# The Generating Function Technique Ameliorates Effective and String Field Theories and Foreshadows Their Linkage to Quantum Information Theory

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Abstract

Many scientists are trying to develop a theory of everything or a supposition to explain all aspects of the physical universe. This paper explores a set of theories called effective and string field theory or *EFT* and *SFT*, respectively. These suppositions can be utilized in both old and possibly new physics. Typically, *EFT* and *SFT* have a mathematical method for solving problems called the perturbation theory (*PT*); the generating function technique or GFT can substitute this means of problem-solving. The latter method is used to solve a few examples of physical problems, such as determining the cause of muon g-2 experimental deviations, the means for the calculation of glueballs via meson decay, the ascertainment of tetraquark mass from their decay products, and the analysis of binary black hole mergers. Ultimately, *GFT*, instead of traditional *PT* methods, is a potent tool for improving our understanding of concepts in contemporary physics, such as in *EFT* and *SFT*. Also, *GFT* shows the existence of a triality between *EFT*, *SFT*, and Quantum Information theory (*QIT*).

## 1.) Introduction

An appropriate theory of everything or *ToE* should adequately combine general relativity and quantum mechanics [2,57,61,62]. Physics significantly grew after Sir Isaac Newton discovered gravity [1,38]. Then, Albert Einstein radically updated humanity's current understanding of gravity with his General Theory of Relativity [13,19,29,37,38,46,91]. General relativity heralded another revolution in physics [1]. Quantum mechanics, the study of quanta or particles, underwent a revolution a little after the world recognized general relativity as being virtually true [18,25,59]. Even though many physicists developed putative mathematical frameworks, there was no provable strong theory combining general relativity and quantum mechanics to date.

Two basic systems of ideas that attempted to explain aspects of the universe involved fields. *EFT* and *SFT* implemented physical body (i.e., particle) and string fields to describe the behavior of facets of classical and quantum physics [21,26,52]. Hypothetically, both theories served as a basis for a new T*oE*.

Calculus was the field of mathematics heavily entrenched in physics; thus, methods for solving such problems incorporating differential equations were essential. Since the inception of calculus, individuals have established various techniques (i.e., perturbation theory or *PT*) for deriving the solutions to differential equations [20]. The latest method for solving physics problems, *GFT*, which this author discovered, involved using several truncated Laurent series of formal power series or generating functions to solve problems known to old physics (i.e., Boussinesq equation, Navier-Stokes problems, etc.) [3,63]. This novel method of accruing solutions to differential equations has a broad reach and is considered capable of solving a wide range of problems in mathematical physics [88].

This paper discusses *EFT* and *SFT* via *PT* as practical *ToEs*. This study is divided into several sections: section 2 provides concepts in perturbation, effective, and string field theories; the next section shows how *GFT* serves as a template for solving problems linked to *EFT* and *SFT*; Section 4 shows how *EFT* and *SFT* via *GFT* can be used in deriving solutions to three issues alluding to old and new physics; finally, the last section gives a quick review of *EFT* and *SFT* that shows *GFT* is a highly effective means for solving problems related to the two suppositions. In addition, the GFT claims at least two significant links are apparent between the two field theories and *QIT* in the last section.

2.) Basics of *EFT* and *SFT* after the consideration of *PT*, then *GFT*

2.1.) *PT* synopsis

In *PT*, an individual finds an approximate solution to a more straightforward defined problem [64]. The solution becomes more accurate as the approximate solution gains terms with decreasing parameters [64,89]. Ultimately, the approximate solution asymptotically approaches the exact solution as the number of terms added to the perturbation series approaches infinity [64]. In other words, the perturbation series becomes a formal power series over time [64].

2.2.) *EFT* synopsis

*EFT* encompasses an extensive array of fields in physics [21]. Therefore, *EFT* covers quantum, classical, and cosmological fields [21]. In this study, we will focus on quantum field theory, or QFT, and the cosmological aspects of *EFT*.

*QFT* combines quantum mechanics, classical field theory, and special relativity [4,24,28,47,60]. It is commonly applied to particle physics and, thus, essential in forming models within subatomic and condensed matter physics [4,24,28,47,60,92]. Since its advent in the 1920s and rebirth in the 1970s, *QFT* has had a prominent role in describing contemporary physics.

*QFT* was divided into at least three branches: quantum electrodynamics (*QED*), quantum flavor-dynamics (*QFD*), and quantum chromodynamics (*QCD*). *QED* was primarily developed by Dirac in 1927 and was built upon canonical quantization [9,32,40,44]. Also, it dealt with the interaction of fermionic and electromagnetic fields. *QFD* studied electroweak nuclear force, such as bosons $Z^0$ and $W^\pm$ activities, while *QCD* involved nuclear solid interactions, generally mediated gluon fields [12,23,47]. It is expected to find situations where certain branches, like *QED* and *QCD*, cross over or encroach on each other.

*PT* can be used to solve many problems in *QFT* [47,60,92]. The interaction between particle fields is treated as small perturbations in a free field. Finally, the integration of *PT* with *QFT* is called perturbative quantum field theory or *pQFT*.

*EFT* can also be applied to cosmology [14,15,29]. Cosmology is the study of the universe and its evolution. This realm of study includes the observations of large-scale structures (*LSS*) and the laws that dictate their behavior [95]. *PT* plays an active role in solving problems associated with cosmology: The combination of *PT* and this area of study is called cosmological perturbation theory [14,15,29]. The practical field theory of large-scale structures or *EFToLSS* is used to enhance the derivation of solutions in this area of science via novel *PT* methodologies [96].

2.3.) *SFT* synopsis

This supposition involves reformulating relativistic strings to *QFT* [12,24,28,60,92]. Unlike *QFT*, which treats an individual field of particles in excited states or quanta, string theory converts point-like particles into one-dimensional entities, referred to as strings. Thus, string field theory uses these one-dimensional strings to define excited particle fields.

Second quantization dictates the type of SFT to be considered, such as open and closed string fields [97]. In the standard model of particle physics, some open string fields are represented by gauge (gluons, photons, W and Z bosons) and quark or lepton fermions [90]. Fields that describe the scattering of open or closed strings are called open or closed SFT, respectively. However, if a field contains a combination of both open and closed strings, it is referred to as an open-closed SFT.

*SFT* possesses advantages over regular string theory. For instance, it permits the calculation of "off-shell" amplitudes and thus provides information about string scattering [52]. In addition to giving an individual the means to calculate the masses of particle systems that obey classical equations of motion, it can be used to determine particle systems that do the contrary. This process is called "off the mass shell" or off-shell of the mass hyperboloid via perturbation methods [52]. In other words, SFT can be used to define attributes of particles that do not follow the equations of motion in a classical sense (i.e., virtual particles, dark sector quanta, etc.) due to its innate ability to depict particles as string fields.

2.4.) Mathematical descriptions of effective and string fields via *GFT*

In *EFT,* an elementary field $\gamma$ is a formal power series of a function $f$. A physical body, such as a particle, can be designated as an exponential function $f$, assuming the auxiliary/characteristic equation is of the first order, takes the following form:

$$f(\xi) = c_1 e^{-\xi},$$

where $c_1$ is an arbitrary constant, and $\xi$ is the ansatz transformed variable. (Note: For *GFT* purposes, the auxiliary function is designated by the Greek letter $\phi$. However, this study will designate the auxiliary function with the alphanumeric letter $f$ due to particle physicists conventional use of $\phi$ for bosons.) The ansatz transformed variable $\xi$ for a (3+1) system was defined as

$$\xi = \alpha t + \beta_1 x + \beta_2 y + \beta_3 z.$$

On the other hand, the expression of a "brane," which can be designated by a sinusoidal wave function $f$, assuming the auxiliary/characteristic equation is of the second order, is expressed as

$$f(\xi) = c_1 \cos(\xi) + c_2 \sin(\xi),$$

or

$$f(\eta) = c_1 cosh(\eta) + i c_2 \sinh(\eta),$$

where $\eta$ is the **complex** ansatz transformed variable, or $\eta = i\xi$, and $c_2$ is another arbitrary constant.

The physical body and wave function $f$ leads to *EFT* and *SFT*, respectively.

Thus, the elementary physical body or string field $\gamma$ can be defined as

$$\gamma(\xi) = \sum_{k=0}^{\infty} p_k f(\xi)^k,$$

where $p_k$ is the $k$-th parameter/coefficient of the formal power series $\gamma$. On the contrary, its conjugate elementary qubit gate $\gamma^*$ is the following expression:

$$\gamma^*(\xi) = -\sum_{k=0}^{\infty} p_k f(-\xi)^k.$$

The formal power series $g$ and $g^*$ are also elementary effective or string fields. It is important to state an elementary string field is an object with an infinite number of branes; one of the branes acts as the "string" while the other branes serve as the "bulk" space or compactified dimensions of the universe [82,87]. Finally, a truncated Laurent series of the elementary effective or string field $g$ or $g^*$ raised by some power $j$ forms a transformed compound effective or string field $U$ if $p_k$ is combinatorial or trigonometric:

$$U(\xi) = \sum_{j=-n}^{n} q_j \gamma(\xi)^j$$

or

$$U^*(\xi) = -\sum_{j=-n}^{n} q_j \gamma^*(-\xi)^j,$$

where $n$ is the absolute integer value of the truncated power and $q_j$ is the $j$-th parameter/coefficient and power. Regarding string field theory, $n$ is equal to the supersymmetry level $N$. Ultimately, the difference between PT and GFT is one method builds upon an approximate solution while the other narrows the general solution to derive the exact solution. The former method would require many steps to achieve its objective due to adding higher-order terms. At the same time, the latter only needs a few steps, like solving the parameter/coefficient and arbitrary constants.

### 3.) *GFT* as a new mathematical basis for garnering solutions in *EFT or SFT*

### 3.1.) *GFT* and general solutions to particles

*GFT*, with some modification, was a method implemented to find the solution of [non]linear PDEs. Its transformed general solution $U$ comprised a Laurent series set of combinatorial or trigonometric-based generating functions [3]. One should consider the transformed general solution U as a transformed effective or string field for our purposes. After one assessed the maximal and minimal power $n$, through which the Laurent series of various types of elementary effective or string field $\gamma$ or $\gamma^*$, their conjugate, is eventually truncated, (s)he plugs in the predefined function $f$ into the transformed general solutions for effective or string fields $\phi_l$, $\phi_l^{\mu}$, $\phi_l^{\mu*}$, $\boldsymbol{\phi}_l$, $\Psi_m$, and $\Psi_m^*$:

$$\Phi_l(\xi) = \sum_{i=1}^{2}\sum_{j=-n}^{n}\left(al_{ij}\left(\sum_{k=0}^{\infty}2f(\xi)^k S_k(0)^i\right)^j + bl_{ij}\left(\sum_{k=0}^{\infty}2C_k(0)^i f(\xi)^k\right)^j\right),$$

$$\Phi_l^{\mu}(\xi) = e^{-\tau}\sum_{i=1}^{2}\sum_{j=-n}^{n}\left(al_{ij}^{\mu}\left(\sum_{k=0}^{\infty}2f(\xi)^k S_k(0)^i\right)^j + bl_{ij}^{\mu}\left(\sum_{k=0}^{\infty}2C_k(0)^i f(\xi)^k\right)^j\right),$$

$$\Phi_l^{\mu*}(\xi) = -e^{\tau}\sum_{i=1}^{2}\sum_{j=-n}^{n}\left(al_{ij}^{\mu}\left(\sum_{k=0}^{\infty}2f(-\xi)^k S_k(0)^i\right)^j + bl_{ij}^{\mu}\left(\sum_{k=0}^{\infty}2C_k(0)^i f(-\xi)^k\right)^j\right),$$

$$\boldsymbol{\Phi}_l(\xi) = \Phi_l^{\mu\nu}(\xi) = \sum_{i=1}^{2}\sum_{j=-n}^{n}\left(\boldsymbol{al}_{ij}\left(\sum_{k=0}^{\infty}2f(\xi)^k S_k(0)^i\right)^j + \boldsymbol{bl}_{ij}\left(\sum_{k=0}^{\infty}2C_k(0)^i f(\xi)^k\right)^j\right),$$

$$\Psi_m(\xi) = e^{-\tau} \sum_{i=1}^{2} \sum_{j=-n}^{n} \left( cm_{ij} \left( \sum_{k=0}^{\infty} 2f(\xi)^k S_k(0)^i \right)^j + dm_{ij} \left( \sum_{k=0}^{\infty} 2C_k(0)^i f(\xi)^k \right)^j \right),$$

and

$$\Psi_m^*(\xi) = -e^{\tau} \sum_{i=1}^{2} \sum_{j=-n}^{n} \left( cm_{ij} \left( \sum_{k=0}^{\infty} 2f(-\xi)^k S_k(0)^i \right)^j + dm_{ij} \left( \sum_{k=0}^{\infty} 2C_k(0)^i f(-\xi)^k \right)^j \right),$$

where $\Phi_l$ are the transformed scalar bosonic effective or string fields, $\Phi_l^{\mu}$ and $\Phi_l^{\mu *}$ are transformed vector bosonic effective or string fields, $\boldsymbol{\Phi}_l$ ($= \Phi_l^{\mu\nu}$) are the transformed tensor bosonic effective or string fields, $\Psi_m$ and $\Psi_m^*$ are the transformed fermionic effective or string fields, and $\tau$ is a specific ansatz transformed time variable or

$$\tau = \alpha_\tau t.$$

The parameter/coefficient $p_k$ was defined as:

$$p_k = 2S_k(0)^i,$$

or

$$p_k = 2C_k(0)^i.$$

Note: the square root of the $k$-th Fibonacci number at/about zero, or $S_k(0)$, was

$$S_k(0) = \sin\left(\frac{\pi k}{2}\right),$$

while the $k$-th Chebyshev $T$ or $U$ (not to be confused with the transformed general solution of the effective or string field) number at/about zero, or $C_k(0)$, was

$$C_k(0) = \cos\left(\frac{\pi k}{2}\right).$$

For this article, the parameter/coefficient $q_j$ was either $al_{ij}$ or $bl_{ij}$ for bosonic effective or string field while the parameter/coefficient $q_j$ was either $cm_{ij}$ or $dm_{ij}$ were used for the fermionic effective or string fields, where $l = 1, 2, \ldots, n_l$ and $m = 1, 2, \ldots, n_m$.

Ultimately, mesonic effective or string fields exhibit as a hyperbolic secant function raised by some power. In contrast, gauge bosonic and fermionic effective or string fields can be expressed as a logistic function raised by some power. Figure 1 claims odd integer spin bosonic or half-spin fermionic string

fields possess open strings whose vibratory modes satisfy solely Dirichlet boundary conditions; the strings, called D-branes, about such fields have endpoints that are fixed in spacetime [83,86,87]. Thus, *GFT* implies the odd integer spin bosonic or half-spin fermionic string field's $cm_{ij}$ are equal to *null*. On the contrary, Figure 1 suggests that other string fields, such as fields that comprise even integer spin bosonic strings (excluding those which describe scalar particles), can be closed or open strings [76,85,86,87]. They possess vibratory modes that satisfy solely Neumann boundary conditions; these fields have string endpoints that are free to roam in spacetime [76,85,86,87]. In short, open string fields that behave like vector bosons, such as photons and gluons, have $al_{ij}^{\mu}$ equal to *null* and $bl_{ij}^{\mu}$ exist as a 4 X 1 vector while open string fields that behave like pseudoscalar mesons have $bl_{ij}$ equal to *null*. On the contrary, open string fields that behave like vector bosons has $\boldsymbol{bl}_{ij}$ equal to *null* and $\boldsymbol{al}_{ij}$ exist as a 4 X 4 matrix. Also, the endpoints of the form open string fields are fermions []. (Note: Scalar Higgs particles or "bundles" can be described by branes that satisfy both Dirichlet and Neumann boundary conditions; these strings are called Nahm-Douglass or ND-branes [].)
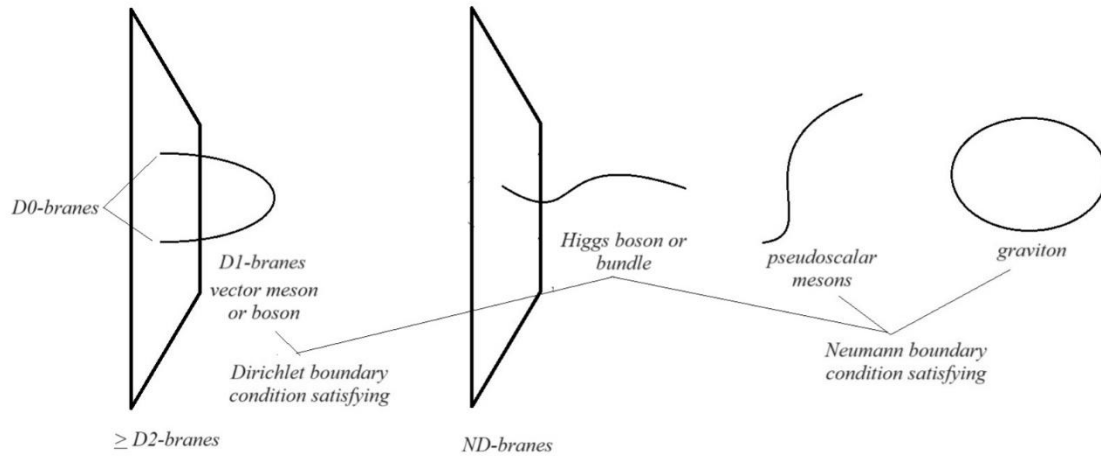


Figure 1: examples of relativistic strings.

3.2.) On- and off-shell rest mass assessment via renormalization

The expression for "self-interacting" renormalization within a volume V was:

$$m_U = \frac{1}{2} \int |U(\xi)U^*(\xi)| dV,$$

where $m_U$ was the mass-energy equivalence for a compound effective or string field U and its conjugate $U^*$. Assuming the spherical volume, using Manhattan/taxicab-like distance $\xi$, for compound effective or string field U was equal to the following expression [75]:

$$V = \frac{\pi \xi^3}{6},$$

the formula for renormalization became the following:

$$m_U = |2 \int_0^\infty \frac{1}{4} \pi \xi^2 U(\xi) U^*(\xi) d\xi|,$$

or

$$m_U = |\int_{-\infty}^\infty \frac{1}{4} \pi \xi^2 U(\xi) U^*(\xi) d\xi|.$$

If the field pertains to either a vector or tensor boson, an individual should divide the final expression by four.


4.) Examples


This section explores and expands upon two basic Lagrangian equations needed to solve the effective and string fields associated with this paper. One equation is the inhomogeneous quantum telegraph for either a fermion or vector boson field is a Dirac-like state with its complex conjugate field, for instance:

$$S[\psi] = h \int dx^4 \left( \partial_\mu \psi \, \partial^\mu \psi^\dagger + g_2 |\psi|^2 \right)$$

or

$$S[\phi^\mu] = h \int dx^4 \left( \partial_\mu \phi^\mu \, \partial^\mu \phi^{\mu*} + g_2 |\phi^\mu|^2 \right).$$

while the inhomogeneous Klein-Gordon equations for a scalar and a tensor boson are

$$S[\phi] = \int dx^4 \left( \partial_\mu \phi \, \partial^\mu \phi + \frac{1}{2} c^2 M^2 \phi^2 + L_{interactions} \right)$$

and

$$S[\boldsymbol{\phi}] = \int dx^4 \left( \partial_\mu \boldsymbol{\phi} \, \partial_\nu \boldsymbol{\phi} + \partial_\nu \boldsymbol{\phi} \, \partial_\mu \boldsymbol{\phi} + \frac{1}{2} c^2 M^2 \boldsymbol{\phi}^2 + L_{interactions} \right),$$

respectively. The values $g_2$ is an interaction constant generally equal to unity while the value $h$ equals a positive integer if the particle/string is a present initially or negative integer if the particle/string is a by-product [16,44]. Also, the number $h$ equals the coefficient associated with a specific term present in the inhomogeneous Klein-Gordon Lagrangian. The principles of least action have several bold Lagrangian terms: $\phi$ represents the set of scalar bosonic effective or string fields, $\phi^\mu$ represents the set of vector-based bosonic effective or string fields, $\boldsymbol{\phi}$ represent the set of tensor bosonic effective or string fields. On the other hand, $\psi_i$ and $\psi_f$ signify the initial and final set of fermionic effective or string fields, respectively. Also, $*$ and $\dagger$ symbolize the conjugate fields.

Supplementary material included with this study are Mathematica® spreadsheets of the following examples of *EFT* solved by *GFT*. Also, the supplementary material contains an instance of linearized gravitational waves solved via *GFT*. **Finally, the results between *EFT* and *SFT* were equal!**

4.1.) The cause of the deviation in the muon g-2 experiment

The muon g-2 experiment is an attempt to measure a muon's magnetic dipole moment or g-factor accurately, and Fermilab is currently conducting it [17,55]. The experiment involves injecting the muons from the decay of pseudoscalar pions into a storage ring and then taking measurements of the muon's g-factor. Theoretically, the strength of the magnetic dipole moment is supposed to be exactly two. Any deviation from this, the latter value claims there are likely additional particles in the standard model in the storage ring.

So far, the laboratory has experimented two times. The results of the first two experiments suggested that the anomalous magnetic moment, which is $a_\mu = \frac{g-2}{2}$, had deviated by a factor of 0.00116592 [33]. Ultimately, this data implied there was likely at least one additional particle that is a by-product of a pion but not currently listed in the standard model.

Assuming the additional particle is a vector boson $\phi_f^\mu$ and the Lagrangian term for interactions is as follow:

$$L_{interactions}[\phi_i, \phi_f^\mu, \psi_{f1}, \psi_{f2}] = g_1\phi_i\left(|\psi_{f1}|^2 + |\psi_{f2}|^2 + 2|\phi_f^\mu|^2\right)$$

then the principle of least action for pion decay is as follows:

$$S[\phi_i, \phi_f^\mu, \psi_{f1}, \psi_{f2}] =$$
$$\int dx^4 \left(-g_1\phi_i\left(|\psi_{f1}|^2 + |\psi_{f2}|^2 + 2|\phi_f^\mu|^2\right) - g_2\left(|\psi_{f1}|^2 + |\psi_{f2}|^2 + 2|\phi_f^\mu|^2\right) - 2\,\partial_\mu\phi_f^\mu\,\partial^\mu\phi_f^{\mu*}\right.$$
$$\left. - \partial_\mu\psi_{f1}\,\partial^\mu\psi_{f1}^\dagger - \partial_\mu\psi_{f2+}\,\partial^\mu\psi_{f2}^\dagger + \frac{1}{2}c^2 M^2\phi_i^2 + \frac{1}{2}\partial_\mu\phi_i\,\partial^\mu\phi_i\right)$$

.

Claiming $\phi_i$ is the pionic effective or string field, $\phi_f^\mu$ is the unknown bosonic effective or string field, and $\psi_{f1}$ is one of the valence muonic effective or string fields while $\psi_{f2}$ is the valence muon neutrino-based effective or string field associated in the decay of the pion, the functional derivative of the principle of least action yields at least four transformed Hamiltonian equations:

$$\alpha i\Psi_{f1\xi} + \alpha_\tau i\Psi_{f1\tau} - \frac{\alpha_\tau^2}{c^2\Psi_{f1\tau\tau}} - \frac{\alpha\alpha_\tau}{c^2\Psi_{f1\xi\tau}} - \left(\frac{\alpha^2}{c^2} - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right)\Psi_{f1\xi\xi} = g_1\Phi_i\Psi_{f1} + g_2\Psi_{f1},$$

$$\alpha i\Psi_{f2\xi} + \alpha_\tau i\Psi_{f2\tau} - \alpha_\tau^2/c^2\Psi_{f2\tau\tau} - \alpha\alpha_\tau/c^2\Psi_{f2\xi\tau} - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right)\Psi_{f2\xi\xi} = g_1\Phi_i\Psi_{f2} +$$
$$g_2\Psi_{f2},$$

$$\left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right)\Phi_{i\xi\xi} - c^2 M^2\Phi_i = g_1\left(\Psi_{f1}\Psi_{f1}^\dagger + \Psi_{f2}\Psi_{f2}^\dagger + 2\Phi_f^*\Phi_f\right),$$

and

$$\alpha i\Phi_{f\xi}^\mu + \alpha_\tau i\Phi_{f\tau}^\mu - \alpha_\tau^2/c^2\Phi_{f\tau}^\mu - \alpha\alpha_\tau/c^2\Phi_{f\xi\tau}^\mu - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right)\Phi_{f\xi\xi}^\mu = g_1\Phi_i\Phi_f^\mu + g_2\Phi_f^\mu.$$

After solving for any arbitrary parameters/coefficients and constants whenever possible and setting $g_1$ and $g_2$ to *unity*, then assuming the self- and mixing interaction mass-energy equivalence equation, one can derive the following mass-energies:

$$m_{\psi_{\text{muon}}} = \frac{1}{72}\pi(-6 + \pi^2)|d1_{12}|^2,$$

$$m_{\psi_{\text{muon neutrino}}} = \frac{1}{288}\pi(-6 + \pi^2)|9\,M^4 - 4\left(2b2_{12}^2 + d1_{12}^2\right)|,$$

$$m_{\phi_{pion}} = \frac{1}{32}\pi(-6 + \pi^2)|M|^4,$$

and

$$m_{\phi_f^\mu} = \frac{1}{72}\pi(-6 + \pi^2)|b2_{12}|^2.$$

After using the three most former expressions and the known rest masses for the particles or relativistic strings of interest, one can solve for the arbitrary parameters/coefficients $b2_{12}$, $d1_{12}$, and constant $M$ using known values for items of interest. Next, the rest mass of the unknown particle or relativistic string is calculated to be $1.7085 \, X \, 10^7$ eV or 17 MeV.


### 4.2.) Glueball estimations


A glueball is a hypothetical particle solely comprised of gluons [58]. Theories in particle physics suggest contemporary colliders should detect them. Even though there is anecdotal evidence pointing to the existence of these particles, they have not been explicitly identified [47,58].


Since a gluon is a vector boson $\phi_i^\mu$ and the Lagrangian terms for interactions of the entanglement type is as follow:

$$L_{interactions}[\phi_f, \phi_i^\mu, \psi_{i1}, \psi_{i2}] = g_1\left(|\phi_i^\mu\psi_{i1}|\phi_f + |\phi_i^\mu\psi_{i2}|\phi_f\right),$$

then the principle of least action for the decay of a meson should be as follows:

$$S[\phi_f, \phi_i^\mu, \psi_{i1}, \psi_{i2}] =$$

$$\int dx^4 \left( -g_1 \left( |\phi_i^\mu \psi_{i1}| \phi_f + |\phi_i^\mu \psi_{i2}| \phi_f \right) - g_2 \left( |\psi_{i1}|^2 + |\psi_{i2}|^2 + |\phi_i^\mu|^2 \right) + \frac{1}{2} c^2 M^2 \phi_f^2 \right.$$

$$\left. + \partial_\mu \phi_i^\mu \, \partial^\mu \phi_i^{\mu*} - \partial_\mu \psi_{i1} \, \partial^\mu \psi_{i1}^\dagger - \partial_\mu \psi_{i2} \, \partial^\mu \psi_{i2}^\dagger - \frac{1}{2} \partial_\mu \phi_f \, \partial^\mu \phi_f \right)$$

.

Suggesting $\phi_f$ is the mesonic effective or string field, $\Phi_i$ is the gluonic effective or string field, and $\Psi_{i1}$ is one of the valence quark effective or string fields while $\Psi_{i2}$ is the other valence quark effective or string field present in the decay of the meson, the functional derivative of the principle of least action yields at least four transformed Hamiltonian equations:

$$\alpha i \Psi_{i1\xi} + \alpha_\tau i \Psi_{i1\tau} - \alpha_\tau^2/c^2 \Psi_{i1\tau\tau} - \alpha \alpha_\tau/c^2 \Psi_{i1\xi\tau} - \left( \alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2) \right) \Psi_{i1\xi\xi} = g_1 \Phi_f \Psi_{i1} +$$
$$g_2 \Psi_{i1},$$

$$\alpha i \Psi_{i2\xi} + \alpha_\tau i \Psi_{i2\tau} - \alpha_\tau^2/c^2 \Psi_{i2\tau\tau} - \alpha \alpha_\tau/c^2 \Psi_{i2\xi\tau} - \left( \alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2) \right) \Psi_{i2\xi\xi} = g_1 \Phi_f \Psi_{i2} +$$
$$g_2 \Psi_{i2},$$

$$\left( \alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2) \right) \Phi_{f\xi\xi} - c^2 M^2 \Phi_i = g_1 \left( \Phi_i^{\mu*} \Psi_{i1} + \Phi_i^\mu \Psi_{i1}^\dagger + \Phi_i^{\mu*} \Psi_{i2} + \Phi_i^\mu \Psi_{i2}^\dagger \right),$$

and

$$\alpha i \Phi_{i\xi}^\mu + \alpha_\tau i \Phi_{i\tau}^\mu - \alpha_\tau^2/c^2 \Phi_{i\tau}^\mu - \alpha \alpha_\tau/c^2 \Phi_{i\xi\tau}^\mu - \left( \alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2) \right) \Phi_{i\xi\xi}^\mu = g_1 \Phi_f \Phi_i^\mu + g_2 \Phi_i^\mu.$$

After solving for any arbitrary parameters/coefficients and constants whenever possible and setting $g_1$ and $g_2$ to *unity*, and using the self- and mixing interaction mass-energy equivalence equation, one can derive the following mass-energies:

$$m_{\psi_{i1}} = \frac{1}{72} \pi (-6 + \pi^2) |d1_{12}|^2,$$

$$m_{\psi_{i2}} = \frac{\pi(-6+\pi^2) \left| \frac{\left( 9\,M^4 + 4b2_{12} d1_{12} \right)^2}{b2_{12}^2} \right|}{1152},$$

$$m_{\phi_f} = \frac{1}{32} \pi (-6 + \pi^2) |M|^4,$$

and

$$m_{\phi_i^\mu} = \frac{1}{72} \pi (-6 + \pi^2) |b2_{12}|^2.$$

One may also assume that the gluonic effective or string field $\phi_i^\mu$ forms a glueball. In other words, the mass of a glueball would constitute the mass-energy equivalence of gluonic effective or string field. After using the three most former expressions and the known rest masses for the particles or relativistic strings

of interest, one can solve for the arbitrary parameters/coefficients $b2_{12}$, $d1_{12}$, and constant $M$ using known values for items of interest. Check the table for calculations of glueballs.

A table of predicted glueball masses derived from various meson decays is featured below:

| | Quark 1/$m_{\psi_{i1}}$ (eV) | Quark 2/$m_{\psi_{i2}}$ (eV) | Meson/$m_{\phi_f}$ (eV) | Glueball/$m_{\phi_i^\mu}$ (eV) |
|---|---|---|---|---|
| charged pion | 2.20*10⁷ | 4.70*10⁶ | 1.40*10⁸ | 1.49*10⁹ [65] |
| neutral pion (pair) | 2.20*10⁶ | 4.70*10⁶ | 1.35*10⁸ | 1.39*10⁹ [65] |
| neutral kaon | 4.70*10⁶ | 9.60*10⁷ | 4.98*10⁸ | 1.72*10⁹ [65] |
| J/psi meson | 1.28*10⁹ | 1.28*10⁹ | 3.10*10⁹ | 1.87*10⁹ [66,67] |

4.3.) Assessment of tetraquarks mass via decay by-products.

Recently, LHC at CERN found that the four charmed tetraquark $c\bar{c}c\bar{c}$, which had a rest mass 6.9 GeV decayed into the vector meson and sigma glueball $\sigma$ [98,99,100].

Assuming the tetraquark is a scalar boson-like entity $\phi_i$ and the Lagrangian term for entanglement interaction is as follow:

$$L_{interactions}[\phi_i, \phi_{f1}^\mu, \phi_{f2}^\mu] = g_1\left(\left|\phi_{f1}^\mu \phi_{f2}^\mu\right|\phi_i\right),$$

then the principle of least action for $c\bar{c}c\bar{c}$ decay is as follows:

$$S[\phi_i, \phi_{f1}^\mu, \phi_{f2}^\mu] =$$
$$\int dx^4 \left(-g_1\left(\left|\phi_{f1}^\mu \phi_{f2}^\mu\right|\phi_i\right) - g_2\left(\left|\phi_{f1}^\mu\right|^2 + \left|\phi_{f2}^\mu\right|^2\right) - \partial_\mu \phi_{f1}^\mu \partial^\mu \phi_{f1}^{\mu*} - \partial_\mu \phi_{f2}^\mu \partial^\mu \phi_{f2}^{\mu*}\right.$$
$$\left. + \frac{1}{2}c^2 M^2 \phi_i^2 + \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i\right)$$

One may also deem that the tetraquark effective or string field $\phi_i$ is comprised the vector meson $J/\psi$ or glueball $\sigma$ effective or string fields $\phi_{f1}^\mu$ and $\phi_{f2}^\mu$, respectively. The functional derivatives of the Lagrangian yields three evolution equations:

$$\left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right)\Phi_{i\xi\xi} - c^2 M^2 \Phi_i = g_1\left(\Phi_{f1}^{\mu*}\Phi_{f2}^\mu + \Phi_{f1}^\mu \Phi_{f2}^{\mu*}\right),$$

$$\alpha i \Phi_{f1\xi}^\mu + \alpha_\tau i \Phi_{f1\tau}^\mu - \alpha_\tau^2/c^2 \Phi_{f1\tau\tau}^\mu - \alpha\alpha_\tau/c^2 \Phi_{f1\xi\tau}^\mu - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right)\Phi_{f1\xi\xi}^\mu = g_1 \Phi_i \Phi_{f1}^\mu + g_2 \Phi_{f1}^\mu,$$

and

$$\alpha i \Phi^\mu_{f2\xi} + \alpha_\tau i \Phi^\mu_{f2\tau} - \alpha_\tau^2/c^2 \Phi^\mu_{f2\tau\tau} - \alpha\alpha_\tau/c^2 \Phi^\mu_{f2\xi\tau} - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right) \Phi^\mu_{f2\xi\xi} = g_1 \Phi_i \Phi^\mu_{f2} + g_2 \Phi^\mu_{f2}.$$

After solving for any arbitrary parameters/coefficients and constants whenever possible and setting $g_1$ and $g_2$ to *unity*, then assuming the self- and mixing interaction mass-energy equivalence equation, one can derive the following mass-energies:

$$m_{c\bar{c}c\bar{c}} = \frac{1}{32}\pi(-6 + \pi^2)|M|^4,$$

$$m_{J/\psi} = \frac{1}{72}\pi(-6 + \pi^2)|b2_{12}|^2,$$

and

$$m_\sigma = \frac{9\pi(-6+\pi^2)|\frac{M^8}{b2_{12}^2}|}{2048}.$$

After using the two latter expressions and the known rest masses for the particles or relativistic strings of interest, one can solve for the arbitrary parameters/coefficients $b2_{12}$ and constant $M$ using known values for the vector meson $J/\psi$ or glueball $\sigma$. Plugging in the value $M$ into the former expression claims, the mass of the four charm tetraquark is $6.90*10^9$ eV.


### 4.4.) Photon analysis linked to a binary black hole merger


Black holes (BH) were objects in the universe with much speculation and excellent study. In the 18th century, Pierre-Simon Laplace and John Michell proposed that the universe possessed objects whose gravitational fields were so intense that light could not escape them [31]. Two centuries later, David Finkelstein and Karl Schwarzschild could make primitive solutions that were used to define such entities [69,70,93] initially. It was not until the 1960s that BH became a regular prediction in the general theory of relativity [37].


While undergoing mergers in spacetime, BH was known to emit gravitational waves (GW) and possibly some forms of light [49,94]. In 1916, Albert Einstein postulated that BH created ripples in spacetime in his great work, the General Theory of Relativity [46]. Before Einstein pinpointed the primary source of GW, Poincare and Heaviside stated there were gravity's equivalent to electromagnetic waves [71,72]. In 2015, the first GW was detected by LIGO gravitational wave detectors. Finally, Atura Tanikawa and associates claimed $\gamma$-ray bursts were also emitted during the BH merger [74].


Quantum entanglement (QE) may be defined as the primary mechanism which allowed BHs to merge. Leonard Susskind and Juan Maldacena generated a conjecture stating BH were like two entangled particles, or Einstein-Podolsky-Rosen pair, connected by a wormhole or Einstein-Rosen bridge [13]. Thus, they established the following relationship: ER = EPR. For this study, we used the previous equation to signify BH-QE.

In the first section of this study, we generate a Lagrangian to define BH-QE. Upon functional differentiation of this Lagrangian, we derive three quantum telegraph equations and two inhomogeneous nonlinear Klein-Gordon equations, or QT-KG. Assuming the three quantum telegraph equations described the two BH associated via ER = EPR and the merged BH while the inhomogeneous Klein-Gordon equations represented GW and photons, we used the generating function technique (GFT) to solve for the three BH, GW, and photons. Then, we try to predict the mass equivalents for photon emission, given we know the values of the three BH and GW. Ultimately, we concluded that population III stars were the source of BH mergers since they produced γ-ray photons predicted via BH-QE system of equations.
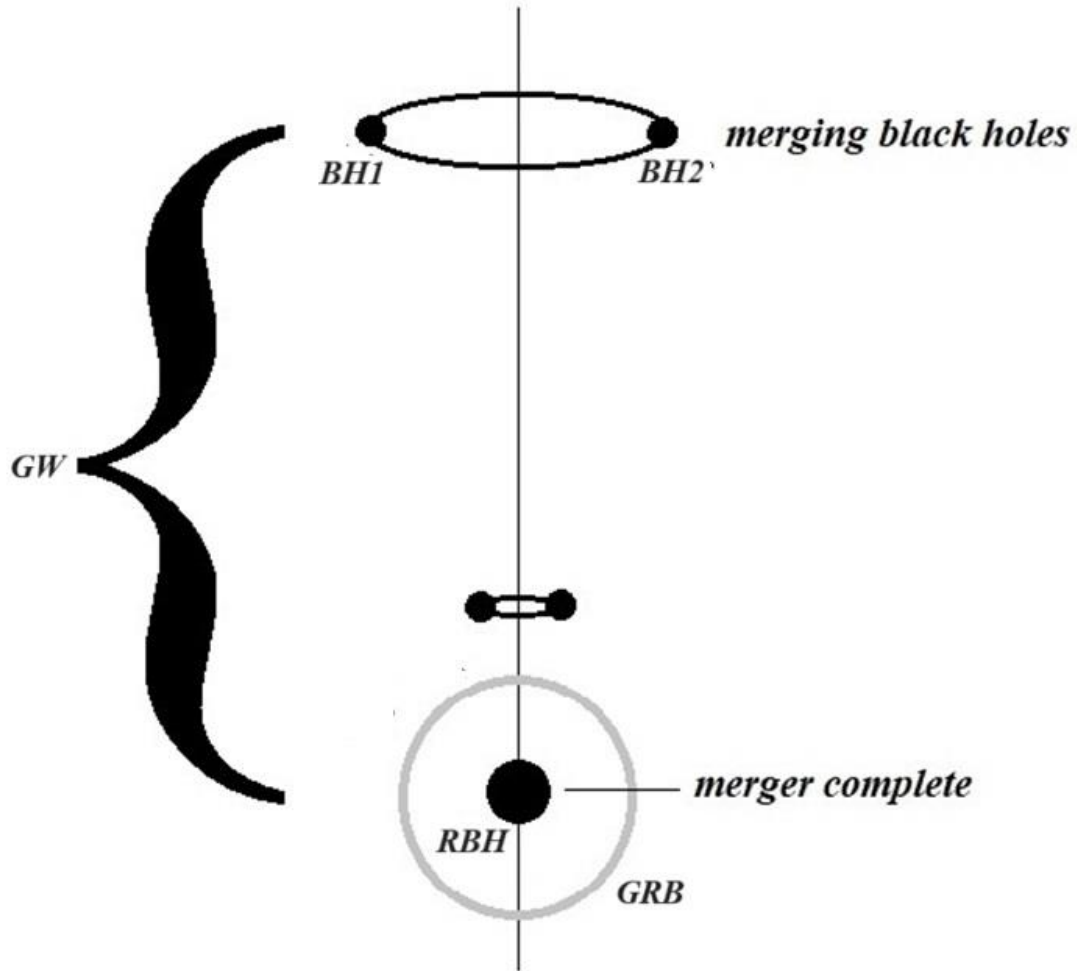


Assuming black holes are vector boson-like and the Lagrangian term for interactions, such as entanglement, is as follow:

$$L_{interactions}[\phi_{i1}^{\mu}, \phi_{i2}^{\mu}, \phi_f^{\mu}, \boldsymbol{\Phi}_G, \phi_\gamma^{\mu}] = g_1 \boldsymbol{\Phi}_G \left( \left| \phi_{i1}^{\mu} \phi_\gamma^{\mu} \right| + \left| \phi_{i2}^{\mu} \phi_\gamma^{\mu} \right| - \left| \phi_f^{\mu} \right|^2 \right),$$

then the principle of least action for the merger of a binary black hole system is as follows:

$$S\left[\phi_{i1}^{\mu}, \phi_{i2}^{\mu}, \phi_{f}^{\mu}, \boldsymbol{\phi}_{G}, \phi_{\gamma}^{\mu}\right] =$$

$$\int dx^4 \left( g_1 \boldsymbol{\phi}_G \left( |\phi_{i1}^{\mu} \phi_{\gamma}^{\mu}| + |\phi_{i2}^{\mu} \phi_{\gamma}^{\mu}| - \left|\phi_{f}^{\mu}\right|^2 \right) + g_2 \boldsymbol{\phi}_G (|\phi_{i1}^{\mu}| + |\phi_{i2}^{\mu}| + |\phi_{\gamma}^{\mu}| - |\phi_{f}^{\mu}|) + \right.$$

$$+ \partial_\mu \phi_{i1}^{\mu} \partial^\mu \phi_{i1}^{\mu*} + \partial_\mu \phi_{i2}^{\mu} \partial^\mu \phi_{i2}^{\mu*} + \partial_\mu \phi_{\gamma}^{\mu} \partial^\mu \phi_{\gamma}^{\mu} - \partial_\mu \phi_{f}^{\mu} \partial^\mu \phi_{f}^{\mu*}$$

$$\left. + \frac{1}{2} c^2 M^2 \boldsymbol{\phi}_G^2 + \partial_\mu \boldsymbol{\phi}_G \partial_\nu \boldsymbol{\phi}_G + \partial_\nu \boldsymbol{\phi}_G \partial_\mu \boldsymbol{\phi}_G \right)$$

Deeming $\boldsymbol{\phi}_G$ is the gravitational wave/gravitonic effective or string field involved in the merger, $\phi_\gamma$ is the photonic effective or string field, $\phi_{i1}^{\mu}$ is the first black hole ,and $\phi_{i2}^{\mu}$ is the second black hole effective or string fields involved in the merger, and $\phi_f$ is the residual black hole effective or string field, the functional derivative of the principle of least action yields at least six transformed Hamiltonian equations:

$$\alpha i \Phi_{i1\xi}^{\mu} + \alpha_\tau i \Phi_{i1\tau}^{\mu} - \alpha_\tau^2/c^2 \Phi_{i1\tau\tau}^{\mu} - \alpha\alpha_\tau/c^2 \Phi_{i1\xi\tau}^{\mu} - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right) \Phi_{i1\xi\xi}^{\mu} = g_1 \boldsymbol{\phi}_G \Phi_{i1}^{\mu} + g_2 \Phi_{i1}^{\mu},$$

and

$$\alpha i \Phi_{i2\xi}^{\mu} + \alpha_\tau i \Phi_{i2\tau}^{\mu} - \alpha_\tau^2/c^2 \Phi_{i2\tau\tau}^{\mu} - \alpha\alpha_\tau/c^2 \Phi_{i2\xi\tau}^{\mu} - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right) \Phi_{i2\xi\xi}^{\mu} = g_1 \boldsymbol{\phi}_G \Phi_{i2}^{\mu} + g_2 \Phi_{i2}^{\mu},$$

$$\alpha i \Phi_{f\xi}^{\mu} + \alpha_\tau i \Phi_{f\tau}^{\mu} - \alpha_\tau^2/c^2 \Phi_{f\tau\tau}^{\mu} - \alpha\alpha_\tau/c^2 \Phi_{f\xi\tau}^{\mu} - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right) \Phi_{f\xi\xi}^{\mu} = g_1 \boldsymbol{\phi}_G \Phi_{f}^{\mu} + g_2 \Phi_{f}^{\mu},$$

$$\alpha i \Phi_{\gamma\xi}^{\mu} + \alpha_\tau i \Phi_{\gamma\tau}^{\mu} - \alpha_\tau^2/c^2 \Phi_{\gamma\tau\tau}^{\mu} - \alpha\alpha_\tau/c^2 \Phi_{\gamma\xi\tau}^{\mu} - \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right) \Phi_{\gamma\xi\xi}^{\mu} = g_1 \boldsymbol{\phi}_G \Phi_{\gamma}^{\mu} + g_2 \Phi_{\gamma}^{\mu},$$

and

$$4 \boldsymbol{AT}\, \boldsymbol{\phi}_{G\xi\xi} + \left(\alpha^2/c^2 - (\beta_1^2 + \beta_2^2 + \beta_3^2)\right) \boldsymbol{\phi}_{G\xi\xi} \delta_{ij} - c^2 M^2 \boldsymbol{\phi}_G \delta_{ij} = g_1 \left((\Phi_{\gamma}^{\mu*} \Phi_{i1}^{\mu} + \Phi_{\gamma}^{\mu} \Phi_{i1}^{\mu*} + \Phi_{\gamma}^{\mu*} \Phi_{i2}^{\mu} + \Phi_{\gamma}^{\mu} \Phi_{i2}^{\mu*}) - \Phi_{f}^{\mu} \Phi_{f}^{\mu*}\right) \delta_{ij},$$

$$\text{where } \boldsymbol{AT} = \begin{pmatrix} \frac{\alpha^2}{c^2} & 0 & 0 & 0 \\ 0 & -\beta_1^2 & 0 & 0 \\ 0 & 0 & -\beta_2^2 & 0 \\ 0 & 0 & 0 & -\beta_3^2 \end{pmatrix} \text{ and } \delta_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$\boldsymbol{AT}$ is the Ansatz transform tensor and $\delta_{ij}$ is Kronecker delta. Before proceeding to the next steps, one must apply a trace to the five equations. After solving for any arbitrary parameters/coefficients whenever possible and constants, setting $g_1$ and $g_2$ to *unity*, and applying the mass-energy equivalence equation to one element of the transformed solutions, assume the following relationships for determining the mass-energy equivalents of fermion and bosonic effective or string fields associated with the BH-QE system of equations were true:

$$m_{\phi_{i1}^{\mu}} = \frac{1}{12}\pi \left(2 - H_{\frac{1}{2}}^{(2)}\right) |b3_{12}|^2$$

$$m_{\phi_{i2}^{\mu}} = \frac{1}{768}\pi\left(2 - H_{\frac{1}{2}}^{(2)}\right)\left|\frac{\left(9\,m^4 - 8b2_{12}b3_{12} + 4\,b5_{12}{}^2\right)^2}{b2_{12}{}^2}\right|,$$

$$m_{\phi_f^{\mu}} = \frac{1}{12}\pi\left(2 - H_{\frac{1}{2}}^{(2)}\right)|b5_{12}|^2$$

$$m_{\phi_G} = \frac{1}{16}\pi\left(2 - H_{\frac{1}{2}}^{(2)}\right)|M|^4,$$

and

$$m_{\phi_\gamma^{\mu}} = \frac{1}{12}\pi\left(2 - H_{\frac{1}{2}}^{(2)}\right)|b2_{12}|^2$$

After using the four most former expressions and the known rest masses for the large-scale structures or relativistic strings of interest, one can solve for the arbitrary parameters/coefficients $b2_{12}$, $b3_{12}$, $b5_{12}$ and constant $M$ using known values for items of interest. Ultimately, (s)he obtained the following table of photons for the first several GWs detected by LIGO:

| Specific GW | BH 1/$m_{\phi_{i1}}$ (solar masses) | BH 2/$m_{\phi_{i2}}$ (solar masses) | Residual BH/$m_{\phi_f}$ (solar masses) | GW/$m_{\phi_G}$ (solar masses) | Photons/$m_{\phi_\gamma^{\mu}}$ (erg) |
|---|---|---|---|---|---|
| GW150914 | 35.6 | 30.6 | 63.1 | 3.1 | $2.62*10^{57}$ |
| GW151012 | 23.3 | 13.6 | 35.7 | 1.5 | $1.30*10^{56}$ |
| GW151226 | 13.7 | 7.7 | 20.5 | 1.0 | $6.65*10^{55}$ |
| GW170104 | 31.0 | 20.1 | 49.1 | 2.2 | $2.67*10^{56}$ |
| GW170608 | 10.9 | 7.6 | 17.8 | 0.9 | $1.31*10^{56}$ |
| GW170729 | 50.6 | 34.3 | 80.3 | 4.8 | $5.31*10^{56}$ |
| GW170809 | 35.2 | 23.8 | 56.4 | 2.7 | $3.80*10^{56}$ |
| GW170814 | 30.7 | 25.3 | 53.4 | 2.6 | $1.35*10^{57}$ |
| GW170818 | 35.4 | 26.7 | 59.4 | 2.7 | $7.24*10^{56}$ |
| GW170823 | 39.5 | 29.0 | 65.4 | 3.3 | $6.74*10^{56}$ |

5.) Conclusion

Via *EFT* and *SFT*, one will likely identify the culprit causing the deviation in the muon g-2 experiment as particle *X17*. The additional by-product from pion decay that coexists with the muons in the Fermilab storage ring has a rest mass of 17 MeV. This is the same mass as particle *X17*, a hypothetical protophobic spin-0 boson first captured by the ATOMKI, then JINR [101,102,103].

*EFT* and *SFT* might provide an accurate means to calculate glueballs derived from the decay of mesons. The rest of the masses of glueballs or condensed gluonic particles or relativistic stings, established from

the decay of pions and kaons, derived in this paper were consistent with CERN data [65].  Therefore, the theories discussed in this study could adequately estimate the masses of glueballs derived from the decay of other mesons.

*EFT* and *SFT* claimed that the initial gravitational waves detected by VIRGO/LIGO were generated from binary black hole mergers probably located in population III stars.  The theories in this study claimed the earliest observed binary black hole mergers likely emitted γ-ray bursts at $\sim 10^{57}$ erg.  It is well known that many cosmological phenomena emit γ-ray bursts.  However, this range of energy for photons suggested the merging BH likely existed in hypothetical population III stars [8].

Establishing a **Triality** between Quantum Information Theory (*QIT*), *EFT*, and *SFT*.  Information theory is known as the study of uncertainty in the quantum realm and its basic unit of information is the qubit, a two-state quantum mechanical system [104].  If an individual applies the complex variable $\eta$ to the exponential function $f$, then (s)he obtains:

$$f(\xi) = c_1 e^{i\eta}.$$

This expression is associated with the exponential map of a qubit [104].  Also, if one applies Euler's formula to the above expression, (s)he obtains the sinusoidal wave function $f$ discussed in section 3.  The above statements imply there is an intimate link between a qubit, physical body/particle, and string.  Another significant association between the three theories involves quantities of information and elementary fields.  The elementary fields with Fibonacci-based parameters produce a hyperbolic secant function comparable to entropy.  On the other hand, cross-entropy loss is a logistic function; thus, KL divergence and the Chebyshev-related elementary fields are equivalent.

Conflicts of Interests

The author of this paper has no conflicts of interest.

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