On quantum-spacetime linelement with scalar curvature from first principles

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Abstract:

In an uncomplicated way there can be derived a quantum-lineelement of local tangent-spacetime of Minkowski-type from a fundamental form of physical anouncements. This term includes a variable, which can be interpreted as combination of Ricci-scalar and cosmological constant. In this method, the Planck-constant is included in the linelement-form in a natural way. If this ansatz is seen as real from first principles, then classical Minkowski-linelement is wrong. So since it is the limiting case for GRT without gravity, quantum gravity-equations also must be false and be modified. Local invariant Lorentz-Planck-transformations can be derived for some special conditions of velocity values. This explorations for quantum in tangential-spacetime may lead to a deeper understanding of quantizing gravity.

Key-words: Planck-Lorentz-transformation; Planck-Minkowski-linelement; quantized flat spacetime; Ricci-scalar; scalar-curvature; cosmological constant; minimal Planck-length; first principles; quantizing uncurved spacetime; advanced Hamilton-function; quantized formulated Lagrangian for SRT.

1.Introduction:

There are two methods constructing a quantum-spacetime-theory from classical limits. Going from $0 \Rightarrow \hbar$ for quantum states against continuity and going from $G \Rightarrow R_k^i$ for gravity and curvature from Newton-model to Einstein Tensor-description and then going from zero to $\hbar$.(G is gravitational constant of Newton-law). The first means going from flat Minkowski spacetime to a sort of quantized tangential spacetime and the second to go from spacetime without curvature to GRT with gravity-force and then to quantum $\hbar$. Mostly the second trying is used. [2.],[3].
This paper here only constructs a quantum flat Planck- Minkowski- spacetime from fundamental Planck-length-squares. If this attempt will be physically, logically and mathematically consistent and succesful, must be seen. From this level then could be constructed a consistent quantum gravity, which limiting case this construction here would be.
Citation Wheeler [1.]: *"Space-time geometry is no longer high above the battle of matter and energy. It takes part in the struggle. Geometry tells matter how it should move, but mass in turn dictates the curvature to geometry."*

This is also a fact for quantum tangential spacetime, where is shown, that a gravityfree local spacetime TM of a manifold M isn't really flat like normally assumed and empty [5.] but depends on some physical variables. The way to calculate this problem in this paper therefore is seen in the following diagram marked in red (left arrow down), where the conventional way is marked in black (middle arrows to the right and then down):

$$\begin{array}{lcl} \text{SRT/TM} & \rightarrow G/R^i_k \rightarrow & \text{GRT} \\ \downarrow \hbar & & \downarrow \hbar \\ \text{QSRT/ } T_Q(M) & \rightarrow G/R^i_k \rightarrow & \text{QGRT} \end{array} \qquad (0.)$$

Diagram 1: relations of SRT and GRT to their quantized systems.

2.Calculation:

From first principles is defined:

$$\text{If} \quad \frac{1}{r^2{}_{PL}} := R_{Fund} \pm \Lambda := \frac{m^2{}_{PL} \cdot c^2}{\hbar^2} \quad , \qquad (1.)$$

where R is the Ricci-scalar curvature and Λ is cosmological constant, then this definition can be developed to its general case of:

$$R \pm \Lambda = \frac{m^2 \cdot c^2}{\hbar^2} \qquad (2.)$$

and of course R is defined over the two curvature-mainaxes (in two dimensions) as scalar invariance of curvature:

$$R := \left| \frac{2}{\rho_1 \cdot \rho_4} \right| \quad \text{resp.} \quad R := R_{i,k} \cdot g^{i,k} \quad . \qquad (3.)$$

This leads to a modification of Einsteins local energy-equation for flat spacetime of:

$$E = m \cdot c^2 - \frac{\hbar^2 \cdot (R \pm \Lambda)}{m} \qquad (4a.)$$

$$\text{with} \quad B := \frac{\hbar^2 \cdot (R \pm \Lambda)}{m} \quad \text{defined.} \qquad (4b.)$$

From this equation (4a.) the advanced Hamilton-function for energy-momentum can be derived in dependence from B :

$$m^2{}_0 \cdot c^4 + p^2 \cdot c^2 - 2 \cdot B \cdot \gamma \cdot m_0 \cdot c^2 \cdot (1 - \frac{v}{c}) = E^2 - 2 \cdot B \cdot (E - p \cdot c) \qquad (4c.)$$

whith the usual notation for Gamma:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad . \qquad (4d.)$$

This equation (4c.) gives the natural completeness of local SRT-Hamiltonian from Pythagoras-set to advanced, developed cosine-set.
Equation (4c.) can be solved for Hamilton function H resp. for energy E :

$$H = E = B \pm \sqrt{m^2{}_0 \cdot c^4 + p^2 \cdot c^2 + B^2 - 2 \cdot B \cdot \left(pc + \gamma \cdot m_0 \cdot c^2 \cdot \left(1 - \frac{v}{c}\right) \right)} \tag{4e.}$$

This result leads directly to corrected line-element of quantized Planck-Minkowski-spacetime in its new form of:

$$ds^2 = x^2 \cdot \frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} - c^2 \cdot t^2 \cdot \left(\frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} - 1 \right) \tag{5a.}$$

which also can be written as:

$$ds^2 = c^2 \cdot t^2 + \frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} \cdot (x^2 - c^2 \cdot t^2) \tag{5b.}$$

As is seen, the timelike coordinate is quantized ***and*** exists in a classical form, which gives this system of description a similiar partial form of a bimetric whereby the spacelike dimension-coordinate appears only in a quantum description.

This equation (5a./5b.) is from now on called „Planck-Minkowski-linelement".

As is seen, this new lineelement is conform to classical Minkowski- description of local spacetime and differs from it only for the conformal term of:

$$K := \frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} \quad . \tag{6.}$$

In this form the new local linelement can be written as:

$$ds^2 = c^2 \cdot t^2 + K \cdot (x^2 - c^2 \cdot t^2) \tag{7a.}$$

or

$$ds^2 = K \cdot x^2 - c^2 \cdot t^2 \cdot (K - 1) \quad , \tag{7b.}$$

which comes from [4.]:

$$ds^2 = d\,x^i \cdot dx^k \cdot \eta_{i,k}$$

with to the classic Minkoski tensor varied compared metric fundamental tensor for uncurved tangential spacetime :

$$\eta_{i,k} = \begin{pmatrix} K & 0 \\ 0 & 1 - K \end{pmatrix} \tag{7c.}$$

This term leads directly to the Planck-Lorentz-transformations, described with its transformation-matrix A (formulated for two dimensions) extended from [10.]:

$$A=\begin{pmatrix} (\sqrt{K}\cdot\gamma\cdot\Gamma) & \Gamma \\ \Gamma & (\sqrt{K}\cdot\gamma\cdot\Gamma) \end{pmatrix} \tag{8.}$$

with $\begin{pmatrix} x' \\ c\cdot t' \end{pmatrix} = \begin{pmatrix} x \\ c\cdot t \end{pmatrix} \cdot A$ (9.)

This shows directly the Planck-Lorentz-transformations for two dimensions of uncurved but quantized local spacetime in its conformal form:

$$x'=\Gamma\cdot(\gamma\cdot\sqrt{K}\cdot x+c\cdot t)$$
$$c\cdot t'=\Gamma\cdot(x+\gamma\cdot\sqrt{K}\cdot c\cdot t)$$

,with $\beta=\frac{v}{c}$ (10a./10b)

where

$$\sqrt{K}:=\frac{\hbar}{c\cdot m_o}\cdot\sqrt{R\pm\Lambda} \tag{11a.}$$

and $\Gamma = \left(\sqrt{K\cdot(1-\beta^2)-1}\right)^{-1}$ (11b.)

where in this notation γ is defined as :

$$\gamma:=\sqrt{\left(1-\frac{v^2}{c^2}\right)} \tag{11c.}$$

This leads in first order to a new formulated, quantized Lagrange-function of SRT resp. quasi-flat tangential spacetime $T_Q(M)$:

$$L = -\frac{m_0\cdot v^2}{\Theta}\cdot\sqrt{1-\Theta} \tag{12a.}$$

where Θ is defined as:

$$\Theta=K\cdot(1-\beta^2) \quad \text{and} \quad \beta=\frac{v}{c} \quad . \tag{12b.}$$

The Lagrange-function also can be written (in more common form) as:

$$L=\frac{-m_0\cdot c^2}{\Theta}\cdot\sqrt{\beta^4\cdot(1-\Theta)} \quad . \tag{12c.}$$

Since there is the Planck- condition $\Gamma^{-1} \geq 0$ as a substitute of new advanced Lorentz-factor to the old factor, which means that this factor itself is quantized and therefore also velocity-term, this then leads finally to the corrected maximal limiting condition in physical velocity for material fermion and boson bodies of:

$$v \leqslant c \cdot \sqrt{1 - \frac{c^2 \cdot m_0{}^2}{\hbar^2 \cdot (R \pm \Lambda)}} \tag{13.a}$$

which also can be written as:

$$v \leqslant c \cdot \sqrt{1 - \frac{m_0 \cdot c^2}{B}} \tag{13b.}$$

instead of:

$$v \leqslant c \quad . \tag{13c.}$$

From this all follows,that new, corrected light-cone conditions can be written as:

$$x = \pm\, c \cdot t \cdot \sqrt{\left(1 - \frac{c^2 \cdot m^2{}_0}{\hbar^2 \cdot (R \pm \Lambda)}\right)} \tag{14a.}$$

which likewise can be written as:

$$x = \pm\, c \cdot t \cdot \sqrt{1 - \frac{E_0}{B}} \quad \text{with} \quad E_0 = m_0 \cdot c^2 \quad . \tag{14b.}$$

instead of:

$$x = \pm c \cdot t \quad . \tag{14c.}$$

Comment:
Since there is the intervall-condition of $K \in [0;1]$, factor K can be used for gauging of probability ψ like Lorentz-factor $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$, which also is defined for $\frac{v}{c} \in [0;1]$. This leads to a natural way of gauging probability. Probability therefore is already inherent build in in formulation of classical SRT resp. tangential spacetime TM and in $T_Q(M)$ also.

3.Conclusion:

It is possible to construct a flat quantum-spacetime in Minkowski-like coordinates with its local quantum invariance conditions for coordinate transformations. If this description would be true, so every real quantum-gravity must lead in its limit without gravity- force to this Planck-Minkowski-metric, like classical gravity of GRT must lead without force in its limit to classical flat spacetime of Minkowski-linelement. In all formulas above either Ricci-scalar term R or cosmological constant Λ can be set to zero. If both are set to zero, classical, local Minkoswki spacetime remains at tangential spacetime to manifold M . Also the local geometric invariance form is reviewed: the light cone will change its form a little bit, which size-changing depends from now on, controlled by R,Λ and m_0 . Not only in gravity-determined spaces [6.],[7.] but even in tangential spacetime, it is no longer a static form [8.],[9.]. At the same foreevent place light-cone differs now only by different m_0 but by constant moving v of a particle with constant m_0 and constant assumed Λ the light-cone form depends at its worldline from changing of local Ricci-scalar R even in tangential spacetime $T_Q(M)$.

4.Summary:

From first principles of a fundamental length-square with identifying square of inverse Planck-length with Ricci-scalar and cosmological constant there can be constructed a local uncurved quantum-spacetime without gravity as a corrected form of tangential spacetime TM_Q for manifold M . But this construction will include Ricci curvature scalar R and may lead to a consistent form of a description of quantum-gravity which in limit for zero gravity is this description of quantum tangential spacetime mentioned above.

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7. Verification:

This paper is written without help from a chatbot like Chat-GPT4 or other AIs. It‘s fully human work.

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