Quantum and Relativistic Physics as Theories of an Internal Observer

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Abstract

We discuss the hypothesis that the special theory of relativity (STR) and quantum mechanics (QM) are descriptions of physical reality from the perspective of an observer who is part of a finite deterministic system.

1. Introduction

H. Everett's idea to place the observer within the observed world represents a kind of «Copernican revolution» that has long been brewing in physics. Before H. Everett, nobody considered the observer as part of the system, let alone as a quantum object. The Copenhagen interpretation excludes the observer from consideration, replacing the measurement process with Born's empirical rule, the meaning of which is unclear. In contrast, in the many-worlds (MW) formulation of quantum mechanics [1], the observer is part of the system. Such an observer can be called "internal" or "subjective" because they are a subject of the quantum system. When such an observer interacts with an object, which can be the rest of the universe, an inseparable quantum state arises, representing a superposition of classical realities, each of which contains a copy of the observer and the object in correlated states.

A vivid example can be tossing a die, where the world splits into six worlds according to the number of die faces. Everett created his theory with the intention of removing from quantum mechanics an element foreign to it - the collapse of the quantum state. He succeeded only partially, as the assertion of the simultaneous existence of all measurement outcomes does not solve the main question - why, when I threw the die, did I end up in this branch of the superposition and not in another? We still do not know what lies behind the mechanism of reduction and how the Born rule, which gives the probability distribution, arises. These questions, related to the so-called measurement problem, can only be answered based on a deeper theory, the possibility of which Einstein believed in.

"God does not play dice!" - said Einstein. "Einstein, quit telling God what to do"- objected Bohr, confident in the fundamental indeterminism of nature. In this article, we will show that despite the polarity of concepts, both geniuses were right. Indeed, quantum mechanics demonstrates a strange hybrid of deterministic and random behavior. We will show that such behavior can be explained based on superdeterminism. Currently, there are many attempts [2,3,4] to reproduce quantum behavior in models based on non-local hidden variables or superdeterminism. It should be noted that superdeterminism is characterized by the non-fulfillment of the condition of statistical independence [5]. This means that in models with hidden variables, there must be a correlation of environment with the observer's state.

In this article, we propose an original approach based on superdeterminism and the so-called physical incompleteness. Physical incompleteness should not be confused with the incompleteness of quantum mechanics that Einstein spoke of. By physical incompleteness, we mean a situation that arises for an observer who is part of the system he observes. This situation is analogous to the limiting theorems of mathematical logic applied to physics.

Formally, the reason for incompleteness is the impossibility of a bijective mapping of the set of world states onto the set of observer states, since the latter is a subset of the former. This leads to the impossibility for an internal observer to have exhaustive knowledge of the world, of which they are a part [6]. This justifies the existence of hidden parameters.

Discussion of hidden parameters is usually encountered only in the narrative of the foundations of quantum mechanics. Our approach reveals hidden parameters in STR and demonstrates a common principle underlying quantum mechanics and special theory of relativity.

2. Projective Interpretation of Quantum Mechanics

G. 't Hooft, while developing an original approach to the foundations of quantum mechanics, considers abstract deterministic models of the world like 'cogwheel' [7]. This is a physical-mathematical metaphor in which the teeth of the 'gear' play the role of ontological states of the world. From a mathematical point of view, such a model is described by a looped space of ontological states.

We will show how to build the simplest superdeterministic model based on this. Let's introduce an observer into 't Hooft's world model, and call it the '2-cogwheel' model. In the simplest case, such a world, denoted as World, is a mechanism consisting of 2 interlocking gears, one of which models the observer Subj, and the other - the object Obj. Let the states of the subject and the object be denoted by ξ and η, respectively. The finiteness of the number of world states is an important condition. By definition, the states of the observer (subject) can be measured and realized by him. In a sense, they are states of consciousness of the observer. The states of the object play the role of hidden parameters.

The state of the system as a whole is uniquely described by the pair ψ={ξ,η}, composed of the observer's state ξ∈Subj (the first gear) and the object's state η∈Obj (the second gear), taken at the point of engagement. Here ξ and η are simply the numbers of the teeth of the respective gears. Formally, this structure is described by the direct product of cyclic groups $\mathbb{Z}_n \times \mathbb{Z}_m$. Let's assume that the orders of the groups are coprime, then the group of ontological states is also cyclic.

The cyclicity of the system's evolution is an inevitable consequence of the underlying finiteness and determinism.

The limitations of incompleteness require distinguishing between physical and ontological time. The time associated with the change of states of the system as a whole (World) is called ontological time. At each moment of ontological time, the system can be in only one ontological state. The time associated with the change of states of the observer Subj is called physical time. Physical time is determined by the change of the observer's state ξ. The system can undergo the evolution of the hidden parameter η without changing the physical time. We will say that such evolution occurs in hidden time.

Examples of evolution in hidden time include stationary quantum processes or a particle passing through two slits simultaneously in the Young's experiment. Let's consider the vector space W over the field of ontological states. Let's call it the ontological space. In order to construct a closed self-consistent model, the foundation must be a closed algebraic system. The requirement of finiteness limits the choice of such systems to the Galois field $\psi \in \mathbb{Z}_{n \cdot m}$ or to a field of roots of 1, isomorphic to it:

$$\psi(\tau) = \left[\hat{U}\right]^{\tau} \psi(0) \qquad (2.1)$$

Here, $\hat{U}$ is a matrix that implements a permutation of the ontological basis. This N×N matrix is from the group of permutations, where N is the number of ontological states. For example, the matrix (2.2) cyclically shifts the components of the vector ψ:

$$\hat{U} = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & \ddots & \\ & & & \ddots & 1 \\ 1 & & & & 0 \end{pmatrix} \qquad (2.2)$$

Such matrices are unitary, $\hat{U}\hat{U}^* = \mathrm{I}$. Unitarity $\hat{U}$, in accordance with Stone's theorem, allows us to express (2.1) in exponential form:

$$\psi(\tau) = e^{-i\hat{H}\tau}\psi(0) \qquad (2.3)$$

Here, ψ is the vector of evolution (a column of ontological states) describing the trajectory of the system in the ontological space $\mathcal{W}$. For the internal observer, ontological dynamics are not accessible because they do not differentiate between ontological states with the same ξ_i but different values of the hidden parameter $\eta_j \neq \eta_k$. These states are equivalent from the point of view of observer:

$$\{\xi_i, \eta_j\} \sim \{\xi_i, \eta_k\}; \qquad (2.4)$$

As a result, the set of ontological states $\psi = \{\xi_i, \eta_j\} \in \mathcal{W}$ is divided into equivalence classes based on the relation of subjective indistinguishability (2.4). Consequently, the ontological space is factorized:

$$P(\mathcal{W}) := \mathcal{W}/\sim. \qquad (2.5)$$

It is worth noting that the obtained structure is very close to the basic structure of quantum formalism - the projective Hilbert space, which is a space of classes of indistinguishable phase states $\psi \sim \psi \cdot e^{i\varphi}$:

$$\mathrm{P}(\mathcal{H}) := \mathcal{H}/\sim \qquad (2.6)$$

Comparison of formulas (2.5) and (2.6) suggests that the phase could be the hidden parameter of quantum mechanics. It is known that the statistical properties of quantum observables do not depend on the absolute phase. However, this does not mean that the outcome of a particular quantum measurement does not depend on it.

In the considered model, the orbit of the system in the ontological space $\mathcal{W}$ 'sweeps' through subspaces (equivalence classes) corresponding to different quantum states ψ_i. This leads to a quantum superposition:

$$\psi = \sum \psi_i \qquad (2.7)$$

The result of a measurement is determined by which subspace the system is in at the current moment of ontological time. However, since the internal observer only distinguishes intervals of physical time and does not have access to the hidden ontological dynamics occurring within these intervals, the result turns out to be random for them.

Let's show how to obtain Born's rule. We will consider the set of ontological states $\{\psi\}$ as a sample space, then the set of all its subsets $\pounds = \mathcal{P}(\{\psi\})$ forms a σ-algebra of events [8]. Here, $\mathcal{P}$ denotes the power set.

In our model, we define the probability of measuring a quantum state as a non-negative additive normalized measure over the σ-algebra. Quantum probabilities in this scheme are elements of the probability space $(\{\psi\}, \pounds, \mathrm{P})$, where P is the probability measure.

In this model, the probability of a quantum state $\psi_i \in \pounds$ is determined by the number of intersections of the orbit $\psi(\tau)$ with the class $\{\psi_i\}$:

$$p_i = \frac{\|\langle\psi_i|\psi\rangle\|^2}{\|\langle\psi|\psi\rangle\|^2} = \frac{|\{\psi_i\} \cap \{\psi\}|}{|\{\psi\}|} \qquad (2.8)$$

Here, curly brackets denote sets, and single straight brackets denote their cardinalities. The normalization condition can be written as $|\bigcup\{\psi_i\}| = |\{\psi\}|$.

Since the phase is fundamentally hidden from the internal observer, the results of quantum measurements appear random to them. We see that incompleteness leads to the fact that despite the deterministic evolution of the 'World' as a whole, the world of the internal observer 'Subj' becomes random. In the considered toy world, a toy 'Copenhagen interpretation' arises, asserting randomness as a fundamental property of the model's nature. Here, we have only outlined the model. A detailed analysis will require separate consideration

3. The Projective Interpretation of Special Theory of Relativity

We have seen that the subject-object model generates a projective structure, where each observable physical state is an equivalence class of corresponding ontological states. In the latter, we see a hint of the common foundation of quantum and relativistic physics. Indeed, following this logic, we can assume that Minkowski space $\{X, Y, Z, T\}$ is also a space of equivalence classes: $\{\mathrm{x}:\mathrm{y}:\mathrm{z}: s: t\}$, where - $\mathrm{x}, \mathrm{y}, \mathrm{z}, s, t$ are homogeneous coordinates:

$$\mathrm{P}^4(\mathbb{R}) \coloneqq \mathbb{R}^5/\sim \qquad (3.1)$$

Here, s is an additional coordinate playing the role of a hidden parameter. It can be interpreted as proper time or action. However, the action is precisely the quantum phase, given by s=$\hbar$φ.

Our approach allows us to derive STR (Special Theory of Relativity) from first principles without resorting to postulates. Attempts to axiomatize STR without the second postulate (the principle of the invariance of the speed of light) began immediately after A. Einstein's creation of the STR in 1905. Five years later, Ignatovsky [9] showed that Lorentz transformations could be derived from the most general considerations of symmetry, independent of the second postulate. Later, Frank and Rothe [10], building on Ignatovsky's work, showed that the most general transformations between inertial reference frames are described by fractional-linear functions. In connection with these works, Wolfgang Pauli [11] wrote: "From the theoretical-group considerations, only the external form of the transformation equations can be obtained, but not their physical content." Let's try to determine this missing physical content.

For this, we need a constructive definition of time, which Aristotle proposed over 2000 years ago. He understood that there is no time, - "time is the number of motion" [12], said Aristotle. The idea is that - things do not change in time, as we are used to thinking, but the change of things is time itself. Applied to our task, this means that the measure of time is the change of coordinates. Let us formalize Aristotle's idea by identifying the metric of the 4-space {x,y,z,s} with time:

$$\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 + s^2 = \mathrm{t}^2 \qquad (3.2)$$

Unlike the pseudo-Euclidean "Minkowski world," we will call such a model the "Aristotelian world." Formally, expression (3.2) describes a 4-dimensional cone in the space of 5 dimensions $\{\mathrm{x}, \mathrm{y}, \mathrm{z}, s, t\}$. The coordinate s is unobservable because the internal observer "lives" in the projective space of Minkowski $\{X, Y, Z, T\}$, where $X = \frac{\mathrm{x}}{\mathrm{s}}; Y = \frac{\mathrm{x}}{\mathrm{s}}; Z = \frac{\mathrm{x}}{\mathrm{s}};\ T = \frac{\mathrm{t}}{\mathrm{s}};\ s \neq 0.$

We will show how to logically transition from Aristotle's representation to Minkowski-Einstein's representation. We will follow F. Klein, who in his Erlangen program showed that any geometry, being a theory of invariants of some group of transformations, can be obtained from the most general group of all fractional-linear transformations by selecting the corresponding subgroups from it [13]. In particular, Klein also pointed out that Minkowski geometry, used in Special Theory of Relativity, is defined by a subgroup of the group of affine transformations that preserves the light cone. In Appendix "A," it is shown that the requirement of preserving the light cone $x^2 + y^2 + z^2 = t^2$ from the group of all linear transformations of homogeneous coordinates $\{x, y, z, s, t\} \to \{x', y', z', s', t'\}$ singles out the Poincaré group, which, in turn, under the condition of cylindricity, narrows down to the Lorentz group.

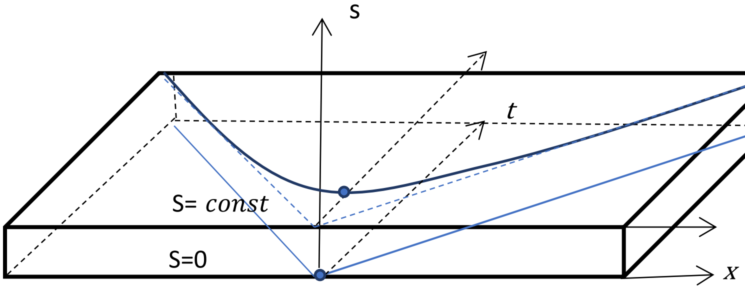

Fig. 1

Without reducing the generality of the conclusions, let's consider the subspace $\{x, s, t\}$. Unlike the 5-dimensional space, this is something we can easily visualize. The fig.1 shows the projective plane $s = const \neq 0$, which consists of an affine part representing the physical space-time $\{X, T\}$, where $X = \frac{x}{s}; T = \frac{t}{s}$ и $s \neq 0$, and the line at infinity, which is formed by all straight lines in the plane $s = 0$. The projective plane is formed by proportional triples - rays $\{x: t: s\}$ passing through the origin. The Lorentz group transforms the affine part of the projective space into itself inside the light cone but leaves the boundary, which is the light cone, unchanged. It is precisely this part of the projective space inside the cone that constitutes the pseudo-Euclidean Minkowski metric space. Outside the light cone, the space remains non-metricized.

Currently, it is believed that the basis of SRT is the postulate of the constancy of the speed of light. We have shown that another approach is possible, according to which the primary ontological structure is the Euclidean space $\{x, y, z, s\}$, and time is its metric. It should be noted that the pseudo-Euclidean nature of space-time $\{X, Y, Z, T\}$ follows solely from the convenience of representing time as a coordinate quantity.

In the Lorentz-covariant formulation, the principle of least action is the primary tool for obtaining equations of motion for both fields and particles. In our formulation, where time is the invariant, the principle of least time becomes such a tool. Its particular case is known from optics and is called Fermat's principle. In the "Aristotelian World," nature "economizes" not action but time. In the most general case, time is a path in space with a metric G_{ik} determined by the acting fields. And this path must be straight because a straight line is the only alternative trajectory that preserves symmetry. Any deviation from the geodesic (from a straight line in curved space) would violate the most important regulative principle - the principle of sufficient reason. In Appendix "B," following the approach outlined by Yu. B. Rumer [14], we provide a scheme for obtaining the Hamilton-Jacobi equation from the generalized Fermat's principle of least time. This example demonstrates the adequacy of the Aristotelian world view.

4. Discussion

- **Implications for physics:**

The approach based on the idea that physical reality is formed as a configuration in the relationships between the subject and the rest of the world is gaining more and more followers. This understanding first appeared in Everett's concept of "relative states" [1]. The same idea in one form or another is found in the works of other researchers. It is worth mentioning the works of C. Rovelli [15] on the relational interpretation of quantum mechanics, as well as the works of D. Page and W. Wootters [16], who, based on the formalism of entangled states, constructed the concept of time for an internal observer.

In the present work, we have raised the question of what physical laws the subject of the finite world will be able to discover by observing it from within. It turns out that the world for such an observer will necessarily be quantum and necessarily relativistic. This conclusion is based on the idea of physical incompleteness, generated by the subject-object structure of the world. By incompleteness, we mean the impossibility (in the case of a finite world) of a bijective mapping of the set of states of the world to the set of states of the observer, since the latter is a subset of the former. Our hypothesis is that this is precisely what underlies the projective structure of the space of quantum states. Recall that quantum states are classes of indistinguishable phase states. In our model, the phase is a hidden parameter that determines the outcome of a quantum measurement, and the cardinality of the class is its probability. This justifies the Born rule.

We have shown that Minkowski space can also be considered as a factor of the ontological 5-space. Thus, physical incompleteness is the common principle that, in our opinion, underlies both quantum mechanics and general relativity. It should be noted that we do not introduce hidden parameters ***ad hoc*** here; we discover, unnoticed previously, their presence in the formalism of QM and STR.

In addition to incompleteness, to justify STR, we use the idea of defining time (in the spirit of constructivism) dating back to Aristotle. Let us ask the question - how do we measure time? Obviously, we judge the passage of time solely by motion (for example, the hands of a clock). We have no other way to measure time. Aristotle defined time as the measure of change in "things". We define time as the measure of change in ontological coordinates in space {x, y, z, s}.

$$\mathrm{dr}^2 + \mathrm{ds}^2 = \mathrm{dt}^2$$

Where $dr^2 = d\mathrm{x}^2 + \mathrm{dy}^2 + \mathrm{dz}^2$, s - here is the hidden coordinate that corresponds to the spacetime interval. It is easy to see that if ds>0, then the velocity $\upsilon=\partial r/\partial t \leq 1$. In the case if ds=0, the velocity $\upsilon=c=1$. That is, such a special definition of the measure of time leads to the existence of a fundamental speed limit.

We have also shown that the parabolic geometry of space-time is a consequence of "squeezing" physical reality into our anthropomorphic picture of the world. In this picture of the world, time is traditionally assigned the role of an independent coordinate quantity, whereas in essence, it is a metric. Indeed, including time as the 5th coordinate and then transitioning to the projective space: P:{x, y, z, s, t}→{X, Y, Z, T} leads to Lorentz invariance. The cost of such "violence" against physical meaning is counterintuitive pseudo-Euclideanness, with all its resulting consequences. The Minkowski and Aristotelian world pictures are isomorphic, so there is no need to abandon the convenience of the Lorentz-covariant approach. It is enough to simply understand its origin. We hope that our analysis will contribute to a deeper understanding of the foundations of quantum mechanics and special theory of relativity.

- **Implications for Philosophy:**

Typically, superdeterminism deprives the observer of freedom in choosing their actions. In this paradigm, Alice and Bob, conducting an EPR experiment, are not free to choose the settings of their filters. Schopenhauer wrote: "we can do what we want, but we cannot want what we want." Does this mean that freedom of choice is an illusion?

In our model, freedom of choice is not an illusion. The point is that the subject whose behavior is supervenient on deterministic world dynamics does not know the reasons for their actions not because they are unavailable or hidden from them, but because in the physical reality of the internal observer they simply do not exist. Being beyond the horizon of incompleteness, they (the reasons) are transcendent to the physical observer. But action in the absence of a cause is precisely what we call free will. In our model, unlike other deterministic models, Buridan's donkey will not die of hunger because it is capable of making unmotivated choices.

Our model can be classified as relativistic compatibilism. It combines determinism with freedom of choice, making these concepts relative.

Now we understand that the question of determinism, which concerned Einstein and Bohr, is a question of choosing the "ontological" coordinate system of the observer (internal or external). Einstein reasoned from the perspective of an external objective observer, while Bohr - from the perspective of an internal one.

Thus, if we are free in our physical world, from the perspective of an external hypothetical observer, we would be puppets. However, since the existence of an external observer is logically contradictory (the very fact of observation inevitably makes it part of the system), the free will of the internal observer must be ontologized.

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Appendix A. Derivation of Lorentz transformations by the Cayley–Klein method

According to Möbius' theorem, projective transformations are defined by linear transformations in the 4+1-dimensional hyper-space. Let's consider linear transformations:

$$(x^{\mu})' = a^{\mu}_{\nu} x^{\nu}, \qquad \text{(A.1)}$$

where:

$$x^{\mu} = \{x^1, x^2, x^3, x^4, x^5\} = \{x, y, z, t, s\}; \quad a^{\mu}_{\nu} = \frac{\partial (x^{\mu})'}{\partial x^{\nu}};$$

In accordance with the condition of cylindricity:

$$a^{i}_{5} = \frac{\partial (x^{i})'}{\partial x^5} = 0; \qquad \text{(A.2)}$$

Let's assume that the additional coordinate x^5 does not depend on the observed coordinates:

$$a^{5}_{i} = \frac{\partial (x^{5})'}{\partial x^i} = 0. \qquad \text{(A.3)}$$

The latter, in the context of Kaluza-Klein theory, implies the absence of electromagnetic fields, since fields are described by components $A_i = \frac{\partial x^5}{\partial x^i}$; Also, let's assume that $a^5_5 = 1$. By setting the coefficients a^5_i to zero, we have narrowed down the group of projective transformations (A.1) to a subgroup of affine transformations:

$$(x^{i})' = a^{i}_{j} x^{j}, \qquad \text{(A.4)}$$

$$(x^{5})' = x^{5}. \qquad \text{(A.5)}$$

Next, we demand the invariance of the following quantity when x^5=0

$$(x^1)^2 + (x^2)^2 + (x^3)^2 - (x^4)^2 = \left(x^{1'}\right)^2 + \left(x^{2'}\right)^2 + \left(x^{3'}\right)^2 - \left(x^{4'}\right)^2. \qquad \text{(A.6)}$$

For the special case where $x^2 = (x^2)'$ and $x^3 = (x^3)'$ it can be easily shown that equality (A.6) is satisfied by the following coefficients a_i^j:

$$a_i^j = \begin{bmatrix} ch(\theta) & -sh(\theta) & 0 & 0 \\ -sh(\theta) & ch(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad \text{(A.7)}$$

This choice of coefficients restricts the group of affine transformations (A.4), (A.5) to a subgroup of Lorentz transformations. By dividing (A.4) by (A.5) and switching to affine coordinates {X,T}, where $X^i = \frac{x^i}{x^5}$, $s \neq 0$,, we obtain the Lorentz transformation: $\left(X^i\right)' = a_j^i X^j$. In the upper left corner of (A.7), we recognize a Lorentz boost. The Lorentz group performs a transformation of the affine part of the projective space (in this case, the plane) into itself (inside the light cone), but leaves the boundary, which is the light cone, unchanged. It is precisely this part of the projective space inside the cone that constitutes the Minkowski pseudo-Euclidean metric space.

Appendix B. Generalized Fermat's principle of least time

We will consider a 4-dimensional space {x, y, z, s}. Taking the path in this space as the measure of time, we have:

$$dt^2 = G_{ik} dx^i dx^k, \qquad \text{(B.1)}$$

where: $x^i = \{x, y, z, s\}$ To derive the equations of motion, as usual, we will use the variational method:

$$\delta t = \delta \int_1^2 G_{ik} dx^i dx^k = 0. \qquad \text{(B.2)}$$

By varying the time δt between events 1 and 2 in the space {x, y, z, s} for the Euclidean metric (+1, +1, +1, +1), we obtain the equation:

$$G^{11}\left(\frac{\partial t}{\partial x}\right)^2 + G^{22}\left(\frac{\partial t}{\partial y}\right)^2 + G^{33}\left(\frac{\partial t}{\partial z}\right)^2 + G^{55}\left(\frac{\partial t}{\partial s}\right)^2 = G^{44}. \qquad \text{(B.3)}$$

This is the equation of the 4-eikonal in the "Aristotelian world" picture. Here, the eikonal is time, since the measure of time is the path traveled by light in the space {x, y, z, s}. Time here is an invariant under orthogonal transformations.

Let's move on to the Minkowski–Einstein picture by considering the function t as the 5th independent variable. That is, we will look for a solution that depends on a single parameter $\Sigma(x, y, z, t, s) = const$. Differentiating the composite function (here, t is an implicit function of $t(x, y, z, s)$), we get:

$$\frac{\partial \Sigma}{\partial x^i} + \frac{\partial \Sigma}{\partial t}\frac{\partial t}{\partial x^i} = 0; \quad \frac{\partial t}{\partial x^i} = -\frac{\frac{\partial \Sigma}{\partial x^i}}{\frac{\partial \Sigma}{\partial t}} . \qquad \text{(B.4)}$$

Substituting into (A.3), we obtain the relativistic 5-eikonal equation [6]:

$$G^{11}\left(\frac{\partial\Sigma}{\partial x}\right)^2 + G^{22}\left(\frac{\partial\Sigma}{\partial y}\right)^2 + G^{33}\left(\frac{\partial\Sigma}{\partial z}\right)^2 + G^{55}\left(\frac{\partial\Sigma}{\partial s}\right)^2 - G^{44}\left(\frac{\partial\Sigma}{\partial t}\right)^2 = 0. \qquad \text{(B.5)}$$

Note that introducing time as a coordinate automatically led to a relativistically covariant equation: a minus sign appeared before the temporal term. In the case of an arbitrary metric $G^{\mu\nu}$, the 5-eikonal equation takes the form:

$$G^{\mu\nu}\frac{\partial\Sigma}{\partial x^\mu}\cdot\frac{\partial\Sigma}{\partial x^\nu} = 0. \qquad \text{(B.6)}$$

In 1926, O. Klein and V.A. Fock showed that the problem of ray propagation in the space {x, y, z, t, s} is equivalent to the problem of the motion of a charged particle in an electro-gravitational field. To transition to the Minkowski spacetime {X, Y, Z, T}, we perform dimensional reduction. We represent the 5-eikonal as the 4-eikonal s plus an addition due to the motion of photons in the extra dimension x^5:

$$\Sigma(x^1, x^2, x^3, x^4, x^5) = \eta x^5 + S(x^1, x^2, x^3, x^4). \quad \text{(B.7)}$$

Here, the capital letter S denotes the 4-action. Substituting (A.7) into (A.6) and taking into account that $g^{55} = 1$, the derivatives of x^5 with respect to the coordinates x^i yield the $g_i^5 = g_i = \frac{e}{mc^2}A_i$, and the derivatives of s with respect to x^5 are zero, we obtain:

$$G^{\mu\nu}\frac{\partial\Sigma}{\partial x^\mu}\cdot\frac{\partial\Sigma}{\partial x^\nu} = g^{ik}\frac{\partial\Sigma}{\partial x^i}\cdot\frac{\partial\Sigma}{\partial x^k} + g^{55}\frac{\partial\Sigma}{\partial x^5}\cdot\frac{\partial\Sigma}{\partial x^5} = g^{ik}\left(\frac{\partial S}{\partial x^i} - \eta g_i\right)\left(\frac{\partial S}{\partial x^k} - \eta g_k\right) + \eta^2 = 0.$$

Identifying $\eta = Zmc$, we obtain:

$$g^{ik}\left(\frac{\partial s}{\partial x^i} \pm \frac{Ze}{c}A_i\right)\left(\frac{\partial s}{\partial x^k} \pm \frac{Ze}{c}A_k\right) + (Zmc)^2 = 0. \quad \text{(B.8)}$$

This is the relativistic Hamilton-Jacobi equation for a particle with mass $|Z|m$ and charge $\pm Ze$.

Quantum and Relativistic Physics as Theories of an Internal Observer

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This article addresses several fundamental ideas in theoretical physics and the philosophy of science. It presents a concept that unifies the realms of quantum mechanics and special relativity through the ideas of superdeterminism and incompleteness. It is demonstrated, that on this basis, special relativity can be derived without resorting to postulates. However, the main idea proposed for discussion by the authors is the hypothesis that the laws of physics, including those described by quantum mechanics and special relativity, are

induced by the ontological status of the observer, who is invariably an integral part of the system they are observing.