# Energy Loss of Electrons in Storage Rings

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**Abstract**

We examine in general the energy loss of electrons caused by the multiple Compton scattering of electrons on black body photons in the storage rings. We derive the scattering rate of electrons in the Planckian photon sea and then the energy loss of electrons per unit length. We discuss possible generalization of our method in particle physics and consider a possible application of our formulas in case of motion of charged particles in the relic cosmological radiation.

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## 1 Introduction

It is well known that storage ring consists of a vacuum pipe passing through a ring of magnets that mantain a constant field so that charged particles may circulate continuously. Two storage rings that intersect at one or more places can be used to study the collisions of two stored beams. For a colliding beam machine to work, the particles must be stored in stable orbits on the time scale of hours compared to the few second they spend in a synchrotron in the acceleration process. This requires an extremely high vacuum compared to that needed in synchrotron. In storage ring, the magnets are continuosly operating, whereas in the normal operation of a synchrotron they are pulsed briefly every few seconds. Also the beams in a storage ring must be focused to a small cross-sectional area and contain a large number of particles in order to have a useful luminosity.

The energy of a charges particle moving in the constant magnetic field is not conserved because of the magnetic bremsstrahlung. In case the motion of a particle is perpendicular to the magnetic field $\mathbf{H}$ the energy loss can be derived in the following form (Landau et al., 1962):

$$-\frac{dE}{dt} = \frac{2e^4H^2}{3m^4c^7}(E^2 - m^2c^4). \tag{1}$$

From the solution of this equation follows the energy dependence of particle on time in the following form:

$$\frac{E}{mc^2} = \cot\left(\frac{2e^4H^2}{3m^3c^5}t + \text{const}\right). \tag{2}$$

For $t \to \infty$ we get $E \to mc^2$ which means that in the classical framework the all energy of particle is spend for the magnetic bremsstrahlung.

The further possibility how can the particle loss its energy is the collisions with the gass molecules in the pipe. However, the situation at present time vacuum beam pipes of modern particle accelerators is a such that the pipes are practically completely without of gas molecules. It seams that the ideal limit of vacuum pipe is achieved.

However, even an ideal vacuum beam pipe in a laboratory at room temperature is filled by the photon gass with the energy given by the Planck distribution law. It was confirmed by experiment in the LEP in CERN where the scattering of electrons on the black body radiation was detected (Bini et al., 1991a; Bini et al., 1991b; Dehning et al., 1990). The long history of the theoretical investigation concerning the scattering of the high-energy electrons on black body photons is summarized by Blumenthal and Gould (1970). More recently, using the numerical Monte Carlo methods, Domenico (1992) has considered this effect for the LEP experiments with the goal of the determination the consequent limit on the beam lifetime. The problem of high-energy electron scattering on black body photons is, no doubt, important and it is worthwhile to study this effect using different approach. Brown and Steinke (1997) computed the scattering rate analytically. They determined it as a function of the prescribed energy loss. This is the important quantity for the beam lifetime. Their results were given analytically excepting a final straightforward numerical integration.

In the following text, we use natural units in which the velocity of light $c = 1$, Planck's constant $\hbar = 1$, and Boltzmann's constant $k_B = 1$, so that temperature is measured in energy units. Later, we use the system MKS with the specificatoion of the corresponding physical constants.

In the next section we rewrite some formulas concerning the scattering rate of electron moving in the Planckian photon sea and in the third section we determine the energy loss of electron caused by the multiple Compton scattering in the Planckian photon sea.

Discussion is devoted to summarizing and to the speculation on the meaning of our formulas in physics of charged cosmic particles which also move in the Planckian sea.

## 2 The scattering rate of an electron moving in the photon sea

The total scattering rate $\Gamma$ of an electron scattering off some photon distribution, is defined by Brown and Steinke (1997) by the following formula in the electron's rest frame:

$$\bar{\Gamma} = \int \frac{(d^3k)}{(2\pi)^3} \bar{f}(\bar{\mathbf{k}})\sigma(\bar{\mathbf{k}}), \tag{3}$$

where $\bar{f}(\bar{\mathbf{k}})$ is the photon number density as a function of the photon momentum and $\sigma(\bar{\mathbf{k}})$ is the scattering cross section, which is similarly a function of the photon momentum. Here all quantities are evaluated in the electron's rest frame as indicated by the over bar. This scattering rate may be viewed as a time derivative

$$\Gamma = \frac{dn}{d\tau}, \tag{4}$$

where $\tau$ is the time in the electron's rest frame. Since numbers are Lorentz invariant and $\tau$ may be defined to be the invariant proper time of the electron, the rate $dn/d\tau$ is, in fact, a Lorentz invariant.

In the non-relativistic limit, $\sigma$ may be replaced with the Thomson cross section, $\sigma_T = 8\pi r_0^2/3$, where $r_0 = e^2/4\pi m$ is the classical electron radius. It was shown by Brown et al. (1997) that for the Planckian thermal photon distribution

$$f(k) = \frac{2}{\exp[\omega/T] - 1} \tag{5}$$

where $\omega = k^0$ is the photon energy and the photon number distribution is isotropic, we have with $\zeta(3) = 1.202\ldots$ being the Riemann zeta function:

$$\Gamma_0 = \frac{2\zeta(3)}{\pi^2} T^3 \sigma_T. \tag{6}$$

The first order relativistic correction to this result can be obtained with the use of corrected cross section as shown in the Brown et al. (1997) article.

For the temperature in the LEP beam pipe we take $T = 291\,\mathrm{K} = 0.0251\,\mathrm{eV}$, which is about room temperature. This gives the leading rate $\Gamma_0 = 9.98 \times 10^{-6}\,\mathrm{s}^{-1}$ corresponding to the mean life $\tau_0 = 1/\Gamma_0 = 28$ hr. A typical LEP beam energy $E = 46.1\,\mathrm{GeV}$ is just above half the $Z^0$ mass — within the width, but on the high side of resonance curve.

## 3 Energy Loss Caused by the Compton Scattering

The total electron scattering rate as observed in the lab frame reads (Brown et al., 1997; Brown, 1992; Sokolov et al., 1983):

$$\Gamma(E) = \frac{1}{2E}\int \frac{(d^3k)}{(2\pi)^3}\frac{1}{2\omega}f(k)\int \frac{(d^3k')}{(2\pi)^3}\frac{1}{2\omega'}\int \frac{(d^3p')}{(2\pi)^3}\frac{1}{2E'}(2\pi)^4\delta^{(4)}(k'+p'-k-p)\frac{1}{2}\sum_{spin}|A_{fi}|^2, \qquad (7)$$

where $(1/2)\sum|A_{fi}|^2$ is the average value obtained by averaging over initial values and summation over final states of polarization of a particle. The quantities $p$ and $p'$ are the initial and final electron four momenta, $k$ and $k'$ the initial and final photon four momenta, with $E = p^0$ , $E' = p'^0$ , $\omega = k^0$ , $\omega' = k'^0$ being the time components of these four vectors.

The phase-space integral is weighted by the function $f(k)$ corresponding to the Bose-Einstein distribution. This function is introduced as a consequence of the situation that the Compton scattering is generated by the thermal photons of the storage pipe. In the final state the photons do not interact with the thermal photons in considered approximation and therefore the Pauli blocking factor is equal to 1. Let us remember that the situation is substantial different if the bremsed medium is composed from electrons. In this case the Pauli blocking factor differs from ours.

In this article, we respect the ideas of Brown et al. (1997), nevertheless our goal will be to determine the energy loss $dE/dx$ in the general form and in the limiting cases.

Except for the initial factor of $1/2E$ which is the lab energy of the initial electron and which converts the invariant proper time into the lab time, the right-hand side of this expression is a Lorentz invariant. The problem proves to be greatly simplified if the integrals are evaluated in the rest frame of the electron, because Compton scattering of a photon on an electron at rest has a very simple nonrelativistic limit. This complicates the initial photon distribution, but, if we introduce according to Pauli (1983) that temperature is dependent on velocity and direction of coming photons, then, in the rest frame of electron, the distribution of photons has the simple form

$$f(k) = \frac{2}{\exp[\frac{\omega}{T(\beta,\chi)}] - 1}, \qquad (8)$$

where $\beta = v/c$, $v = |\mathbf{v}|$, $\mathbf{v}$ being the velocity of particle and we suppose that $\mathbf{v} \parallel z$ and oriented in the positive direction of $z$ axis. Symbol $\chi$ is the spherical angle. Temperature $T(\beta,\chi)$ is defined as follows:

$$T(\beta,\chi) = \frac{T}{1+\beta\cos\chi}. \qquad (9)$$

A quantum field theoretic definition of the energy loss of an electron moving in the photon sea can be obtained as follows (Pauli, 1983). The mean distance between two collisions of an electron moving with velocity $v$ in the Planckian photon sea is given by formula $\Delta x = v/\Gamma$. We assume that the dimension of the photon sea is large compared to the mean free path of an electron, corresponding to a large number of collisions, and that the energy transfer per collision is small compared to the energy of an electron. The energy loss of an electron per collision is given by formula $\Delta E = E - E' = \omega' - \omega$, where $\omega$, $\omega'$ denotes the energy photon with four momentum $k = (\omega, \mathbf{k})$ and $k' = (\omega', \mathbf{k}')$. The mean energy loss is now obtained by averaging over the interaction rate $\Gamma$ times the energy transfer $\Delta E$ and dividing by the velocity $v$ of an electron. It means that the energy loss can be expressed by the formula

$$-\frac{dE}{dx} = \frac{1}{v}\int d\Gamma(\omega' - \omega). \qquad (10)$$

The equation means that $\omega' - \omega$ has to be inserted under integrals that define the interaction rate, i.e. (7). Later we shall find the explicit formula for the energy loss.

Using the identity (Sokolov et al., 1983)

$$\int \frac{(d^3k')}{2\omega'}\int \frac{(d^3p')}{2E'} = \int (d^4k')(d^4p')\delta(k'^2)\delta(p'^2 - m^2), \qquad (11)$$

we get instead of eq. (7)

$$\Gamma(E) = \frac{1}{2E}\int \frac{(d^3k)}{(2\pi)^3}\frac{1}{2\omega}f(k)\int \frac{(d^3k')}{(2\pi)^3}\frac{1}{2\omega'}\delta((p+k-k')^2 - m^2)4\alpha^2 C, \qquad (12)$$

where $\alpha$ is the fine structure constant and (Sokolov et al., 1983)

$$C = 2\left[\frac{pk}{pk'} + \frac{pk'}{pk} + m^4\frac{(kk')^2}{(pk)^2(pk')^2} - 2m^2\frac{kk'}{(pk)(pk')}\right]. \tag{13}$$

Using

$$u = \frac{kk'}{kp'}; \quad \kappa = \frac{2kp}{m^2}, \tag{14}$$

we get with $kp' = kp - kk'$:

$$C = C(\kappa, u) = 2 + \frac{u^2}{1+u} - 4\frac{u}{\kappa}(1 - \frac{u}{\kappa}). \tag{15}$$

Let us introduce

$$dL = \frac{(d^3k')}{2\omega'}\delta(x); \quad x = (p + k - k')^2 - m^2 = 2(pk - pk' - kk') \tag{16}$$

or,

$$dL = \frac{(d^3k')}{2\omega'}\frac{1}{2}\delta(pk - pk' - kk'). \tag{17}$$

Then, we can write for the energy loss per unit length with regard to the Braaten and Thoma article (1991):

$$-\frac{dE}{dx} = \frac{1}{v}\frac{1}{2E}\int dL(E - E')\frac{(d^3k)}{(2\pi)^3}\frac{1}{2\omega}f(k)\int 4\alpha^2 C(\kappa, u). \tag{18}$$

Now, we are prepared to evaluate the energy loss of electron moving in the photon sea of the black body via the storage pipe. First, let us perform the $k'$-integration in the CMS system and later we perform the determination of the energy loss in the rest system of electron.

In the CMS system, we have:

$$\mathbf{p} + \mathbf{k} = \mathbf{p}' + \mathbf{k}' = 0; \quad pk = E\omega - \mathbf{pk} = E\omega + \omega^2 = \omega(E + \omega); \quad k'(p + k) = \omega'(E + \omega);$$

$$kk' = \omega^2(1 - \cos\Theta) \quad \delta(x) = \delta(2(E + \omega)(\omega - \omega')). \tag{19}$$

Using $(d^3k') = (-1)(\omega')^2 d\omega' d(\cos)\Theta d\varphi'$, we get for $\delta(x)$:

$$\delta(x) = \frac{1}{2}\delta((E + \omega)(\omega - \omega')), \tag{20}$$

and for $dL$

$$dL = (-1)\frac{1}{2\omega'}\frac{1}{2(E + \omega)}{\omega'}^2 d\omega' d(\cos\Theta)d\varphi\delta(\omega - \omega'). \tag{21}$$

Using

$$d(\cos\Theta) = (-1)\frac{kp}{\omega^2}\frac{du}{(1+u)^2}, \tag{22}$$

we get

$$dL = \frac{1}{4}d\varphi d\omega'\delta(\omega - \omega')\frac{du}{(1+u)^2}, \tag{23}$$

and after some algebraic operation,

$$\Gamma(E) = \frac{\alpha^2}{2E}\int\frac{1}{2\omega}\frac{(d^3k)}{(2\pi)^2}\frac{2}{\exp(\omega/T) - 1}\int_0^\kappa\frac{du}{(1+u)^2}\left[2 + \frac{u^2}{1+u} - \frac{4u}{\kappa}\left(1 - \frac{u}{\kappa}\right)\right], \tag{24}$$

where the last integral is considered in the CMS system. The determination of the energy loss in the CMS system is not possible because of the term $E - E' = \omega' - \omega$ in the formula for the energy loss with the simultaneous $\delta(\omega' - \omega)$ in $dL$. So, we write first the reaction rate in the rest system of electron where

$$\mathbf{p} = 0; \quad \kappa = \frac{2\omega}{m}; \quad \omega' = \frac{\omega}{1+u}. \tag{25}$$

Then, using $(d^3k) = (-1)\omega^2 d\omega dx d\varphi; x = \cos\chi$ and some integration, we have

$$\frac{d\Gamma}{d\omega d\omega'} = \frac{\alpha^2}{2E}\frac{1}{2\pi}\int_1^{-1}\frac{dx}{\exp[\frac{\omega}{T}(1+\beta x)]-1}\left[\frac{\omega}{\omega'}+\frac{\omega'}{\omega}+\left(\frac{m}{\omega'}-\frac{m}{\omega}\right)^2 - 2m\left(\frac{1}{\omega'}-\frac{1}{\omega}\right)\right]. \tag{26}$$

Now, using the definition of the energy loss formula with variables of $\omega$ and $\omega'$,

$$-\frac{dE}{dx} = \frac{1}{v}\int d\Gamma(\omega'-\omega), \tag{27}$$

we get the analytical form of the energy loss:

$$\frac{dE}{dx} = \frac{1}{v}\int_0^\infty d\omega\int_{\omega_1}^{\omega_2} d\omega'(\omega'-\omega)\frac{\alpha^2}{2E}\frac{1}{2\pi}\int_{-1}^{1}\frac{dx}{\exp[\frac{\omega}{T}(1+\beta x)]-1}$$

$$\left[\frac{\omega}{\omega'}+\frac{\omega'}{\omega}+\left(\frac{m}{\omega'}-\frac{m}{\omega}\right)^2 - 2m\left(\frac{1}{\omega'}-\frac{1}{\omega}\right)\right], \tag{28}$$

where the integral $\omega'$-limits are as follows:

$$\omega_2 = \frac{\omega}{1+\frac{2\omega}{m}} \le \omega' \le \omega = \omega_1. \tag{29}$$

The energy loss formula (28) can be evaluated explicitely using the expansion

$$\frac{1}{e^\delta - 1} = \sum_{n=1}^{\infty} e^{-n\delta}. \tag{30}$$

Then, we have:

$$\int_{-1}^{1}\frac{dx}{\exp[\frac{\omega}{T}(1+\beta x)]-1} = \sum_{n=1}^{\infty}\left(\frac{T}{n\omega\beta}\right)\left\{e^{-\frac{n\omega}{T}(1-\beta)} - e^{-\frac{n\omega}{T}(1+\beta)}\right\}. \tag{31}$$

For the energy loss eq. (28) we finally have:

$$\frac{dE}{dx} = \frac{1}{v}\int_0^\infty d\omega\int_{\omega_1}^{\omega_2} d\omega'(\omega'-\omega)\frac{\alpha^2}{2E}\frac{1}{2\pi}\sum_{n=1}^{\infty}\left(\frac{T}{n\omega\beta}\right)$$

$$\left\{e^{-\frac{n\omega}{T}(1-\beta)} - e^{-\frac{n\omega}{T}(1+\beta)}\right\}\times\left[\frac{\omega}{\omega'}+\frac{\omega'}{\omega}+\left(\frac{m}{\omega'}-\frac{m}{\omega}\right)^2 - 2m\left(\frac{1}{\omega'}-\frac{1}{\omega}\right)\right]. \tag{32}$$

# 4 Discussion

We have seen that the energy loss of electron moving in the pipe of accelerator where at the same time the black body radiation is present can be described in the framework of the quantum field theory. The derived formula (32) describes the energy loss caused by the multiple Compton scattering of electrons when moving in the black body radiation. This formula can be applied not only in the situation of accelerator, however also in case of the charged particle moving in the cosmical space where the relic photons are present. These photos are thermal with the Planckian distribution. Although the energy loss of charged particle caused by the presence of the relic radiation is very small, nevertheless it enables new look on the motion of the charged cosmical particles in the cosmical space (Heer et al., 1968), (Henry et al., 1968), (Partovi, 1993).

It is possible to consider also the the quantum correction to the magnetic bremstrahlung (Schwinger et al., 1978) and also the corrections involving the modified propagator of photon (Pardy, 1994a; 1994b) and (Pardy, 1997). However, the significant influencees on motion of electrons in accelarator are caused by magnetic bremstrahlung and by the Compton multiple interaction with the Planckian photon gas.

The derived theory can be considered as an introduction to theory of interaction of elementary particles not only with the Planckian photon sea but also with the arbitrary media such as quark-gluon plasma, nuclear matter and so on (Braaten et al., 1991). Specially, the interaction of neutrino with the photon sea can be very atractive problem. The Landau-Pomerančuk-Migdal effect, which is at present time one of the most important processes in particle physics, can be also investigated by the methods described in this article. In such a way, the ideas of the present article can be considered as the integral part of the today particle physics.

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