L[n]garithms

Juan Elias Millas Vera

Zaragoza (Spain) February (2024)

Dedicated to Arya, a cat with a tiger soul.

0- Abstract

I introduce in this paper a new perception of the concept of logarithm. Generalization of the log_a(b) and doing some combinations we can assume that the log concept is just a case of a more abstract idea.

1- Introduction

The log concept was developed by John Napier in 1614. The main idea of logarithms is to be the inverse of exponential function so

$$e^{f(x)} \Leftrightarrow \log_e f(x) \quad (1)$$

but we can carry this concept to 3-variable base-power being this variables numerical

$$b = a^c \Leftrightarrow \log_a b = c \quad .(2)$$

One simple application of that is the riemannian surfaces in which we could see a difference of concepts between the exponent concept and the power of a function concept, the idea could be very similar but topologically speaking there exists particularities.

2- L[n]garithms

Following my paper about algebraic operation notation[1], we can establish a more complex concept of logarithm, from the basic arithmetic operations, we define the mathematical expression

$$l[n]g_a b = c \quad (3)$$

As a the expanded concept of the canonical logarithm. And we assign this different behaviors:

$$l[1]g_a b = c \Leftrightarrow b = a + c \quad (4)$$
$$l[2]g_a b = c \Leftrightarrow b = a \cdot c \quad (5)$$
$$l[3]g_a b = c \Leftrightarrow b = a^c \quad (6)$$
$$l[-1]g_a b = c \Leftrightarrow b = a - c \quad (7)$$
$$l[-2]g_a b = c \Leftrightarrow b = a \div c \quad (8)$$
$$l[-3]g_a b = c \Leftrightarrow b = \sqrt[c]{a} \quad (9)$$

and with null concept

$$l[0]g_a b=c \Leftrightarrow a=0, b=0, c=0 \Leftrightarrow 0=0 \quad (10)$$

to the more recent concepts of Hyperoperations

$$l[4]g_a b=(c_1, c_2) \Leftrightarrow b=a^{(c_1 \uparrow (c_2))} \quad (11)$$
$$l[5]g_a b=(c_1, c_2, c_3) \Leftrightarrow b=a^{(c_1(\uparrow c_2 \uparrow (c_3)))} \quad (12)$$
$$l[-4]g_a b=(c_1, c_2) \Leftrightarrow b=\sqrt[c_2]{\sqrt[c_1]{a}} \quad (13)$$
$$l[-5]g_a b=(c_1, c_2, c_3) \Leftrightarrow b=\sqrt[c_3]{\sqrt[c_2]{\sqrt[c_1]{a}}} \quad . (14)$$

I used Knuth's up arrow to the optimal visualization of the concept, not using exponent towers cause the font problems.
So as we can see the classic logarithm is just a case of this more global concept, being the classic logarithm tool just a part in a bigger set of mathematical tools

$$l[3]g_a b=\log_a b \quad .(15)$$

3- Numerical examples.

Lets do some numbers to help the memorization of concepts.

$$l[1]g_4 6=2 \Leftrightarrow 6=4+2 \quad (16)$$
$$l[2]g_5 10=2 \Leftrightarrow 10=5 \cdot 2 \quad (17)$$
$$l[3]g_4 16=2 \Leftrightarrow 16=4^2 \quad (18)$$
$$l[-1]g_{17} 12=5 \Leftrightarrow 12=17-5 \quad (19)$$

*The positive value in the numerical result should be done, remember to apply negative sign as indication of operation not as indication of negativeness.

$$l[-2]g_8 4=2 \Leftrightarrow 4=8 \div 2 \quad (20)$$
$$l[-3]g_9 3=2 \Leftrightarrow 3=\sqrt[2]{9} \quad (21)$$

The hyperoperations constructions are left to the reader.

4- Conclusions.

As we could see the l[n]garithm concept is very near to algebraic structures of the lineal equations and a very interesting aspect of functional analysis which can be applied to number theory for example. The main conclusion is that l[n]g tool are just a reconsideration of the equation problem solving.

5- References:

[1]**Millas Vera, Juan Elias. Number Notation for Operations and Hyperoperations (https://vixra.org/abs/2311.0112)**

Ideas for the paper from:

- Ramanujan, Srinivasa. HIGHLY COMPOSITE NUMBERS. Proceedings of the London Mathematical Society, 2, XIV, 1915, 347-409.
- Zalamea, Fernando. YouTube video: Seminario de Filosofía Matemática – RIEMANN (3) – 2020-II