed that U and V are zero at the boundary of a rectangle with length l_u resp. l_v.

Then for the general solutions $U = a\cos(k_2u) + \bar{a}\sin(k_2u)$ and $V = b\cos(k_3v) + \bar{b}\sin(k_3v)$ in which $a, \bar{a}, b$ and $\bar{b}$ give the amplitudes of the specific modes in the total wave function for $u = 0$ it holds $a = 0$ and for $v = 0$ accordingly $b = 0$. For $u = l_u$ results $k_2l_u = n_u\pi$ with $n_u = 1, 2\ldots$ and for $v = l_v$ at last $k_3l_v = n_v\pi$ with $n_v = 1, 2\ldots$.

Particle mass or Compton wavelength become discrete

$$m_0 = \frac{\hbar}{\lambda_C c} = \frac{\pi\hbar}{c}\sqrt{\frac{n_u^2}{l_u^2} + \frac{n_v^2}{l_v^2}}. \tag{15}$$

The possible values of particle mass correspond to the eigenvalues of an oscillating plate with edge length roughly the Compton wavelength.

The so found characteristics of mass are different to those of theories with compactification. In these the additional dimensions must be extremely small what generates a huge mass of possible particles whereas in UR l_u and l_v can become also very large so that particle mass can become very small.

This result opens up a way to explain in part 4 of the series dark matter.

In the standard model of particle physics the Higgs mechanism destroying the symmetries of the Lagrangians is needed to allow particles with mass. UR needs no additional symmetry breaking field as the transition from six to four dimensions destroys the SU(4) symmetry. The approach also gives no indication of the existence of symmetry breaking fields.

Introducing particle mass this way is compatible with the demanded gauge invariance. In the 6d Lagrangian no mass term exists so it is gauge invariant without restraint. Only the transition produces in the 4d Lagrangian the mass term forbidden for gauge invariance. Because there is however no need for the 4d Lagrangian to be gauge invariant the generated mass term there makes no trouble.

2.2 Maxwell's equations

The deduction given in the last section is simplified by setting all Yang-Mills fields to zero. A more comprehensive derivation has to start with the 6d Lagrangian without free boson field part and the Yang-Mills fields being introduced as parameters given by external fields. From this a 4d Lagrangian with Yang-Mills fields must be deduced.

Whether a solution of the resulting equation of motion in general is possible is not yet clear. But there exists a special case in which the calculations can be realized.

Among the generators of SU(4) given in table 1 of part 1 of the series with $\hat{\lambda}_3$ there is one, that has elements unequal to zero only in his diagonal. We examine the situation that all Young-Mills fields besides $W_{\alpha 3}\,(\alpha = 1 \ldots 6)$ related to this generator are zero. The only elements of the generator unequal to zero are $\left(\hat{\lambda}_3\right)_{11} = 1$ and $\left(\hat{\lambda}_3\right)_{22} = -1$. This simplifies the 6d Lagrangian to

$$\begin{aligned} \mathcal{L}6_{KG} = &\left(\left(\partial_\alpha + igW_{\alpha 3}\right)\phi_1^\star\right)\left(\partial_\alpha - igW_{\alpha 3}\right)\phi_1 \\ &+ \left(\left(\partial_\alpha - igW_{\alpha 3}\right)\phi_2^\star\right)\left(\partial_\alpha + igW_{\alpha 3}\right)\phi_2. \end{aligned} \tag{16}$$

The resulting equation of motion decomposes into

$$\partial_\alpha\left(\partial_\alpha - igW_{\alpha 3}\right)\phi_1 - igW_{\alpha 3}\left(\partial_\alpha - igW_{\alpha 3}\right)\phi_1 = 0$$

and the conjugate complex equation for ϕ_2. It is to sum over α.

Introducing x, T, u, v, ψ, U and V as for free particles and assuming that the Young-Mills fields connected to the derivatives with respect to x and T depend only on x and T and the Young-Mills fields connected to the derivatives with respect to u and v depend only on u resp. v each equation by a produktansatz can be split into three equations depending on x, T, u or v. This gives e.g.

$$\partial_u\left(\partial_u - igW_{u3}\right)U - igW_{u3}\left(\partial_u - igW_{u3}\right)U + k_2^2 U = 0.$$

The index u means the α related to the variable u.

This equation is solved by $U = C \exp\left(ik_2 u + ig \int W_{u3} du\right)$. An accordant solution is found for V.

Inserting these solutions into $\hat{\mathcal{L}}4$ and integrating over u and v we get as for free particles the 4d Lagrangian of a particle or its antiparticle with rest mass in an electromagnetic field. The four components of the vector potential of the electromagnetic field are given by the four Young-Mills fields connected with the derivatives with respect to x and T.

As the 4d vector potential is equivalent to Maxwell's equations they can be deducted.[13]

2.3 Grand Unified Theory (GUT) and Higgs particle

As mentioned, one of the open questions of the standard model is why there are different symmetries needed for the three forces. The various GUT approaches try to find a common symmetry allowing to deduct all three forces in one step.

For UR, the question is exactly the opposite. In 6d there is one force and one symmetry. This means that the goal pursued by GUT is already answered in the basis of the approach. Instead of looking for a unification of forces, it must be shown that the one force existing in 6d leads to the correct forces in 4d.

The $SU(4)$ symmetry of the 6d Lagrangian in UR demands a boson field with 15 spin 1 gauge bosons.

Calculating the transition from six to four dimensions for the bosons means:

1. Deriving the equation of motion from the Lagrangian of the free boson field part $\mathcal{L}6_B$ given in equation (5)
2. Solving the system of coupled nonlinear partial differential equations for the 90 components of the Yang-Mills fields, implementing the found solutions in $\mathcal{L}6_B$, integrating over u and v.
3. Arranging the various terms found under item 2 in the schemes as given by equation (5) for free boson field Lagrangians in 4d with appropriate unitary symmetries.
4. Assigning suitable terms not fitting in the scheme to mass terms.

This calculation still has to be done. But also without overcoming the mathematical problems associated with that procedure some important conclusions can be drawn by symmetry considerations.

The transition from six to four dimensions destroys the SO(6) symmetry. Because of the close relation between SO(6) and SU(4) this also causes distortions of the SU(4) symmetry and hence influences the character of the force. Depending on the respective Yang-Mills fields the distortion shows itself more or less pronounced. In the 4d Lagrangian we therefore can expect as modified symmetries besides an SU(4) symmetry the symmetries of the subgroups of SU(4) i.e. SU(3), SU(2) and U(1).

So the symmetry considerations allow choosing the possible symmetries of the 4d Lagrangians in which the various terms of $\mathcal{L}6_B$ are to be sorted. The free boson fields in 4d must have SU(4), SU(3), SU(2) and U(1) symmetry. For each one a coupling factor can be extracted. Remaining terms can be assigned to particle mass of some gauge bosons.

Besides the 4d force with SU(4) symmetry the other found forces are just those of the standard model. Their deduction here can serve as a justification of the there ad hoc introduced symmetries.

As the Yang-Mills fields of the standard model have all together 48 components it can be seen that the 4d SU(4) symmetry cannot be mapped in its entirety. It can exist only for particular Yang-Mills fields. Experimental results strongly suggest that a 4d force with SU(4) symmetry exists.[14]

Unification of the three forces is not achieved by choosing a superior symmetry in 4d as GUT is trying to do but by ascribing them to a unique root.

It needs solving the mathematical problems associated with the transit procedure to get a deeper insight allowing to understand how the 15 gauge bosons of the 6d force are converted into the in total 12 gauge bosons of the standard model. But it is obvious that representatives of the three extra gauge bosons must exist in 4d. One of them could be the at CERN found heavy boson interpreted as Higgs boson, what means that two others could be found.

3 Systematics of fermions

3.1 Deducting the 4d Dirac equation

To further develop the standard model of particle physics we have to examine the behavior of particles with spin ½. Taking effects generated by spin into account the 6d Lagrangian based on the Dirac equation (3) has to be used. Setting as for spinless particles all Yang-Mills fields to zero this Lagrangian also is split in four equal Lagrangians each for one spinor and each with an equation of motion given by a Dirac equation.

The question arises how the 4d Dirac equation is related to the 6d one. Two approaches to an answer will be demonstrated.

1. The 6d Dirac equation is written with (8x8)-Dirac-type matrices and a spinor with eight components. Splitting the spinor in an upper spinor ϕ^t and lower one ϕ^b, both with four components, and the (8x8)-matrices as given in part 1 section 3.2 of the series in four (4x4)-matrices the 6d Dirac equation can be transformed in a coupled equation for the two spinor parts. Thus we find

$$\begin{aligned}\gamma^\alpha \partial_\alpha \phi = (a\partial_2 + b\partial_3 + c\partial_5 + d\partial_6)\,\phi^t + E4\,(\partial_1 - i\partial_4)\,\phi^b \\ + E4\,(\partial_1 + i\partial_4)\,\phi^t - (a\partial_2 + b\partial_3 + c\partial_5 + d\partial_6)\,\phi^b = 0\end{aligned} \tag{17}$$

$E4$ is a unit (4x4) matrix.

The parts $(a\partial_2 + b\partial_3 + c\partial_5 + d\partial_6)\,\phi^t$ resp. ϕ^b of the equation have the structure of 4d Dirac equations. Setting $E4\,(\partial_1 - i\partial_4)\,\phi^b = m\phi^t$ and $E4\,(\partial_1 + i\partial_4)\,\phi^t = -m\phi^b$ the equations decouple to two common Dirac equations with a mass term.

2. The deduction of the 4d from the 6d Klein-Gordon equation indicates another way how we can transform a 6d into a 4d Dirac equation. As already used when constructing the 6d Dirac equation each of the eight components of its spinor obeys a 6d Klein-Gordon equation. For each of these equations as shown the transition to a 4d Klein-Gordon equation with particle mass can be carried out. Following the procedure of Dirac we can construct thereof a 4d spinor equation. The 6d Dirac equation without mass term so is related to a 4d Dirac equations with one.

3.2 The SU(4) ordering principle of particles

Experimental results of particle physics demand an SU(4) symmetry that within the scope of 4d physics cannot be deducted. Besides some exotics the elementary particles can be classified according to their properties into families assigned to the multiplets of the SU(4) group.

To explain this behavior the standard model assumes an inner structure of the elementary particles build up by quarks. These fermions are generating the SU(4) symmetry. It is an ad hoc assumption to describe the observations.

UR is able to justify the occurrence of this symmetry. It is a consequence of the SU(8) symmetry of the 6d Dirac equation.

The procedure given under 2. in section 3.1 shows that the resulting 4d spinor is rather loosely connected to the starting 6d one. As the transformation relates not spinor to spinor but components only, the SU(8) symmetry of the 6d spinor is not reflected by the 4d Dirac equation and must be introduced by additional assumptions.

The SU(4) group used in the standard model does not describe the full symmetry of the quarks. As they also have the SU(2) symmetry of the spin their full symmetry is given by SU(4)⊗SU(2) (⊗ means the Kronecker product). This means that every element of the (4x4) matrices describing SU(4) is replaced by a (2x2) matrix. Together a subset of the unitary (8x8) matrices demanded by the SU(8) symmetry of the 6d Dirac equation is generated what explains the ordering principle.[15]

3.3 SU(8) symmetry and flavour quantum numbers

Groups can be classified by representation-independent parameters. These can be determined by the eigenvalues of Casimir operators. To define a group SU(n), the eigenvalues of n-1 Casimir operators are required. This means that the Casimir operators of the SU(8) group of the 6d Dirac equation generate a seven dimensional multiplet structure.

To fix a position in the multiplets seven parameters must be defined. Taking spin as one of these, associating the six remaining parameters with the six flavour quantum numbers characterizing the six quarks and six leptons of the standard model seems to be likely.

That does not mean that the Casimir operators and the flavour quantum numbers are the same, but they can be mapped on each other.

This allows the conclusion that besides the known six quarks and leptons there exist no others.

It shows that the standard model by introducing quarks is transmitting the SU(8) symmetry of the 6d Dirac equation to 4d. So UR justifies the ad hoc made assumptions.

Furthermore, the SU(8) structure provides space for the inclusion of the exotic particles found for example by the Belle or BES collaborations in recent years and certainly also in the future, particles which cannot be explained within the framework of the standard model. [16]

4 Fermions in an external field

As with the procedure for the introduction of Maxwell's equations from the Lagrangian based on the Klein-Gordon equation a deduction of a comprehensive spinor-equation should start with the 6d Lagrangian without free boson field but including all Yang-Mills fields as parameters given by external fields. From this a 4d Lagrangian with Yang-Mills fields must be deducted. Its equation of motion is a generalized Dirac equation.

The wave function – still to be found – solving it would be characterized by the seven quantum numbers given by the Casimir operators. These discrete quantum numbers dictate in which steps change by interaction is possible.

In the interpretation of fundamental physical structures as proposed by UR given in part 4 of this series this fact will prove to be crucial. It makes the SU(8) symmetry to a key for understand 4d physics.

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