Fundamental Force Coupling Constants Are Wavenumbers

Brian Beverly

February 22, 2024

This started as a joke derivation for force coupling constants but it is the simplest derivation of alpha, the fine structure constant, using Coulomb's law and the Planck-Einstein relations. The derivation shows that coupling constants are wavenumbers. Another derivation using Newton's law of gravity also provides the coupling constant for the gravitational force. I argue that alpha and the coupling constants represents the minimum uncertainty between wavenumber and radial distance. This is like the uncertainty between momentum (wavelength) and position. Wavenumber is defined as the inverse of the wavelength per unit distance. This is equivalent to saying that alpha is about 137 wavelengths per unit distance of radius. I go on to show this provides the correct ionization wavelengths of light for the hydrogen atom. Using whole integers, n number of energy levels, allowed me to derive the Rydberg formula. Alpha is nearly an integer number because we are using a wavenumber. This derivation is equivalent to that of the Bohr model but without needing to use classical ideas of electrons in orbit around the nucleus like planets in orbit around the sun.

1 Deriving Coulomb's Law

The intensity a distance r away from a spherically symmetric point source is defined as

$$I = \frac{P}{A} \tag{1}$$

I is the intensity, P is the power and A is the area the power is spread over. Many physical phenomena can be described as spherically symmetric radiating point sources. Newton's law of gravity, Coulomb's law and the power emitted from a light source all follow the same inverse square law. Substituting in the surface area of a sphere of radius r for the area in the intensity equation gives

$$\boxed{I = \frac{P}{4\pi r^2}} \tag{2}$$

The power distributed over an area is analogous to the probability density created by the wavefunction of two charges, ψ_1, ψ_2, which is equal to the coupling constant squared, the Born rule, spread over the surface area of a sphere or radius r

$$P =< \psi_1|\psi_2 >= e^2 \tag{3}$$

Coulomb's law is beginning to take shape

$$I = \frac{e^2}{4\pi r^2} \tag{4}$$

In electromagnetism the permittivity is defined as the ability of the electric field to penetrate a substance. The greater the permittivity the more difficult it is for the E-field to penetrate the material. The permittivity of the vacuum has been measured experimentally and is denoted by ϵ_o. We find Coulomb's law for two point charges by taking I/ϵ_o

$$\boxed{F = \frac{e^2}{4\pi\epsilon_o r^2}} \tag{5}$$

Coulomb's law replaces point charges with the total sum of the point charges in each object.

2 Deriving α

The force carrying particle of the electromagnetic force is the photon. The intensity of the probability density for the location of the particles relative to each other will move the particles closer together or farther away as the charged particles interact with the photon. The closer the charges are the higher the probability density is for them to be located closer or farther apart. The momentum and energy of a photon is defined by the Planck-Einstein relations in terms of the wavelength λ and frequency ν

$$P = \frac{h}{\lambda} \tag{6}$$

$$E = h\nu \tag{7}$$

The angular form with $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi\nu$, and $\hbar = \frac{h}{2\pi}$ is

$$P = \hbar k \tag{8}$$

$$E = \hbar\omega \tag{9}$$

The speed a photon travels at is a constant, c, which is the wavelength of the photon multiplied by the frequency

$$\lambda\nu = c \tag{10}$$

We can rewrite the energy using the wavenumber as

$$E = \hbar ck \tag{11}$$

The integral of the force provides us the potential energy between two charges separated by a distance r

$$U = -\int F dr \tag{12}$$

This provides us with an equation expressing the wavelength of the photon in terms of the distance between the source and destination as

$$U = -\int \frac{e^2}{4\pi\epsilon_o r^2} dr \tag{13}$$

$$U = \frac{e^2}{4\pi\epsilon_o r} \tag{14}$$

The energy of the photon that bounds an electron to an orbit is defined by the Einstein-Planck relations. When an electron drops an orbital it emits a photon and when the potential energy increases the electron absorbs a photon. Energy is conserved and if the potential energy is zero then the kinetic energy is $E = \hbar ck$

$$\hbar ck = \frac{e^2}{4\pi\epsilon_o r} \tag{15}$$

Solving for k

$$k = \frac{e^2}{4\pi\epsilon_o \hbar cr} \tag{16}$$

α is defined as

$$\boxed{\alpha = \frac{e^2}{4\pi\epsilon_o \hbar c} \approx \frac{1}{137}} \tag{17}$$

Therefore the wavenumber of the photon between two charges is

$$\boxed{k = \frac{\alpha}{r}} \tag{18}$$

Setting $\alpha = 1$ gives us the Planck charge

$$e = \sqrt{4\pi\epsilon_o\hbar c} \tag{19}$$

Since $\alpha = 1$ we should rename the Planck charge from the electric charge, e, to P_c

$$\boxed{P_c = \sqrt{4\pi\epsilon_o\hbar c}} \tag{20}$$

3 Newton's Law of Gravity

The same derivation for Coulomb's law leads to an interesting conclusion if we use the same method for deriving Newton's law of gravity. We use the concept to explain the force of gravity by using a force carrying particle, the graviton, exchanged between two equal masses. As before we find

$$P =< \psi_1|\psi_2 >= m^2 \tag{21}$$

Again we substitute this into equation the intensity equation for a spherically symmetric radiating point source. Plugging in the experimentally determined constant that dampens the probability amplitude of the graviton, G, and dividing by the surface area of the sphere gives us Newton's law of gravity.

$$\boxed{F = \frac{Gm^2}{r^2}} \tag{22}$$

The energy of a force carrying particle is defined as $E = \hbar ck$ and we plug it into the equation above after integrating it to find the force

$$\hbar ck = \frac{Gm^2}{r} \tag{23}$$

Solving for k

$$k = \frac{Gm^2}{\hbar cr} \tag{24}$$

The gravitational coupling constant is defined as

$$\boxed{\alpha_G = \frac{Gm^2}{\hbar c}} \tag{25}$$

This simplifies the wave number of force carrying graviton as

$$\boxed{k = \frac{\alpha_G}{r}} \tag{26}$$

Setting α_G to 1 gives us the planck-mass

$$\boxed{m = \sqrt{\frac{\hbar c}{G}}} \tag{27}$$

4 Deriving the Hydrogen Ionization Wavelength Using the Generalized Uncertainty Principle

The equation $2\pi r = \alpha\lambda$ is telling us that there may be an orbital with the circumference equivalent to the coupling constant, α, multiplied by a photon with wavelength λ. Our assumption of a spherically symmetric radiating point source with an energy that varies as the inverse of the radius is nearly universal.

$$2\pi r = \alpha\lambda \tag{28}$$

For instance, the classical electron radius is defined as

$$2\pi r_e = \alpha\lambda_e \tag{29}$$

where λ_e as the Compton wavelength $\lambda_c = \frac{h}{m_e c}$

This is identical to our derived $k = \frac{\alpha}{r}$. The ground state energy of the electron in the hydrogen atom is $-13.6eV$. The Bohr radius, the distance from the nucleus to the ground state orbital is 5.29110^{-11}m. Setting r to the Bohr radius and solving for lambda gives us a wavelength of 4.55513610^{-8}m.

The ionization energy of hydrogen, the energy to remove the electron from the atom requires a photon with a wavelength of 9.1110^{-8}m. Multiplying our $\lambda = 4.55510^{-11}$ by 2 gives the correct ionization wavelength for the photon that can ionize hydrogen. The factor of two is not an accident and that is due to the uncertainty principle. The uncertainty principle tells us that the relationship between position and momentum is

$$\Delta X \Delta P \geq \frac{\hbar}{2} \tag{30}$$

The generalized uncertainty principle for two observables is

$$\sigma_a \sigma_b \geq \frac{[A, B]}{2} \tag{31}$$

We apply the same idea using our wavenumber k for the momentum and r as the distance which is equal to a constant α

$$kr = \alpha \tag{32}$$

We set $\Delta k = \Delta P$ and $\Delta r = \Delta X$

$$\Delta k \Delta r \geq \frac{\alpha}{2} \tag{33}$$

The generalized uncertainty principle provides the missing factor of 2 and hints that the distance r and wavenumber k provide us with α when we find,

$$[r, k] = rk - kr = \alpha \tag{34}$$

If this is true then alpha may be due to the uncertainty between two observables the momentum and radius. It also hints there may be a connection to Fourier series which also have an uncertainty principle.

This leads to

$$\lambda = \frac{4\pi r}{\alpha} \tag{35}$$

Using the Bohr radius $a_o = 5.291772 \times 10^{-11}$m we find the correct wavelength of a photon that ionizes hydrogen with an electron in the ground state

$$\lambda = \frac{4\pi a_o}{\frac{1}{137}} = 9.1103 \times 10^8 \text{ m}$$

5 Deriving Rydberg's Formula

From the generalized uncertainty principle between the wave number and radius we found $\Delta k \Delta r \geq \frac{\alpha}{2}$. We quantize by using discrete steps in the change of wavelength and radius. By increasing Δk by n the radius must also increase by n. Our change in radius and wavelength can only occur in whole integers.

$$(nr)(nk) \geq \frac{\alpha}{2} \tag{36}$$

Setting k to $\frac{2\pi}{\lambda}$ leads to our previous result but is also quantized for the energy levels.

$$n^2 r \frac{2\pi}{\lambda} \geq \frac{\alpha}{2} \tag{37}$$

$$4\pi n^2 r \geq \alpha\lambda \tag{38}$$

The ground state, the lowest possible state, is the Bohr radius, a_o. Every additional increase in size is an integer factor of that radius.

$$4\pi n^2 a_o \geq \alpha\lambda \tag{39}$$

If we are finding the difference between an initial orbital and a final orbital then we do this by

$$4\pi n_f^2 a_o - 4\pi n_i^2 a_o = 4\pi a_o(n_f^2 - n_i^2) \tag{40}$$

We can now find Rydberg's formula

$$4\pi a_o(n_f^2 - n_i^2) \geq \alpha\lambda \tag{41}$$

$$\frac{1}{\lambda} \geq \frac{\alpha}{4\pi a_o} \frac{1}{n_f^2 - n_i^2} \tag{42}$$

Using Rydberg's constant $R = \frac{\alpha}{4\pi a_o}$ completes the Rydberg formula derivation

$$\frac{1}{\lambda} \geq R \frac{1}{n_f^2 - n_i^2} \tag{43}$$