Critical Explorations of the Relativity Theory

Jorma Jormakka
jorma.o.jormakka@gmail.com

10. March 2024

Abstract

This paper contains a collection of my critical explorations of the relativity theory. The reader is encouraged to check the calculations.

Introduction: the two basic errors in the Relativity Theory

Let P be a point in an oriented 4-manifold M. We inherit local Cartesian coordinates and the inner product $(|)$ from R^4. The point P can be expressed in local coordinate systems as $P = X = (x_0, x_1, x_2, x_3)$ and the norm is

$$r^2 = (r\bar{e}_r|r\bar{e}_r) = (\sum x_i\bar{e}_i|\sum x_i\bar{e}_i) = \sum x_i^2. \tag{1}$$

We do not get any cross terms x_ix_j, $i \neq j$, because Cartesian local coordinates are orthogonal in the in herited metric induced by the inner product. Differentiating

$$2rdr = \sum 2x_idx_i$$

we get

$$dr^2 = \left(\sum \frac{x_i}{r}dx_i\right)^2. \tag{2}$$

Let us also define a line element

$$ds^2 = \sum dx_i^2. \tag{3}$$

Notice that this line element is not the square of the differential of the norm. This line element is the square of the norm of the following vector:

$$d\bar{s} = ds\bar{e}_r = \sum dx_i\bar{e}_i. \tag{4}$$

There also cannot be any cross terms dx_idx_j, $i \neq j$, in (3) because local Cartesian coordinates are orthogonal in the inherited metric. If we get cross terms dx_idx_j, $i \neq j$, then the local coordinates are not orthogonal. The coordinates X are coordinates of an inertial frame of reference R.

Let us take a frame of reference R' that moves with the constant velocity v with respect to R to the direction x_i. We assume that all inertial coordinate systems are in a similar position. Therefore we can select local Cartesian coordinates for R' inheriting the coordinates and the inner product from R^4. We can express $P = X' = (x_0', x_1', x_2', x_3')$. We can define a line element in these coordinates

$$ds'^2 = \sum dx'^2_i \tag{5}$$

and again we cannot have any cross terms $dx_i'dx_j'$, $i \neq j$ because the local coordinates are orthogonal.

We will change the Riemannian metric to a Minkowski metric by defining $t = c^{-2}ix_0$ and $t' = c^{-2}ix'_0$. The line elements are

$$ds^2 = -c^2dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \tag{6}$$

$$ds'^2 = -c^2dt'^2 + dx'^2_1 + dx'^2_2 + dx'^2_3$$

where we have the sign convention (-,+,+,+) because it naturally came from this derivation. Einstein preferred to put the signs as (+,-,-,-). Notice that in this Minkowski metric we do get cross terms $dx'_i dx'_j$, $i \neq j$, in (6) in these local orthogonal coordinates.

Let now X and X' be related by a Lorentz transform

$$x'_i = \gamma(x_i - vt) \qquad x_i = \gamma(x'_i + vt) \tag{7}$$

$$t' = \gamma(t - (v/c^2)x_i) \qquad t = \gamma(t' + (v/c^2)x'_i)$$

$$x'_j = x_j \text{ if } j \neq i.$$

Then

$$dx_i = \gamma dx'_i + \gamma v dt' \qquad dt = \gamma dt' + \gamma(v/c^2)dx'_i. \tag{8}$$

Inserting to (6) we get

$$\begin{aligned} ds^2 &= -c^2\gamma^2 dt'^2 - c^2 2\gamma^2 \frac{v}{c^2} dt' dx'_i - c^2 \frac{v^2}{c^4}\gamma^2 dx'^2_i \\ &\quad + \gamma^2 dx'^2_i + 2\gamma^2 v dt' dx'_i + \gamma^2 v^2 dt'^2 + \sum_{j\neq i} dx'^2_j \\ &= \gamma^2\left(1 - \frac{v^2}{c^2}\right)\left(-c^2 dt'^2 + dx'^2_i\right) + \sum_{j\neq i} dx'^2_j. \end{aligned} \tag{9}$$

Thus

$$ds^2 = -c^2 dt'^2 + dx'^2_1 + dx'^2_2 + dx'^2_3 = ds'^2. \tag{10}$$

So, in these coordinates the line element is invariant in a Lorentz transform. Let us look at the local speed of light (in vacuum). We set $dx_j = 0$ if $j \neq i$ and require that light travels on light-like world paths. Light-like world paths have $ds^2 = 0$ in R. In R' the condition is $ds'^2 = 0$. Thus

$$0 = -c^2 dt^2 + dx_i^2 \tag{11}$$

$$0 = -c^2 dt'^2 + dx'^2_i.$$

We see from (11) that the speed of light is constant in these coordinates

$$c = |\frac{dx_i}{dt}| \tag{12}$$

$$c = |\frac{dx'_i}{dt'}|.$$

Equation (10) seems to be the source of the first error, the false belief that the Lorentz transform makes the speed of light constant in every inertial frame R'. It seems to do so in these coordinates, but these coordinates are not independent coordinates. The condition for coordinates to be independent means that the projection of the n-tuple $(x_1, x_2, x_3, ..., x_n)$ in the ith coordinate axis is x_i. Notice that this is a different condition that the condition that the coordinate vectors are linearly independent. Coordinate vectors can very well be linearly independent though the coordinates are not independent in the above sense. Consider our familiar timezone time. It is an example of coordinates that are not independent. Travel from London to Beijing and you have to turn your watch to the local time. Timezone time depends on the location. The time coordinate axis of this system is a Greenwitch. If you want to know the flight time from London to Beijing you cannot subtract the local times, you have to project the Beijing time to the Greenwich time (i.e., London time) and then subtract the times. This does not in any sense mean that time does not tick forward in Beijing. It is not so that the time and space coordinates are linearly dependent. Were that the case, then if you fix your location to Beijing, you would also fix the time to one single value. But time certainly can have any values. The dependency in coordinates that are dependent means that if you fix the time coordinate time to some value, then the local times depend on the location. In the timezone system they depend on the location noncontinuously, while in the Lorentz transform the time t' depends on the location x' continuously if you keep t fixed.

In order to calculate the speed of light in R' two points ${X'}_1 = (t'_1, x'_{1,1}, x'_{1,2}, x'_{1,3})$ and ${X'}_2 = (t'_2, x'_{2,1}, x'_{2,2}, x'_{2,3})$ are needed and we have to calculate the time and space difference between these two points. The space difference is obtained correctly even without a projection on the space coordinate, but the time difference requires taking the projection. Taking the projection shows that the speed of light in R' is not c. We can do the projection in at least three ways: we can remove the time offset like when we change the local time to Greenwich time in our timezone coordinates, or we can draw the coordinate axes $({x'}_i, t')$ on the coordinates of (x, t), which gives two non-orthogonal lines, and make the projection by drawing parallel lines to the coordinate axes $({x'}_i, t')$, or we can take a point X', find its preimage X, project X to the t-coordinate and find X_1 that maps to ${X'}_1$ on the t'-coordinate axix. All three ways give the same result: the projection of (x', t') where $t' = \gamma(t - (v/c^2)x)$ on the coordinate axis t' is ${t'}_1 = \gamma^{-1}t$. The coordinate system of R' with independent coordinates comes from the transform

$$x' = \gamma(x - vt) \qquad t' = \gamma^{-1}t \qquad {x'}_j = x_j \;\; j \neq i. \tag{12}$$

The line element ds^2 is not invariant under the transform (12). Let us see this in a simple case of the Galileo transform: we set $\gamma = 1$ in (12), $dx = dx_1$ and $dx_2 = dx_3 = 0$. Then

$$\begin{aligned} d\bar{s} &= dt\bar{e}_0 + dx\bar{e}_1 \qquad (13) \\ d\bar{s}' &= dt'\bar{e}_0 + dx'\bar{e}_1 \end{aligned}$$

Then

$$\begin{aligned} ds^2 &= dt^2 + dx^2 \\ {ds'}^2 &= {dt'}^2 + {dx'}^2 = dt^2 + (dx + vdt)^2 \\ &= dt^2 + dx^2 - 2vdxdt + dt^2 = ds^2 - 2vdxdt + dt^2. \qquad (14) \end{aligned}$$

Notice that we get a cross term $dxdt$ in (14). It is not possible to cancel this cross term by any other means than by the time transform in the Lorentz transform, but then the coordinates are not independent. The local coordinates of R and R' must be independent as they are inherited from the Cartesian coordinates of R^4 and in Cartesian coordinate system coordinates are independent. Thus, it is impossible to get (10) if we use correct coordinates.

One may ask if it would not be possible to define that in the Special Relativity Theory (SRT) we use the coordinates X' even thought the coordinate system does not have independent coordinates. It is not possible. It leads to contradictions. For instance, if we define that we use coordinates X', then we can set the universal constant c to any chosen value $c' \leq c$ simply by selecting the rest frame of the laboratory as R' and selecting (i.e., imagining) R that moves with the speed $-v$ with respect to the laboratory. We can e.g. set c' to one meter per second in the laboratory and measure if the speed of light is now smaller than walking speed. It is not. Therefore it is not possible to define that we use coordinates X' in SRT. This is so because even if we define that we use the coordinates X', there remains the valid way of going from X' to X, making a projection in X to X_1 and coming back to X'_1. This is what creates the contradiction.

Let us now take a metric where there is a field:

$$ds^2 = -c^2 A_0(X)dt^2 + A_1(X)dx_1^2 + A_2(X)dx_2^2 + A_3(X)dx_3^2. \tag{15}$$

Making the Lorentz transform we get

$$ds^2 = \gamma^2 \left(A_0(X(X')) - \frac{v^2}{c^2} A_i(X(X')) \right) \left(-c^2 dt'^2 + dx'^2_i \right) \tag{16}$$

$$+ \sum_{j \neq i} dx'^2_j + 2\gamma^2 \frac{v}{c} \left(-A_0(X(X')) + A_i(X(X')) \right) dt' dx'_i.$$

From (16) follows that the cross term $dt'dx'_i$ disappears for every $i = 1, 2, 3$ only if the field is induced by a scalar field $A_k(X) = \phi^2(X)$, $k = 0, 1, 2, 3$. If this condition is true, then (16) simplifies to

$$ds^2 = -c^2 \phi^2 dt'^2 + \phi^2 dx'^2_1 + \phi^2 dx'^2_2 + \phi^2 dx'^2_3 = ds'^2 \tag{17}$$

and

$$ds^2 = -c^2 \phi^2 dt^2 + \phi^2 dx_1^2 + \phi^2 dx_2^2 + \phi^2 dx_3^2. \tag{18}$$

Then also the speed of light in vacuum is c to each direction in both coordinates X and X', but notice that the coordinate system X' has non-independent coordinates.

Consider calculating the Christoffel symbols from the two metrics (17) and (18). The numerical values are not the same, as in the metric of R we derivate with respect to components of X and in R' with components of X', but the form of the equations for Christoffel symbols are the same. Then calculate the Ricci tensor entries and the Ricci scalar. Again, the numeric values are not the same, but the form of the equation is the same. With the Ricci entries and the Ricci scalar we can form the left side of Einstein equations

$$R_{\mathrm{ab}} - \frac{1}{2}Rg_{\mathrm{ab}} = k_0 T_{\mathrm{ab}} - \lambda g_{\mathrm{ab}} \tag{19}$$

The right side does not have the same numeric value because the numeric values are not the same in (17) and (18), only the form. This means that if in coordinates X the right side of (19) is zero, it does not need to be zero in coordinates X'. However, the form is the same. The Einstein equations are invariant (in fact, covariant) under the Lorentz transform assuming that the field is scalar.

Pay attention: the Einstein equations are Lorentz covariant only if the cross term in (16) disappears and this happens only if the field is a scalar field ϕ. I have mentioned to many people that the metric in the General Relativity Theory (GRT) has locally constant speed of light only if the field is scalar, which is true and easily verified from (16), compare it to (11)-(12), but these people have not understood that this is a mandatory condition in GRT: the speed of light must be locally constant. Now, notice that if the field is not scalar, then the Einstein equations are not Lorentz covariant. That certainly is a mandatory condition in GRT. Let us simply add that if the field is scalar, then the Einstein equations do not have any solutions that approximate Newtonian gravity in the simplest possible case, the one in the Schwarzschild solution: a point mass in empty space. This is the second major error. It is a fatal error of GRT.

As some people think that the Schwarzschild solution does approximate Newtonian gravitation field sufficiently well, and as Einstein used the Schwarzschild solution in his verifications of GRT, e.g, in the precession of Mercuryäs perihelion, let us mention one fatal problem in the Schwarzschild solution. More problems are shown in [1]. The one we can mention here is that if the Schwarzschild metric, which is expressed in spherical coordinates, is expressed in Cartesian coordinates, then there are cross terms. This is impossible meaning that the Schwarzschild metric is not a valid metric. There cannot be cross terms in any metric expressed in local orthogonal coordinates. If is a different issue if we express a metric in R with coordinates of a moving coordinate system of R'. Then we do get cross terms. But if we change orthogonal spherical coordinates of R to equally orthogonal Cartesian coordinates of R, then there cannot be any cross terms in either coordinate system. This is simply the meaning of orthogonal coordinates.

These are the two major error in the Relativity Theory. The first is a fatal error in SRT, the second is a fatal error in GRT. There are more errors in the Relativity Theory. We can mention the fatal error in the relativistic mass and Einstein's proof of $E = mc^2$. Einstein used the formula how to get force from work

$$F = \frac{d}{ds} W \tag{20}$$

in order to get force from kinetic energy. The way to get force from kinetic energy is

$$F = \frac{d}{dt} \frac{\partial}{\partial \dot{s}} E_k(\dot{s}). \tag{27}$$

Force can be obtained from kinetic energy by the formula (20) only if the kinetic energy has the Newtonian form $E_k = (1/2)mv^2$ where the mass is constant. Einstein uses the formula when the mass is not constant, which is a fatal error.

There is also a fatal error in Einstein's geodesic Lagrangean. It does not make a stone falling freely in a gravitational field to accelerate, thus it fails Galileo's Pisa tower stone dropping test.

The following is a selection of my papers from 2022-2024 proving beyond any doubt that the Relativity'Theory has all these errors and many more errors. In some cases, as in 3.5, it is clear that Einstein cheated intentionally.

As for the question how this could happen, how can a hundred years old theory be wrong, your guess is as good as mine. Anyway, it did happen. It is impossible to get the main stream to accept this truth. I tried and got the following answer (as the only one, from other journals I did not get any answers): "It is not the role of the referee to pinpoint errors and misunderstandings in this paper." Many people, also competent people, have tried to pinpoint errors in my claims of serious errors in Einstein's relativity theory. They have all failed.

CONTENTS

PART 1. RELATIVISTIC MASS AND $E = mc^2$

Paper 1.1 is my final argument refuting the relativistic mass formula. Papers 1.2 and 1.3 are earlier versions of the argument, but they include material that is not in Paper 1.1. Papers 1.4 and 1.5 are attempts to find the real explanation to the apparent increase of mass by velocity in the experiments that first were made by Lorentz. The mass does seem to increase according to the relativistic mass formula, but it is only apparent, in reality the inteaction force gets weaker when the relative speed of the test mass and the field grows.

In this introductory part I give the strongest argument from Paper 1.1 refuting relativistic mass. The main error in Einstein's calculations and his proof of $E = mc^2$ is that he did not understand that energy is not work and force is not obtained from kinetic energy with the formula $F = d/dsE_k$. Force is obtained from work with the formula $F = d/dsW$ and we can use this formula by replacing W by E_k only if the kinetic energy is of the form $(1/2)mv^2$ where m is constant.

The error in Einstein's proof of $m = \gamma m_0$ and his proof of $E = mc^2$ is that he gets to the equation

$$\frac{d}{ds}(m - m_0)c^2 = \frac{d}{dt}(mv) \tag{1}$$

where $m = \gamma m_0$. The right side Einstein presents as a force. If it is a force, then it comes from kinetic energy $E_k(v)$ that has the form

$$\frac{d}{dt}\frac{d}{dv}E_k(v). \tag{2}$$

This is so because consider the Lagrange-Euler equation. A test mass falls in a gravitational field along a minimum energy path and this minimum energy path minimizes the action

$$S = \int L dt \tag{3}$$

where

$$L = E_p(s) + E_k(v). \tag{4}$$

The Euler-Lagrange equation is

$$\frac{\partial}{\partial s}E_p(s) - \frac{d}{dt}\frac{\partial}{\partial v}E_k(v) = 0. \tag{5}$$

Notice that the left term in (1.4) is the force given by the potential energy. Therefore the second term is also a force. It is the force given by the kinetic energy.

Notice that the force given by the kinetic energy does not equal

$$F = \frac{d}{ds}E_k(v) \tag{6}$$

except in the special case $E_k(v) = Cv^2$ where C is constant. That is, let us solve

$$\frac{d}{ds}E_k(v) = \frac{d}{dt}\frac{d}{dv}E_k(v) \tag{7}$$

$$\frac{dv}{ds}\frac{d}{dv}E_k(v) = \frac{d}{dt}E_k'(v) \tag{8}$$

where $E_k'(v) = dE_k(v)/dv$. We multiply both sides with $v = ds/dt$

$$\frac{ds}{dt}\frac{dv}{ds}E_k'(v) = v\frac{dv}{dt}\frac{d}{dv}E_k'(v) \tag{9}$$

$$\frac{dv}{dt}E_k'(v) = v\frac{dv}{dt}E_k''(v) \tag{10}$$

$$E_k'(v) = vE_k''(v) \tag{11}$$

$$\frac{E_k''(v)}{E_k'(v)} = \frac{1}{v} \tag{12}$$

$$\ln E_k'(v) = \ln v + \ln 2C \tag{13}$$

where C is a constant

$$E_k'(v) = 2Cv \tag{14}$$

$$E_k(v) = Cv^2 + B \tag{15}$$

where B is a constant that we must choose as zero so that $E_k(0) = 0$.

The kinetic energy that creates the force at the right side of (1) is

$$E_k(v) = (1 - \gamma^{-1})m_0c^2 \tag{16}$$

because

$$\frac{d}{ds}(1 - \gamma^{-1})m_0c^2 = \frac{d}{dt}(\gamma m_0 v) = \frac{d}{dt}(mv). \tag{17}$$

In (16) we have added a constant so that the kinetic energy has the leading term $(1/2)m_0v^2$. The work that the force at the right side of (1) makes is the left side of (1) because

$$W = \int F ds \qquad \frac{d}{ds}W = F. \tag{18}$$

The kinetic energy (16) is not of the form (15). Therefore the work Ws that the force F coming from the kinetic energy (16) does not equal the kinetic energy. Indeed

$$F = \frac{d}{dt}(mv) \tag{19}$$

$$F = \frac{d}{dt}\frac{d}{dv}E_k(v) \tag{20}$$

$$F = \frac{d}{ds}W \qquad (21).$$

Then

$$E_k(v) = (1 - \gamma^{-1})m_0c^2 \tag{21}$$

$$W = (m - m_0)c^2 \tag{22}$$

$$E_k(v) < W. \tag{23}$$

The result (23) is impossible and shows that $m = \gamma m_0$ is impossible. The kinetic energy that creates the force cannot be smaller than the work that the force makes. As (1) is also the basis of Einstein's proof if $E = mc^2$, this proof is wrong. The formula $m = \gamma m_0$ is equivalent with the so called total energy formula

$$mc^2 = \sqrt{(m_0c^2) + (pc)^2}. \tag{24}$$

The equation (24) is simply a rewrting of $m = \gamma m_0$

$$m^2\gamma^{-2} = m_0^2 \tag{25}$$

$$m^2c^4 - m^2v^2c^2 = m_0c^4 \tag{26}$$

$$(mc^2)^2 = (m_0c^2)^2 + (mv)^2c^2. \tag{27}$$

Therefore this total energy formula is also wrong. The error made by Einstein is to think that the force from kinetic energy E_k is obtained as

$$F = \frac{d}{ds}E_k \tag{28}$$

and that the right side of (1) is kinetic energy. The right side of (1) is work. The force is obtained from kinetic energy by the formula

$$F = \frac{d}{dt}\frac{\partial}{\partial v}E_k(v). \tag{29}$$

1.1 Irrefutable proof that the relativistic mass formula is wrong and Einstein did not prove $E = mc^2$

Abstract:

I was challenged with a proposed "irrefutable proof" to the relativistic mass formula. It was a fake proof, essentially the same Einstein's proof, but this "proof" could be turned into an irrefutable proof that the relativistic mass formula is false and that Einstein did not prove $E = mc^2$.

1. The "irrefutable proof" that the relativistic mass is real

Let us first calculate a mathematical identity for the Lorentz factor γ.

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{1}$$

$$\gamma^2 \left(1 - \frac{v^2}{c^2}\right) = 1 \tag{2}$$

$$\frac{v}{c^2}\gamma^3 \left(1 - \frac{v^2}{c^2}\right) = \gamma \frac{v}{c^2} \tag{3}$$

$$\frac{d\gamma}{dv} \left(1 - \frac{v^2}{c^2}\right) = \gamma \frac{v}{c^2} \tag{4}$$

$$\frac{d\gamma}{dv} = \frac{d\gamma}{dv}\frac{v^2}{c^2} + \gamma \frac{v}{c^2} \tag{5}$$

$$\frac{c^2}{v}\frac{d\gamma}{dv} = \frac{d\gamma}{dv} v + \gamma \tag{6}$$

$$\frac{dv}{dt}\frac{d\gamma}{dv}\frac{c^2}{v} = \frac{dv}{dt}\left(\frac{d\gamma}{dv} v + \gamma\right) \tag{7}$$

$$\frac{dv}{dt}\frac{d\gamma}{dv}\frac{c^2}{v} = \frac{dv}{dt}\frac{d}{dv}(\gamma v) \tag{8}$$

$$\frac{d\gamma}{dt}\frac{c^2}{v} = \frac{dv}{dt}\frac{d}{dv}(\gamma v) \tag{9}$$

$$\frac{d\gamma}{dt}\frac{dt}{ds} c^2 = \frac{dv}{dt}\frac{d}{dv}(\gamma v) \tag{10}$$

$$\frac{d\gamma}{ds} c^2 = \frac{d}{dt}(\gamma v). \tag{11}$$

$$\frac{d}{ds}(\gamma c^2) = \frac{d}{dt}(\gamma v). \tag{12}$$

Thus, (12) is a mathematical identity. It is an equation that γ satisfies. It does not have any physical meaning as such. We can derive mathematical identities for any functions.

In Newton's mechanics the force that a mass m_0 with the speed v creates when it slows down when meeting an obstacle is

$$F = \nabla E_{kinetic} = \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) \tag{13}$$

this force is against the obstacle. This force must equal the deacceleration of the mass m_0

$$F = m_0 a = \frac{d}{dt}(m_0 v). \tag{14}$$

Indeed, they do equal

$$\frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) = m_0 v\frac{dv}{ds} = m_0\frac{ds}{dt}\frac{dv}{ds} = m_0\frac{dv}{dt} = \frac{d}{dt}(m_0 v), \tag{15}$$

This must be so, the only energy that the moving mass can release when slowing down is its kinetic energy. It does not release it's heat energy, or its caloric value if it is edible, or its rest energy $E_{rest} = m_0 c^2$ while slowing down when hitting an obstacle. You can try, a mass, like a stone, hitting an obstacle in a slow speed really does not start any atomic reactions. So, he equation (14) works quite well in small speeds.

The "irrefutable proof" goes like this:

In the Special Relativity Theory the mass of a moving object is the relativistic mass $m = \gamma m_0$. Multiplying both sides of the mathematical identity (12) by the constant m_0 we get

$$\frac{d}{ds}(\gamma m_0 c^2) = \frac{d}{dt}(\gamma m_0 v). \tag{16}$$

$$\frac{d}{ds}(mc^2) = \frac{d}{dt}(mv). \tag{17}$$

This is supposed to be a proof. You are supposed to be impressed by the fact that (12) is a mathematical identity.

The error is the step (16). It is actually: assuming that mass grows in movement and assuming that the mass grows by the formula $m = \gamma m_0$, then by multiplying (12) by m_0 we get (17). But (17) does not prove the assumption that mass grows in movement and that mass grows with the formula $m = \gamma m_0$. If we assume that mass does not grow, or that it grows according to some other function $m = f(v)m_0$, then multiplying (12) by m_0 does not give a force F because γm_0 is not a mass.

What is even more weird is that (17) and its variants are thought to prove $E = mc^2$. There is nothing resembling a proof of this formula in (17). The equation (17) simply says that the left side equals the right side. It does not say that any relativistic mass is created by movement or that the rest mass of m_0 should be $E = m_0 c^2$. It says nothing of the energy released in atomic reactions as in those reactions the mass is not moving.

2. The irrefutable proof that the relativistic mass is not real

The result (17) however does prove that the relativistic mass is not real under the condition that the kinetic energy is $(1/2)mv^2$ or $(1/2)m_ov^2$.

We can calculate how to get (17) from the Newtonian equation (15):

$$F = \frac{d}{dt}(m_0 v) = \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) \tag{18}$$

Adding to both sides the positive force that we express in several equivalent forms

$$F_1 = \gamma^2 \frac{v^2}{c^2}\frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) \tag{19}$$

$$F_1 = \gamma^{-1}\gamma^3 \frac{v}{c^2} v \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) \tag{20}$$

$$F_1 = \gamma^{-1}\frac{d\gamma}{dv} v \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) \tag{21}$$

$$F_1 = \gamma^{-1}\frac{d\gamma}{dv}\frac{dv}{ds} v m_0 v \tag{22}$$

$$F_1 = \gamma^{-1}\frac{d\gamma}{ds} v m_0 v \tag{22}$$

$$F_1 = \gamma^{-1}\frac{d\gamma}{ds}\frac{ds}{dt} m_0 v \tag{24}$$

$$F_1 = \gamma^{-1}\frac{d\gamma}{dt} m_0 v \tag{25}$$

we get by adding F_1 to the left side of (18) in the form of (25)

$$F + F_1 = \frac{d}{dt}(m_0 v) + \gamma^{-1}\frac{d\gamma}{dt} m_0 v = \gamma^{-1}\left(\gamma \frac{d}{dt}(m_0 v) + \frac{d\gamma}{dt} m_0 v\right) \tag{24}$$

$$= \gamma^{-1}\frac{d}{dt}(\gamma m_0 v) \tag{26}$$

and by adding F_1 to the right side of (18) in the form (21)

$$F + F_1 = \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) + \gamma^{-1}\frac{d\gamma}{dv} v \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) \tag{27}$$

$$= \gamma^{-1}\left(\gamma \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) + \frac{d\gamma}{dv} v \frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right)\right) \tag{28}$$

$$= \gamma^{-1}\frac{d}{ds}\left(\frac{1}{2}\gamma m_0 v^2\right) \tag{29}$$

Multiplying both sides by γ which is larger than one we get

$$\frac{d}{dt}(\gamma m_0 v) = \frac{d}{ds}\left(\frac{1}{2}\gamma m_0 v^2\right) \tag{30}$$

In order to get (30) we first added a positive force F_1 to both sides of (18) and then multiplied both sides by a function γ which is larger than one. That means, we increased the force on both sides of (18). What force allows us to increase the force on both sides?

The only force we have is from the kinetic energy. Let us assume the kinetic energy of the relativistic mass is

$$F_2 = \frac{d}{ds}\left(\frac{1}{2}\gamma m_0 v^2\right). \tag{31}$$

We can applie the same proof to the case that the kinetic energy is Newtonian.

We can express this force as:

$$F_2 = \gamma\frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) + \frac{1}{2}\frac{d\gamma}{ds}m_0 v^2 \tag{32}$$

$$F_2 = \gamma\left(\frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) + \frac{1}{2}\gamma^{-1}\frac{d\gamma}{ds}v m_0 v\right) \tag{33}$$

From (22) we recognize F_1 in the second term

$$F_2 = \gamma\left(\frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) + \frac{1}{2}F_1\right) \tag{34}$$

Notice that the left side of (30) is according to (18) and the steps we did:

$$F_3 = \frac{d}{dt}(\gamma m_0 v) = \gamma\left(\frac{d}{dt}(m_0 v) + F_1\right) \tag{35}$$

$$F_3 = \frac{d}{dt}(\gamma m_0 v) = \gamma\left(\frac{d}{ds}\left(\frac{1}{2}m_0 v^2\right) + F_1\right). \tag{36}$$

Comparing (33) and (36) we have a conflict. The mass $m = \gamma m_0$ having the velocity v and hitting an obstacle cannot give a bigger force than F_2 in (33). Yet, the decelerating mass m has the acceleration that gives the force F_3, which is strictly larger than F_2. This is impossible. The relativistic mass cannot release the energy of its rest mass, or its heat energy, or its caloric energy if it is food, or its spiritual energy if the mass belongs to some religious supporter of the relativity theory. The mass m can only release the kinetic energy and the force it can give is F_2. This force already has the energy that can be released by the decreasing relativistic mass.

The only way the mass m could give a bigger force, under the assumption on the kinetic energy, is that the relativistic mass has in addition to its kinetic energy also the energy $E = (m - m_0)c^2$ of the mass $m - m_0$. Most people working on the relativity theory deny that there is such additional energy, and for good reason: if the mass has its kinetic energy $E_k = (1/2)mv^2$ and the energy

$E = (m - m_0)c^2$ of the mass $m - m_0$, then it must release over twice as much energy when it hits an obstacle than the Newtonian mechanics tells. This would also happen with small speeds v, though the mass $m - m_0$ is very small, the energy $E = (m - m_0)c^2$ is not at all small, it is larger than the kinetic energy $E_k = (1/2)mv^2$ at any speed v. This is why most supporters of the relativity theory must claim that there is no additional energy $E = (m - m_0)c^2$ of the mass $m - m_0$, the mass m hitting an obstacle can only release its relativistic kinetic energy $E_k = (1/2)mv^2$ and that energy includes the released energy of the additional mass.

But this explanation means exactly that the force that the moving mass can release is F_2 and it is strictly smaller than the force F_3 that is obtained from the deceleration of the mass m. This is an irrefutable proof that the relativistic mass concept is incorrect. There is no relativistic mass. And as there is no relativistic mass, Einstein's proof of $E = mc^2$, which is nothing else than (17), is invalid.

Some relativity theory people probably would like to take the way out that is discussed in the next section: redefining the kinetic energy formula.

3. The last way out: the relativistic mass formula is a definition

In this and the next section we refute all attempts to redefine the kinetic energy.

In this way out of the problem the idea is that Newton's kinetic energy formula is incorrect and the correct kinetic energy formula is

$$E_{kinetic} = (\gamma - 1)m_0c^2 \tag{37}$$

This means that the equation

$$\frac{d}{dt}(mv) = \frac{d}{ds}(m - m_0)c^2 \tag{38}$$

is a definition stating that the gradient of the kinetic energy equals the force accelerating the mass.

Notice that in this way there is still no argument that shows correct the assumption that the mass grows with velocity and that it grows with the formula $m = \gamma m_0$. A proof that the relativistic mass does exist requires such an argument. The experiments where it appears that mass does grow as $m = \gamma m_0$ do not make a sufficient argument because there are other explanations for those experiments, we will briefly discuss this issue in the last section. Additionally, in this last way out the kinetic energy formula is redefined in an effort to get the kinetic energy equal to the work that the force makes.

There are arguments against this way out. A definition must make sense and be without contradictions. In this section we give two common sense arguments, but they are also strong. In the next section we give first a physical and then a mathematical argument proving that the kinetic energy cannot be redefined.

Firstly, we ask if the kinetic energy formula looks like it can be redefined. Consider the following differentials. The first one (39) is the differential of work done by the force corresponding to $F = ma$ when the mass is $m = \gamma m_0$

$$\frac{d}{dt}(mv)ds = v^2 dm + mvdv \tag{39}$$

The second one is the differential of the Newtonian kinetic energy when the mass is $m = \gamma m_0$

$$d\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2 dm + mvdv \tag{40}$$

These differentials are not equal, (39) is strictly larger. If the Newtonian kinetic energy formula still holds, it means that some work is lost. Can this happen? I think it can very well happen in the equation (39).

New mass $dm = d(\gamma m_0)$ is created in (39). Does it have the velocity v or does it need to be accelerated to the velocity of the rest of the mass? If dm needs to be accelerated to the velocity of the rest of the mass, then the work cannot be conservative and the kinetic energy of the mass at the end must be smaller than the work used to accelerate the mass. Thinking of an analogous case of accelerating a car and small stones jump up from the ground, they hit the car, get stuck to the car, and get very fast accelerated to the velocity of the car, usually doing considerable damage to the car. In this case the situation is not energy conserving. I do not see any reason why this situation would not be the same here. This is because the mechanism how new mass is created in (39) is not described. Mass simply appears from nowhere in (39). No energy is spent on making this mass dm. I would say that without spending the energy $dE = dmc^2$ the mass cannot appear to the formula. I would also say that the mass dm, as it appears from nowhere, probably does not have the velocity v. It looks like some daemon creating mass from nowhere is throwing stones on the mass and they get stuck to it.

As a conclusion from this consideration: the process where new mass is created and it is given the velocity of the moving mass cannot be energy conserving and the kinetic energy cannot equal the work used. An effort to define kinetic energy so that the process would be energy conserving cannot work.

Secondly, we ask if mass can grow by velocity. This is a killing argument. Though it is simple and common sense, there is no way to refute it.

Velocity is velocity with respect to some coordinate system In the Special Relativity Theory there is no preferred inertial frame of reference. We can select whatever frame of reference we want. If we have a mass m_0 that is not moving in a frame of reference R and we select a frame of reference R' that is moving with the speed v with respect to R, then the mass m_0 moves with the speed v in R'. If the relativistic mass formula holds, then the mass in R' is $m = \gamma m_0$. We can make the mass m as large as we want simply by selecting the speed v. Mass creates gravitational effects and changes energy levels of the electron belt in atoms, and many other observable effects. Thus, in R' we should see some of these effects. For instance, if we select v so large that m_0 surely must turn onto a black hole in R', then in R' it will draw to itself all nearby masses. Thus, I put a kilo of flour on a table and call it m_0. Then I select v to be extremely close to c and imagine the frame R'. In R' I must be drawn to the black hole creates by m. But whatever happens in R' must also happen in R because we are seeing the same world in both frames of reference, only from a different viewing point. So, I look around. I am still here. This means that the kilo of flour cannot be a black hole in R'. This means that mass cannot grow because of velocity as in the formula $m = \gamma m_0$.

These two considerations prove that the definition that the kinetic energy is (37) do not make sense and they lead to contradictions. The first argument shows that (37) does not make sense. The mass dm is appearing from nowhere in (39) and the work should not be conservative, so (37) cannot be the correct kinetic energy. The second argument proves that the relativistic mass idea contradicts common experience and cannot be correct. As a conclusion from these two common sense arguments, it is not possible to define the kinetic energy as in (37).

4. The Euler-Lagrange equations refute relativistic mass

In this section we give both a physical and a mathematical argument that redefining kinetic energy does not work.

The question is the equation (17), which we rewrite here

$$\frac{d}{ds}(m - m_0)c^2 = \frac{d}{dt}(mv) \tag{17'}$$

We will first take the physical argument. Is the entity $(m - m_0)c^2$ work or energy? If it is work, then the expression in the left side of (17') is a force because

$$W = \int F ds \qquad \frac{d}{ds}\, W = F. \tag{41}$$

Then there is no problem, but work is not energy. Both work and energy have the unit Joule, but the concepts are different. Energy is ability to do work, but in order to get work out of energy, there must be energy gradient. What the gradient is depends on the type of energy. If energy is heat energy, then there must be a temperature gradient, else we cannot get any work from the energy, We must take the gradient, or partial derivative, of the heat energy by T, the temperature. If the energy is field energy and the field ϕ is a function of space coordinates q_i only, then we must take the gradient of the field by space coordinates, $F = \nabla\phi$. Especially, if we are interested in one dimension only in our case, we take the partial derivative

$$F = \frac{\partial}{\partial q_i}\phi(q). \tag{42}$$

If ϕ depends only on $q = s$, i.e., there is only one dimension

$$F = \frac{d}{ds}\phi. \tag{43}$$

If the energy is kinetic energy, then the energy depends on v, and usually only on $v = \dot{q}$, or if there are several dimensions, on $v_i = \dot{q}_i$ where $\dot{q} = dq/dt$. In order to get work out of kinetic energy, there must be a veocity gradient. That is, if all masses move with equal speed, kinetic energy cannot do any work, but if a moving mass hits a mass that is not moving, then it does make work and creates a force on the impact. Thus, we must take a partial derivative with respect to v, but the result of the partial derivative does not have the units Newton, it is not a force. It has the units Newton times second. Clearly, we have to divide with time in some sense. What the division with time must be can be seen in the Euler-Lagrange equations: it is not actually division with time, it is a total derivative with respect to time, which will make the units into Newton. The force that kinetic energy produces if there is a speed gradient is (and can only be)

$$F = \frac{d}{dt}\frac{\partial}{\partial \dot{q}_i} E_{kinetic}. \tag{44}$$

In case when there is only one dimension and $E_{kinetic}$ is a function of v only, we get

$$F = \frac{d}{dt}\frac{d}{dv} E_{kinetic}. \tag{45}$$

This is the only possible way to get a force out of kinetic energy. When this force affects for some distance, we get work as in (41). The entity X in the following expression

$$X = \frac{d}{ds}\ E_{kinetic} \tag{46}$$

has no physical sense. If we take a partial derivative with respect to $s = q$ from kinetic energy, we get zero because $E_{kinetic}$ is a function of $\dot{s} = \dot{q}$ and q and $\dot{q}$ are independent variables. The only reason why we could replace a partial derivative $\partial/\partial s$ with a total derivative d/ds is that the function we derivate is a function of s only. This was the case in (43). With kinetic energy this is not the case: even in a one dimensional case, kinetic energy is not a function of $q = s$. It is a function of $\dot{q} = \dot{s}$. We have no justification of taking a total derivative with respect to t. The force can only be (45).

We see now that the left side of (17') is a force only if $(m - m_0)c^2$ is work. It is work done by the force at the right side of (17'). The right side must be force because the left side is force. The force in the right side must come from some kinetic energy. We can easily find the kinetic energy $E_k(v)$ that gives this force

$$\frac{d}{dt}\frac{d}{dv}E_k = \frac{d}{dt}(mv). \tag{47}$$

$$\frac{d}{dv}E_k(v) = mv + C. \tag{48}$$

$$\acute{E}_k(v) = \int mv + C = -\gamma^{-1}m_0c^2 + Cv + B. \tag{49}$$

In order to have the leading term of the kinetic energy as Newtonian kinetic energy, we set $C = 0$ and $B = m_0c^2$, thus

$$E_k(v) = (1 - \gamma^{-1})m_0c^2 \tag{50}$$

Notice that

$$\frac{d}{ds}\ E_k(v) = m\frac{dv}{dt} \neq F = \frac{d}{dt}(mv). \tag{51}$$

Especially we point out that in (17') the left side is not

$$\frac{d}{ds}E_{kinetic} = \frac{d}{ds}E_k(v) = m\frac{dv}{dt}. \tag{52}$$

We see that it is not possible to redefine kinetic energy to be $(m - m_0)c^2$ in (17'). This last way out fails. The entity $(m - m_0)c^2$ can only be work. The equation (17') does not say that the kinetic energy of a moving mass is $(m - m_0)c^2$. It says that the kinetic energy of the moving mass is $E_k(v)$ and the work that the force makes in (17') is larger than the kinetic energy

$$(m - m_0)c^2 > (1 - \gamma^{-1})m_0c^2 \tag{53}$$

$$(\gamma - 1) > (\gamma - 1)\gamma^{-1}. \tag{54}$$

This physical argument proves beyond any doubt that (17') is impossible. There cannot be any relativistic mass.

The following argument is mathematical. The primary dynamic equation for Newtonian mechanics is not the one in (15) that we for convenience rewrite

$$\frac{d}{ds}\left(\frac{1}{2}m_0v^2\right) = \frac{d}{dt}(m_0v). \tag{15$'$}$$

The dynamic equation for Newtonian mechanics is derived from the Euler-Lagrange equation minimizing total energy

$$E = E_{potential} + E_{kinetic} = m\phi + \frac{1}{2}mv^2 \tag{55}$$

The total energy is taken as the Lagraqngean as

$$L = E = m\phi + \frac{1}{2}m\dot{q}^2 \tag{56}$$

where q is the position, $\dot{q} = v$ is the velocity and $\ddot{q} = a$ is the acceleration. In (56) the mass m is constant and ϕ depends on q. We calculate

$$\frac{\partial L}{\partial q} = \partial_q\phi \tag{57}$$

$$\frac{\partial L}{\partial \dot{q}} = m\dot{q} \tag{58}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = m\ddot{q} \tag{59}$$

The Euler-Lagrange equation for q is

$$0 = \frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \tag{60}$$

$$0 = \partial_q \phi + m\ddot{q} \tag{61}$$

which is the Newtonian equation of motion $F = m\nabla\phi = ma$.

If (17') makes any sense, then it should be derived from an Euler-Lagrange equation minimizing the total energy. Let us skip the calculations (they are straight-forward, though give a bit long expressions) and only give the Lagrangean and the Euler-Lagrange equation.

If the kinetic energy is $(m - m_0)c^2$, then the Lagrangean is

$$L = E = m\phi - (m - m_0)c^2 \tag{62}$$

Here $m = \gamma m_0$. The Euler-Lagrange equation is (dividing m away)

$$0 = \partial_q \phi - \ddot{q}\gamma^4 \left(1 + \frac{\phi}{c^2}\right)\left(1 + 2\frac{\dot{q}^2}{c^2}\right) - \frac{\dot{q}}{c^2}\gamma^2\dot{\phi}. \tag{63}$$

The first order approximation is

$$0 = \partial_q \phi - \ddot{q} - \ddot{q}\left(\frac{4\dot{q}^2}{c^2} + \frac{\phi}{c^2}\right) - \frac{\dot{q}}{c^2}\dot{\phi} + O(c^{-4}). \tag{64}$$

If the kinetic energy is $\frac{1}{2}mv^2$, then the Lagrangean is

$$L = E = m\phi - \frac{1}{2}mv^2 \tag{65}$$

The Euler-Lagrange equation is (dividing m away)

$$0 = \partial_q \phi - \ddot{q}\left(1 + \frac{\gamma^2}{c^2}\left(\phi + \frac{5}{2}\dot{q}^2\right) + \frac{\gamma^4}{c^4}3\dot{q}^2\left(\phi + \frac{\dot{q}^2}{c^2}\right)\right) - \frac{\dot{q}}{c^2}\gamma^2\dot{\phi}. \tag{66}$$

The first order approximation is

$$0 = \partial_q \phi - \ddot{q} - \ddot{q}\frac{1}{c^2}\left(\phi + \frac{5}{2}\dot{q}^2\right) - \frac{\dot{q}}{c^2}\dot{\phi} + O(c^{-4}). \tag{67}$$

The force in (17') has the following expression

$$\frac{d}{dt}(mv) = m\ddot{q}\gamma^2 \tag{68}$$

The dynamic equation this force gives is

$$0 = \partial_q \phi - \ddot{q}\gamma^2 \tag{69}$$

The first order approximation is

$$0 = \partial_q(-\phi) - \ddot{q}\left(1 + \frac{\dot{q}^2}{c^2}\right) + O(c^{-4}). \tag{70}$$

We see that equation (17') is not a valid dynamic equation at all if the mass is not constant. There is no sense in saying that the kinetic energy could be $(m - m_0)c^2$ and then equation (17') is satisfied and the theory is fine with this new definition of kinetic energy. The equation (17') does not minimize the total energy and therefore the equation is wrong and satisfying it is incorrect and the new definition of kinetic energy is also wrong. Let us explain this a bit more.

The term in the Euler-Lagrange equation that gives the dynamic equation $F = ma$ is

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}} E_{kinetic} \tag{71}$$

If $E_{kinetic}$ only depends on $v = \dot{q}$, we can write the equation as

$$\frac{d}{dt}\frac{d}{dv} E_{kinetic}. \tag{72}$$

In Newtonian mechanics m is constant and $E_{kinetic} = (1/2)mv^2$, then

$$\frac{d}{dt}\frac{d}{dv}\frac{1}{2}mv^2 = m\frac{dv}{dt}. \tag{73}$$

For v^2 holds

$$\frac{d}{ds}v^2 = \frac{d}{dt}\frac{d}{dv}v^2 \tag{74}$$

but this holds only for v^2. In general

$$\frac{d}{ds} \neq \frac{d}{dt}\frac{d}{dv}. \tag{75}$$

You cannot use this identity for $m = \gamma m_0$ or for kinetic energy $(m - m_0)c^2$. It only works for the Newtonian kinetic energy.

If the kinetic energy is $(m - m_0)c^2$, then the force is

$$\frac{d}{dt}\frac{d}{dv}(m - m_0)c^2 = m\frac{dv}{dt}\gamma^4\left(1 + 2\frac{v^2}{c^2}\right) \tag{76}$$

and the right side is your dynamic equation replacing $F = ma$. While

$$\frac{d}{dt}(mv) = m\frac{dv}{dt}\gamma^4\left(1 + \frac{v^2}{c^2}\right) \tag{77}$$

is not the correct force. Einstein made an incorrect generalization thinking that this should be the force for a changing mass. It is not the force. The term

$$\frac{d}{ds}(m - m_0)c^2 = m\frac{dv}{dt}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\gamma^2\right) \tag{78}$$

is again a different function. It is not the correct force (76) for this kinetic energy, and it is not the force (77) that Einstein thought was the force.

The issue is that Einstein mixed up Newtonian formulas, changed the mass to be non-constant, did not check how this changes the dynamic equation, and those who think they can redefine the kinetic energy do an even bigger error. If the kinetic energy formula changes, then the dynamic equation formula also changes.

Trying for some time with the Euler-Lagrange equations you will soon be convinced that the only thing that makes any sense is that the mass is constant. Else you get the time derivatives of the field ϕ and other unwanted things. The conclusion from Euler-Lagrange equations is that the mass is not changing. Newtonian mechanics has the correct formulas, all Newtonian mechanics needs is an interaction mechanism between a force field and a test mass. This mechanism, if worked out, see [4][5], should give the gravitational time dilation, time dilation in acceleration, $E = mc^2$, the relativistic mass formula, and all what you need to upgrade Newtonian mechanics. Einstein's approach is fundamentally flawed, as will be seen in the next section when the references of this article are outlined. For more discussion on the error in the relativistic mass formula, see [2][3].

5. Comment on the relativistic mass

The relativistic mass formula appears in some experiments, as was noticed before Einstein. When electrons are accelerated, they behave as if their masses were $m = \gamma m_0$. This does not mean that the mass actually has grown. It means that the force that tries to accelerate the mass (to any direction) when the electron is moving fast needs to be stronger than when the electron is not moving fast. This phenomenon can be explained by the force becoming weaker if the relative speed between the field and the mass is higher, see [4][5]. The force is obtained from a field ψ and is $F = m_0\nabla\psi$. In slow speeds it seems that the force that the moving mass m_0 feels is the Newtonian force

$$F = m_0\frac{dv}{dt} \tag{79}$$

but the interaction needs some time and the interaction gets slower when the mass moves. The force actually is

$$F = m_0\frac{d\tau}{dt}\frac{dv}{d\tau} \tag{80}$$

The mass feels the force as

$$F_1 = m_0\frac{dv}{d\tau} \tag{81}$$

while the external force sees the mass as

$$m = \frac{d\tau}{dt} m_0 \tag{82}$$

Geometric considerations show that we get the Lorentz factor

$$\frac{d\tau}{dt} = \gamma. \tag{83}$$

There is no mystery in the experiments where the mass of an electron seems to be growing. The force becomes weaker because the interactions takes a longer time.

In a similar way one can at least partially explain the formula $E = mc^2$ as coming from the interaction of exchanging messages that contain a negative or positive impulse $\pm\Delta mc$ and are moving with the local speed c.

The relativistic mass idea is only one of the serious errors in the relativity theory. The references of this article are not literature references, except for [1] just to honor Einstein. They are a list of preprints that prove beyond any doubt that there are very many serious errors in the relativity theory, thus, they are a continuation of this article, but left outside the article because of space constraints. As the character of this article is to point out that the relativity theory is completely wrong, there is no need to refer to published literature on the relativity theory as this literature accepts the theory, or parts of it, as correct. It is all wrong and beyond any repair.

The main error in the Special Relativity Theory (SRT) is that the main claim of SRT is wrong: the speed of light is not constant in every inertial frame. This error is caused by Einstein not taking a projection on the t' axis when calculating the time difference of two points $(x'_1.t'_1)$ and $(x'_2.t'_2)$ in the moving frame R'. He calculates $T' = t'_2 - t'_1$, but the coordinate system (x', t') is a time-shift coordinate system where

$$t' = \gamma^{-1} t - \frac{v}{c^2} x' \tag{84}$$

The second term to the right is the time shift and it depended on x'. The time in R' depends on the place in the same way as local time depends on the place in the timezone system. The projection on the t'-axis is made by removing this timeshift term, just like in the timezone system we get the Greenwich time by removing the timezone offset. The difference in time in R' of two points is calculated as the difference of the projections on the t' axis. When this is done, the speed of light is not constant. As a consequence of this, whole SRT collapses. See [6], [7], [8].

The main error in the General Relativity Theory (GRT) is that if the speed of light is constant c in vacuum at every point to every direction, as it must be if the tangent space is a Minkowski space at each point, then there are no solutions to the Einstein equations that approximate the Newtonian gravitation potential, see [7],[9], [10], [11]. This means that all experiments that claim to verify GRT are invalid because GRT cannot provide a solution to the field equations that can be used in these experiments.

A special concern is the Schwarzschild solution. The Schwarzschild metric is not a valid metric, the speed of light is not constant in that solution, but also, the geodesics that have been calculated for this solution are incorrectly calculated, see [12][13][14]. Let us especially mention what happens to a test mass that falls freely in the gravitational field created by a point mass in this Einstein's geometric explanation of gravitation: the test mass practically does not accelerate, see [14]. We know that the test mass does accelerate, so the whole approach is wrong. When the geodesic concept is corrected in this respect, we notice that according to this concept light should bend around the world, which is does not do, see [13]. It is all totally wrong, Einstein even calculated the geodesic equation incorrectly, see [14].

Thus, no experiments that have been explained with the Schwarzschild metric can be considered as verification of GRT. For a discussion on the precession of the perihelion of Mercury, see [15] and for the Shapiro delay see [16].

For the problems in the field equation in the geometrization idea see [17][18], the first one also shows errors in Einstein's presentation of Friedman's cosmological results in chapter 5 of [1]. Quantization of gravitation is briefly discussed in [19] and gravitational time dilation in [20], but my ideas have changed since that time towards [4][5]. The relativity theory and also Nordström's ideas are incorrect.

Let us still explain what is wrong in Einstein's proof of $E = mc^2$, his most famous result. Einstein's proof is that he derived the equation

$$(m - m_0)c^2 = \int_0^s \frac{d}{dt}(mv)ds \tag{85}$$

by inserting the formula $m = \gamma m_0$. Derivating (85) gives (17), which for clarity we repeat here

$$\frac{d}{ds}(m - m_0)c^2 = \frac{d}{dt}(mv). \tag{17$''$}$$

There are several problems with (17") that were mentioned in previous sections. The main ones are the following: there is no argument why $m = \gamma m_0$ should hold, mentioned in section 1 under (16). If the kinetic energy is taken as $(1/2)mv^2$ or $(1/2)m_0v^2$, then the kinetic energy is not enough for giving the force in the right side of (17") proven in section 2. The mass cannot depend on velocity, proven in section 3 at the end. The physical argument in section 3 in (54) is that it is not possible to redefine the kinetic energy to $(m - m_0)c^2$ and the kinetic energy that we get is smaller than the work done by the force in (17"). Finally, the mathematical argument in section 4 shows that (17") gives a dynamic equation that does not minimize total energy and cannot be a correct dynamic equation.

As a conclusion, equation (17") is pure nonsense. It is Einstein's $E = mc^2$ proof. The equation $E = mc^2$ itself is correct and can be attributed to Olento di Pretto.

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1.2 Einstein did not prove $E = mc^2$ and the relativistic mass formula is wrong

Abstract: It is generally believed that Einstein proved Olinto di Pretto's formula $E = mc^2$ but Einstein's calculation does not show anything like that. Einstein's proof of $E = mc^2$ only shows that if a mass m_0 that is originally at rest is accelerated by force F and the mass grows by the relativisic mass formula $m = \gamma m_0$, then the work W made by the force F accelerating the mass equals the energy $(m - m_0)c^2$, but in the calculation the force F does not create new mass of the size $m - m_0$. This mass simply appears in the calculation without being created by any energy. This is demonstrated in the first section by an example where the speed v of the mass is small. The rest mass m_0 originally has potential energy E_p in the Earth's gravitational field, then the mass falls freely under the gravitationa force F. Practically all potential energy E_p goes to the kinetic energy of m_0 and there is no energy to create the new mass $m - m_0$. Though the mass $m - m0$ is tiny, creating this mass would require the same amount of energy as W. If Einstein's relativistic mass formula were correct, his calculation would show that no energy is needed for making the mass $m - m_0$ as there is no energy for making it. Mass $m - m_0$ just appears from nowhere. Such from nowhere appearing mass would make it possible to construct a perpetual motion machine. But no perpetual motion machine is pssible and the relativistic mass formula is wrong because it comes from the Lorentz transform which is wrong. The fourth section of the paper shows that the Lorentz transform is seriously wrong. Thus, Einstein's calculation proves nothing at all.

1. The fake formula $E^2 = m_0c^2 + (pc)^2$

Some people think that the total energy E, which they identify with $E = mc^2$ satisfies the following formula:

$$E^2 = m_0c^2 + (pc)^2. \tag{1.1}$$

Let us see what is the error in this formula and why it is identical to the relativistic mass formula

$$m = \gamma m_0 \tag{1.2}$$

which is also wrong.

We start from the equation of motion of a mass m that can change in time. Then Newton's formula $F = ma$ can be written as

$$F = d/dt(mv) \tag{1.3}$$

The work done by this force is

$$W = \int F ds. \tag{1.4}$$

The differential of the work W can be written as

$$dW = \frac{d}{dt}(mv)ds = \frac{dm}{dt}vds + m\frac{dv}{dt}ds \tag{1.5}$$

$$= \frac{ds}{dt} v dm + m \frac{ds}{dt} ds = v^2 dm + mv dv \tag{1.6}$$

$$= \frac{1}{2m} d(m^2 v^2) = \frac{1}{2mc^2} d((pc)^2). \tag{1.7}$$

Thus, we can write an equation

$$\int 2mc^2 dW = C + (pc)^2. \tag{1.8}$$

Here C is an integration constant. By, and only by, setting

$$dE = dW \qquad E = mc^2 \tag{1.9}$$

we get the equation to the form

$$\int 2E dE = C + (pc)^2 \tag{1.10}$$

$$E^2 - E_0^2 = (pc)^2 \tag{1.11}$$

and if $E = mc^2$, then $E_0 = m_0 c^2$. We have the equation (1.1)

$$E^2 = E_0^2 + (pc)^2 = (m_0 c^2)^2 + (pc)^2. \tag{1.12}$$

Notice the error in (1.9). The mass is growing in the equation (1.3), but there is no work in the total work differential for the growing mass in (1.9). As the mass is growing by dm, we have to write

$$dE = dW + c^2 dm \tag{1.13}$$

instead of (1.9), but then the equation cannot be integrated as E is not mc^2.

Notice also that the equation (1.1) is simply a form of (1.2). Squaring (1.2) and multiplying both sides by c^4, we get from (1.2)

$$m^2 c^4 \gamma^{-2} = m_0^2 c^4 \tag{1.14}$$

$$m^2 c^4 (1 - v^2/c^2) = (m_0 c^2)^2 \tag{1.15}$$

$$(mc^2)^2 = (m_0 c^2)^2 + m^2 v^2 c^2 = (m_0 c^2)^2 + (pc)^2. \tag{1.16}$$

That is, $m = \gamma m_0$ is equivalent with (1.1). (1.1) comes from (1.16) and mass in (1.3) appears from nowhere. No energy is used for making the mass $m - m_0$. Einstein's proof that $E = mc^2$ is that if $dE = dW$ and $m = \gamma m_0$, then $E = mc^2$. There are two errors: dE is not dW and m is not γm_0 as will soon be seen.

2. Why (1.2) is not experimentally verified and why mass cannot grow with speed?

Some people claim that there are several experiments where the mass of an electron has been measured to grow according to the formula $m = \gamma m_0$. They say that an engineers designing television catode tubes have been using this formula and it works, so it must be correct. They also say that the phenomenon was observed already by Kaufmann before Einstein and $m = \gamma m_0$ is a verified formula that has been known for a hundred years.

This all is in a sense true, but the formula is still wrong. The experiments are correct, but they do not verify $m = \gamma m_0$, they falsify it. An engineer who uses this formula must set the velocity v of the electron with the (rest) mass m_0 to exactly one number if he wants a correct result. This correct number is the speed of the electron with respect to the field. Thus, the phenomenon is a relation involving the field and the mass. The relation between the field and the mass is an interaction, a force. It is the force that changes when the speed between the field and the mass changes. If the relative speed is very high, the interaction is more difficult. We can easily think of some reasons for this. The interaction takes some small time and progresses with the speed of light. If the mass moves with nearly the speed of light, the interaction takes more time and if the time for the interaction is limited, the interaction may fail. The force gets weaker. Instead of having the mass transform $m' = \gamma m$ between two inertial frames R' and R where R' moves with the speed v with respect to R, we can get the exactly same dynamical equation (1.3) in R' by using a force transform $F' = \gamma^{-1}F$. As the dynamical equation is the same, the measurements seem to verify the formula $m = \gamma m_0$ but in fact they verify $F' = \gamma^{-1}F$ and falsify $m = \gamma m_0$.

The equation (1.2) where everything is in one inertial frame R is directly obtained from the transform equation of the mass between R and R' by considering the acceleration of a mass in R from rest to the speed v. In (1.3) the speed v is a variable speed in R. It is not the constant speed of the frame R'. Therefore we denote the constant speed of R' by w. Then $\gamma_w = (1 - w^2/c^2)$ is a constant in the equation (1.3). The mass transform formula between the frames R and R' is thus

$$m' = \gamma_w m. \qquad (2.1)m$$

We consider two rest frames of the mass that is being accelerated in R as the frames R and R'. The first rest frame R is when the acceleration starts and the mass is at rest in R. Thus, $m = m_0$ in (2.1). The second rest frame is when the accelerated mass has reached the speed $v = w$ in R. This rest frame is R'. The mass in R' is given as m', but it is m at the speed v. The transform (2.1) takes the form

$$m = \gamma_w m_0 \qquad (2.2)$$

and as $w = v$, we can write this as the relativistic mass formula

$$m = \gamma m_0. \qquad (2.3)$$

We see that (1.2) is simply the same as (2.1). We can reverse engineer how the formula (2.1) must have been derived and what it actually means. The equation (1.2) has certainly not been derived from considerations that the total energy

E should satisfy (1.1) because nobody in his right mind would suggest that mass is created from nowhere and would not impose the equation $dE = dW$ in (1.9). The derivation of (1.2) comes from something else. Clearly, the Lorentz transform must have been used to derive (1.2) because there is the Lorentz factor γ. The Lorentz transform only deals with transforms of space and time coordinates. In order to derive a transform for the mass there is only one way: one must take some equation containing mass and to require that the equation is Lorentz invariant. Einstein did want equations of motion to be Lorentz invariant. He especially demanded that the Einstein equations in the General Relativity Theory are Lorentz invariant, i.e., Lorentz covariant in those tensor equations. We can be sure that he wanted (1.3) to be Lorentz invariant and as he uses $m = \gamma m_0$ in the equation (1.3), we can conclude that the mass transform formula (2.1) comes from making (1.3) Lorentz invariant and (1.2) is derived from (2.1) in the way given above.

In order to make (1.2) Lorentz invariant, we need the transforms for length s, time t and the force F. Then we get the transform for m. The transform for s is from the Lorentz transform $s' = \gamma s$. The Lorentz transform actually gives $t' = \gamma^{-1}t$, but Einstein used the transform $t' = \gamma t$. This we know because this formula is in his book The Meaning of Relativity. The form $t' = \gamma^{-1}t$ is impossible in a Minkowski space, though it is what the Lorentz transform gives, see Section 4. The transform that Einstein selected for the force F is easy to find: there is no transform formula for force in the Special Relativity Theory. Therefore F is invariant: $F' = F$. Now we get the mass transform formula (2.1)

$$F' = d/dt'(m'v') = d/dt'(m'ds'/dt') = d/d(\gamma_w t)(m'd(\gamma_w s)/d(\gamma_w t)) \qquad (2.4)$$

Notice that γ is γ_w in this calculation where w is the constant speed of R and not to be confused with the variable speed v. Simplifying

$$F' = \frac{d}{dt}\left(\frac{m'}{\gamma_w}v\right) \qquad (2.5)$$

$$F = \frac{d}{dt}(mv) \qquad (2.6)$$

Setting $F' = F$ we get

$$m' = \gamma_w m. \qquad (2.6)$$

Writing $w = v$ in the coordinate transform we have (2.1)

$$m' = \gamma m. \qquad (2.7)$$

This is the way how (1.2) has been derived. There is no other way to get correctly to this formula, ignoring fake derivations that use steps like (1.9). We see now that there are two errors in this derivation. Firstly, $t' = \gamma t$ is not the equation from the Lorentz transform. However, this error can be accepted because the formula $t' = \gamma^{-1}t$ is impossible and it cannot be accepted. Instead we have to remove the term $-(v/c^2)x$ from the time transform in the Lorentz transform.

Thus, this error is not serious, it merely shows that the Lorentz transform is wrong.

The second error is serious. There is no sense at all to choose that the force F is invariant. It is exactly the force that should change if the speed between a field and a mass changes. We can now see that we can equally well get (1.3) Lorentz invariant by requiring $m' = m$ and $F' = \gamma^{-1}F$. Then

$$F' = \frac{d}{dt}\left(\frac{m}{\gamma_w}v)\right) = \gamma_w^{-1}\frac{d}{dt}(mv) \tag{2.8}$$

$$F' = \gamma^{-1}F. \tag{2.9}$$

Now we have explained why (1.2) has not been experimentally verified, indeed, it has been falsified by these experiments because a very simple argument shows that the mass cannot change with the speed. The argument is the following.

Consider a frame R and some masses and fields in this frame. There are some observable phenomena in this frame R, like the field interacts with a mass in some way. Next select another frame R' that moves with a constant speed w with respect to R and look at what happens with your fields and masses. You see exactly the same observable phenomena, only you see them from a moving coordinate system: everything is moving with the same additional speed that comes from adding the constant speed w (according to some formula) to the speeds that these objects have in R. It is the same world. There cannot be any new observable phenomena. You cannot e.g. by selecting w to be very close to c make some mass m_0 that was at rest in R to grow to so enormous quantity that it would turn into a black hole. You cannot with any experiment notice that the very high speed of any mass in R' changes anything that you can measure in R. Expecially, you cannot have any experiment that verifies the formula $m = \gamma m_0$ by looking at R' because everything you can see in R' is exactly the same that you can see in R. This means that the speed of a mass does not cause any observable changes because the speed of a mass is determined by the coordinate system that you choose to measure the speed of the mass.

But what does not change when you change your coordinate system from the coordinates of R to the coordinates of R' is the relative speed between your objects. The speed between the field and the mass does not change when changing the coordinate system to any moving coordinate system of an inertial frame of reference. This means that the observable difference that has been measured and what is claimed to verify $m = \gamma m_0$ is caused by the speed between the field and the mass and therefore it is a phenomenon that derives from the interaction between the field and the mass. The force between the fiend and the mass becomes weaker if the relative speed is higher.

3. The relativistic mass formula gives a perpetual motion machine

Consider driving a car of mass m_0 on a cliff of height h with respect to the ground level. The car has the potential energy $E_p = ghm_0$. Then push the car off the cliff. The gravitational force F accelerates the car to the velocity v before it crashes on the ground and makes a hole. In Newtonian physics the equation of motion is

$$F = m_0 a = m_0 \frac{dv}{dt} \tag{3.1}$$

and the work is

$$W = \int F ds = \int m_0 \frac{dv\{dt}{d} s = m_0 \int \frac{ds}{dt} dv = m_0 \int v dv = \frac{1}{2} m_0 v^2 = E_k. \tag{3.2}$$

The work done by the force F equals the kinetic energy the mass m_0 gets. The work also equals the potential energy E_p and setting $E_p = E_k$ we get the velocity as $v = \sqrt{2gh}$.

In the relativity theory the equation of motion is $F = d/dt(mv)$, thus, the mass is allowed to change. Notice that this equation does not include any mechanism for making new mass. We can use this equation for instance if we model an airplane that is tanked in the air, the mass increases, or a rocket that is burning its fuel, the mass decreases. There is no term in this equation that corresponds to making the new mass. If the mass grows as in the formula $m = \gamma m_0$, it simply appears to the equation from nowhere.

Let us calculate in a similar way as for $F = ma$ what the work W is

$$W = \int F ds = \int v \frac{dm}{dt} ds + \int m \frac{dv}{dt} ds \tag{3.3}$$

$$= \int v \frac{ds}{dt} dm + \int m \frac{ds}{dt} dv \tag{3.4}$$

$$= \int v^2 dm + \int m v dv \tag{3.5}$$

Thus, $dW = v^2 dm + mvdv$. The right side can be obtained from the relativistic mass formula

$$m = \gamma m_0 \tag{3.6}$$

$$m^2(1 - (v - c)^2) = m_0^2$$

$$m^2c^2 - m^2v^2 = m_0^2 c^2 \tag{3.7}$$

Differentiating

$$2mdmc^2 - 2mdmv^2 - m^2 2vdv = 0 \tag{3.8}$$

which is

$$c^2 dm = v^2 dm + mvdv. \tag{3.9}$$

The fake proof of $E = mc^2$ is obtained by writing

$$\int c^2 dm = \int v^2 dm + \int mvdv = W \tag{3.10}$$

$$(m - m_0)c^2 = W \tag{3.11}$$

and then setting the initial energy to $E_0 = m_0c^2$. The equation (3.11) is correct, but it does not in way say that W is the energy content of the mass $m - m_0$. The work W is almost completely used for giving the mass m_0 kinetic energy and all of W is used to give kinetic energy to the mass m. Notice especially that there is no energy to make the mass $m - m_0$.

Assuming that the mass indeed grows and the car has the mass $m = \gamma m_0$ when it hits the ground, then the energy that is released is the whole kinetic energy, i.e., practically all of W, and the energy contained in $m - m_0$, i.e., another energy of the size of W. This is so because the car cannot have either kinetic energy, or relativistic mass when m_0 is not moving as it is in the hole in the ground. We see that the hole must be twice as big as it should be according to Newton's mechanics, but Newton's mechanics works well in small speeds v. Alternatively, we can collect the energy when the car crashes to the ground and get about twice the potential energy. Thus, we not only have a perpetual motion machine, but a perpetual motion machine that produces lots of energy from nothing. Clearly, the mass of the car cannot increase as in Einstein's relativistic mass formula.

Let us reflect a bit where the error is.

The mass $m - m_0$ must come from somewhere. Either this mass is taken from another mass or it is made from energy. We can think of two scenarios to demonstrate these alternatives. The car is on the cliff and falls down. In the first scenario there is a rope from the cliff to the ground and mountain climbers are hanging from the rope, spaced by one meter. When the car falls passing the level of the mountain climber, he throws a stone through the car window to the car. In this way the mass of the car grows and by suitably selecting the stones the mass can grow as $m = \gamma m_0$. In the second scenario there is a nuclear physicist in the car. He has two high power photon guns and it using them to make particle-antiparticle pairs. He throws the antiparticle off the window, so the mass of the car grows. But the energy he needs is at least $(m - m_0)c^2$. Clearly, as we do not have either of these cases, the mass of the car cannot grow as $m = \gamma m_0$ because there is no energy for it. All potential energy the car had goes to making kinetic energy.

Here is a small but important point. The the relativity theory the energy equivalent $E - E_0 = (m - m_0)c^2$ of the new mass $m - m_0$ is called relativistic kinetic energy. It is not the kinetic energy $E_k = (1/2)mv^2$ of the mass relativistic m. Indeed, from $m = \gamma m_0$ we get

$$mc^2 = m_0c^2 + (1/2)mv^2 + A \tag{3.12}$$

where

$$A = m_0c^2 \sum_{n=1}^{\infty} (1 * 3 * 5 * \cdots * (2n-3)(n-1)n!^{-1}2^{-n}(v^2/c^2)^n \tag{3.13}$$

Here A is $O(v^4c^{-2})$, A is positive and not zero.

This A is a part of the work W that the force F makes, $W = (m - m_0)c^2$ and $W = (1-2)mv^2 + A$, but this work is not in the energy of the mass when it falls to the ground. What is this work A? It is easy to explain with the mountain climber example. When a stone is thrown to the car from the level l, $0 < l < h$, it trushes to the back window of the car because it does not have the speed that the car has on that level. When it is on the window, it gets accelerated to the same speed as the car. This takes some additional work and A is the sum of this work on all levels l. The potential energy equals the work W, but it is necessary to count to the potential energy also the potential energy the additional mass has on the levels l. However, the kinetic energy that the mass m has when it crashes to the ground is not W, it is a bit smaller than W. We cannot say that the energy $(m - m_0)c^2$ to make the mass $m - m0$ is the kinetic energy $(1-2)mv^2$ of the relativistic mass m. This is false.

We also cannot say that the energy $(m - m_0)c^2$ to make the mass $m - m0$ includes the kinetic energy $(1-2)m_0v^2$ of the rest mass m_0. We can only say that the first term of the Taylor series of $(m - m_0)c^2$ has the same expression as the kinetic energy $(1/2)m_0v^2$ of the rest mass m_0. In order to see that the kinetic energy of m_0 really is not included in $(m - m_0)c^2$ and it is simply a coincidence that the expressions are the same, try changing the formula $m = \gamma m_0$ slightly. Replace it e.g. with $m = (1 - av^2/c^2)^{-1/2}m_0$ where a is a constant very close to 1. You see that the first term of the Taylor series of $(m - m_0)c^2$ is $a(1/2)m_0v^2$ while the term $(1/2)m_0v^2$ is still in the Taylor series of W and a only influences the terms $O(v^4c^{-2})$. This kinetic energy term in W is there whether m changes or does not change at all and it does not change if the formula for the mass change is modified.

In the end of this section, we make some calculations to better see what happens.

We can calculate what W is

$$W = \int v^2 dm + \int mv dv \tag{3.14}$$

$$= \int v^2 dm + \int (m - m_0)v dv + \int m_0 v dv \tag{3.15}$$

$$= \int v^2 m_0 d\gamma + \int (m - m_0)v dv + \frac{1}{2}m_0v^2. \tag{3.16}$$

First

$$\frac{d\gamma}{dv} = \frac{v}{c^2}\gamma^3 \tag{3.17}$$

thus

$$d\gamma = \frac{v}{c^2}\gamma^3 dv \tag{3.18}$$

and

$$\int v^2 m_0 d\gamma = \frac{m_0}{c^2}\int v^3\gamma^3 dv \tag{3.19}$$

$$= \frac{m_0}{c^2}\frac{1}{4}v^4 + O(v^6c^{-4}) \tag{3.20}$$

where we used the Taylor expansion

$$\gamma = (1-(v/c)^2)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + O(v^6c^{-6}) \tag{3.21}$$

We obtained

$$\int v^2 dm = \frac{1}{2}\left((1/2)m_0\frac{v^2}{c^2}\right)v^2 + O(v^6c^{-4}) \tag{3.22}$$

The second term in (3.16) is

$$\int (m-m_0)vdv = m_0\int(\gamma-1)vdv = m_0\int\frac{1}{2}\frac{v^3}{c^2}dv + O(v^6c^{-4}) \tag{3.23}$$

$$= \frac{1}{4}\left(\frac{1}{2}m_0\frac{v^2}{c^2}\right)v^2 + O(v^6c^{-4}). \tag{3.24}$$

Thus

$$W = \frac{1}{2}m_0v^2 + \frac{3}{4}\left(\frac{1}{2}m_0(v/c)^2\right)v^2 + O(v^6c^{-4}). \tag{3.25}$$

As

$$m - m_0 = m_0(\gamma-1) = (1/2)m_0(v/c)^2 + O(v^4c^{-4}) \tag{3.26}$$

we can insert

$$(1/2)m_0(v/c)^2 = (m-m_0) + O(v^4c^{-4}) \tag{3.27}$$

to (3.25) and get the final result

$$W = \frac{1}{2}m_0v^2 + \frac{3}{2}\left(\frac{1}{2}(m-m_0)v^2\right) + O(v^6c^{-4}). \tag{3.28}$$

The entity in the parenthesis is the kinetic energy of the new mass $m - m_0$. This mass is very small because in our car example v is very small compared to c. The term $O(v^6c^{-4})$ is still much smaller. We see that W is almost completely the first term in the right side of (3.28) the kinetic energy of m_0.

The calculation that $W = (m - m_0)$ is correct, indeed from (3.21)

$$(m-m_0)c^2 = m_0(\gamma-1) = \frac{1}{2}m_0v^2 + \frac{3}{4}\left((1/2)m_0(v/c)^2\right)v^2 + O(v^6c^{-4}) \tag{3.29}$$

and by inserting (3.25) we get the result that has an identical right side as in (3.28)

$$(m-m_0)c^2 = m_0(\gamma-1) = \frac{1}{2}m_0v^2 + \frac{3}{2}\left(\frac{1}{2}(m-m_0)v^2\right) + O(v^6c^{-4}) \tag{3.30}$$

but the result $(m - m_0)c^2 = W$ does not say anything of the kind that the energy contained in the mass $m - m_0$ would equal $(m - m_0)c^2$. It says that the kinetic energy of m_0 is almost all of W in this example when v is small, and that terms corresponding to the increased mass in W are very small, they mostly correspond to the small kinetic energy of $m - m_0$. There are no terms in W that correspond to creating the mass $m - m_0$ because the energy for creating this mas is as large as W itself.

4. Error in the Lorentz transform

As argued in section 2, the relativistic mass formula must have been derived by requiring Lorentz invariance of the equation of motion $F = d - dt(mv)$. There is no reason to demand Lorentz invariance of any equations of motion because the Lorentz transform is wrong.

In order to see why the Lorentz fransform

$$x' = \gamma(x - vt) \qquad\qquad t' = \gamma(t - (v/c^2)x) \tag{4.1}$$

is wrong, think about flying a plane from New York to London. The distance is 5700 km and the time zone difference is five hours. The flight time is seven hours and from that we get the average speed of the plane as 812 km/h.

There usually is a panel showing the local time in over-Atlantic planes. Let us assume that the programmer who made this application was a bit lazy and did not care of time zones, after all, the plane mostly flies over the ocean. Thus, he wrote the program that the local time in the panel is

$$t' = t - ax \qquad\qquad \text{where} \qquad\qquad \text{a} = \text{5h/5700km}. \tag{4.2}$$

The time t is the London time and x is the distance from London. This equation works fine in both London and New York. In London $x = 0$ and $t' = t$, the London time. In New York $x = 5700km$ and $t' = t - 5h$, the New York time. The programmer also wanted to define a space coordinate x' for the plane, so that x' would give the location of the seat of a passenger. He lazily ignored that the plane does not always fly exactly $v = 812$ km/h and defined

$$x' = x - vt. \tag{4.3}$$

The coordinates (x', t') can be used as a coordinate system for the plane and the transform from (x, t) coordinate system is very much like the Lorentz transform, only with $\gamma = 1$

$$x' = x - vt \qquad\qquad t' = t - (v/c^2)x. \tag{4.4}$$

Assume you get to the plane in New York at the local time 12 AM, and you sit on a seat on the last row so that your x'-coordinate is $x'_1 = 0$. The t'-coordinate is $t'_1 = 12$ AM as it shows the local time. Thus, in the plane coordinates your point is $(x', t') = (0, 12AM)$. The x-coordinate is 5700 km as it is the distance from London and t is the time in London, 5 PM. The plane arrives to London at the local time 12 PM. The plane time shows $t'_2 = 12$ PM and if you are sitting on the same seat, then $x'_2 = 0$. The flight time is seven hours. How do you get the flight time? The flight time is the time difference between starting time and ending time, but it is not the time difference between the local time 12 AM when the plane leaves New York and the local time 12 PM when you arrive to London. Indeed, $t'_2 - t'_1 = 12PM - 12AM = 12$ hours. You must project the point (x'_1, t'_1) to the t'-axis, which is the Greenwich time i.e., the London time. The projection of $(0, 12AM)$ on the t'-axis is 5 PM. The point $(0, t'_2)$ already has the time in the Greenwich time. The flight time is 12 PM-5 PM=7 hours. Notice that if you move your seat to another row, then your t' time changes because a seat that is more in the front is a bit closer to London. Thus, your time t' in the plane depends on your x'-value. The coordinates x' and t' are not independent.

Compare this situation to the Lorentz transform. The time defined by the formula $t' = \gamma(t - ax)$ is a local time at the place x. The coefficient a gives the ration of time offset in time units divided by the space distance. In the plane coordinate system $a = 5h/5700km$, in the Lorentz transform $a = v/c^2$, that is, it depends on v, which means that this timeoffset coordinate system works in the same way as the plane timeoffset coordinate system (only) if v is always the same, but in our example v is constant.

Also in the Lorentz transform the coordinates x' and t' are not independent. Indeed, the line $x = \gamma^{-1}x' + vt$ maps in the Lorentz transform to $x' = \gamma((\gamma^{-1}x' + vt) - vt)$ and the corresponding time is

$$t' = \gamma(t - (v/c^2)(\gamma^{-1}x' + vt)) = \gamma(t - (v/c^2)vt) - (v/c^2)x' \tag{4.5}$$

We have a linear dependence between t' and x'

$$t' = \gamma^{-1}t - (v/c^2)x'. \tag{4.6}$$

This linear dependency means that in order to take the projections on the x'-axis and t'-axis from a point (x', t') we cannot simply take the number in the 2-tuple (x', t'). We must project to point to the coordinate axis. Let us do the projections, first the projection on the x'-axis.

The projection on the x'-axis.

1. The projection is some point x'_1 on the x'-axis. The preimage of $(x'_1, 0)$ is a point on the line $t = (v/c^2)x$

2. The line $x = \gamma^{-1}x' + vt$ maps to a point that has the x'-coordinate value x'.

3. The intersection point of the lines $t = (v/c^2)x$ and $x = \gamma^{-1}x' + vt$ is $(x, t) = (\gamma x', (v/c^2)\gamma x')$. The image of this intersection point is $(x', 0)$. This is a point on the t'-axis. The projection of (x', t') on the x'-axis is x'.

The projection of the t'-axis. We try to do the same as for the x'-axis.

1. The projection is some point t'_1 on the t'-axis. The preimage of $(0, t'_1)$ is the line $x = vt$.

2. The line $x = \gamma^{-1}x' + vt$ maps to a point that has the x'-coordinate value x'.

3. There is no intersection point of the lines $x = vt$ and $x = \gamma^{-1}x' + vt$. They are parallel lines meaning that each line gives a clock that has a different time offset. In order to project the point (x', t') on the t'-axis, we must use the projection in the coordinates (x, t). We project $(\gamma^{-1}x' + vt, t)$ on the t-axis. It gives the point $(0, t)$. We project (vt_1, t_1) on the t'-axis. It gives the point $(0, t_1)$. In order that the image $(0, \gamma^{-1}t_1)$ of (vt_1, t_1) is the projection of $(x'.t')$ on the t'-axis, we must have $t = t_1$. Thus, the projection of (x', t') on the t'-axis is $t'_1 = \gamma^{-1}t$.

Doing the projections on the x' and t'-axes we notice that the speed of light is not c in $(x'.t')$ coordinates of the moving frame.

Indeed, let the frame of reference R have the independent coordinates x and t, let light have the speed c in R, and let there be a rod of (moving) length L in R. At the time $t = 0$ the left end of the rod is at $x = 0$ and the rod moves to the right with the speed v. Let R' be the moving frame of the rod and let R' have the coordinates (x', t') from the Lorentz transform. We sent light in R from the left end of the rod to the right end. Light arrives to the right end at the time T. Then light moves as $x = ct$ and the right end of the rod as $x = L + vt$. Light arrives to the right end when $cT = L + vT$ giving $L/T = c - v$. The Lorentz transforms of the points are as follows. The starting point $(0, 0)$ maps to $(0, 0)$. The ending point $(L + vT, T)$ maps to $(L', \gamma(1 - v/c)T)$ where $L' = \gamma L$. The speed of light in R' is NOT as Einstein calculated

$$c' = L'/(\gamma(1 - v/c)T) = \frac{c}{c - v}\frac{L}{T} = c. \tag{4.7}$$

The time in R' is not $\gamma(1 - v/c)T = \gamma^{-1}T - (v/c^2)L'$. It is $T' = \gamma^{-1}T$ and the speed of light in R' is

$$c' = L'/T' = \gamma^2(c - v). \tag{4.8}$$

We conclude that the rest time of R' from the Lorentz transform is $\tau = \gamma^{-1}t$, but Einstein has more errors. The rest time, or proper time, of R' cannot be $\tau = \gamma^{-1}t$ because in a Minkowski space the speed of light can be read from the line element:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 \tag{4.9}$$

If light is sent to the direction x, then $dy = dz = 0$ and for light $ds = 0$. Thus

$$c^2 = \frac{dx^2}{dt^2} \tag{4.10}$$

must hold. It also must hold in R', thus

$$c^2 = \frac{dx'^2}{dt'^2} \tag{4.11}$$

Here (x', t') are obtained from (x, t) through a transform. If $x' = \gamma x$, then necessarily the transform must give $t' = \gamma t$. This is why the Lorentz transform cannot have the timeshift term $-(v/c^2)x$ in the time transform. We must drop this term and the transform can only be

$$x' = \gamma(x - vt) \qquad\qquad t' = \gamma t. \tag{4.12}$$

But this is also wrong because the transform relates two frames of reference A and B that can be billions of light years apart and their world paths never need to meet, yet the transform allows us to compare the rest times of the frames. We can make the twin paradox. Assume that the frames are identical and they have mutual velocity v. Both frames have their rest times t_a and t_b respectively and nothing we can do by selecting which frame to take as the frame R or R' can change these rest times. If we choose $R = A$ and $R' = B$, then $t_b = \gamma t_a$ while if we choose $R = B$, $R' = A$, then $t_a = \gamma t_b$. Necessarily $\gamma = 1$. Einstein cheated when he claimed that the world paths must meet in order to compare the times in the twin paradox. Naturally they do not need to meet as the transform gives the comparison.

We are now left with the Galileo transform.

$$x' = x - vt \qquad\qquad t' = t. \tag{4.13}$$

but this is still not correct. It is widely believed (and seems correct) that if we have two planets far away from each other and with mutual speed, then the speed of light is c close to both planets. This can easily be so if we e.g. assume that the speed of light is set by the local gravitational field. But it means that we cannot have global coordinate systems where the speed of light is c everywhere and a global coordinate transform relating these coordinate systems. The whole idea of having such a transform like the Lorentz transform (with whatever changes you might imagine) is false. There is no such global transform, though we can use the transform $x' = \gamma(x - vt)$, $t' = \gamma t$ in a smaller environment and get (2.9).

Consequently, there is absolutely no justification for requiring Lorentz invariance from equations of motion. This means that the relativistic mass formula is completely unjustified, and it is wrong as the car example of Sections 1-3 show.

5. Ten serious errors in the Relativity Theory

One should not get the false impression that Einstein's only serious errors are giving a fake proof of $E = mc^2$, a false relativistic mass formula and making false claims of the Lorentz transform. The whole of the Relativity Theory in [4] and [5] is false. I give ten errors, and these are all serious:

1. Einstein defines infinite number of times to the moving frame R' though R' has only one time. That is, Einstein treats the t' value in (c', t') as a time value in the time of R'. The t' value is a local time. There are infinitely many local times in a continuous timeshift coordinate system, like the coordinates (x', t') from the Lorentz transform are. These local times have the same length of seconds, but a different starting time. But Einstein makes it worse than this

by treating the time difference of two different local times as a time value in the time of R'. This way he gets infinitely many times that have different seconds. This is explained in [7] and [6].

2. Einstein does not take the projection on the t'-axis and therefore falsely claims that the speed of light is constant in all inertial frames of reference. When this error is corrected, whole Special Relativity Theory falls down. It is not possible to DEFINE that the time should be treated as Einstein does. Such an attempt leads to a contradiction: simply by imagining a moving frame you can set the universal constant c to any value you want, like to 1 m/s. This is explained in [7] and [6].

3. The Lorentz transform gives the proper time for R' as $t' = \gamma^{-1}t$ while from the definition of the proper time and from Minkowski space we get $t' = \gamma t$. This means that the timeshift term $-(v/c^2)x$ must be dropped. Without dropping the term, the speed of light has no upper bound: if light is sent backward, the speed of light in R' is $c^2/(c-v)$. See Section 4 and [6].

4. The twin paradox shows that $\gamma = 1$ and the transform is the Galileo transform, but finally we notice that there cannot be any global coordinate transform where the speed of light is constant in the coordinate system. This is explained in Section 4. Notice that when the Lorentz factor is dropped because of the twin paradox, as it has to be, Einstein has not proof that c is the maximal speed.

5. Requiring Lorentz invariance (like covariance) from any equation is a grave error because there is no justification for it. It gives a false formula for the relativistic mass. This error alone invalidates the General Relativity Theory (GRT) as that theory is Lorentz covariant. See Section 4 and [7]. There are valid nontrivial transforms of coordinates, e.g. when there is a gravitational field, so one may get a mass transform formula, but it is better to place the growing mass to the rest coordinates of the mass if such changing mass is needed.

6. Einstein did not prove $E = mc^2$. The whole proof is fake. See Sections 1-3.

7. The field equation of GRT does not have any solutions which have speed of light constant c in vacuum at every point to every direction and which approximate Newtonian gravitation field. This invalidates GRT in all applications in our solar system and it invalidates all experiments that are claimed to verify GRT. See [6].

8. This is a stronger form of error 6: even if we allow the field to have speed of light that differs in the gradient of the gravitational field, there is no solution for the case of a point mass in empty space that has the speed of light c to every direction that is orthogonal to the gradient of the gravitational field. See [13] and [12]. The calculation in [13] is just like in the Schwarzschild solution, but there is no solution.

9. Einstein's calculation that GRT satisfies the Shapiro delay test is wrong. This is because the Schwarzschils solution (a convenient example for doing this calculation) does not have a constant speed of light. Einstein also calculated that Nordström's theory (see [1]-[3], [10]-[12]) fails the Shapiro test. It does

not fail the test, it passes it. See [10]. But as GRT does not approximate the Newtonian gravitation field and we know that it is a good approximation to the reality, GRT cannot pass any tests anywhere in our solar system.

10. Einstein gave a false calculation of the precession of the perihelion of Mercury. GRT cannot correctly predict anything in our solar system as it cannot approximate Newtonian gravitation. Therefore the calculation is false. Einstein also gave a false calculation that Nordström's gravitation theory predicts the precession incorrectly. The calculation is nonsense. Nordström's theory does not predict anything as the situation is very complicated to model. See [8].

I guess that is enough for one theory. Of course one could add problems in the theory, like that GRT is not quantized. (One can quantizise a scalar gravitation theory, like Nordström's theory, see [9]), but such are merely problems. It is more relevant that the Relativity Theory has at least ten fatal errors. Those errors show that Einstein cheated intentionally, and that his theory is totally wrong.

There is an old saying that nobody ever convinces the supporters of the old theory. The old die and the new generation accepts the new theory. This may be so, and in most cases the new theory is better. But this saying can also be applied to a situation when the new theory is completely false. It gets accepted by this mechanism and a better old theory is discarded. This is what happened with the Relativity Theory. I very much doubt such can happen unless several high level journals and academic positions and media support the false new theory and their pressure gets the theory to schools and universities as the scientific truth. And this is what must have happened. This has nothing to do with common and healty reservation towards new theories.

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1.3 A calculation showing that the relativistic mass formula is wrong

Abstract: This is a calculation showing that the relativistic mass formula $m = \gamma m0$ is wrong. In case it seems that I am adding too many intermediate steps, then the reason is that without all these steps the supporters of the relativity theory are unable (or unwilling) to follow the argument. Naturally, they are unwilling even when all steps are written down, but that's another thing.

1. The calculation

Let us consider the situation where a mass m_0 is on the height h in the gravitational field of the Earth. The place is chose so that the acceleration of a mass under the gravitational force is very precisely g, or we measure g in this place. The height h is ten meters and the gravitational acceleration can be assumed constant in this short distance. From Newtonian mechanics we get an approximation to the speed of the mass when it falls from the height h to the ground: the mass m_0 hits the ground with the speed that is closely approximated by $v = \sqrt{2gh}$.

The Special Relativity Theory (SRT) claims that the mass grows because of the speed v according to the formula $m = \gamma m_0$. The speed of the falling mass is so small in this case that the increase of the mass is very small

$$m = m_0 + O(v^2/c^2) \tag{1}$$

and the approximation of the potential energy of the mass when it is on the height h is very good. This approximation of the potential energy is

$$E_p = ghm_0. \tag{2}$$

When the mass falls, the gravitational force F accelerates it. We do not need to model the gravitational force. It suffices to agree that the following two formulas are correct and accepted both in SRT and in Newtonian mechanics:

$$F = \frac{d}{dt}(mv) \tag{3}$$

$$W = \int F ds \tag{4}$$

Then

$$dW = F ds = \frac{d}{dt}(mv) ds = \frac{dm}{dt} v ds + \frac{dv}{dt} m ds \tag{5}$$

$$= \frac{ds}{dt} v dm + \frac{dv}{dt} m ds \tag{6}$$

$$= v^2 dm + \frac{dv}{dt} m(s) ds. \tag{7}$$

The potential energy of the mass at the height $h - s$ is

$$E_p(s) = g(h - s)m \tag{8}$$

where m now can change because of speed v. As v depends on s, we can consider m as a function of s, $m = m(s)$. Differentiating (8) we get

$$dE_p(s) = E_p(s + ds) - E_p(s) = g(h - s - ds)m(s + ds) - g(h - s)m(s) \tag{9}$$

$$= g(h - s)(m(s + ds) - m(s)) - g\,ds\,m(s + ds) \tag{10}$$

Writing $dm = dm(s) = m(s+ds) - m(s)$ and taking the first order approximation of

$$gdsm(s + ds) = g\,ds\,m(s) + O(ds^2) \tag{11}$$

we get

$$dE_p(s) = g(m - s)dm - g\,m(s)ds \tag{12}$$

If the mass dm is real mass, it just cannot appear from nowhere. Making this new mass takes the energy $dE = c^2dm$. There are so far no terms in any of the given equations for making this new mass. Let us consider three cases.

Case 1 is that there is no energy used for making the mass dm and this mass is real mass.

Case 2 is that the mass dm is real mass and potential energy is used for making this mass. The relativistic mass formula holds: $m = \gamma m_0$.

Case 3 is that the mass dm is not real mass. It is some effective mass, no energy is needed for making this mass. The mass does not in reality increase, the relativistic mass formula is just one way of expressing that the force gets weaker when the relative speed between a mass and the field grows larger. The relativistic mass formula holds: $m = \gamma m_0$.

Case 1

The mass dm is real mass and there is no term containing the energy to make it. The term $g(h - s)dm$ contains the potential energy of the mass dm and the term v^2dm comes from the calculation of the kinetic energy, therefore it is kinetic energy of dm even though it is not of the classical form $(1/2)dmv^2$. There is no term for $dE = c^2dm$. Therefore the mass cannot grow and we must set $dm = 0$. Then

$$dE_p(s) = -gm(s)ds \tag{13}$$

$$dW(s) = \frac{dv}{dt}m(s)ds \tag{14}$$

The energy conservation equation is

$$0 = dE_p(s) + dW(s). \tag{15}$$

The equation is not $dE(s) = dW(s)$ because the variable s appears as negative in $E_p(s) = g(h-s)m$. The solution is the classical solution

$$0 = \left(\frac{dv}{dt} - g\right) m(s)ds \tag{16}$$

thus $dv/dt = g$, as it is in Newtonian mechanics, and $W = (1/2)m_0 v^2$.

Case 2

The mass dm is real mass and potential energy is used to create this mass. We divide the potential energy differential to two parts

$$dE_p(s) = dE_{p,1}(s) + dE_{p,2}(s) \tag{17}$$

where $dE_{p,1}(s)$ is used to create the kinetic energy W and $E_{p,2}(s)$ is used to create the mass dm. We also assume that $m = \gamma m_0$ holds.

The energy conservation equations are

$$0 = dE_{p,1}(s) + dW(s) \tag{18}$$

$$dE_{p,2} = dE = c^2 dm \tag{19}$$

The reason why in (19) the sum of the differentials is not zero is that in (19) there is no negatively signed s that changed the sign of $dE_{p,1}$ to negative.

Thus

$$dE_{p,1}(s) = dE_p(s) - E_{p,2}(s) = (g(h-s) - c^2)dm - g\,m(s)ds$$

and the equation is

$$0 = (g(h-s) + v^2 - c^2)dm + \left(\frac{dv}{dt} - g\right) m(s)ds \tag{20}$$

which simplifies

$$\left(g - \frac{dv}{dt}\right) m(s)ds = (g(h-s) + v^2 - c^2)dm \tag{21}$$

$$g - \frac{dv}{dt} = \frac{1}{mds}(g(h-s) + v^2 - c^2)dm\frac{dv}{dv} \tag{22}$$

$$g - \frac{dv}{dt} = \frac{1}{mds}(g(h-s) + v^2 - c^2)\frac{dm}{dv}dv \tag{23}$$

Using $m = \gamma m_0$ we have $dm/dv = m_0(v/c^2)\gamma^3$. Inserting to (23) and multiplying the equation by dt

$$gdt - dv = \frac{1}{\gamma m_0}\frac{dt}{ds}(g(h-s) + v^2 - c^2)m_0\frac{v}{c^2}\gamma^3 dv \tag{24}$$

$$gdt - dv = \frac{1}{v}(g(h-s) + v^2 - c^2)\frac{v}{c^2}\gamma^2 dv \tag{25}$$

$$gdt - dv = \frac{1}{c^2}\gamma^2(g(h-s) + v^2 - c^2)dv \tag{26}$$

$$g - \frac{dv}{dt} = \frac{1}{c^2 - v^2}(g(h-s) + v^2)\frac{dv}{dt} - \gamma^2\frac{dv}{dt} \tag{27}$$

Notice that we have a contradiction in (27). The term

$$\frac{1}{c^2 - v^2}(g(h-s) + v^2)\frac{dv}{dt} \tag{28}$$

is very small as v is very small compared to c. The Lorentz factor γ is practically one. The equation (27) claims that

$$g - \frac{dv}{dt} = -\frac{dv}{dt} + O(c^{-2}) \tag{29}$$

which is not possible: $g = 9.81m/s^2$ and not close to zero. We can solve (27) exactly, but because there is a contradiction with the physical reality, the result is absurd. Simplifying

$$g = \frac{dv}{dt}\left(1 + \frac{1}{c^2 - v^2}(g(h-s) + v^2 - c^2)\right) \tag{30}$$

$$g = \frac{dv}{dt}\frac{c^2 - v^2 + g(h-s) + v^2 - c^2}{c^2 - v^2} \tag{31}$$

$$g = \frac{dv}{dt}\frac{g(h-s)}{c^2 - v^2} \tag{32}$$

$$\frac{dv}{dt} = \frac{c^2 - v^2}{h - s}. \tag{33}$$

Of course (33) is wrong, a result of a contradiction. It is not possible to treat dm as real mass and assume that $m = \gamma m_0$ holds.

Case 3 The mass dm is not real mass. It is effective mass and actually describes that the interaction force depends on the relative speed of the mass and the field. As the mass dm is not mass, we will not include any term containing the energy to make it. Instead, we assume that $m = \gamma m_0$ holds.The potential energy differential is

$$dE_p(s) = g(h-s)dm - gm(s)ds \tag{34}$$

and the equation is

$$0 = (g(h-s) + v^2)dm + \left(\frac{dv}{dt} - g\right)mds. \tag{35}$$

The calculation is very similar to the Case 2. The equation corresponding to (31) is

$$g = \frac{dv}{dt}\frac{c^2 - v^2 + g(h-s) + v^2}{c^2 - v^2} \tag{36}$$

there is just missing the term $-c^2$ from $-c^2dm$ in Case 2. The result is

$$\frac{dv}{dt} = g\gamma^{-2}\frac{1}{1 + g(h-s)/c^2} \tag{37}$$

Clearly, the acceleration of v is very close to g for small values of v. This result is possible but what is really says is that $m = \gamma m_0$ is possible only if the mass is not really growing. The relativistic mass is simply one way to calculate the change of the interaction strength when the relative speed grows. Thus also means that Einstein's proof of $E = mc^2$ is not any proof or anything. The mass that he has in his proof is not mass, it is virtual mass that does not even need energy to be made. The formula $E = mc^2$ does hold and the mass there is real mass.

Conclusions

Mass is not growing with velocity. There are certain cases when the formula $m = \gamma m_0$ seems to work, especially in electro-magnetism. The real explanation of such phenomenon is not that the mass grows. The natural explanation is that the interaction force depends on the relative speed of the particle and the field. Let us add that the energy-momentum formula

$$E^2 = (m_0c^2)^2 + (pc)^2 \tag{38}$$

where $E = mc^2$ is equivalent with $m = \gamma m_0$ and it is also only a phenomenological formula, or maybe heuristic is a better word as the formula is known to be wrong. But such formulas can be used where they work. At some point somebody should revise the theory of relativity, it has too many errors.

1.4 Explanation of the Lorentz factor and the relativistic mass formula

Abstract:

H.A. Lorentz derived the relativistic mass formula $m = \gamma m_0$ for the mass of an electron and it was verified in β-radiation. Therefore is formula does work, but it does not mean that the mass in reality grows. A. Einstein confused the issue in his Special Relativity Theory. The article recovers the correct understanding of this formula and also what the Lorentz factor really is.

1. The derivation of $m = \gamma m_0$

Consider the interaction of a field with a mass. The mass m_0 is moving with the speed v with respect to the field. There is an interaction between the field and the mass.

A common understaiding of an interaction is that it is an exchange of virtual interaction bosons. We assume that this exchange requires a two-way exchange of interaction bosons. There is a time constraint in virtual bosons, therefore the interaction may take too long and fail. If so, it consumes energy from field but does not change the orbit of the mass. We also assume that the boson is massless and travels with the speed of light c.

The interaction time T' (i.e., the roundtrip time) over the distance L is

$$T' = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c}\gamma^2 = \gamma^2 T \tag{1}$$

Here T is the interaction time if the mass is not moving. Light sent to any direction from the mass makes the distance $X' = cT'$ assuming that the speed of light sent from the mass is c. Then X transforms like

$$X' = \gamma^2 X \tag{2}$$

where X is the distance if the mass is not moving.

We assume that the force F decreases as r^{-2}, which is true for the Coulomb force and for the gravitational force. It is not true for all forces. As the distance is transforming X to X', the force is transforming

$$F' = C(X')^{-2} \qquad \text{and} \qquad \mathrm{F = CX^{-2}} \tag{3}$$

where C is a constant. The force transform must be

$$F' = \gamma^{-4} F \tag{5}$$

The energy differential transform is

$$dW' = \gamma^{-3} dW. \tag{6}$$

This is not derived analytically, it follows from the empirically verified formula that Lorentz used for eletron's mass:

$$m = \gamma m_0 \tag{7}$$

This formula and a reference to Lorentz is in Einstein's book [1] as the formula (46). Thus, Einstein also justified this formula with the work of Lorentz.

The derivation of (6) is simple. Starting from

$$dW = Fds = d/dt(mv)ds = \frac{dm}{dt}vds + \frac{dv}{dt}mds = v^2dm + mv\,dv \tag{8}$$

and inserting $m = \gamma m_0$, $dm = (v/c^2)\gamma^3 dvm_0$ we get

$$dW = v^2\frac{v}{c^2}\gamma^3 dvm_0 + mv\,dv \tag{9}$$

$$= mv\,dv\left(\frac{v^2}{c^2}\gamma^2 + 1\right) \tag{10}$$

$$= \gamma^3 m_0 v\,dv = \gamma^3 dW' \tag{11}$$

As

$$dW' = F'ds' = \gamma^{-4}Fds' = \gamma^{-3}dW = \gamma^{-3}Fds \tag{12}$$

we must set the transform of ds' as $ds' = \gamma ds$. Then $dt' = \gamma dt$ because we want the speed v to be the same

$$v = \frac{ds'}{dt'} = \frac{ds}{dt} \tag{13}$$

to have the same orbit for the mass in both coordinate systems ds, dt and ds', dt'.

2. The meaning of the Lorentz transform

The term γ^2 comes naturally from a two-way exchange of interaction bosons in equation (1).

How to understand what ds' and dt' are? One way of understanding them is that they are infinitesimal coordinates of the frame of the moving mass. The transform giving $ds' = \gamma ds$, $dt' = \gamma dt$ is

$$x' = \gamma(x - ct) \qquad\qquad t' = \gamma t \tag{14}$$

It is the Lorentz transform that Einstein uses in SRT when discussing Minkowski spaces, and it is the transform the Einstein uses in the General Relativity Theory (GRT) when requiring Lorentz covariance.

It is not the Lorentz transform that Einstein gave in the Special Relativity Theory (SRT) by which he claims (incorrectly) that the speed of light is c in all inertial frames. That Lorentz transform is

$$x' = \gamma(x - ct) \qquad\qquad t' = \gamma(t - (v/c^2)x) \tag{15}$$

The coordinates x' and t' in (15) are not independent and cannot be used as local coordinates in the Minkowski space. When the coordinates are made independent by projecting (x', t') on the t'-axis, the differential coordinates we get are $ds' = \gamma ds$, $dt' = \gamma^{-1} dt$. Einstein never noticed that he is using two different transforms that he calls with the same name, the Lorentz transform, which incidentally is not the transform Lorentz used. That transform was

$$x' = \gamma(x - ct) \qquad\qquad t' = \gamma^{-1} t \tag{16}$$

the only transform that gives c as the roundtrip speed of light in the moving frame.

There are other understandings of ds' and dt' and we can define them differently. A rather natural definition would be $dt' = \gamma^2 dt$, $ds' = \gamma^2 ds$ because it corresponds to the interaction time delay transform. The negative result of this choice is that then the force transfrom is

$$F' = \gamma^{-5} F \tag{17}$$

Another possible choice is $dt' = dt$, $ds' = ds$, which has the advantage of having only one coordinate system. Then the force transform formula would be

$$F' = \gamma^{-3} F \tag{18}$$

The energy differential transform formula is always (6). The issue with force transform formulas like (17) and (18) is that then the force does not grow as r^{-2}. This inverse square law is in reality a statement that the space is flat and the surface of a sphere grows to square as a function of the radius. We can naturally select a different geometry.

There are still different ways of understanding ds' and dt'. The interaction of the field and the mass is expected to be very short range and the growth of the surface of a sphere in the space geometry probably plays no role. The interactions can well fail because of time constraints of virtual bosons and the force relation $F' = \gamma^{-5} F$ need not have anything to do with geometry. Indeed, interactions must be very short range and very fast because if something is exchanged between the Sun and planets in gravitational interaction with the speed of light, then the solar system becomes unstable. We may not need the r^{-2} behavior for the force in the situation of the interaction between the field and the mass and the Lorentz-type transforms of ds' and dt' can very well be wrong. A model for the interaction would be needed. Maybe something new would arise from such a model.

3. Relativistic mass $m = \gamma m_0$ is not mass

It should be clear from section 1 that m is not mass. It is simply a way to express that the force becomes weaker if the relative speed between the field and the mass is larger. The equation $m = \gamma m_0$ fits to measurements: we can measure the energy used and we need the energy that is sufficient for the mass m, and we can measure the orbit of the mass and obtain v. But both measurements give the same numbers whether the real reason is that the mass really grows or that the force gets weaker.

There are several reasons why the mass does not really grow with speed.

1. The speed v of a mass m depends on what coordinates we use. We can take a mass m_0 that is at rest in our coordinates, for instance, m_0 can be one kilo of flour on the table. Then we select a coordinate system that is moving with the speed $-v$ where v is so close to c that according to the formula $m = \gamma m_0$ the mass in this moving coordinate system is so enormous that it must collapse to a black hole. Looking at the same world from two different coordinate systems we must see the same behavior. There cannot be any observable differences because all that changes is our view point. It m_0 is a black hole in one coordinate system, then it is a black hole in every coordinate system. We just look around and notice that the kilo of flour is still just a kilo of flour. Therefore it cannot be a black hole in the moving coordinate system. The mass cannot grow with speed.

2. In the experiments, like that of Lorentz, where the relativistic mass formula has been verified, the speed v can only be the relative speed of the field and the mass. If we select a moving coordinate system where the mass moves very fast and insert this velocity to the formula, then the result is wrong. This means that the measured phenomenon is not about the speed of the mass. It is about the relative speed between the mass and the field. It is a relation though $m = \gamma m_0$ does not show that it is a relation. The relation between the field and the mass is the interaction, the force. This phenomenon is caused by the force changing. The force becomes weaker when the relative speed between the field and the force grows.

3. Consider a nuclear experiment where the amount $2(m - m_0)$ of mass turns into energy. Half of this energy, $E_1 = (m - m_0)c^2$ escapes as radiation, half $E_2 = (m - m_0)c^2$ escapes as kinetic energy of the moving particle with the rest mass m_0. The energy E_2 gives the mass m_0 the velocity v and the work needed to give the mass this kinetic energy is $W = (m - m_0)c^2$. Now, if this mass m_0 did really grow to m because of the velocity v, then how much matter was turned to energy? First $2(m - m_0)$ mass turned to energy, then $m - m_0$ new mass was created, so only $m - m_0$ mass was turned to energy. It released the energy $(m - m_0)c^2$, which is just enough to explain the energy E_1 that escaped in raditation. So, where did the kinetic energy to accelerate the mass m_0 to the speed v come from?

It is easy to invent more similar arguments and though Einstein's book [1] quite clearly says that mass grows with speed under the equation (44), most researchers of relativity try to explain this problem off by stating that the mass actually does not grow, the relativistic mass is the kinetic energy of the moving mass.

This explanation is wrong because the kinetic energy of the moving mass is smaller than the energy equivalent of the mass $m - m_0$. When the moving mass stops, it must release the energy $(m - m_0)c^2$ but this energy cannot be released from the kinetic energy $(1/2)mv^2$ that the moving mass has because $(1/2)mv^2$ is strictly smaller than $(m - m_0)c^2$. The energy cannot be released from the energy m_0c^2 that the rest mass contains because the rest mass is still there when the moving mass stops. Therefore the moving mass would have to turn some of its moving mass to energy in order to release the energy $(m - m_0)c^2$. But if the moving mass turns some of the mass $m - m_0$ to energy, then certainly it must turn all of the mass $m - m_0$ to energy. What would be the sense of turning only a small bit of this mass to energy? If the mass turns all of the mass $m - m_0$ to energy and additionally releases the kinetic energy $(1/2)mv^2$, then we have a perpetual motion machine. We get more energy than we put it. Consider dropping mass m_0 from height h in the Earth's gravitational field. The potential energy of this mass very closely approximates the kinetic energy of the mass when it hits the ground, as Newtonian mechanics works quite well in this situation. The energy of the additional moving mass $m - m_0$ in this case equals the potential energy and is a bit larger than the kinetic energy of the falling mass when it hits the ground. We collect this energy and get about twice the potential energy. We can make a hydropower plant without a river: simply have a pool of water, let it fall to the turbines, with half of the collected energy pump the water back to the pool and sell the rest to the national grid. It would be nice, but naturally, this does not work.

Let us show for those who claim that the relative mass is or is contained in the kinetic energy that this is not the case. The kinetic energy is smaller than $(m - m_0)c^2$. We do this first with a simpler example. The mass m_0 is on the height h and falls. We will not approximate the potential energy at all in order to avoid all approximations, but us select the height h to be so small that we can ignore all relativistic effects. The mass m_0 falls to the level $s = s_0 < h$. The kinetic energy that the mass gains on the trip $s \in [0, s_0)$ is solved from

$$dW = v^2 dm + mvdv \tag{19}$$

by setting $dm = 0$ and $m = m_0$. Then

$$W_1 = (1/2)m_0 v(s_0)^2 \tag{20}$$

Then we add the mass Δm, which is small in this example and can be considered as dm in the equation (19) as

$$\Delta W = v(s_0)^2 \Delta m \tag{21}$$

On the trip $s \in (s_0, h]$ the mass is constant $m = m_0 + \Delta m$. The kinetic energy is solved from (19) as

$$W_2 = (1/2)(m_0 + \Delta m)(v^2 - v(s_0)^2) \tag{22}$$

The total energy used to give the mass its kinetic energy is

$$W = W_1 + \Delta W + W_2 = (1/2)(m_0 + \Delta m)v^2 + (1/2)\Delta m v(s_0)^2 \tag{23}$$

The kinetic energy of the mass when it hits the ground is

$$W_k = (1/2)(m_0 + \Delta m)v^2 \tag{24}$$

Notice that $W > W_k$. This is so because the mass Δm must be accelerated to the speed $v(s_0)$ very fast. This mass is not accelerated by the gravitational force, it is accelerated by the falling mass that already has quite high velocity.

We can do a similar calculation from the formula $m = \gamma m_0$. Here there are small additions of mass dm all the time when s grows from zero to h.

$$dW = c^2 dm = v^2 dm + mv dv \tag{25}$$

Notice that

$$c^2 dm = v\, m\, dv \sum_{k=0}^{\infty} (v^2/c^2)^k \tag{26}$$

$$v^2 dm = v\, m\, dv \sum_{k=1}^{\infty} (v^2/c^2)^k \tag{27}$$

The first term in $c^2 dm$ gives the kinetic energy $(1/2)mv^2$ and the other terms are in $v^2 dm$. Indeed

$$E_k = (1/2)mv^2 = \gamma \int \gamma^{-1}(c^2 dm - v^2 dm) \tag{28}$$

We get

$$W = (m - m_0)c^2 \tag{29}$$

$$W_k = (1/2)mv^2 \tag{30}$$

and again

$$W_k < W \tag{31}$$

The relativistic mass is not in the kinetic energy of the moving mass. The mass $m - m_0$ does not exist. It is simply a mathematical trick to calculate with an effective mass and to get correct results in some situations. The real explanation of the mass $m - m_0$ is that the force gets weaker if the relative velocity of the field and the mass grows.

4. References

[1] A. Einstein, The Meaning of Relativity, Princeton University Press, last edition 1955.

1.5 Gravitational time dilation and weakening of the interaction force

Abstract:

The article explains why there cannot be speed dependent time dilation or mass change and how all measured effects come from a change of interaction forces. Then the article gives a simple model for an interaction and from this model derives explanations for the relativistic mass formula and gravitational time dilation.

1. A model for the interaction force

Speed and acceleration of an object are determined by our choice of the coordinate system. Our choice of the coordinate system cannot in any way change anything that happens in the world, it only shows the same behavior from a different coordinate system. Therefore no property that can be observed to change can depend on the velocity or acceleration of the object. Especially this means that the mass cannot depend on velocity as in the relativistic mass formula and time cannot depend on the velocity of an inertial frame as it is in the Lorentz transform. What has been measured and seems to verify formulas of this type has alternative explanations.

One explanation possibility is that there is a preferred frame of reference, but there seems to be no single preferred frame of reference. The speed of light on the Earth seems to be constant to each direction and the inertial frame of reference of the Earth hardly can be the single preferred frame of reference in the Universe. Instead, there seem to be many local frames of reference moving with large mass bodies and these frames of reference may therefore depend on the rest frame of the gravitational field of the mass body.

A simple explanation is that the speed and acceleration of an object that can cause observable changes can only be the relative speed or acceleration with respect to a force field.

The Lorentz transform comes naturally if we assume that the cause of mass change is the weakening of a force if the object moves very fast. First we need a model for an interaction.

In Newton's gravitation law the interaction is between two masses. An interactions between the Sun and the Earth should take eight minutes, or sixteen minutes if the interaction is two-ways. Such a long delay should give a different orbit for a planes than what comes from Newton's gravitation law where the effect of the force is immediate from any distance. Yet Newtonian physics predicts the orbit of a planet rather well. This means that the action of the force must be very fast, yet gravitation should proceed with a finite speed, probably with the speed of light. This apparent problem is solved by assuming that a gravitational interaction is local between the field and the test mass, and we will make this assumption.

Let us assume that in one chosen interaction the interaction distance is some short distance ΔL. We will propose a model for a gravitational interaction, but it can also be used for attractive Coulomb force.

Some point of the field that is close to the test mass m contains a small mass that sends to the mass a message with the content

1) a mass at P sends $(-\Delta m)(\bar{c})$ —> m

The mass m absorbs this message and gets the momentum $\Delta p = -(\Delta m)c$, which causes the mass m to move towards the point P in the field where this message came from.

2) the test mass m receives the message and moves towards P

The mass responds by sending two messages, one towards the point where the message from the field came from and the other to the opposite direction. The sum of the momentums of these two messages is zero. We need these messages because the mass m must respond to the mass at P:

3) <—- $(-\Delta m)(-\bar{c})$ m sends —-> $(-\Delta m)(\bar{c})$

Sending the message in the step 1) takes the time ΔT and sending left the message in the step 3) takes the time ΔT if the field and the mass m do not have relative speed.

The field passes the message 3) forward in the following way. The field has very many small masses, one of them being at the point that generated message 1). The field is basically incompressible but for a short time it can be compressed

4) the mass at P absorbs $(-\Delta m)(-\bar{c})$ and moves to the right

but very soon it returns to where it was and sends the message

5) <—- $(-\Delta m)(-\bar{c})$ mass at P returns to its place and sends.

The mass at P also sends two messages, similarly as m sends the messages in 3), because it is a mass and all masses should behave in the same way:

6) <—- $(-\Delta m)(-\bar{c})$ mass at P sends —-> $(-\Delta m)(\bar{c})$

The interaction passes many similar points P, always staying as a local two-way exchange of messages. Finally the interaction comes to the mass M that created the gravitational field.

The mass M absorbs this message and gets the momentum $\Delta p = -(\Delta m)c$, which causes the mass to move towards the point in the field where this message came from.

7) M receives the message and moves towards the last point P

The mass M sends two messages, one towards the point where the message from the field came from and the other to the opposite direction. The sum of the momentums of these two messages is zero.

8) <—- $(-\Delta m)(-\bar{c})$ M sends —-> $(-\Delta m)(\bar{c})$

The two-way interaction between the field and the test mass m is 1) and the left-side message in 3). The two-way exchange of messages is because a force always causes a counter force and because if two masses move because of gravitation attraction between them, they both move. The force is both ways. We must make this two-way exchange local, not between the end-points, because the interaction must be very fast. This is why there are the sendings of pairs of messages to both directions.

The interaction can go through all possible paths: in Newtonian gravitation theory the field created by a point mass M at the origin at the distance r is the same as a field created at the distance r by the same size mass M which is distributed with constant density over a ball of radius R centered at the origin. Iteratively this result shows that the field is the same if we let the force lines come from the mass M to the test mass m at the distance r through all possible paths.

The additions of momentum in this interaction model have negative masses $-\Delta m$. This is necessary because the force is attractive. It does not need to mean anything mystical, like negative masses or negative frequencies. If there is a baseline frequency and the momentum is e.g. $p = h\lambda$, then λ that is sufficiently much above the baseline means negative momentum if we assume that a baseline momentum does not make a test mass move, e.g., the energy is consumed by the process and atoms can buffer the baseline momentum packets. Something of this type could be proposed, but at the moment the model does not have such details.

2. The explanation of the relativistic mass formula

Let the mass move with the speed v with respect to the field in two situations. In the first case the distance ΔL is in the same direction as $\bar{v}$. Then the time ΔT_1 for sending 1) is solved from $c\Delta T_1 = \Delta L + v\Delta T_1$, i.e., $\Delta T_1 = L/(c-v)$. The time ΔT_2 for sending 3) comes from $c\Delta T_2 = \Delta L - v\Delta T_2$, i.e., $\Delta T_2 = L/(c+v)$. The other messages need not be considered. The interaction time is

$$\Delta T_v = \Delta T_1 + \Delta T_2 = \frac{2\Delta L}{c}\gamma = \gamma \Delta T$$

where $\gamma = 1/\sqrt{1-(v/c)^2}$ is the Lorentz factor.

In the second case ΔL is orthogonal to $\bar{v}$. Then

$$(c\Delta T_1)^2 = (\Delta L)^2 + (v\Delta T_1)^2$$

$$(c\Delta T_2)^2 = (\Delta L)^2 + (v\Delta T_2)^2$$

Again

$$\Delta T_v = \Delta T_1 + \Delta T_2 = \frac{2\Delta L}{c}\gamma = \gamma \Delta T$$

In both cases we get the same result. If we imagine that the force F_v that is applied in the situation that the mass m moves with the speed v with respect to the field is the same F that is applied when there is no relative speed, then we will conclude that the mass m has grown to m_v because we get the same trajectory. One interaction adds v by Δv. The acceleration is

$$a = \frac{\Delta v}{\Delta T}$$

in the case that there is no relative speed. The force is

$$F = ma = m\frac{\Delta v}{\Delta T}$$

If there is relative speed, the acceleration is

$$a_v = \frac{\Delta v}{\Delta T_v} = \gamma^{-1} a$$

The force is

$$F_v = m_v a = m_v \gamma^{-1} a$$

The equation gives

$$m_v = \gamma m$$

the relativistic mass.

Notice that all forces in the rest frame of the field will see the mass as the relativistic mass. Above we have two situations: one where the force accelerates the mass (v points to the same direction as ΔL) and one where the force deviates the trajectory of the mass (v is orthogonal to ΔL). Both cases give the same relativistic mass.

3. The explanation of the gravitational time dilation formula

All clocks do not slow down in a stronger gravitational field. A pendulum clock speeds up, but an atomic clock does slow down. We will show in this section that gravitational time dilation does not mean that a local time is different in a stronger gravitational field. The question is of the interaction between the gravitational field and an atomic clock.

An atomic clock measures time from resonant frequencies of atoms. Electron states in an atom are on different energy levels and electrons make transitions between states. One can calculate the energy levels e.g. from the Schrödinger equation. The Schrödinger equation includes the mass m. For an atomic clock the mass m that is in the equation is the mass of the nucleus. If the mass m changes, the time that the clock is measuring changes.

Let us consider two identical atomic clocks. One is on the Earth surface at the distance R from the center of the Earth and the other one is at the distace r from the center of the Earth. The second atomic clock can e.g. be a GPS satellite clock. We will denote the masses of the atomic nucleus in these two clocks by $m = m_R$ and m_r and the energy levels by E and E_r.

In simple applications of the Schrödinger equation the energy levels are inversely proportional to the mass of the nucleus m and m_r. Therefore the ratio of the energy levels E an E_r is inversely proportional to the ratio of the masses

$$\frac{E_r}{E} = \frac{m}{m_r}$$

It follows that the energy level difference is also be inversely proportional to the ratio of the masses. The energy level differences determine the resonance frequency and the resonance frequency divided by a large constant is the ticking time of the clock.

Fortunately, gravitational time dilation has been measured in many experiments and we know that the result in the General Relativity Theory is correct. The correct time dilation is

$$t_r = t\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}$$

and the resonance frequency therefore is

$$f_r = \frac{1}{t_r} = f\sqrt{1-\frac{2GM}{rc^2}}$$

Assuming that the energy levels are inversersely proportional to the masses m and m_r (as it seems to be in this case), we get the correct result if

$$m_r = m\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}.$$

Thus, we should get a kind of a relativistic mass formula.

In the Schrödinger equation potential energy is changed to kinetic energy. One can say that an atomic clock has a mass, the mass of the nucleus, and a spring. The spring is the electrostatic force between the positive charge of the nucleus and the negative charge of the electron belt. We can express this spring as a force that moves the mass of the nucleus. The question is what mass of the nucleus this force sees.

The force that moves the mass in an atomic clock is in the same frame of reference as the gravitational field, both rotate once in 24 hours. The gravitational force acts as acceleration. A stronger gravitational field means stronger acceleration. Stronger acceleration makes the relativistic mass bigger, thus the mass appears bigger when there is stronger gravitational field. It appears bigger to all forces in that same frame of reference, especially the mass of the nucleus seems bigger for the oscillating force of the atomic clock, the electrostatic spring. In the GPS satellite clock the acceleration is smaller. For force that makes the mass of the nucleus oscillating in the GPS satellite clock the mass seems lighter.

The gravitational accelerations are:

$$g = G\frac{M}{R^2}$$

$$g_r = G\frac{M}{r^2}$$

We can treat the situation as if the mass of the nucleus were accelerating because of gravitation. Using the same method as in Section 2, we can calculate the forces

$$F = ma = m\frac{2\Delta L}{\Delta T}$$

$$F_r = m_r a_r = m_r\frac{2\Delta L}{\Delta T_r}$$

where ΔL is he interaction distance and ΔT and ΔT_r are the interaction times. We set $F_r = F$ in order to get the relativistic mass m_r.

$$\frac{m_r}{m} = \frac{\Delta T_r}{\Delta T}$$

It remains to find ΔT_r. In Section 2 we derived

$$\Delta T_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta T$$

but now we do not have velocity. We have potential.

This problem can be solved with a special application of the equivalence principle. Einstein wanted that the equivalence principle should hold always. An example at the end of this section shows that it does not hold always, but it is very convincing that it must hold in the lift example: if a mass is placed on a lift, the mass cannot know if the lift is going up in accelerated motion or if the lift is in a gravitational field, provided that the acceleration equals the gravitational acceleration.

The mass m_r is in a lift in a gravitational field with the gravitational acceleration g. The potential energy of the mass m_r of the nucleus on the distance r from the center of the Earth with mass M is

$$E = G\frac{Mm}{r}.$$

If the mass m_r would be in an accelerated lift, then instead of potential energy, the mass would have equal amount of kinetic energy:

$$\frac{1}{2}mv^2 = G\frac{Mm}{r}.$$

This gives us v^2

$$v^2 = \frac{2GM}{r}.$$

We insert this v^2 in the expression of $\Delta T_r = \Delta T_v$. Then

$$\frac{\Delta T_r}{\Delta T} = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}$$

We get the same expression for gravitational time dilation as in the General Relativity Theory

$$\frac{E_r}{E} = \frac{m}{m_r} = \frac{\Delta T_r}{\Delta T}$$

$$\frac{f}{f_r} = \frac{t_r}{t} = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}$$

$$t_r = t\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}.$$

Notice that the gravitational time dilation is also phenomenological. The time is not in reality changed. The force is weakened. The mass growth is not real. But what is completely real is the changed energy levels. The gravitational force keeps the nucleus mass is its grep so that the mass cannot oscillate as it used to. Therefore the gravitational time dilation is a real measurable behavior. Only it does mot mean that in high gravitational field time is actually any different, only masses are more difficult to move, so the oscillation time is different and an atomic clock shows a different time.

In the General Relativity Theory explanation of the GPS clock dilation the dilation is the sum of the gravitational time dilation and the Lorentz transform time dilation. The latter time dilation cannot exist, but there is the acceleration time dilation because the orbit of the satellite is curved. In order to get the acceleration dilation we only need to derive v from acceleration and insert it to the expression of $\Delta T_r = \Delta T_v$.

In the muon in laboratory experiment there cannot be Lorentz transform time dilation for the muon, but the muon is created and accelerated before it comes to the bubble chamber. Because of acceleration time dilation the muon is younger and lives longer.

Consider two identical masses that exercise to each others an attractive force and both have already reached a high velocity because of this force. Which mass is larger? Of course, the situation is fully symmetric and the masses must be equal. Both masses feel that the other mass has grown and has higher both inertial and gravitational masses. But at the same time, the gravitational force coming from the other mass is smaller, so its gravitational mass is larger only when it is seen as the test mass, but smaller when it is seen as the mass creating the field. This example shows that the equivalence principle does not hold always. It is always better just to think of how the force has changed and not to go to changes of masses or times or lengths. All that is only an illusion.

Mass means many different things. There is mass that turns to energy when a nucleus is splitted. That is binding energy. It appears as gravitational mass when the results of a reaction are weighted. There is relativistic mass, which is an illusion of inertial mass. But there are masses of elementary particles. They are discrete, but a particle, antiparticle pair can turn to energy. Mass also appears as residues in the Laplace operator equation. There it is singularities and if the singularities are replaced by a continuous function, then there are no residues and the Laplace operator gives zero. Mass is something real and active, it sends messages, i.e., force lines. Therefore mass does not change, but mass may seem to be changing.

Time is not a coordinate of our space. The space that we have is a three-dimensional space, and there is additionally time. Space and time should not be treated as space-time. This is why time does not have real dilation.

This short note is a part of the series [1]-[3].

3. References

[1] J. Jormakka, Failure of the geometrization principle and some cosmological considerations, preprint, ResearchGate, 2023.

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PART 2. SPECIAL RELATIVITY THEORY

Paper 2.1 shows that the Lorentz transform does not make the speed of light constant in the moving frame of reference. Einstein forgot to take a projection of the points in the moving frame coordinates to the time axis when calculating the time difference. As a consequence of this error. whole SRT is false. Section 4 of rPaper 2.1 includes my first proof that the General Relativity Theory cannot approximate Newtonian gravitation field of a spherical mass, meaning that no empirical tests of GRT can possibly verify GRT as the theory cannot even produce a field that could apply to those experiments.

2.1 Essential Questions in the Relativity Theory

Abstract: Though the Relativity Theory is over 100-years old, some questions still remain. Section 2 asks if the time values in the moving frame of reference R' in the Lorentz Transform should not be taken by projecting the values on the t'-axis because the coordinates of R' have a time shift. Section 3 asks if the Lorentz Transform leads to a contraction: if the Earth is taken as the moving frame R' and a fixed frame R is imagined, then apparently the speed of light c can be set to any chosen value. This seems curious. Section 4 asks if the General Relativity Theory actually can approximate the Newtonian gravitation potential in our solar system because locally constant speed of light does imply that the gravitation field is a scalar field and the Einstein equations do not allow any solutions that approximate the Newtonian gravitation field in the case of a single point mass in an empty space. This also seems a bit strange. Finally Section 5 asks if the Relativity Theory really is verified by several experiments and if the reason that the Relativity Theory is considered verified could be that manuscripts that raise questions like the presented one are not given a careful review.

1. Introduction

The Lorentz Transform in two dimensions is

$$x' = \gamma(x - vt), t' = \gamma\left(t - \frac{v}{c^2}x\right), \qquad \text{where} \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}. \tag{1}$$

Let us take two points $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (L+\mathrm{vT}, T)$. The coordinates x and t are orthogonal, thus the projection to the x-axis is simply the first number x in the pair (x, t), and the projection to the t-axis is simply the second number t. Therefore the difference of the two points in the projection to the x-axis is $L + \mathrm{vT} = x_2 - x_1$ and difference of the two points in the projection to the t-axis is $T = t_2 - t_1$. The coordinates x' and t' obtained from the Lorentz Transform for the moving frame of reference R' by transforming the normal orthogonal coordinates (x, t) of the fixed frame R are not normal orthogonal coordinates: they have a time shift. This time shift means that the time difference between points (x'_1, t'_1) and (x'_2, t'_2) where $x'_1 \neq x'_2$ must be done by first projecting both points to the same vertical line $x' = a$, usually to the t'-axis, $x' = 0$. When the projections are made, the speed of light of light is not constant c in R'. Section 1 Figure 1 shows how to do the projection in R and Figure 2 shows how the projection looks like in R'.

One may think that the Lorentz Transform could still be used without taking these projections as one possible way to define a coordinate system in R'. Section 3 shows that this is not possible. The time shift $-(v/c^2)x$ in the time transform formula causes a contradiction. This time shift cannot be in the transform and as it is the only possible time shift that makes the speed of light constant in R'

(albeit with a wrong calculation that is not taking the projections correctly), the speed of light in R' cannot be constant c in any linear transform.

Section 4 shows two issues in the General Relativity Theory (GRT). The first is that the Newtonian gravitation potential does not nearly satisfy field equation of GRT. This is not necessarily an error because there still might be a solution for the GRT field equation that is close to the Newtonian gravitation potential, that is, it might work in the other direction. This is not the case. The second issue shown in Section 4 is that if light has locally constant speed c, then the gravitation field must be a scalar field and GRT does not have scalar field solutions that are close to the Newtonian gravitation field.

Section 5 discusses briefly why many experiments that seem to show that the Relativity Theory is verified do not prove the theory correct. In general, experiments where a theory predicts correctly do not prove the theory: there can always be alternative theories that also predict all these cases correctly, but a single experiment where a theory fails does refute the theory. Section 5 gives such a refuting experiment for the Special Relativity Theory (SRT): from SRT follows that it is possible to set c, the speed of light in vacuum, to any chosen value, like to 1 m/s, simply by imagining a frame R. If this experiment is made, the prediction from SRT is certainly not fulfilled: imagination does not change the speed of light. In the case of the General Relativity Theory it is shown in Section 4 that GRT cannot give solutions that are close to the Newtonian gravitation potential in a situation that is close to the situation in our solar system. Therefore none of the experiments of GRT that have been done in our solar system and are claimed to verify GRT can do it: GRT cannot be used in our solar system as it cannot give the gravitation field that there is in our solar system. It is the single experiment that refutes the theory.

2. Projections of time shifted coordinates in the Lorentz Transform

Let us set $x' = 0$ as a vertical line. Thus, the line $t = v^{-1}x$ is shown as a vertical line from the origin, it is the t'-axis. The line when $t' = 0$ is line $t = (\frac{v}{c^2})x$. It is the x'-axis and shown as a horizontal line in the (x', t')-plane.

Figure 1 shows in the (x, t)-plane the lines $t = v^{-1}x$, $t = (\frac{v}{c^2})x$, the line of light sent to the positive x-axis from the origin: $t = c^{-1}x$, and the line $t = v^{-1}(x - L)$ of the receiver of light starting at the position $(L, 0)$ and ending to the position $(L + vT, T)$ at the time T when light sent from the origin arrives to the receiver. As $L + vT = cT$ we get $L = (c - v)T$.

Additionally Figure 1 shows point $P1$ where the preimage of the t'-axis, i.e., the line$t = v^{-1}x$, intersects with a line parallel to the preimage of the x'-axis, i.e., the line $t = (\frac{v}{c^2})x$, going through the point $(L + vT, T)$. The line parallel to the line $t = (\frac{v}{c^2})x$ and going through the point $(L+vT, T)$ is $t = (\frac{v}{c^2})x + \gamma^2 T - (\frac{v}{c^2})L$. Intersecting it with $t = v^{-1}x$ gives the point $P1$ as $\left(\frac{cv}{c+v}T, \frac{c}{c+v}T\right)$. Its image in the (x', t')-plane is $P1'$ which is $\left(0, \gamma\left(1 - \frac{v}{c}\right)T\right)$.

Figure 1 still shows one point, $P2$, which is the intersection of a line parallel to the preimage of the x'-axis, i.e., the line $t = (\frac{v}{c^2})x$, going through the point $(L, 0)$ and the the preimage of t'-axis, i.e., the line $t = v^{-1}x$. The line parallel to the line $t = (\frac{v}{c^2})x$ going through $(L, 0)$ is $t = (\frac{v}{c^2})(x - L)$. Intersecting it with $t = v^{-1}x$ gives $P2$ as $(-(\frac{v^2}{c^2})\gamma^2 L, -(\frac{v}{c^2})\gamma^2 L)$ and its image is the point $P2'$ which is $(0, -\gamma\frac{v}{c^2}L)$. Figure 2 displays the points $P1'$ and $P2'$ in the (x', t')-plane.

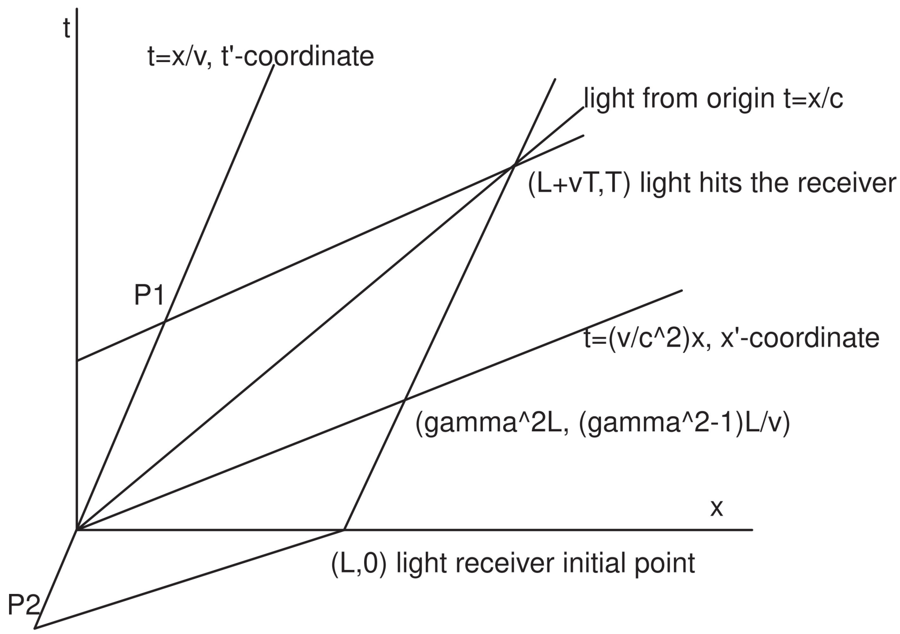

Figure 1. The points and lines shown in the (x, t)-plane.

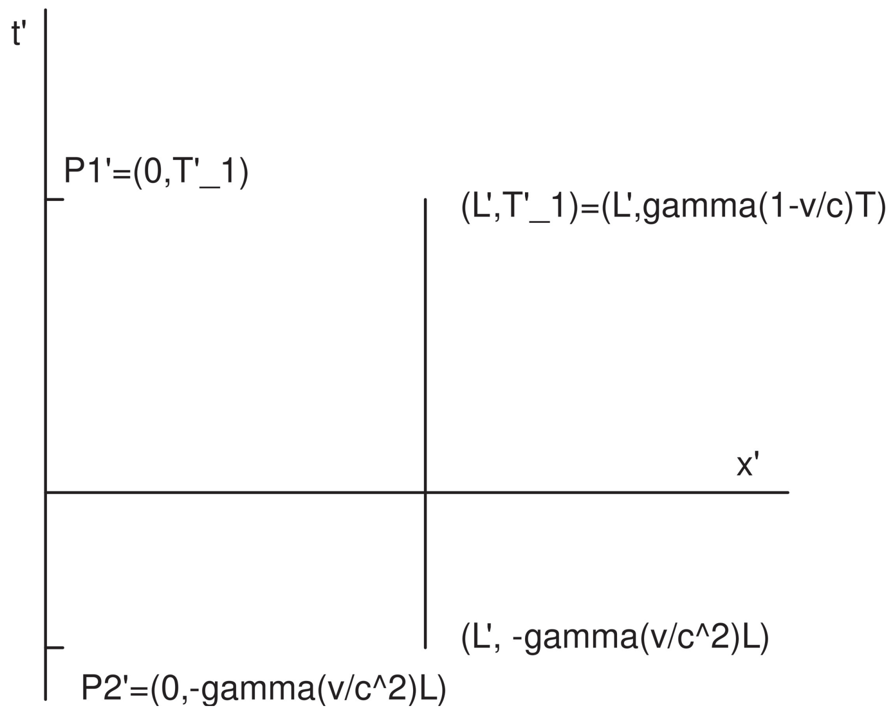

Figure 2. The points $P1'$ and $P2'$ shown in the (x',t')-plane. The time light travels in R' is the time from $P2'$ to $P1'$.

In Figure 2 we have denoted

$$T'_1 = \gamma(1-\frac{v}{c})T = \sqrt{\frac{c-v}{c+v}}T. \tag{2}$$

Light starts in $(x.t)$-coordinates at the point $(0,0)$ at the time $t=0$, but Figure 1 shows that the projection of $(L,0)$ is $P2$ and the projection of $(L+vT,T)$ is $P1$ in (x,r). The projection $P2$ means that according to a clock placed in $x=L$ the starting time on the t'-axis (i.e., the line $t=x/v$) is the time coordinate in P, which is $t=-\gamma^2(v/c^2)(c-v)T$. This shows that fixed clocks placed in R' to $x'=L'$ and $x'=0$ do not have the same originl in their times. There is a time shift.

We are familiar with time shifted coordinate systems, they appear in our everyday life as timezone differences between countries and regions. As an example, flying from Helsinki to Warsaw takes two hours and there is one hour timezone difference. If time is measured by local clocks at the end points, then the flight takes three hours one way and one hour the other way. The flight time is not three hours one way and one hour the other way. In order to calculate the flight time we must project both times to a single time, for instance to the Greenwich time, and only then subtract them. Should the flight from Helsinki to Warsaw some day take two hours both ways, then we can be certain that there is very heavy wind and the flight time is not the same both ways.

Let us measure the travel time of light by using one clock that is stationary in R'. We can use a clock that is stationary in R' at $L' = \gamma L$. According to this clock light starts at $x' = 0$ at the time $-\gamma(\frac{v}{c^2})L'$. According to this clock light is received at $x' = L'$ at the time T'_1. The time T' that light travels in (x', t')-coordinates is the time difference between the points $P2'$ and $P1'$. We get

$$T' = \gamma(1 - \frac{v}{c})T + \gamma(\frac{v}{c^2})L = \gamma(1 - \frac{v}{c})T + \gamma(\frac{v}{c^2})(c - v)T = \gamma^{-1}T. \qquad (3)$$

Thus, the speed of light in R' to the positive x'-axis is

$$c' = \frac{L}{T'} = \frac{\gamma L}{\gamma^{-1}T} = \gamma^2 \frac{L}{T} = \gamma^2(c - v). \qquad (4)$$

The time T' is the same as what we get by measuring the time in (x', t')-plane with a clock fixed at the origin of (x', t'), i.e., having the equation $x = vt$. Then

$$T' = t' = \gamma(T - (\frac{v}{c^2})vT) = \gamma(1 - \frac{v^2}{c^2})T = \gamma^{-1}T. \qquad (5)$$

This is natural, in time shifted coordinates we can measure the travel time by using any clock that is stationary in R'.

The error in the Lorentz Transform is to think that the projection on the t'-axis is $T' = t'_2 - t'_1$. If this were the case, then we would get

$$T'_1 = \gamma \left(t - \frac{v}{c^2}x\right) = \gamma \left(T - \frac{v}{c^2}cT\right) = \gamma(1 - v/c)T. \qquad (6)$$

as the time light travels in R'. Then the speed of light would be

$$c' = \frac{L}{T'_1} = \frac{\gamma L}{\gamma(1 - \frac{v}{c})T} = \frac{\gamma(c - v)T}{\gamma(1 - v/c)T} = c. \qquad (7)$$

However, this is wrong. Figure 1 shows that the line from $(0, 0)$ to $P1$ has the same length and direction as the line from $(\gamma^2 L, (\gamma^2 - 1)\frac{L}{v})$ to $(L + vT, T)$. Both have the x-coordinate difference $\frac{Tcv}{c+v}$ and the t-coordinate difference $\frac{Tc}{c+v}$. Thus, the time value $T'_1 = \gamma(1 - \frac{v}{c})T$ in the time coordinate of $P1'$ is the time light travels from the image of $(\gamma^2 L, (\gamma^2 - 1)\frac{L}{v})$ to the image of $(L + vT, T)$.

Yet, some supporters of SRT may argue that we can in some way ignore the projections and calculate as in SRT. This is not possible as the next section shows: it leads to a serious contradiction.

3. The time shift in the Lorentz Transform gives a contradiction

The contradiction is derived in Steps 1-4.

Step 1. The one-way speed of light has been directly measured in the close vicinity of the Earth e.g. by relaying them through a satellite. We know that the speed of light c' in the close vicinity of the Earth is about $3 * 10^8$ m/s. Let us select orthogonal (x', t') coordinates on the Earth and and send a microwave signal from the origin $(0, 0)$ to a receiver that is at $(-L', 0)$ in the time $t' = 0$. The length L' can be for instance 30 km. Light travels the path $(0, 0)$ to $(-L', T')$ and T' is about 100 microseconds, quite in a measurable range. The speed of light $c' = L'/T'$ is about $3 * 10^8$ m/s. It is in fact the speed of light in the atmosphere, but does not much differ from the speed of light c in vacuum.

Step 2. Next we consider the Earth as the frame R' for some other frame R that we can either imagine or select. R has normal orthogonal coordinates (x, t) and the Lorentz Transform gives the transform $(x, t) \to (x', t')$. Thus, the origin of (x, t) is mapped to the origin (x', t') and so on, all as it is in the Lorentz Transform. Notice that the (x', t') coordinates of R' are not the same as the (x', t') coordinates of the Earth in Step 1. In the (x', t') coordinates of the Earth in Step 1 there is no time shift and the signal travels the path $(0, 0)$ to $(-L', T')$. In the (x', t') coordinates of R' the light beam (i.e., the microwave signal) travels the path from $(0, 0)$ to $(-L', T' - (-(v/c^2)(-L'))$ because this coordinate system has a time shift. On the t'-axis there is no time shift and we select the (x', t') coordinates of R' so that both (x', t') coordinate systems completely agree on the t'-axis. Thus, the line segment $(0, 0)$ to $(0, T')$ in the (x', t') coordinates of the Earth maps to the line segment $(0, 0)$ to $(0, T')$ in the (x', t') coordinates of R'. We notice that L' of the coordinates of the Earth maps to L' of the coordinates of R', i.e., on the Earth the wave is received is at the point $(-L', T')$ and in R' the wave is received at the point $(-L', T'_2)$, $T'_2 = T' - (v/c^2)L'$ giving $T'_2 = \gamma(1 + v/c)T$. The x'-coordinate of these points is the same. Thus, we have transformed L' and T' from the (x', t') coordinates of the Earth to the (x', t') coordinates of R' and $L'/t' = c'$ is about $3 * 10^8$ m/s.

Step 3. In this step we consider only R. The preimage of the line segment $(0, 0)$ to $(0, T')$ in R', i.e., the sending and receiving times of the wave on the Earth, is the line segment $(0, 0)$ to (vT, T) in R. The coordinates (x, t) of R are normal orthogonal coordinates, there is no time shift: the time difference between the points $(0, 0)$ and (vT, T) is T. It is the time the wave travels in R. From the Lorentz Transform we get $T' = \gamma(T - (v/c^2)vT) = \gamma^{-1}T$. In R the length L' corresponds to a moving rod with (moving) length L. The left end of the rod is at the point $(-L, 0)$ at the time $t = 0$ and moves with the speed v to the right, i.e., has the equation $x = -L + vt$. Light is sent from $(0, 0)$ at the time $t = 0$ and it moves to the left. According to the Special Relativity Theory the speed of light in R is c, thus the equation of light is $x = -ct$. The receiver meets the light beam in R at the time T which satisfies $-L + vT = -cT$, thus $L = (c + v)T$. The meeting is at the x value $x = -cT$. Transfering the line segment $(0, 0)$ to $(-cT, T)$ to R' gives the line segment $(0, 0)$ to (x', t') where

$x' = \gamma(-cT - vT) = -\gamma L$ and $t' = \gamma(T - (v - c^2)cT) = \gamma(1 + v/c)T$. Thus, $L' = \gamma L$ and we can verify that $\gamma(1 + v/c)T$ is indeed $T'_2 = T' - (v/c^2)L'$, as it is.

Step 4. We already have all that is needed for the contradiction: $c' = L'/T'$, $T' = \gamma^{-1}T$, $L' = \gamma L$ and $L = (c + v)T$. Thus

$$c' = L'/T' = \gamma^2 L/T = \gamma^2(c + v) = c^2/(c - v) = kc$$

where $k = c/(c - v)$ i.e., $v = v(1 - 1/k)$. The value c' is measured on the Earth and is about $3 * 10^8$ m/s. We can by selecting k set the speed of light to any chosen value $c = c'/k$. Considering that in SRT, the speed of light in vacuum is a universal constant, we hardly should be able to set it to any value simply by imagining frame R.

This contradiction shows that it is not possible to have a theory with the time offset $-(v/c^2)x'$ in the transform formula for t'. Therefore it is not necessary to argue at all with people who maintain that the time values in the time shifted coordinate system R' do not need to be projected to the time axis and there is some mathematically sound way to calculate in SRT. There is no such sound way, there is a serious contradiction in SRT.

4. The General Relativity Theory does not approximate Newtonian gravitation

The gravitation field on the Earth and close to the Earth appears to be very close to the Newtonian gravitation potential. We will first show that the Newtonian potential does not nearly satisfy the field equation of GRT as one might expect if Newtonian gravity is an approximation of the more correct theory GRT. The Newtonian potential is

$$\varphi = \varphi(r) = -\frac{G\rho}{r}. \tag{8}$$

It is the solution to the Newtonian field equation

$$\Delta\varphi = -4\pi G\rho. \tag{9}$$

Let us assume that the mass is a spherical mass with a finite radius. In the empty space outside this radius ρ is constant and we have

$$\frac{\partial^2}{\partial x^2}\varphi = G\rho\frac{1}{r^5}(r^2 - 3x^2). \tag{10}$$

By symmetry

$$\Delta\varphi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi = G\rho\frac{1}{r^5}(3r^2 - 3x^2 - 3y^2 - 3z^2) = 0. \tag{11}$$

Thus, in the empty space outside the mass the solution fulfills the equation

$$\Delta\varphi = 0. \tag{12}$$

In the same way, if we solve the field equation of GRT

$$R_{\mathrm{ab}} - \frac{1}{2}Rg_{\mathrm{ab}} = k_0 T_{\mathrm{ab}} - \lambda g_{\mathrm{ab}} \tag{13}$$

in the empty 4-space outside a spherical mass. The equation reduces to

$$R_{\mathrm{ab}} = \left(\frac{1}{2}R - \lambda\right) g_{\mathrm{ab}}. \tag{14}$$

The Newtonian gravitational potential (8) yields in orthogonal local coordinates the following metric

$$ds^2 = c^{-2}g_{00}\phi^2 - g_{11}\phi^2 - g_{22}\phi^2 - g_{33}\phi^2 \tag{15}$$

For easier notations in equations of GRT c is usually set to one by rescaling seconds and meters. In (13) and (14) $c = 1$. We will follow this convention, but in (23) c is shown as (23) is needed later. For any scalar potential field ψ, the metric g_{ab} corresponding to the field ψ in Cartesian coordinates (t, x, y, z) where $x^0 = t$, $x^1 = x$ $x^2 = y$, $x^3 = z$ and the signs are (+,-,-,-), is given by

$$g_{00} = \psi^2,\ g_{11} = -\psi^2,\ g_{22} = -\psi^2,\ g_{33} = -\psi^2, \quad \text{and} \quad g_{\mathrm{ab}} = 0 \text{ if } a \neq b. \tag{16}$$

In spherical coordinates (t, r, θ, ϕ), where $x^0 = t$, $x\acute{z} = r$, $x^2 = \theta$, $x^3 = \phi$ and the signs are (+,-,-,-), the metric is given by

$$g_{00} = \psi^2,\ g_{11} = -\psi^2,\ g_{22} = -\psi^2 r^2,\ g_{33} = -\psi^2 r^2 \sin^2\theta, \quad \text{and} \quad g_{\mathrm{ab}} = 0 \text{ if } a \neq b. \tag{17}$$

For any orthogonal metric (i.e., $g_{\mathrm{ab}} = 0$ if $a \neq b$) holds

$$g^{\mathrm{aa}} = \tfrac{1}{g_{\mathrm{aa}}} \tag{18}$$

and the Christoffel symbols satisfy (the notation $g_{\mathrm{ab},c} = \partial_c g_{\mathrm{ab}}$) the following:

$$\Gamma^a_{\mathrm{aa}} = \tfrac{1}{2}g^{\mathrm{aa}}g_{\mathrm{aa},a}, \tag{19}$$

$$\Gamma^a_{\rm ba} = \tfrac{1}{2} g^{\rm aa} g_{{\rm aa},b} \text{ , if } a \neq b$$
$$\Gamma^a_{\rm bb} = -\tfrac{1}{2} g^{\rm aa} g_{{\rm bb},a} \text{ , if } a \neq b$$
$$\Gamma^a_{\rm bc} = 0 \text{ , if } a \neq b,\ a \neq c \text{ and } c \neq b.$$

In order to get the Einstein equations we calculate the Christoffel symbols for the metric in both coordinate systems. Then we calculate the Ricci entries

$$R_{\rm bd} = R^a_{\rm bad} = \Gamma^a_{{\rm bd},a} - \Gamma^a_{{\rm ba},d} + \Gamma^e_{\rm bd}\Gamma^a_{\rm ae} - \Gamma^e_{\rm ba}\Gamma^a_{\rm ed}. \tag{20}$$

All ways to do the calculation are tedious. One way to calculate is the following, it may be easier to program as an algorithm than some others. In an orthogonal metric

$$R_{\rm jj} = \sum_{\substack{i=0\\ i\neq j}}^{4} \left\{ \frac{1}{4} g^{\rm ii} g_{{\rm ii},j} \left(g^{\rm jj} g_{{\rm jj},j} - g^{\rm ii} g_{{\rm ii},j} \right) - \frac{1}{2} \partial_j \left(g^{\rm ii} g_{{\rm ii},j} \right) \right\} \tag{21}$$

$$- \sum_{\substack{i=0\\ i\neq j}}^{4} \left\{ \tfrac{1}{4} g^{\rm ii} g_{{\rm jj},i} \left(\sum_{\substack{k=0\\ k\neq j}}^{4} g^{\rm kk} g_{{\rm kk},i} - g^{\rm jj} g_{{\rm jj},i} \right) + \tfrac{1}{2} \partial_i \left(g^{\rm ii} g_{{\rm jj},i} \right) \right\},\ j = 0,1,2,3$$

In an orthogonal metric the off-diagonal Ricci entries have the equation

$$R_{\rm ij} = \tfrac{1}{4} \sum_{\substack{k=0\\ k\neq i, k\neq j}}^{4} g^{\rm kk} g_{{\rm kk},i} \left(g^{\rm ii} g_{{\rm ii},j} - g^{\rm kk} g_{{\rm kk},j} \right) + \tfrac{1}{2} \partial_i \left(g^{\rm ii} g_{{\rm ii},j} \right)$$

$$-\tfrac{1}{2} \partial_j \left(\sum_{\substack{k=0\\ k\neq j}}^{4} g^{\rm kk} g_{{\rm kk},i} \right) + \tfrac{1}{4} g^{\rm jj} g_{{\rm jj},i} \sum_{\substack{k=0\\ k\neq i, k\neq j}}^{4} g^{\rm kk} g_{{\rm kk},j}. \tag{22}$$

The off-diagonal Ricci entries are zero both in Cartesian and in spherical coordinates. After a fairly long calculation the result is as follows.

For Cartesian coordinates the nonzero Ricci entries are:

$$R_{00} = -\frac{1}{c^2} \psi^{-1} \Box \psi + 3\psi^{-2} \left(\frac{\partial \psi}{\partial t} \right)^2 + \frac{1}{c^2} \psi^{-2} \sum_{i=1}^{3} \left(\frac{\partial \psi}{\partial x_i} \right)^2 - 2\psi^{-1} \frac{\partial^2 \psi}{\partial t^2} \tag{23}$$

$$R_{\rm ii} = \psi^{-1} \Box \psi + c^2 \psi^{-2} \left(\tfrac{\partial \psi}{\partial t} \right)^2 - \psi^{-2} \textstyle\sum_{i=1}^{3} \left(\tfrac{\partial \psi}{\partial x_i} \right)^2 - 2\psi^{-1} \tfrac{\partial^2 \psi}{\partial x_i^2} + 4\psi^{-2} \left(\tfrac{\partial \psi}{\partial x_i} \right)^2$$

for $\quad i = 1,2,3$

where we wrote x^i as x_i in order not to confuse an index with a power. The box $\Box$ is the D'Alembertian and the signs (i.e., η^{ab}) are (+,-,-,-). In Cartesian coordinates

$$\Box = c^2\partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2. \tag{24}$$

The Ricci scalar

$$R = g^{\mathrm{aa}}R_{\mathrm{aa}} \tag{25}$$

for the metric given by a scalar field is

$$R = c^2\psi^{-2}R_{00} - \psi^{-2}R_{11} - \psi^{-2}R_{22} - \psi^{-2}R_{33} = -6\psi^{-3}\Box\psi. \tag{26}$$

For spherical coordinates the nonzero Ricci entries are (when $c = 1$)

$$R_{00} = \frac{1}{2}\psi^{-2}\left\{\frac{\partial^2\psi^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi^2}{\partial\theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi^2}{\partial\phi^2} - 3\frac{\partial^2\psi^2}{\partial t^2}\right\} \tag{27}$$

$$+\frac{1}{2}\psi^{-2}\left\{\frac{2}{r}\frac{\partial\psi^2}{\partial r} + \frac{1}{r^2}\cot\theta\frac{\partial\psi^2}{\partial\theta}\right\} + \frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial t}\right)^2$$

$$R_{11} = \frac{1}{2}\psi^{-2}\left\{-3\frac{\partial^2\psi^2}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2\psi^2}{\partial\theta^2} - \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi^2}{\partial\phi^2} + \frac{\partial^2\psi^2}{\partial t^2}\right\}$$

$$+\frac{1}{2}\psi^{-2}\left\{-\frac{2}{r}\frac{\partial\psi^2}{\partial r} - \frac{1}{r^2}\cot\theta\frac{\partial\psi^2}{\partial\theta}\right\} + \frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial r}\right)^2$$

$$R_{22} = \frac{1}{2}\psi^{-2}\left\{-r^2\frac{\partial^2\psi^2}{\partial r^2} - 3\frac{\partial^2\psi^2}{\partial\theta^2} - \frac{1}{\sin^2\theta}\frac{\partial^2\psi^2}{\partial\phi^2} + r^2\frac{\partial^2\psi^2}{\partial t^2}\right\}$$

$$+\frac{1}{2}\psi^{-2}\left\{-4r\frac{\partial\psi^2}{\partial r} - \cot\theta\frac{\partial\psi^2}{\partial\theta}\right\} + \frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial\theta}\right)^2$$

$$R_{33} = \frac{1}{2}\psi^{-2}\left\{-r^2\sin^2\theta\frac{\partial^2\psi^2}{\partial r^2} - \sin^2\theta\frac{\partial^2\psi^2}{\partial\theta^2} - 3\frac{\partial^2\psi^2}{\partial\phi^2} + r^2\sin^2\theta\frac{\partial^2\psi^2}{\partial t^2}\right\}$$

$$+\frac{1}{2}\psi^{-2}\left\{-4r\sin^2\theta\frac{\partial\psi^2}{\partial r} + 3\sin\theta\cos\theta\frac{\partial\psi^2}{\partial\theta}\right\} + \frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial\phi}\right)^2.$$

The Ricci scalar in sperical coordinates gives the same equation also in spherical coordinates

$$R = g^{\mathrm{ab}} R_{\mathrm{ab}} = \psi^{-2} R_{00} - \psi^{-2} R_{11} - \psi^{-2} r^{-2} R_{22} - \psi^{-2} (r^{-2} \sin^{-2}\theta) R_{33}$$
$$= -6\psi^{-3} \Box\psi \tag{28}$$

when the D'Alembertian is expressed in spherical coordinates and the signs are set to (+,-,-,-). Ricci entries in Cartesian coordinates are easier to work with, and in a small area in the empty space outside the mass we certainly can use Cartesian coordinates.

The Newtonian gravitational potential (8) is a solution to the case of empty space with a point mass at the origin. The Ricci tensor entries for the metric are obtained from the potential field ϕ in (8) in Cartesian coordinates from (23). $\partial_0 \phi = 0$ and $\Box\phi = -\Delta\phi = 0$, thus

$$R_{00} = \frac{1}{c^2} \sum_{j=1}^{3} (\partial_j \phi)^2 = \frac{1}{c^2} \left(\left(\frac{\partial r^{-1}}{\partial x} \right)^2 + \left(\frac{\partial r^{-1}}{\partial y} \right)^2 + \left(\frac{\partial r^{-1}}{\partial z} \right)^2 \right) = \frac{1}{c^2} \frac{1}{r^2}$$

$$R_{\mathrm{ii}} = \frac{1}{r^2} + 2\frac{x_i^2}{r^4} \quad \text{for} \quad i = 1, 2, 3. \tag{29}$$

We notice that these entries are not zero, but the Ricci scalar is zero. In the classical limit $R_{00} = 0$ but $R_{\mathrm{ii}} \neq 0$ for $i = 1, 2, 3$. Especially $R_{\mathrm{ii}} - c^{-2} R_{00}$ is not zero and it does not tend to any small number in the classical limit $c \to \infty$.

In the case of a scalar field ψ and Cartesian local coordinates as in (16) and a point mass in an empty space as in (14) any solution of the Einstein equations (13) satisfies

$$R_{00} - c^{-2} R_{\mathrm{ii}} = \left(\frac{1}{2} R - \lambda \right) \left(c^2 g_{00} - g_{\mathrm{ii}} \right) = \left(\frac{1}{2} R - \lambda \right) \left(\psi^2 - \psi^2 \right) = 0. \tag{30}$$

Next we show that if the speed of light is constant, then the metric must necessarily be induced by a scalar field. We assume that light travels along geodesics of the gravitational field. From the place (x, y, z) and the time t light can move an infinitesimally small distance to any direction. We can consider the 3-space and the time as a four-dimensional Euclidian space. In the Euclidean 4-space the infinitesimal movements would be (dx, dy, dz, dt), but we want to give this 4-space a different metric. Therefore let the infinitesimal movements at the place (x, y, z) and the time t be (Adx, Bdy, Cdz, Ddt) where A, B, C, D are functions of the place (x, y, z) and the time t. The 3-dimensional sphere of the metric at the place (x, y, z) and the time t is

$$dr^2 = (Adx)^2 + (Bdy)^2 + (Cdz)^2. \tag{31}$$

The requirement that light has the same speed c to each direction implies that in any place (x, y, z) and at any time t holds

$$c = \frac{Adx}{Ddt} = \frac{Bdy}{Ddt} = \frac{Cdz}{Ddt}. \tag{32}$$

Thus $Adx = Bdy = Cdz$. In the Euclian metric the infinitesimals in each direction are equally long, i.e., $|dx| = |dy| = |dz|$. It means that $A = B = C$, that is, the infinitesimal spheres of our 3-dimensional metric are round:

$$dr^2 = A^2dx^2 + A^2dy^2 + A^2dz^2 \quad \text{and} \quad D = \frac{1}{c}A. \tag{33}$$

Let us write $A = -\psi$, where $\psi < 0$ is a scalar gravitational field. The location of our light spot in an infinitesimal sphere of the 4-dimensional space in the geometry for gravitation field metric is

$(-\psi dx, -\psi dy, -\psi dz, -c^{-1}\psi dt)$ and the line element is

$$dr^2 = \psi^2dx^2 + \psi^2dy^2 + \psi^2dz^2 + c^{-2}\psi^2dt. \tag{34}$$

In the Minkowski space there is a pseudometric where $\eta_{\mathrm{aa}} = (+, -, -, -)$ the line element in Cartesian local coordinates is

$$ds^2 = -\psi^2dx^2 - \psi^2dy^2 - \psi^2dz^2 + c^{-2}\psi^2dt^2. \tag{35}$$

Identifying g_{ii} for a general metric in Cartesian local coordinates in a Minkowski space

$$\mathrm{ds}^2 = -g_{11}\mathrm{dx}_1^2 - g_{22}\mathrm{dx}_2^2 - g_{33}\mathrm{dx}_3^2 + g_{00}\mathrm{dx}_0^2. \tag{36}$$

we see that $g_{00} = c^{-2}\psi^2$ and $g_{\mathrm{ii}} = \psi^2$, $i = 1, 2, 3$. There is no other metric that can yield the velocity of light as a scalar constant in the local environment. If the speed of light is not a scalar constant, then it becomes a vector function $c_i(x_0, x_1, x_2, x_3)$, $i = 1, 2, 3$. It is necessary that the speed of light is locally constant in Einsteinian relativity theory as c appears in many equations in that theory.

We notice that the metric that gives equal speed c of light to each direction in any place (x, y, z) and any time t is necessarily created by a scalar field ψ and the geometry is conformal: the space can expand and contract in any place, but the infinitesimal sphere, if taken as a sphere of a 4-dimensional manifold, is always perfectly round. This condition implies that angles are preserved.

Let us proceed to solve the Einstein equations in the case of an empty space with a point mass at the origin. As the metric is induced by a scalar field, the case with a point mass in the origin gives $\Box\psi = 0$. Therefore also holds $R = -6\psi^{-3}\Box\psi = 0$. The Einstein equations reduce to (30), which can be written as

$$c^2 R_{00} = R_{\mathrm{ii}}. \tag{36}$$

For every$i \in \{1,2,3\}$ in the expression of R_{ii} in (23) there are only two terms that are different for different i. We move all other terms in the expression (23) for R_{ii} to the left side of (36). Then the left side is the same for every i:

$$\begin{aligned} c^2 R_{00} - \psi^{-1}\Box\psi - c^2\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2 + \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \\ = -2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2} + 4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \end{aligned} \tag{37}$$

and the right side can be written more compactly by using

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = \psi^{-2}\left(\frac{\partial^2\psi}{\partial x_i^2} - 2\left(\frac{\partial\psi}{\partial x_i}\right)^2\psi^{-1}\right) \tag{38}$$

The right side of (37) for $i, j \in \{1,2,3\}$ gives the equations which lead to the contradiction in finding an approximation for the Newtonian gravitation potential:

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial\psi}{\partial x_j}\psi^{-2}\right). \tag{39}$$

Equations (39) are solved by any function the form $\psi = \psi(\rho)$ where $\rho = \sum x_j$, but the solution we need is close to the radially symmetric Newtonian gravitation field in this special case of a single point mass in the origin. We will first show that there is no solution $\psi = \psi(r)$. Inserting $\psi(r)$ to the left side of (39) gives

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = r^{-3}\psi'\psi^{-3}(-1 + x_i^2 r^{-3}(3r - f + f'/f)) \tag{40}$$

where $\psi' = \psi'(r) = d\psi(r)/dr$ and $f = f(r) = \psi(r)'/\psi(r)$. The only way (39) can hold is that the coefficient of x_i^2 is zero. Thus

$$3r = f - f'/f = \psi'/\psi - f'/f. \tag{41}$$

Integrating gives

$$\frac{3}{2}r^2 + c = \ln\psi - \ln f = -\ln(\psi'/\psi^2) \tag{42}$$

i.e.,

$$\psi'(r) = \psi(r)^2 C e^{-\frac{3}{2}r} \tag{43}$$

where c and $C = \exp(-c)$ are intergration constants.

Equation (43) is not possible. As ψ approximates the Newtonian gravitation potential $-GM/r$, it grows as r^{-1}. Therefore ψ' grows as r^{-2} and not with exponential dumping as in(43). Next, let usconsider if the solution ψ can be something else than $\psi = \psi(r)$. If the solution has cylinder symmetry, like the Schwarzschild solution, the proof above still works as we only need a plane (x_i, x_j) where the solution depends on the radius on that plane $r = (x_i^2 + x_j^2)^{-1/2}$. The Schwarzschild solution is not a scalar field solution and does not have locally constant speed of light. Let us consider a more general solution $\psi = a/r$ where a is a function that is close to $-GM$. Inserting $\psi = r^{-1} + a$ to the left side of (39) gives

$$\frac{\partial}{\partial x_i}\left(\frac{\partial \psi}{\partial x_i}\psi^{-2}\right)$$
$$= r^{-1}\psi^{-1}\left(-1 + \frac{x_i^2}{r^2} + \frac{\partial a}{\partial x_i}\frac{x}{a}\left(1 - r^{-2}\right) - \frac{2}{a^2}\left(\frac{\partial a}{\partial x_i}\right)^2 + \frac{1}{a}\frac{\partial^2 a}{\partial x_i^2}\right) \tag{44}$$

The term that should be cancelled is x_j^2/r^2. Keeping other x_j constant, a is a function only of x_i.

If the term x_j^2/r^2 is obtained by the $\partial a/\partial x_i$ term, then a as a function of x_i grows as

$$\frac{x}{a}\frac{da}{dx_i} \sim -\frac{x_i^2}{r^2} \tag{45}$$

giving for the leading term of a

$$\frac{1}{a}\frac{da}{dx_i} \sim -\frac{x_i}{r^2}. \tag{46}$$

Integrating yields for the leading term of a

$$\ln(a) = -\ln(r^2) \qquad \text{i.e.,} \qquad a = Cr^{-2} \tag{47}$$

where C is an integration constant. This is impossible as a is close to $-GM$.

If the x^2/r^5 term is obtained from the $(\partial a/\partial x_i)^2$, then a as a function of x_i grows as

$$\frac{2}{a^2}\left(\frac{da}{dx_i}\right)^2 \sim \frac{x_i^2}{r^2} \tag{48}$$

giving for the leading term of a

$$\frac{\sqrt{2}}{a}\left(\frac{da}{dx_i}\right) = -\frac{x_i}{r}. \tag{49}$$

Integrating yields the leading term of a

$$\ln(a) = -2^{-1/2}\ln(r) \quad \text{i.e.,} \quad a = Cr^{-\sqrt{1/2}} \tag{50}$$

where C is an integration constant. This is impossible as a is close to $-GM$.

If the x^2/r^5 term is obtained from the $\partial^2 a/\partial x_i^2$ term, then a as a function of x_i grows as

$$\frac{1}{a}\frac{d^2a}{dx_i^2} \sim \frac{x_i^2}{r^2}. \tag{51}$$

Writing

$$\frac{1}{a}\frac{d^2a}{dx_i^2} = \frac{1}{a^2}\frac{da}{dx_i} + \frac{d}{dx_i}\left(\frac{1}{a}\frac{da}{dx_i}\right) \tag{52}$$

$$\int -\frac{x_i^2}{r^2}dx_i = -\frac{x_i^2}{r^2} + \int 2\ln(r)dx_i$$

and integrating gives the leading term of a

$$-\frac{1}{a} + \frac{1}{a}\frac{da}{dx_i} = -\frac{x_i^2}{r^2} + \int 2\ln(r)dx_i. \tag{53}$$

We get

$$a = \exp\left(\int\left(\frac{1}{a} - \frac{x_i^2}{r^2} + C\right)dx_i\right)\exp\left(2\int \ln(r)dx_i\right). \tag{54}$$

Here C is an integration constant. The function a is almost constant. The first integral can stay near a constant value when we let x_i grow to large values where x_i/r is nearly one, or the first integral may grow or decrease as $\exp(C_1x_i)$, but the second integral grows with a smaller speed than $\int C_1x_i$ for any C_1. It cannot be compensated by the first integral, threfore a cannot stay close to $-GM$ when x_i grows. Either a goes to zero, or a grows without limit.

All three cases fail to cancel the x^2 term while keeping a close to $-GM$. This means that ψ is not an approximation of the Newtonian gravitation field. This result is natural. In Cartesian coordinates the terms R_{ii}, $i = 1, 2, 3$, are all symmetric but they depend on x_i. If the solution is a function of r, we get the term x_i^2 from R_{ii}. Summing all terms R_{ii}, $i = 1, 2, 3$, we get a function of r^2. This happens in the Ricci scalar. But in Einstein's equations there is an equation for each i. We can form the equation (40) relating i and j. The x_i^2 term cannot be cancelled with the term x_j^2 and they cannot make r^2. Thus, (40) cannot be satisfied.

We conclude that there are no solutions to the Einstein equations (13) that approximate the Newtonian gravitation potential in the single point mass situation and that have locally constant speed of light in vacuum.

5. About experiments that verify the Relativity Theory

There are many experiments that claim to verify the Special Relativity Theory, but no finite number of experiments where a theory predicts correctly can ever prove a theory correct, because there can be an alternative explanation that also satisfies all these experiments and we can never know all possible alternative theories. A single experiment where a theory predicts incorrectly refutes the theory. Let us give an experiment that refutes the Lorentz Transform and the Special Relativity Theory. It is the contradiction from Section 3 formulated as a physical experiment.

An experimenter has tools to measure speed of light in vacuum. Then he considers the Earth as R', the moving frame in the Lorentz Transform, and imagines that there is frame R such that the frames have the mutual speed $v = 3 * 10^8(1 - 1/3 * 10^8)$ m/s. He can just imagine the frame R, the Lorentz Transform states that now the speed of light c is about 1 m/s, see Section 3. The experimenter checks if his imagining the frame R really did change the speed of light in vacuum to 1 m/s. Needless to say, it did not. This experiment can be repated: any number of experimenters can try to set the speed of light c to their chosen value by imagining frame R. The speed of light never changes.

This impossible result comes because the Lorentz Transform has the term $-(v/c^2)x$ in the time transform. This term must have this exact value it has or else the speed of light is not constant in R', but this time offset term cannot be in the transform because if it is in the transform, then we can prove that $c = 1$ m/s. The proof does not violate or modify anything in the Special Relativity Theory (SRT), Steps 1-4 in Section 3 are in accordance with SRT as can be checked by the reader. Notice that the proof that $c = 1$ m/s does not need the knowledge that times must be projected on the t'-axis in R'. It does not make any projections in R' and it does not need the traveling time of light in R'. The proof uses the formula $T' = \gamma T$. This formula comes because there is the time shift and this formula is very, very wrong. The length $L' = \gamma L$ and the time $T' = \gamma^{-1}T$ are transforming in inverse way: the infinitesimal unit sphere is deforming. This cannot be so. It is clear that Einstein knew this problem because he defined the proper time as

$$t' = \gamma t$$

and not as $t' = \gamma^{-1}t$ which would come from the Lorentz Transform as the time experienced by an observer that is not moving in R'. The formula $t' = \gamma t$ comes only if you drop the term $-(v/c^2)x'$. So, Einstein obviously did drop this impossible term, but he forgot (or did not want to) to tell anybody that the Lorentz Transform is wrong, and because the Lorentz Transform is wrong, all Lorentz invariant theories, like the General Relativity Theory, are also wrong.

Notice also that the twin paradox between the frames R and R' does show that $\gamma = 1$. Some physicists have claimed to me that the twin paradox has been explained in numerous textbooks and it does not imply that $\gamma = 1$. This is not so. The twin paradox in the basic form is that if there are two identical systems in fully symmetric positions (like R and R' in the Lorentz Transform), then whatever is true to one system is equally true to the other system. As an example, consider two planets that are identical to the Earth and have mutual constant velocity v. In which planet time ticks slower? If it tick slower in the first planet, then just as correctly it ticks slower in the second planet. Thus, $\gamma = 1$, the times of the planets must tick with the same speed. There are these numerous textbooks that try to claim otherwise. This is easily explained: on one planet there is a group of physicists who have numerous textbooks that explain how time ticks slower on one of the planets. On the other planet there is an identical group of physicsts with identical numerous textbooks that explain how time ticks slower on the other planet. Clearly, the explanations are wrong: fully identical systems in fully symmetric positions are really fully identical systems in fully symmetric positions: whatever is true for one is equally well true for the other. The only way for breaking full symmetry is a spontaneous breakdown of logical thinking.

Indeed, the twin paradox does work. When we drop the time shift term from the Lorentz Transform, we get $x' = \gamma(x - vt)$ and $t' = \gamma t$. The twin paradox implies that $\gamma = 1$, thus the transform reduces to the Galileo transform $x' = x - vt$, $t' = t$ and this is all that can be obtained in the setting of SRT.

If we go to the setting of GRT we can include gravitation fields and accelerated motions. Then the transform upgrades to $x' = \alpha(x - vt)$, $t' = \alpha t$ where α depends on the gravitation field and the derivative of v. The Pound-Rebka experiment shows that there is gravitational redshift and by the equivalence principle (which does seem like a good assumption) the transform should also depend on acceleration. We can additionally impose a rule that the speed of light in vacuum is locally always constant c to each direction. This locally constant speed of light is then caused by the local gravitation field. Locally constant speed of light does not mean that ligth seen from a longer distance should have the constant speed c, thus light in a frame R or R' need not have the speed c everywhere. Indeed, it cannot because we must drop the time shift from the linear transform.

The condition that the speed of light is locally constant implies that the gravitation field is scalar, as was shown in Section 4. Solutions of Einstein's field equations for GRT, such as the Schwarzschild solution, do not have a round infinitesimal sphere. Therefore the speed of light is different in different places (x, y, z) and times t.

When the gravitation field is scalar, the field equation of GRT does not have any solutions that are close to the Newtonian gravitation potential in a situation of one point mass in the empty space. This situation is closely approximated by our solar system. Thus, GRT does not work in our solar system. Therefore all experiments that claim to verify GRT and have been done in our solar system must necessarily be wrong: they do not use a valid solution of the field equations of GRT. All experiments that claim to verify GRT by observations that have been done outside our solar system must depend on some theory, as we have never been outside our solar system. The theory behind these observations usually derives from GRT and SRT, thus it may not be able to verify GRT.

A note on references: it is customary to have many references in a published article, though Einstein did not have many references in his SRT paper and interestingly Henry Poincaré, who should have been referreed to and who would have been the best referee for the SRT paper, never accepted SRT. I agree with this principle in a typical case, but do not want to include references to numerous textbooks that claim that the fully symmetric situation in the twin paradox is in some sense not fully symmetric, or that SRT and GRT have been verified. I also cannot include references to any papers written by people who accept what these numerous textbooks say.

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2.2 The relativity theory needs some fixing

Abstract: Section 2 refutes the basic claim of the Special Relativity Theory: that the speed of light is constant in all frames of reference moving with a constant speed with respect to each others. The Michelson-Morley experiment is shown to be flawed in Section 3. Section 4 demonstrates that there are issues in the derivation of the moving mass formula. Section 5 shows that if the gravitation field is Newtonian and the moving mass formula is true, then the trajectory of a freely falling mass is unphysical. A physical solution is obtained if the field has an additional term preventing the speed of a moving mass from exceeding the speed of light and assuming that the mass does not depend on the velocity. The proposal is that the moving mass formula is dropped as unjustified. If this formula is dropped, then Einstein's proof of $E = \mathrm{mc}^2$ *does not work. As this equation is correct and should be preserved, Section 6 gives a very simple proof of* $E = \mathrm{mc}^2$ *for a discrete space model.*

Keywords: Special Relativity Theory, Energy=Mass equation, Michelson-Morley experiment.

1. Introduction

This article was peer-reviewed for a long time, accepted and published by IntechOpen in a book edited by Professor Brian Robson. Then IntechOpen cancelled the publication of the book and did not inform Robson or any authors of the articles. I noticed that there is a big text Cancelled with my name in a IntechOpen page. They had nothing against my article, so they claimed, the problem was that Robson had reused some of his articles, so IntechOpen claimed. Robson denied this. In any case my article was cancelled. I had great trouble getting the publication fee back because IntechOpen sent it to an inactive PayPal account that PayPal once had created for me, but I had neved had a password for it. I did finally manage the get the fee back.

However, Robson required that I should not write that relativity theory is seriously wrong. I should write that the relativity theory is incomplete to get the paper accepted. Now the reviewed, accepted, paid and published article is cancelled, therefore I write as it is in reality. The relality theory is not incomplete. It is all wrong and it is hopeless to try to fix it. Everything in relativity theory is wrong, except for gravitational time dilation, which is given a wrong explanation, and apparent relativistic mass growth, which is also explained in a wrong way.

The presented article proves the following claims:

It is not true that the speed of light is constant in every frame of reference which move with a constant speed with respect to each other. The argument that this should be the case is based on the Lorentz transform. A direct calculation from the Lorentz transform does give the constant speed c in the moving frame of reference to every direction, but only by defining a different time for every direction. The moving frame of reference must have a single time only, therefore the Lorentz transform does not define a valid (proper) time for the moving frame.

Because the Lorentz transform does not define a valid time, the transform between two frames of reference moving with a constant speed with respect to each other can only be a conformal transform. The twin paradox shows that the time cannot be faster in either one of the frames, therefore the time is the same for all frames of reference that move with a constant speed. (There is no need in the twin paradox to have accelerating motion, it is sufficient to ask in which frame time goes slower and notice that the situation is fully symmetric.) Consequently there is no time dilation or length dilation in movement with a constant speed. However there is time and length dilation in accelerated motion as is shown by the measurement of longer half-time for muons that are speeded to very high velocities. This longer half-time is caused by the acceleration phase. When muons move with a constant speed they have the same time as in the laboratory.

As the Lorentz transform does not give a valid time, there is no reason to require that equations of motion should be Lorentz invariant. They should be invariant in conformal mappings. As a consequence, the moving mass concept is not justified. A similar observable behavior than from a moving mass is obtained by adding to the force a second component that guarantees that the speed of a moving mass does not exceed the speed of light in the local environment. The second term can be understood as the space-time slowing down a mass that without this term would exceed the speed of light. The speed of light is only locally constant, i.e., in every point of the gravitational field the speed of light is constant c to each direction, but as every mass has a gravitational field and masses have relative movement, this does not imply that the speed of light is constant in any fixed coordinate system. Indeed, consider two masses having a constant relative speed. Close to either mass the speed of light is c, but in the space between these masses the gravitational field is changing and as at each point the momentary speed of light is always c, the speed of light is also changing when the field changes.

In the geometric paradigm a gravitation field is modeled as curvature. If the speed of light is constant c to each direction in each point, then the gravitation field is scalar. Calculating Einstein equations for a scalar field shows that Einstein equations of the General Relativity Theory do not have any solutions that are close to the Newtonian gravitation potential. This means that the Einstein equations would have to be replaced by the field equations of Nordtröm's scalar gravitation theory, but notice that also this theory is hopelessly wrong. Another reason why these equations cannot be used is that they are Lorentz covariant and therefore do not hve a valid time in moving frames of reference. Experiments that claim to prove the relativity theory must be reconsidered.

Einstein's famous equation $E = \mathrm{mc}^2$, which actually should be called Olinto De Pretto's equation, is confirmed by experiments, but Einstein's proof for this equation is not valid as he uses the moving mass formula that is derived from the Lorentz transform. The later proof by Max von Laue has the problem that his proof discusses Lorentz invariant equations and dropping the Lorentz transform also this proof loses its validity. A very simple proof of the $E = \mathrm{mc}^2$ is added just to demonstrate that this equation can be derived in many ways, also without the relativity theory and the Lorentz transform.

The Special Relativity Theory claims that a mass m_0 moving with velocity v has the moving mass $m = \frac{m_0}{\sqrt{1-\beta^2}}$ where $\beta = \frac{v}{c}$. From the formula follows that if a particle with the mass m_0 is accelerated to a velocity v close to the speed of light c, the mass that needs to be accelerated is the moving mass m. As m tends to infinity when v approaches c, the energy required becomes infinite. This is given as the reason why a massive particle cannot be speeded to c. The presented article argues that this moving mass formula should be dropped. The reasons are that Sections 2-4 show that the Lorentz transform does not work as it is claimed to work and therefore demanding Lorentz invariance is not justified. Section 5 demonstrates that instead of having a moving mass formula, a freely falling mass gets the same equation of motion if we assume that the mass remains constant but the gravitation field has an additional term that prevents the mass from exceeding the speed of light. This additional term corresponds to the space not allowing speeds exceeding the speed of light. I still think this calculation can be interesting and I keep it in the article, though I have found a better way: the interaction force weakens when the relative speed between the field and a test mass increases.

The moving mass formula is needed in Einstein's proof of $E = \mathrm{mc}^2$, see e.g. [1]. In Einstein's proof the mass is growing in the frame of reference where the mass is moving. If the mass transformation formula is dropped, then Einstein's proof of $E = \mathrm{mc}^2$ does not work. Section 6 proposes a simple proof for $E = \mathrm{mc}^2$ in a discrete space model as a replacement. I do not think proving this formula is any difficult task. It can be a nice exercise for some beginning student.

2. The error in Einstein's usage of the Lorentz transform

The problem in Einstein's usage of the Lorentz transform is that though the transform does give the constant speed of light in any frame of reference $R^{'}$ moving with a constant speed v with respect to a fixed frame of reference R, it uses a time concept for $R^{'}$ which is not a valid scalar time (a proper time). The time in the Lorentz transform is a scalar variable, but it is not valid as a scalar time because it demands that there is a different time in $R^{'}$ for sending light to different directions. In fact, the Lorentz transform only describes how the Doppler effect shows in new coordinates. Therefore the Lorentz transform cannot be applied in the situation that Einstein had.

Consider the following experiment. A square box with each side having the length L is moving with a constant speed$\overline{v} = v\overline{e}_v$ with respect to a rest frame of reference R. From the midpoint of one of the sides of the square light is emitted. It shines on the opposite side, reflects from it and returns to the starting point. In the frame R this roundtrip time is $T = T_1 + T_2$ where T_1 is the time for the light beam to reach the mirror and T_2 is the time for the light beam to travel from the mirror back to the starting point. The velocity $\overline{v}$ is in the direction of the positive x-axis. We assume that light travels with a constant speed c to each direction in R. In the Special Relativity Theory light has always the speed c, therefore the assumption is fulfilled in that theory. In the Ether Hypothesis, which was the hypothesis Michelson and Morley tested, we select R so that the ether is at rest in R.

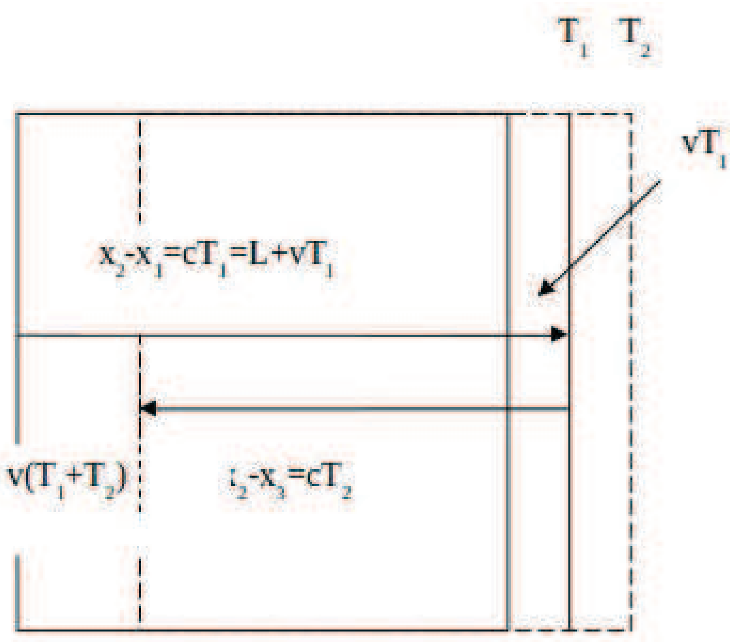

Figure 1. The scenario of the experiment.

The left side of the box is in x_1 at the time t_1. Light is sent from this point and it arrives in the time $T_1 = t_2 - t_1$ to the right side of the box. The right side has moved in this time to $x_2 = x_1 + L + \mathrm{vT}_1$. The velocity of light is c in R, thus

$$c = \frac{x_2-x_1}{t_2-t_1} = \frac{L+\mathrm{vT}_1}{T_1}, \text{ we get } T_1 = \frac{L}{c-v}. \tag{2.1}$$

The return trip of the light is from the right side of the box inx_2 to the left side of the box in x_3. Light arrives tox_3 at the time t_3. Then $x_3 = x_1 + v(t_3 - t_1)$. Let $T_2 = t_3 - t_2$. Thus $x_2 - x_3 = L - \mathrm{vT}_2$ and the speed of the light in R is c for the return trip:

$$c = \frac{x_2-x_3}{t_3-t_2} = \frac{L-\mathrm{vT}_2}{T_2}, \text{ we get } T_2 = \frac{L}{c+v}. \tag{2.2}$$

Next, we make the Lorentz transform$x^{'} = \gamma(x - \mathrm{vt})$, $t^{'} = \gamma(t - (\frac{v}{c^2})x)$. Let us define (2.3)

$$L_1^{'} \equiv x_2^{'} - x_1^{'} = \gamma((x_2 - x_1) - v(t_2 - t_1)) = \gamma(L + \mathrm{vT}_1 - \mathrm{vT}_1) = \gamma L$$

$$L_2^{'} \equiv x_2^{'} - x_3^{'} = \gamma((x_2 - x_3) - v(t_2 - t_3)) = \gamma(L - \mathrm{vT}_2 + \mathrm{vT}_2) = \gamma L.$$

We notice that the length of the box in $R^{'}$ is the same to both the positive and the negative x-direction and that it is L multiplied by a scaling factor γ . However, let us calculate the time intervals in $R^{'}$

$$\begin{aligned}
T_1^{'} &\equiv t_2^{'} - t_1^{'} = \gamma((t_2 - t_1) - (\tfrac{v}{c^2})(x_2 - x_1)) \\
&= \gamma(T_1 - (\tfrac{v}{c^2})(L + \mathrm{v}\mathrm{T}_1)) = \gamma \tfrac{c-v}{c} T_1 \qquad (2.4) \\
T_2^{'} &\equiv t_3^{'} - t_{21}^{'} = \gamma((t_3 - t_2) - (\tfrac{v}{c^2})(x_3 - x_2)) \\
&= \gamma(T_2 + (\tfrac{v}{c^2})(L - \mathrm{v}\mathrm{T}_2)) = \gamma \tfrac{c+v}{c} T_2,
\end{aligned}$$

where we eliminated L by using $L = (c - v)T_1 = (c + v)T_2$. Inserting T_1 and T_2 we get $T_1^{'} = T_2^{'} = \gamma \frac{L}{c}$, thus we get the speed of light in $R^{'}$ as $c_i^{'} = \frac{L_i^{'}}{T_i^{'}} = c$, $i = 1, 2$.

For the roundtrip we get directly from the Lorentz transform

$$\begin{aligned}
x_3^{'} - x_1^{'} &= \gamma(x_3 - x_1 - v(t_3 - t_1)) \\
T^{'} &= t_3^{'} - t_1^{'} = \gamma(t_3 - t_1 - (\tfrac{v}{c^2})(x_3 - x_1)) \\
&= \gamma(T - (\tfrac{v}{c^2})\mathrm{v}\mathrm{T}) = \gamma(1 - (\tfrac{v^2}{c^2}))T \qquad (2.5)
\end{aligned}$$

The length of the roundtrip in $R^{'}$ is not $x_3^{'} - x_1^{'}$, it is $L^{'} = L_1^{'} + L_2^{'} = \gamma 2L$. The time in R is

$$T = T_1 + T_2 = \tfrac{L}{c-v} + \tfrac{L}{c+v} = 2L \tfrac{c}{c^2 - v^2} \qquad (2.6)$$

The speed of light for the roundtrip in $R^{'}$ is $\frac{L^{'}}{T'} = c$. The Lorenz transform does give the same constant speed of light in all frames of reference, but (2.4) is highly problematic. What are the two different times in (2.4)? They are not the times experienced by a photon moving to either direction. As a photon is moving with the speed of light, in the Lorentz transform the time does not move at all for a photon. They are two times for $R^{'}$: there is a different time in $R^{'}$ for sending light to the positive x-direction and to the negative x-direction. We can see what these times are in reality by a simple thought experiment. Split light from a single oscillator to both directions and let some information be sent as bits in the light beam. If the sending times in $R^{'}$ are different to the two directions, the transmission time of the signal is different. Bits get buffered to some invisible buffer in the direction where the transmission speed is lower. Obviously this does not happen: bits leave $R^{'}$ with the same transmission speed. Let us have two receivers in the fixed frame of reference R. If the transmission times were different to the two directions, these receivers would receive signal with two different frequencies. They of course do receive the signal with two different frequencies, but it is caused by the Doppler effect. The Doppler effect it is all we see. It perfectly matches the effect that we should see with the times in (2.4). This shows that what (2.4) describes is simply the Doppler effect as it appears in the coordinate system $x^{'}, t^{'}$. The whole reason why the Lorentz transform makes the speed of light constant in all moving frames $R^{'}$ is in the formulas in (2.4). As it in reality is a description of the Doppler effect, the times $T_i^{'}$ are not anything real. The Doppler effect does not require changing the absolute Newtonian time concept. By directly calculating $T^{'}$ from the Lorentz transform we do not get the real time in $R^{'}$, we get purely mathematical constructs that describe the Doppler effect. There must be a single time in $R^{'}$, not several different times as in (2.4) an (2.5).

Is the time $T^{'}$ calculated from the Lorentz transform as in (2.3)-(2.4) the time $T^{'}$ for $R^{'}$in the Special Relativity Theory? It is not, we can show it by examples. Let us consider a muon and speed it to the velocity of $0.9c$. The moving frame $R^{'}$ is the rest frame of the muon and the fixed frame R is the rest frame of the laboratory. Muon is unstable and has a halftime Tin the laboratory. We ask what is the halftime of the moving muon. Or we can consider a spaceship speeded to a velocity v close to the speed of light c. The moving frame $R^{'}$ is the rest frame of the spaceship and the fixed frame R is the rest frame of the Earth. The time $T$is one year. What is the time $T^{'}$ in the spaceship? How much slower does the astronaut age than the people on the Earth? Is the answer by the Special Relativity Theory to the second question perhaps that the time $T^{'}$ depends on what the spaceship is doing? Is the answer that if the spaceship is sending light to the direction of movement, then $T^{'} = \gamma(1 - \frac{v}{c})T$, while if it is sending light to the backward direction, then the time $T^{'}$ is $T^{'} = \gamma(1 + \frac{v}{c})T$ (from (2.4)), and if the light makes a roundtrip, then the time is $T^{'} = \gamma(1 - \frac{v^2}{c^2})T$(from (2.5))? Clearly, this is not how the Special Relativity Theory would answer the second question. The theory answers that the time in $R^{'}$ is always the proper time. The halftime of the muon in $R^{'}$ is the proper time $T^{'} = \gamma T$and the astronaut ages by the proper time $T^{'} = \gamma T$. This is because Einstein did understand that there cannot be multiple different times in $R^{'}$. This shows clearly that Einstein did not want to calculate $T^{'}$ directly from the Lorentz transform. Had he done so, he would have got times $T^{'}$ that too obviously show that $T^{'}$ is not a real time, it is a mathematical construct that describes the Doppler effect in the coordinates $x^{'}, t^{'}$. Therefore Einstein cheated: on one hand he claimed that he is using the Lorentz transform and this transform guarantees that the speed of light is always constant c in all moving frames $R^{'}$, but he in reality used the proper time $\tau = \gamma t$ for calculating the time in $R^{'}$ because the time $T^{'}$ from the Lorentz transform is different in different situations.

Thus, in the Special Relativity Theory the intended time is the proper time. Therefore the length and time intervals transform as

$$L^{'} = \gamma L, T^{'} = \gamma T. \tag{2.7}$$

Let us apply this transform of intervals to the roundtrip delay in Figure 1. The total time in $R$is

$$T = T_1 + T_2 = \frac{L}{c-v} + \frac{L}{c+v} = 2L\frac{c}{c^2-v^2}. \tag{2.8}$$

The roundtrip length in $R^{'}$is $2L^{'} = \gamma 2L$. The roundtrip time in $R^{'}$is $T^{'} = \gamma T$, thus the speed of light for the roundtrip in $R^{'}$is

$$c^{'} = \frac{\gamma L}{\gamma L}\frac{c}{c^2-v^2} = \frac{c}{c^2-v^2} \neq c. \tag{2.9}$$

We can get the roundtrip speed to the value c by defining $T^{'} = \gamma^{-1}T$, which is also a proper time definition for $R^{'}$. However, Einstein's proper time and length transformation formulae do not give c as the roundtrip speed of light in Figure 1 for $R^{'}$, and in fact, no definition of the type $T_i^{'} = \gamma_1 T_i$ (for any γ_1 depending only on v) can make the speed of light in $R^{'}$the same to the positive and negative x-axis in Figure 1, in one way it will be smaller than c, in one way higher than c.

Notice that not only the times $T_i^{'}$ in (2.4) and (2.5) are nothing real, Einstein's proper time is also nothing real. This is shown by the twin paradox. This paradox can be presented as the muon-laboratory paradox to avoid the de-acceleration/acceleration issue with a spaceship that needs to turn: there is no sense to say that the time in the muon's rest frame goes slower than the time in the laboratory's rest frame because equally well we can say that the muon's rest frame is the fixed frame and the laboratory is moving.

3. The error in Michelson-Morley experiment

Figure 1 gives the setting of a simplified Michelson-Morley experiment. This experiment inspired Einstein to propose the Special Relativity Theory and that the time and coordinates in a moving frame should be calculated from the Lorentz transform. In the Ether Hypothesis time and space are universal and there is substance called ether where light undulates. In the simplified experiment in Figure 1 the ether is assumed to be at rest in the fixed frame R. Therefore, with absolute time and space, in the moving frame R' the speed of light calculated from the time light takes in the roundtrip must depend on the direction of the vector $\overline{v}$: if $\overline{v}$points to the positive x-axis, as in Figure 1, we get a different speed of light in R' than if $\overline{v}$points to the positive y-axis. Michelson and Morley had two light beams, one pointing to the x-axis and the other to the y-axis and in both directions light made a rountrip, first to the positive axis, then returning to the negative axis. Each beam makes a roundtrip that has the length $2L$ in $R^{'}$assuming that space and time are universal. These roundtrips are not the same and if the Ether Hypothesis holds, the roundtrip time is different in the two paths because the speed of light is different in each path. Michelson and Morley thought that there should be an interference picture when these two light beams are added at the end of the roundtrip. The error in this logic is that Michelson and Morley did not have the same starting time for the two beams: the beams had the same finishing time but a different starting time. The two beams had the same frequency and they were in the same phase at any chosen time. There could not be any interference picture when the beams are combined.

In order to see this with simple equations, let the roundtrip times on the two paths be denoted by T_i, $i = 1, 2$. The roundtrip times are different if the speed of light is different in the two paths. But the frequency is the same on both paths and only the wavelength is different on each path, thus a frequency component f has the same oscillation time $T_f = \frac{1}{f}$ on both paths. In order to interfere the two beams must be taken to the same place (the end of the roundtrip) at the same time T_F. In order to be at the end of the roundtrip at the time T_F, the beam i must have left the splitter at the time $T_F - T_i$. That is, the beams have a different starting time. Before the beam left the splitter it had made $(T_F - T_i)/T_f$ wavelengths on the frequency f. On the roundtrip the beam i made T_i/T_f oscillations. Thus, at the time T_F both beams made in total $(T_F - T_i)/T_f$+T_i/T_f=T_F/T_f oscillations in the frequency f. The situation is the same for every frequency and we notice that both beams are in exactly the same phase when the researchers try to make them interfere. Naturally there is no interference picture. For some reason this obvious logical error was not noticed. The result of the Michelson-Morley experiment was unexpected and Einstein proceeded to solve the problem how the speed of light can be the same in all moving frames by defining new coordinates for the moving frame R'.

That the Michelson-Morley experiment was flawed does not mean that their result was wrong. The result was correct, but for a different reason. The "ether" where light undulates may well be the gravitational field. This much of Einstein's theory may be correct (but does not need to be). The gravitational field of the Earth follows the Earth. Thus, the speed of light would be the same to all directions on the Earth. Even if the experiment had been made correctly, the result had very possibly been the same, but this result cannot be interpolated to a situation where a bus is moving on the Earth. A bus creates a very weak gravitational field and the field inside the bus is essentially the gravitational field of the Earth. We should not see equal speed of light to all directions in the rest frame of a moving bus (or a muon in a laboratory).

4. Invariant equations of motion

Maxwell equations are invariant in the Lorentz transform (but only if E and B are required to transform in a special way to make the equations Lorentz invariant) and Einstein proceeded to require that all equations of motion must be invariant in the Lorentz transform. Later the equations were required to be covariant, but this concept only applies to tensor equations. There is no difference between these concepts in ordinary partial differential equations. What is meant is that if (x', t') of the moving frame of reference is inserted to the equations of (x, t) for the rest frame of reference, then the equation of motion for (x', t') has the same form as for (x, t).

The basic equation of movement in Newtonian mechanics is $F =$ ma. Let the frame R' move with a constant speed $\overline{v}$ and let the mass m move in the same direction as $\overline{v}$. Initially the velocity of the mass in the frame R seems to be

$$w = \tfrac{dx}{dt}, \tag{4.1}$$

but this is not so for the following reason. In the Lorentz transform x and t are independent coordinates in R. In R' the coordinates x' and t' are independent. Thus

$$x = \gamma(x' + vt') \text{ gives } \tfrac{dx}{dt'} = \gamma v, \tag{4.2}$$

$$t = \gamma \left(t^{'} + \frac{vx^{'}}{c^2} \right) \text{ yields } \frac{dt}{dt'} = \gamma$$

and $x^{'} = \gamma(x - \text{vt})$ leads to the velocity in the frame R'

$$w^{'} = \frac{dx^{'}}{dt'} = \gamma \left(\frac{\text{dx}}{dt'} - v \frac{\text{dt}}{dt'} \right) = \gamma(\gamma v - v\gamma) = 0. \tag{4.3}$$

This is not correct: the mass is not at rest in R'. The velocity of the mass is not obtained by derivation from the Lorentz transform. Again, the Special Relativity Theory does not use the Lorentz transform directly also here. The velocity is actually

$$w = \lim \frac{\Delta x}{\Delta t}, \tag{4.4}$$

and both Δx and Δt are intervals. Therefore they transfer as (2.7)

$$\Delta x^{'} = \gamma \Delta x \text{ and } \Delta t^{'} = \gamma \Delta t. \tag{4.5}$$

We get the velocity as

$$w^{'} = \lim \frac{\Delta x^{'}}{\Delta t'} = \lim \frac{\gamma \Delta x}{\gamma \Delta t} = w. \tag{4.6}$$

If we instead of (4.5) use the transform that gives the speed of light as c in $R^{'}$ for the roundtrip in Figure 1

$$\Delta x^{'} = \gamma \Delta x \qquad \text{and} \qquad \Delta t^{'} = \gamma^{-1} \Delta t \tag{4.7}$$

we get

$$w^{'} = \lim \frac{\Delta x^{'}}{\Delta t'} = \lim \frac{\gamma \Delta x}{\gamma^{-1} \Delta t} = \gamma^2 w. \tag{4.8}$$

Notice that $\beta = \frac{v}{c}$ is constant as the velocity $\overline{v}$ is constant. Only the mass m can accelerate in this case, not the frame R'. In R the equation of motion (allowing the mass to change and what Einstein used) is

$$F = \frac{d}{dt} \left(m \frac{dx}{dt} \right) . \tag{4.9}$$

In R' , following the transform (4.5), we get

$$F^{'} = \frac{d}{dt'} \left(m^{'} \frac{dx^{'}}{dt'} \right) = \frac{1}{\sqrt{1-\beta^2}} \frac{d}{dt} \left(m^{'} \frac{dx}{dt} \right). \tag{4.10}$$

If we take the transform (4.7) in R' , we have

$$F^{'} = \frac{d}{dt'} \left(m^{'} \frac{dx^{'}}{dt'} \right) = \frac{1}{\gamma^{-1}} \frac{d}{dt} \left(m^{'} \gamma^2 \frac{dx}{dt} \right) = \frac{1}{(1-\beta^2)\sqrt{1-\beta^2}} \frac{d}{dt} \left(m^{'} \frac{dx}{dt} \right). \tag{4.11}$$

Assuming that the force $F^{'}$ equals force F, we notice that there is a solution for (4.10) where m and $m^{'}$ do not depend on the time t. For

$$F = m \frac{d^2 x}{dt^2} \tag{4.12}$$

$$F = F^{'} = \frac{m^{'}}{\sqrt{1-\beta^2}} \frac{d^2 x}{dt^2}.$$

The equations (4.9) and (4.10) are identical (the equation is invariant in the Lorentz transform) if

$$m = \frac{m^{'}}{\sqrt{1-\beta^2}}. \tag{4.13}$$

This is enough for the equation of motion to be invariant, but Einstein went further and decided that the mass changes in the frame R as

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \tag{4.14}$$

and in a frame that is accelerating with the mass he defined $m^{'} = m_{0.}$ Notice this very odd choice. In the cases of $T^{'}$ and $L^{'}$ the properties in R are kept as they were and the modified versions are in R', but the mass changes in R. This is essential for Einstein's proof of E=mc^2, the proof fails if the mass change is in R'.

If we use the transform (4.7), which gives the speed of light in R' in the roundtrip in Figure 1 as c, then the time independent value of $m^{'}$ that makes (4.9) and (4.10) identical if $F^{'} = F$ is

$$m = \frac{m^{'}}{(1-\beta^2)\sqrt{1-\beta^2}}. \tag{4.15}$$

This mass change formula would ruin Einstein's proof of E=mc^2, but notice that (4.5) does not give the speed of light in R' in the roundtrip in Figure 1 as c while (4.7) does, though neither transform gives the speed of light in R' as c for the first part of the roundtrip in Figure 1.

From [1] we see that Einstein derived his formula$m = \frac{m_0}{\sqrt{1-\beta^2}}$ from the equation $F = \frac{d}{dt}(m(t)\frac{ds}{dt})$ basically in the way we did it in (4.12)-(4.14). This derivation raises some questions. What is the sense of requiring invariance of $F =$ ma under the Lorentz transform, especially in the form $F = (\frac{d}{\mathrm{dt}})m(t)(\frac{\mathrm{ds}}{\mathrm{dt}})$? Why Fshould transfer to itself in the Lorentz transform, as it does in (4.12), since force is not a conserved quantity. If e.g., $E = \frac{\mathrm{mv}^2}{2}$ or $W = \mathrm{Fs} = \mathrm{mas}$ is required to be Lorentz invariant, then m is not changing. The equation for the moving mass (4.14) is derived in the situation of an accelerated frame of reference, not when a frame of reference has a constant velocity. What is the justification of extending the Lorentz transform to accelerating situations? In fact, what is the justification of using the Lorentz transform in any situation? After all, the Michelson-Morley experiment is flawed, the Lorentz transform does not make the speed of light in the roundtrip in the Michelson-Morley experiment of Figure 1 equal to c (as the time used in the Special Relativity Theory actually is the proper time), and the transform that makes the roundtrip speed of light equal to c cannot make the speed of light in a one-way trip equal c.

5. A test mass falling in the gravitational field of a point mass

Consider a mass m falling freely in a gravitational field created by a point mass M in Newton's gravitation theory. The movement of the test mass m is in the radial direction: the test mass falls from the initial place r_0 towards the origin set to the position of the mass M. The equation of the motion is

$$F = \frac{GmM}{r^2} = ma = m\frac{d^2(r_0-r)}{dt^2}. \tag{5.1}$$

Writing this with the Newtonian gravitational potential field $\phi = -\frac{GM}{r}$ we get

$$m\frac{d}{dr}\left(-\frac{GM}{r}\right) = m\frac{d}{dr}\phi = -m\frac{d^2r}{dt^2}. \tag{5.2}$$

The equation of motion of the test mass m is

$$m\frac{d}{dr}\phi = -m\frac{d^2r}{dt^2} \text{ , simplifying to } \qquad \frac{d}{dr}\phi = -\frac{d^2r}{dt^2} \tag{5.3}$$

that is, the mass cancels out. This is the equivalence principle in the Newtonian gravitation theory: m in the equation $F = ma$ (the inertial mass) and m in the equation $F = \frac{GmM}{r^2}$ (the gravitational mass) is the same mass. Einstein wanted this equivalence principle to hold also in a relativistic theory of gravitation. Then it gets some new content: the principle implies that local time also transforms the same way for the inertial mass and gravitational mass. As in a gravitational field a clock slows down (as is verified by the Pound-Rebka experiment), then also in accelerating motion a clock slows down. Let us accept this equivalence principle.

Inserting the Newtonian potential $\phi = -\frac{GM}{r}$, we can solve the equation of motion. The solution in the rest frame of the mass M is

$$r = \left(GM\tfrac{9}{2}\right)^{\frac{1}{3}} t^{\frac{2}{3}} \tag{5.4}$$

implying that the mass m is in the place $r_0 - r = r_0 - \left(\mathrm{GM}\frac{9}{2}\right)^{\frac{1}{3}} t^{\frac{2}{3}}$ at the time t.

The solution is easily checked:

$$\begin{aligned} -\frac{d^2r}{dt^2} &= -\left(GM\tfrac{9}{2}\right)^{\frac{1}{3}} \tfrac{2}{3}\left(-\tfrac{1}{3}\right) t^{-\frac{4}{3}} = (\mathrm{GM})^{\frac{1}{3}}\left(\tfrac{2}{9}\right)^{1-\frac{1}{3}} t^{-\frac{4}{3}} \\ &= (GM)^{\frac{1}{3}}\left(\tfrac{2}{9}\right)^{\frac{2}{3}} t^{-\frac{4}{3}} \\ \frac{d}{dr}\phi &= GM\frac{1}{r^2} = GM\left(GM\tfrac{9}{2}\right)^{-\frac{2}{3}} t^{-\frac{4}{3}} = (GM)^{\frac{1}{3}}\left(\tfrac{2}{9}\right)^{\frac{2}{3}} t^{-\frac{4}{3}}. \end{aligned} \tag{5.5}$$

If we want the solution to be in a familiar form, then we must change the parameters. The solution can be expressed with the radius R of the Earth and $g = 9.81m/s^2$ as

$$r = R - \tfrac{1}{2}g\tau^2 \mp \tfrac{2g}{9T}\tau^3 + \cdots$$

where $\tau = T \mp t$ and

$$T = R^{\frac{3}{2}}\left(GM\tfrac{9}{2}\right)^{-\frac{1}{2}},\ g = \frac{GM}{R^2}. \tag{5.6}$$

If T is large, the trajectory of the falling mass (mass falls to a well from the surface of the Earth) is exactly what we expect it to be.

In the Newtonian gravitation theory the test mass m does not change size. In Einstein's relativity theory a mass becomes larger if it is moving with a velocity close to the speed of light. Einstein wrote the Newtonian equation of motion $F = ma$ in the form

$$F = \frac{d}{dt}\left(m(t)\frac{dx(t)}{dt}\right). \tag{5.7}$$

This form allows the mass m to increase as a function of the time. We find this formula e.g. as the equation before the equation (51) in Einstein's *The Meaning of Relativity* (1922) [1], the lectures he gave in Princeton. It is a better source than Einstein's papers because Einstein edited the book still on his late years. There are errors in Einstein's theories and the book shows clearly that Einstein had no intention of admitting or fixing any of these errors. He did not. forinstance. in 1952 modify the Special Relativity Theory. It is all the same in his book right to the end, only more errors have been added.

In Einstein's proof of $E = mc^2$ the mass m to increases in the frame where the mass m moves, i.e., the test mass m increases in the rest frame of the mass M that creates the field.

I have better arguments to show that the relativistic mass formula is patently false, but I want to keep this calculation because is has a remarkable cancellation at one point and I think it has something relevant to say and should be looked at. Let us investigate if the test mass m can grow in the rest frame of M: we now assume that this is possible and we will see where it leads. As the mass is moving in the radial direction towards the origin, the equation (5.5) has the form:

$$F = -\frac{d}{dt}\left(m(t)\frac{dr(t)}{dt}\right). \tag{5.8}$$

As m is a function of time in (5.8), the equivalence principle requires that $m(r) = m(r(t)) = m(t)$ is a function of r. Consequently, we have to write the gravitational force as

$$F = \frac{d}{dr}\left(m(r)\phi(r)\right). \tag{5.9}$$

The equation of motion (5.3) gets the form

$$\frac{d}{dr}\left(m(r)\phi\right) = -\frac{d}{dt}m(t)\frac{dr}{dt}. \tag{5.10}$$

This yields

$$\frac{dm(r)}{dr}\phi + m(r)\frac{d}{dr}\phi = -\frac{dm(t)}{dt}\frac{dr}{dt} - m(t)\frac{d^2r}{dt^2}. \tag{5.11}$$

As $m(r) = m(r(t)) = m(t)$ we can write

$$m\left(\frac{d\phi}{dr} + \frac{d^2r}{dt^2}\right) = -\frac{dm(t)}{dt}\frac{dr}{dt} - \frac{dm(r)}{dr}\phi \tag{5.12}$$

and since in this case $r = r(t)$, we have

$$m\left(\frac{d\phi}{dr} + \frac{d^2r}{dt^2}\right) = -\frac{dr}{dt}\frac{dm(r)}{dr}\frac{dr}{dt} - \frac{dm(r)}{dr}\phi \tag{5.13}$$

which is simplified to

$$m\left(\frac{d\phi}{dr}+\frac{d^2r}{dt^2}\right)=-\frac{dm}{dr}\left(\left(\frac{dr}{dt}\right)^2+\phi\right) \tag{5.14}$$

and finally to the form

$$m^{-1}\frac{dm}{dr}=-\left(\frac{d\phi}{dr}+\frac{d^2r}{dt^2}\right)\left(\left(\frac{dr}{dt}\right)^2+\phi\right)^{-1}. \tag{5.15}$$

Let us denote

$$f=\left(\frac{dr}{dt}\right)^2+\phi \tag{5.16}$$

then

$$\frac{df}{dr}=\frac{d}{dr}\left(\left(\frac{dr}{dt}\right)^2+\phi\right)=\frac{d\phi}{dr}+2\frac{d^2r}{dt^2}. \tag{5.17}$$

We can write the equation of motion (5.10) as

$$m^{-1}\frac{dm}{dr}=-\left(\frac{d\phi}{dr}+2\frac{d^2r}{dt^2}\right)\left(\left(\frac{dr}{dt}\right)^2+\phi\right)^{-1}+\frac{d^2r}{dt^2}\left(\left(\frac{dr}{dt}\right)^2+\phi\right)^{-1} \tag{5.18}$$

and if $\frac{dm}{dr}$ is not zero we can write (5.18) in the form

$$\frac{d}{dr}\log(mf)=\frac{d^2r}{dt^2}f^{-1}. \tag{5.19}$$

Let $\left|\frac{dr}{dt}\right|$ and $\left|\frac{d^2r}{dt^2}\right|$ be so small that the mass m does not move with a speed close to the speed of light and the increase of the mass m can be ignored. This does not imply that $\left|\frac{dr}{dt}\right|$ and $\left|\frac{d^2r}{dt^2}\right|$ are very small. They are not infinitesimally small, they are only small compared to c. We say that they are sufficiently small. According to Einstein's relativity theory, the increase of mass m must be very small, thus

$$m\left(\frac{d\phi}{dr}+\frac{d^2r}{dt^2}\right)=-\frac{dm}{dr}\left(\left(\frac{dr}{dt}\right)^2+\phi\right)\approx 0 \tag{5.20}$$

and the solution must be very close to the solution in the Newtonian gravitation theory

$$r\approx\left(\mathrm{GM}\tfrac{9}{2}\right)^{\frac{1}{3}}t^{\frac{2}{3}}\ . \tag{5.21}$$

However, if $\left|\frac{dr}{dt}\right|$ and $\left|\frac{d^2r}{dt^2}\right|$ are sufficiently small, the value of the mass m cannot have any effect to the right side in the equation (5.19) because mass m cancels out in the Newtonian gravitation theory: in Newtonian mechanisms all masses fall in a gravitation field with the same speed. Integrating (5.19) with respect to r

$$\log(mf)=\int^r\frac{d^2r}{dt^2}f^{-1}dr\ ,\ f=\left(\frac{dr}{dt}\right)^2+\phi=\left(\frac{dr}{dt}\right)^2-\frac{GM}{r}. \tag{5.22}$$

We see that f does not depend on m. We also see that the total energy is

$$E=E_k+E_p=\frac{1}{2}m\left(\frac{dr}{dt}\right)^2+m\phi=mf-\frac{1}{2}m\left(\frac{dr}{dt}\right)^2. \tag{5.23}$$

If the total energy is constant, mf depends linearly on m, so log(mf) depends on mass. The right side (5.18) does not depend on mass. This is not because there is no parameter m in the right side. It is because Galileo showed that all masses fall in the same way: the equation of r in the field ϕ does not depend on the mass m of the test particle. This mass cancels out in (5.3). In order to make it clear that the right side of (5.19) does depend on m, compose the mass m from N small parts Δm, $m = N\Delta m$ and let N depend on r, so that $m(r) = N(r)\Delta m$. Every small mass Δm falls in the same way, so they all give the same function in the right side of (5.19). The dependence of $m(r)$ on r means that $(\frac{d}{dr})\log(m)$ is not zero and it depends on $m(r)$.

We assume that $\frac{dm}{dr} \neq 0$. One way to remove the dependency of the left side of (5.19) from mass is to set $f = 0$ exactly (and not approximately) if $\left|\frac{dr}{dt}\right|$ and $\left|\frac{d^2r}{dt^2}\right|$ are sufficiently small. This is because even a small nonzero value of f lets a large value of m influence the right side. We notice that

$$f = \left(\frac{dr}{dt}\right)^2 + \phi = 0 \tag{5.24}$$

has a solution that is similar to the previous exact solution, but not the same

$$r = (GM)^{\frac{1}{3}}\, t^{\frac{2}{3}} \text{ for } \phi = \frac{-GM}{r}. \tag{5.25}$$

Inserting this solution we get

$$\left(\frac{d\phi}{dr} + \frac{d^2r}{dt^2}\right) = (GM)^{\frac{1}{3}}\, t^{-\frac{4}{3}} - (\mathrm{GM})^{\frac{1}{3}}\, \frac{2}{9} t^{-\frac{4}{3}} = \frac{7}{9}\, (GM)^{\frac{1}{3}}\, t^{-\frac{4}{3}} \tag{5.26}$$

Thus (5.20)

$$m\left(\frac{d\phi}{dr} + \frac{d^2r}{dt^2}\right) \approx 0 \tag{5.27}$$

is not satisfied. This way of solving the problem in (5.19) is not possible, yet it had to be checked.

Let us now assume that $\frac{dm}{dr} \neq 0$ and $f \neq 0$. There is still one way left to try to satisfy (5.19). If

$$m = \exp\left(\int_a^r h(r)dr\right) = e^{H(r)}e^{-H(a)} \tag{5.28}$$

for some smooth function $h(r) = dH(r)/dr$, then

$$\frac{d}{dr}\log(m) = h(r) \text{ and we can set } m_0 = e^{-H(a)}. \tag{5.29}$$

The $H(r)$ can depend only on the trajectory which is the same for all masses. Explicitly, we can demand that

$$m = m_0\left(1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right)^{-\alpha}. \tag{5.30}$$

For $\alpha = \frac{1}{2}$ we have Einstein's formula for moving mass. Let us assume this is the case, then

$$h(r) = \frac{d}{dr}H(r) = -\alpha\frac{d}{dr}\log\left(1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right) = \alpha\frac{1}{1-\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2}\frac{d}{dr}\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 \tag{5.31}$$

$$= \alpha\left(c^2 - \left(\frac{dr}{dt}\right)^2\right)^{-1}\frac{dt}{dr}\frac{d}{dt}\left(\frac{dr}{dt}\right)^2 = 2\alpha\left(c^2 - \left(\frac{dr}{dt}\right)^2\right)^{-1}\frac{d^2r}{dt^2}.$$

From (5.15)

$$h(r) = m^{-1}\tfrac{dm}{dr} = -\left(\tfrac{d\phi}{dr} + \tfrac{d^2r}{dt^2}\right)\left(\left(\tfrac{dr}{dt}\right)^2 + \phi\right)^{-1} \tag{5.32}$$

we get an equation

$$2\alpha\tfrac{d^2r}{dt^2}\left(\left(\tfrac{dr}{dt}\right)^2 + \phi\right) = -\left(\tfrac{d\phi}{dr} + \tfrac{d^2r}{dt^2}\right)\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right). \tag{5.33}$$

We make a small calculation

$$0 = 2\alpha\tfrac{d^2r}{dt^2}\phi + (2\alpha-1)\tfrac{d^2r}{dt^2}\left(\tfrac{dr}{dt}\right)^2 + \tfrac{d\phi}{dr}\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right) + c^2\tfrac{d^2r}{dt^2} \tag{5.34}$$

$$0 = 2\alpha\tfrac{d^2r}{dt^2}\tfrac{dr}{dt}\phi + (2\alpha-1)\tfrac{d^2r}{dt^2}\left(\tfrac{dr}{dt}\right)^3 + \tfrac{dr}{dt}\left(\tfrac{dt}{dr}\tfrac{d\phi}{dt}\right)\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right) + c^2\tfrac{d^2r}{dt^2}\tfrac{dr}{dt}$$

$$0 = \alpha\phi\tfrac{d}{dt}\left(\tfrac{dr}{dt}\right)^2 + \tfrac{d}{dt}\left(\tfrac{1}{4}(2\alpha-1)\left(\tfrac{dr}{dt}\right)^4\right) + \tfrac{d\phi}{dt}\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right) + \tfrac{1}{2}c^2\tfrac{d}{dt}\left(\tfrac{dr}{dt}\right)^2$$

and finally we have

$$\tfrac{d}{dt}\left\{c^2\left(\phi + \tfrac{1}{2}\left(\tfrac{dr}{dt}\right)^2\right) + \tfrac{1}{2}\left(\alpha - \tfrac{1}{2}\right)\left(\tfrac{dr}{dt}\right)^4\right\} = \tfrac{d\phi}{dt}\left(\tfrac{dr}{dt}\right)^2 - \alpha\phi\tfrac{d}{dt}\left(\tfrac{dt}{dt}\right)^2. \tag{5.35}$$

Let us insert $\alpha = \frac{1}{2}$. Then the equation is easily solved:

$$c^2\tfrac{d\phi}{dt} + \tfrac{1}{2}c^2\tfrac{d}{dt}\left(\tfrac{dr}{dt}\right)^2 = \tfrac{d\phi}{dt}\left(\tfrac{dr}{dt}\right)^2 - \tfrac{1}{2}\phi\tfrac{d}{dt}\left(\tfrac{dt}{dt}\right)^2 \tag{5.36}$$

gives

$$\tfrac{d\phi}{dt}\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right) = -\tfrac{1}{2}\tfrac{d}{dt}\left(\tfrac{dt}{dt}\right)^2\left(c^2 + \phi\right) \tag{5.37}$$

and

$$\tfrac{d\phi}{dt}\left(c^2 + \phi\right)^{-1} = -\tfrac{1}{2}\tfrac{d}{dt}\left(\tfrac{dt}{dt}\right)^2\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right)^{-1}, \tag{5.38}$$

which can be integrated

$$\log(c^2 + \phi) = \tfrac{1}{2}\log\left(c^2 - \left(\tfrac{dr}{dt}\right)^2\right) + \log B \tag{5.39}$$

where B is an integration constant. Thus,

$$c^2 + \phi = B\sqrt{c^2 - \left(\tfrac{dr}{dt}\right)^2}. \tag{5.40}$$

We must set $B = c$ in order to cancel the leading term in

$$c^4 + 2c^2\phi + \phi^2 = B^2c^2 - B^2\left(\tfrac{dr}{dt}\right)^2. \tag{5.41}$$

The final equation of movement is

$$2\phi + \tfrac{1}{c^2}\phi^2 = -\left(\tfrac{dr}{dt}\right)^2. \tag{5.42}$$

This equation looks rather strange, but we can put it to a more familiar form by differentiating it with respect to time:

$$2\tfrac{d\phi}{dt}\left(1 + \tfrac{1}{c^2}\phi\right) = -2\tfrac{d^2r}{dt^2}\left(\tfrac{dr}{dt}\right) \tag{5.43}$$

and writing it as

$$\frac{dt}{dr}\frac{d\phi}{dt}\left(1+\frac{1}{c^2}\phi\right)=-\frac{d^2r}{dt^2}$$

that is

$$\frac{d\phi}{dr}\left(1+\frac{1}{c^2}\phi\right)=-\frac{d^2r}{dt^2}. \qquad (5.44)$$

From this form it is clear that the classical limit is (5.3). Notice that this formula does not have mass. The rest mass of the test mass is in $m_0 = \exp(-H(a))$. What (5.44) is claiming is that all sizes of test masses fall according to (5.44). The situation is spherically symmetric. We notice that the equation (5.44) can be written as

$$\frac{d\Psi}{dr}=-\frac{d^2r}{dt^2} \qquad \text{where} \qquad \Psi=\phi+\frac{1}{2c^2}\phi^2 \qquad (5.45)$$

and be interpreted as an equation of two forces

$$F_{\text{field}}=m\nabla\Psi=F_{\text{acceleration}}=ma=m\frac{d^2(r_0-r)}{dt^2} \qquad (5.46)$$

where the mass m stays constant. This understanding is possible only if $\alpha=\frac{1}{2}$ in (5.35), which seems to mean that $\alpha=\frac{1}{2}$ is the correct value. There is a remarkable cancellation only for this value of α, which is the reason I have preserved this calculation even though I have shown that the mass cannot grow. Indeed, the mass does not really grow by velicity, but already Lorentz showed experimenatally that mass appears to grow with velocity and exactly by the relativistic mass formula. There is something in the value $\alpha = 1/2$ and the relativistic mass formula, as an apparent growth, is a true and verified fact. But it has some other explanation than the one that Einstein gave.

Equation (2.46) agrees with the basic concepts of $F=$ ma and $F=m\nabla\Psi$. Thus, in a certain sense the gravitation field created by the mass M is not $\phi=\frac{-\text{GM}}{r}$ but

$$\psi=-\frac{\text{GM}}{r}+\frac{1}{2c^2}\left(\frac{\text{GM}}{r}\right)^2. \qquad (5.47)$$

This sense is not that the Newtonian potential is wrong, it is that something in the space does not allow a test mass to exceed the speed of light.

We have a normal Newtonian equation of motion and a field that stops the test mass from reaching the speed of light: in small values of r the second term in Ψ gives a negative force and slows down the test mass. This is correct: if a test mass is accelerated to speeds close to c, it cannot increase its speed above c, assuming, as we do, that the speed of light is the maximal speed. A falling test mass loses potential energy, but cannot gain equally much kinetic energy. If there is an energy difference, then it must go somewhere. If it does not go into building moving mass, it goes into some other form of energy. However, in (5.46) the test mass does not lose much potential energy when it is falling: the energy stays in the form of potential energy. Equation (2.47) also implies that there is a radius

$$r=\frac{\text{GM}}{2c^2} \qquad (5.48)$$

where the field Ψis zero.

Notice that the derivation from (5.10) and (5.30) to (5.44) does not anywhere use any explicit form of ϕ. From (5.10) to (5.44) there is no assumption that ϕ is the Newtonian gravitation field as in (5.1). One way of understanding (5.44) is to say that (5.30) is only an apparent dependency of the mass from the velocity, in reality the mass is constant while (5.10) is incorrect: field ϕ does not act as in (5.10). The space does not allow the mass to exceed the velocity c and there comes an additional term to the equation of movement: the correct form is (5.44).

The other way of understanding (5.44) is that the equation of motion is (5.10) and the mass grows as in (5.30). This is Einstein's understanding. It leads to an unphysical solution if ϕ is the Newtonian gravitation field. Equation (5.44) is a direct consequence of (5.10) and (5.30) with $\alpha = \frac{1}{2}$. We can solve (5.42) exactly for the Newtonian gravitation potential:

$$\begin{aligned}\tfrac{dr}{dt} &= \pm\left(-2\phi - \tfrac{1}{c^2}\phi^2\right)^{-\frac{1}{2}} = \left(\tfrac{2GM}{r}\right)^{-\frac{1}{2}}\left(\tfrac{GM}{2c^2 r}\right)^{-\frac{1}{2}}\left(\tfrac{2c^2 r}{GM} - 1\right)^{-\frac{1}{2}} \\ &= \tfrac{cr}{GM}\left(\tfrac{2c^2 r}{GM} - 1\right)^{-\frac{1}{2}} \end{aligned} \tag{5.49}$$

where we selected + from ± because both r and t are positive and inserted $\phi = -\frac{GM}{r}$. Writing

$$t = \left(\tfrac{2}{9\mathrm{GM}}\right)^{\frac{1}{2}} r^{\frac{3}{2}} + g(r) \tag{5.50}$$

for some smooth function $g(r)$ we get from (5.49)

$$g'(r) = \tfrac{cr}{GM}\left(\tfrac{2c^2 r}{GM} - 1\right)^{-\frac{1}{2}} - \tfrac{\sqrt{r}}{\sqrt{2GM}} \tag{5.51}$$

which is integrated to

$$g(r) = r^{\frac{3}{2}}\sqrt{\tfrac{2}{GM}}\left\{\sqrt{1 - \tfrac{GM}{2c^2 r}} - \tfrac{2}{3}\left(1 - \tfrac{GM}{2c^2 r}\right)^{\frac{3}{2}} - \tfrac{1}{3}\right\}. \tag{5.52}$$

The integration constant is zero because if $c \to \infty$, then $g(r) = 0$. Thus, (5.50) and (5.52) give the exact trajectory of the test mass m in Einstein's understanding of (5.10) and (5.30). We can see from (5.49) that if

$$\begin{aligned} &r = \tfrac{GM}{c^2}, \text{ then } \tfrac{dr}{dt} = c \text{ and } F = m\nabla\psi = \tfrac{d}{dr}\left(m(r)\phi(r)\right) = 0 \\ &r = \tfrac{2}{3}\tfrac{GM}{c^2}, \text{ then } \tfrac{dr}{dt} = \tfrac{\sqrt{3}}{2}c \\ &r = \tfrac{GM}{2c^2}, \text{ then } \tfrac{dr}{dt} = 0 \text{ and } \psi = 0. \end{aligned} \tag{5.53}$$

Firstly, according to (5.43), the mass m does reach the speed c at one point. The inertial mass does grow to infinity at this point if (5.30) is assumed to describe the physical reality, but the gravitational mass in the left side of (5.10) also grows to infinity at the same point: the attraction force also becomes infinite. Secondly, after this point the mass slows down and its velocity goes to zero at the radius (5.48). In Einstein's understanding the gravitational attraction force in (5.10) always increases when r decreases, so there is no reason why the mass should start slowing down. We conclude that Einstein's understanding is not possible if ϕ is the Newtonian gravitation field.

We cannot based on this calculation only exclude the possibility that the moving mass formula (5.30) is correct and the field ϕ is not the Newtonian gravitation field (yet, I now do have stronger arguments to exclude the possibility), but a natural way to understand what happens is in (5.45)-(5.47): the gravitational force induced by the field ϕ has two components and $F = m\nabla\Psi$, not $F = m\nabla\phi$. The reason for this is that the field geometry does not allow a test mass to exceed the speed of light.

The field Ψ can be continued as zero, or some other function, inside the radius (5.48). Therefore there need not be any singularity in the gravitational field, which is good as singularities should not appear in physical systems. The field Ψ is a spherically symmetric scalar field. Such a field cannot be obtained from the field equation of the General Relativity Theory, but it comes naturally from Nordström's first gravitation theory where the field equation is

$$\Box\phi = 4\pi G\rho \text{ when } \eta = (+,-,-,-). \tag{5.54}$$

with

$$F = m\nabla\Psi \text{ where } \Psi = \phi + \tfrac{1}{2c^2}\phi^2. \tag{5.55}$$

Applied to electromagnetism, this would be similar to a case where the Maxwell equations (the field equation) are not changed, but the Coulomb force is changed.

Gunnar Nordström collaborated with Einstein, finally Einstein got him confused with the stress tensor and Nordström's gravitation theory was discarded. Nordström wrote the equation of motion of the test mass with the proper time of the Special Relativity Theory. The proper time does work as the gravitational local time and it explains correctly the Pound-Rebka experiment. The equation of motion is simply the Newtonian formula written with the proper time:

$$F = \text{ma} = m\tfrac{d^2s}{d\tau^2}. \tag{5.56}$$

Yet, I must add a note to this text that I wrote long ago: though I still think Nordström's theory can be a starting point for someone wanting to create a better theory than GRT, Nordström's theory is also incorrect: the geometrization idea fails.

We can now look at Einstein's proof of $E = \text{mc}^2$ as it is often presented in modern times.

6. The error in Einstein's proof of $E = mc^2$

The usual proof for $E = mc^2$ is very simple. From $m_0 = m\sqrt{1-\beta^2}$ we get by squaring

$$m_0^2c^2 = m^2c^2 - m^2v^2 \tag{6.1}$$

Assuming that this equation holds when v is not constant, we can differentiate

$$0 = 2mc^2dm - 2mv^2dm - 2m^2vdv \tag{6.2}$$

and obtain

$$c^2dm = v^2dm + mvdv. \tag{6.3}$$

Inserting the equation of motion

$$F = \frac{d}{dt}(mv) = \frac{dm}{dt}v + m\frac{dv}{dt} \tag{6.4}$$

to

$$dW_K = Fds = v\frac{dm}{dt}ds + m\frac{dv}{dt}ds = v\frac{ds}{dt}dm + m\frac{ds}{dt}dv$$

$$= v^2dm + mvdv = c^2dm \tag{6.5}$$

and confusing energy with work we get

$$E = \int dW_K = \int_{m_0}^{m} c^2dm = mc^2 - m_0c^2. \tag{6.6}$$

When I wrote this article I did not realize that Einstein confused energy with work, I only had the following objections: If $m_0 = m\sqrt{1-\beta^2}$is not correct, this proof fails. Notice that this proof works in the frame of reference where the mass is moving. It is not in the rest frame of the mass. In that frame $v = 0$.

Section 5 shows in the discussion after (5.44) that there is a more reasonable explanation to (5.44) where the mass is not growing with velocity, but the field created by a force has an additional component which slows the mass down and stops it from exceeding the speed of light. There seems to be no experimental way to tell the difference between this case and the case where the mass grows. Considering that Section 2 proves that the Special Relativity Theory is wrong and Section 4 demonstrates that there is no reason to require Lorentz invariance from the equation of motion, the natural conclusion is that the mass does not grow and Einstein's proof of $E = mc^2$ is incorrect.

The equation $E = mc^2$ is not any deep result and it has nothing to do with the mass growing when the velocity is increasing. This equation was first published by Olinto de Pretto and should be called De Pretto's equation. In a discrete model it is trivial to derive it. Consider mass m being originally at rest and then speeded in the time Δt to the velocity c. Thus, the velocity difference is $\Delta v = c$ and the acceleration is $a = \frac{\Delta v}{\Delta t} = c(\frac{1}{\Delta t})$. The force needed for giving the mass m this acceleration is $F = ma = mc(\frac{1}{\Delta t})$. In the time Δt the object moves a distance Δs. The force acts for this distance Δs, thus the work is $W = F\Delta s$ and we get

$$W = F\Delta s = mc\frac{\Delta s}{\Delta t}. \tag{6.7}$$

The term $\frac{\Delta s}{\Delta t}$ is a velocity. In a continuous space-time this velocity would be the average velocity where the velocity increases linearly from zero to c. Thus, we would get the usual formula for the kinetic energy for mass m moving with speed c

$$W = mc\frac{\Delta s}{\Delta t} = mc\frac{1}{2}c = \frac{1}{2}mc^2. \tag{6.8}$$

In a discrete model this is different. The mass accelerates in one discrete time unit Δt and the space unit is $\Delta s = c\Delta t$. The velocity in a space unit can be either zero or c and nothing between. Then we do get

$$E = W = mc\frac{\Delta s}{\Delta t} = mc^2. \tag{6.9}$$

Ups, do I here confuse energy with work? It does not matter here. This is simply a trivial calculation, not a proof of anything,

Speeding a mass to a velocity c in (6.9) must be understood in the sense that what gets this velocity c must be massless. The baryon number must be conserved in any nuclear reaction where mass changes to energy. If the sum of mass before and after the reaction does not match, then the missing mass has turned to energy.

A discrete space-time model also explains why the maximum speed is c: it is the lattice speed of the space. A discrete model with a lattice speed is usually discarded as such a model is not Lorentz invariant. Sections 2 and 4 show that the Lorentz transform is incorrect and there is no reason to demand a model to be Lorentz invariant. The correct demand is that the geometry is conformal. Indeed, if we replace the Minkowski space with a 4-dimensional Euclidean space, the transform of space and time intervals as (2.7) means a conformal transform of $R^4$to R^4 and such a mapping defines a conformal geometry to the target space. Fortunately, demanding that an equation is Lorentz invariant often implies that it is invariant under conformal mappings and there should be little need to make changes to existing gauge field theories because of dropping the Lorentz invariance.

7. Conclusions

Einstein understood that the Michelson-Morley experiment, which actually only tested if the Ether Hypothesis is correct, showed that the speed of light is constant in every frame of reference moving with a constant speed. The Michelson-Morley experiment has a flaw as shown in Section 3, but the result what they found is correct - in the experimental setting they should measure the same speed of light in all directions. Yet, this result does not imply that the speed of light is constant in every frame of reference moving with a constant speed. An alternative explanation is that light travels along the geodesics of the gravitational field. As the gravitational field in the experiment is mainly caused by the Earth, Michelson and Morley do not have the different directions where light might have different speed: the gravitational field of the Earth is spherically symmetric. But I looked at this explanation long after writing this article. This alternative is also false: light does not follow a geodesic of the gravitational field, but I am still in the belief that the speed of light in vacuum is indeed locally constant.

Because of misunderstanding what the Michelson-Morley experiment shows (or did not show as it had a flaw), Einstein decided to require Lorentz invariance. The problem is that the time that the Lorentz transform gives to the moving frame is not a valid time. This is because the Lorentz transform is nothing but explanation of the Doppler effect with a new set of coordinates: as the Doppler effect is different to backward and forward directions, Einstein got a time for the moving frame that is different to backward and forward directions. He did notice the problem and defined the proper time, which is a valid time, but he forgot to mention that if the time is the proper time, then the theory is not Lorentz invariant.

Einstein never gave a mathematically fully satisfactory proof of De Pretto's theorem $\mathrm{E{=}mc^2}$, see the discussion in [3]. An accepted proof of the theorem was given by Max von Laue in 1911 [4], but von Laue's theorem deals with Lorentz invariant tensors and from that we can conclude that the proof is not valid: any Lorentz invariant theory necessarily has a time in the moving frame that cannot be a valid time. The field equations in the General Relativity Theory (GRT) are Lorentz invariant. Therefore they do not give a valid time and especially they do not give Einstein's proper time. This alone implies that GRT is flawed and von Laue's proof applies to field equations that cannot describe gravitation in our world. Especially von Laue's theorem does not explain why nuclear reactions release energy according to the formula $E = mc^2$, as this energy is released in out real world and not the world of tensor equations having an invalid time and for that reason being impossible in our world.

There is another serious flaw in The General Relativity Theory and the reason for this flaw is also that GRT is Lorentz invariant: GRT does not have constant speed of light in a gravitation field. It is easy to see why this is so: the infinitesimal line element in GRT satisfies $\mathrm{ds}^2 = g_{\mathrm{ab}}(x)\mathrm{dx}^a\mathrm{dx}^b$. The square of the speed of light in the direction of the spatial dimension x^i, $i = 1, 2, 3$, is $|g_{\mathrm{ii}}(x)\mathrm{dx}^i \div g_{00}(x)\mathrm{dx}^0|$. The infinitesimals $(\mathrm{dx}^0, \mathrm{dx}^1, \mathrm{dx}^2, \mathrm{dx}^3)$ have the same absolute values as the infinitesimals $(\mathrm{dt}, \mathrm{dx}, \mathrm{dy}, \mathrm{dz})$ of an Euclidian 4-space, thus $|\mathrm{dx}^i \div \mathrm{dx}^0| = 1$ for every $i = 1, 2, 3$. If the speed of light is constant c, then $|g_{\mathrm{ii}}(x)| = c^2\,|g_{00}(x)|$. The element ab of the metric tensor $g_{\mathrm{ab}}(x)$ can be a complex function, but because ds^2 is a real number as the square of the length of the line element and dx^i is a real number, $g_{\mathrm{aa}}(x)$ is a real function and the eigenvalue is simply $|g_{\mathrm{aa}}(x)| = \pm g_{\mathrm{aa}}(x)$. The sign is determined by η_{ab}. We get the result $g_{\mathrm{ab}} = \eta_{\mathrm{ab}}\varphi^2$, $\alpha\beta \neq 00$, $g_{00} = \eta_{00}\frac{\varphi^2}{c^2}$ for some scalar gravitational field $\varphi(x)$. The result is that only a scalar gravitational field has a constant speed of light. As the General Relativity Theory satisfies the equivalence principle, also accelerating frames do not have constant speed of light. However, a scalar gravitation theory is not Lorentz invariant: it has a proper time, time intervals between a fixed and moving frame of reference transform as $T^{'} = \gamma_1 T$ for some γ_1.

We see that the Lorentz invariance is the reason for both problems: that the Special and General Relativity Theories do not have a valid time and that the General Relativity Theory does not have a constant speed of light in a gravitational field or in accelerating frames of reference. Einstein did know that there was this problem because he defined the proper time, but he intentionally confused the issue and claimed that the relativity theory is Lorentz invariant. In fact, some parts are, some not, the theory is not consistent.

Einstein did not have any references in his article 1905 of special relativity, therefore a reference to his book [1] should suffice in an article refuting the Special Relativity Theory. Reference [2] is a longer and older unpublished version of the presented article (obtainable by request). In Section 8 paper [2] has calculations of what GRT field equations give for a scalar gravitation field. These calculations refute the General Relativity Theory. [2] also includes a discussion of the experimental "proofs" by which the Special Relativity Theory is "verified". Theoretical proofs of any formula in the Special of General Relativity Theory are typically of two types: either they directly use the Lorentz transform to calculate the time in the moving frame and therefore they do not have a valid time for the moving frame, which is a reason for discarding the proof, or they use the proper time as the time for the moving frame and in that case the speed of light is not constant in the moving frame, which is also a reason to discard the proof. All empirical results that claim to verify the Relativity Theory seem to have a flaw, or they equally well verify some other theory, like Nordström's gravitation theory, as is the case of the test of gravitational refshift. Some empirical papers seem to refute or question the Special or General Relativity Theory. One paper that can be mentioned is [11] by Reginald T. Cahill. It throws some suspicion on Einstein's results. The problem with such empirical results is that they are often disputed and not accepted. Einstein's results can be better refuted by mathematical arguments, like in this presented paper. There is no need for an empirical refutation of a theory with logical errors.

As SRT and GRT have serious errors but something in the relativity theory is true, like the gravitational redshift (cannot find anything else to mention), it once seemed to me that GRT could be replaced by some quantized version of Nordström's gravitation theory, but I do not think so any more. Nordström's gravitation theory is a scalar theory, and it also satisfies the equivalence principle [6]. As a scalar theory it is easily quantized, see [7] how to quantize a scalar field. My unpublished paper [8] has a proposal how to connect this scalar gravitational field to the Higgs field. Something of this type might work and Nordström's gravitation theory can be a starting point, but notice that Nordström's theory is not correct. The whole geometrization idea is wrong.

Many papers discussing Nordström's gravitation theory, like [6], repeat false arguments against this theory. A typical ones are the claims that light does not bend in Nordström's gravitation theory, that it fails the Shapiro delay test and that it does not explain the perihelion of Mercury. All of these claims are false. Light bends in every theory where light follows the geodesics of the space-time geometry, as we can define it to do in Nordström's gravitation theory should we so want. Because GRT does not have a constant speed of light in a gravitation field, the Shapiro delay test fails in GRT, while Nordstr/"om's gravitation theory passes this test, see [9] for my calculations. As for the perihelion of Mercury, see my arguments in [10], GRT is not needed in [10] for explaining the perihelion of Mercury and Einstein knew what the measurement oddity was and could tune his theory to give a suitable correction. In reality the field equations of GRT, if we require that the speed of light is constant, do not yield any solutions that are close to Newton's gravitation theory and as Mercury circulates close to the Sun, certainly any theory that explains issues in the perihelion of Mercury must give a close approximation to Newtonian gravitation theory, see [2] for the calculations. The Swarzschild solution seems to give an approximation to Newton's gravity force, but the speed of light is not constant in the Swarzschild solution and it must be discarded.

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2.3 Still one more time: the error in the Lorentz Transform

Abstract: this short calculation shows that the error in the Lorentz Transform is in taking projections from non-orthogonal coordinates that this transform gives. The result is that the Lorentz Transform does not make the speed of light in vacuum constant in each inertial frame.

1. Introduction

The Lorentz Transform in two dimensions is

$t' = \gamma(t - (\frac{v}{c^2})x)$, where $\gamma = (1 - \frac{v^2}{c^2})^{\frac{-1}{2}}$.

Let us take two points $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (L+\mathrm{vT}, T)$. The coordinates x and t are orthogonal, thus the projection to the x-axis is simply the first number x in the pair (x, t), and the projection to the t-axis is simply the second number t. Therefore the difference of the two points in the projection to the x-axis is $L + \mathrm{vT} = x_2 - x_1$ and difference of the two points in the projection to the t-axis is $T = t_2 - t_1$. The coordinates x' and t' obtained from the Lorentz Transform are not orthogonal and this causes the error in the Lorentz Transform.

2. The correct projections

Let us set x'=0 as a vertical line. Thus, the line $t = v^{-1}x$ is shown as a vertical line from the origin, it is the t'-axis. The line when t'=0 is $t = (\frac{v}{c^2})x$. It is the x'-axis and shown as a horizontal line in the (x',t')-plane.

In Figure 1 are shown in (x,t)-plane the lines $t = v^{-1}x$, $t = (\frac{v}{c^2})x$ and the line of light sent to the positive x-axis from the origin: $t = c^{-1}x$, and the line $t = v^{-1}(x - L)$ of the receiver of light starting at the position $(L, 0)$ and ending to the position $(L + \mathrm{vT}, T)$ at the time when light sent from the origin arrives to the receiver. As $L + \mathrm{vT} = \mathrm{cT}$ we get $L = (c - v)T$.

Additionally Figure 1 shows point P1 where the preimage of t'-axis, i.e., the line$t = v^{-1}x$, intersects with a line parallel to the preimage of the x'-axis, i.e., the line $t = (\frac{v}{c^2})x$, going through $(L + \mathrm{vT}, T)$. The line parallel to the line $t = (\frac{v}{c^2})x$ and going through $(L+\mathrm{vT}, T)$ is $t = (\frac{v}{c^2})x + \gamma^2 T - (\frac{v}{c^2})L$. Intersecting it with $t = v^{-1}x$ gives the point P1 as $(\frac{\mathrm{cvT}}{(}c - v), \frac{\mathrm{cT}}{(}c - v))$. Its image in the (x',t')-plane is P1' which is $(0, \gamma(1 - \frac{v}{c})T)$.

Figure 1 still shows one point, P2, which is the intersection of a line parallel to the preimage of the x'-axis, i.e., the line $t = (\frac{v}{c^2})x$, going through the point $(L, 0)$ and the the preimage of t'-axis, i.e., the line$t = v^{-1}x$. The line parallel to the line $t = (\frac{v}{c^2})x$ going through $(L, 0)$ is $t = (\frac{v}{c^2})(x - L)$. Intersecting it with $t = v^{-1}x$ gives P2 as $(-(\frac{v^2}{c^2})\gamma^2 L, -(\frac{v}{c^2})\gamma^2 L)$ and its image is the point P2' which is $(0, -\gamma(\frac{v}{c^2})L)$. Figure 2 displays the points P1'and P2'in the (x',t')-plane.

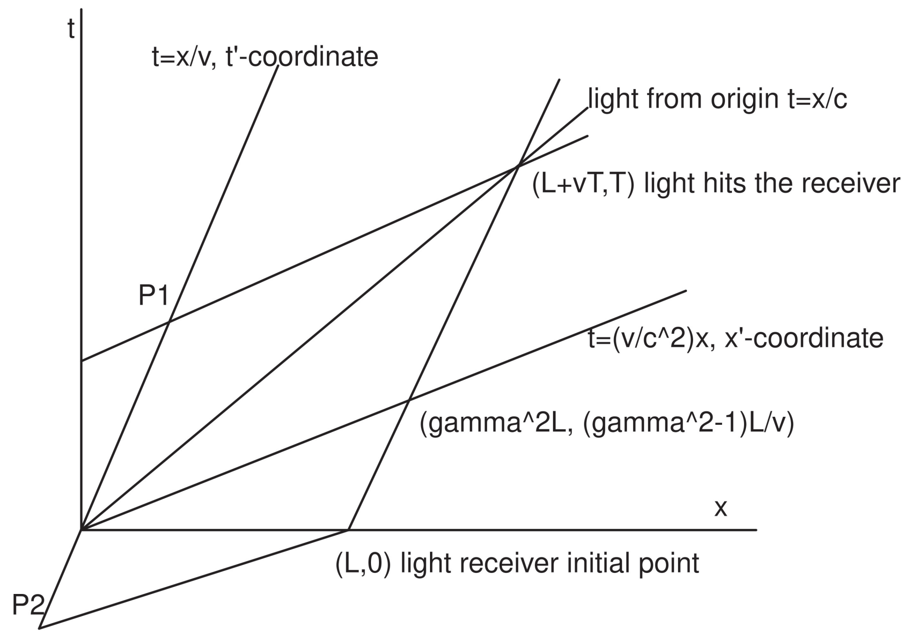

Figure 1. The points and lines shown in the (x,t)-plane.

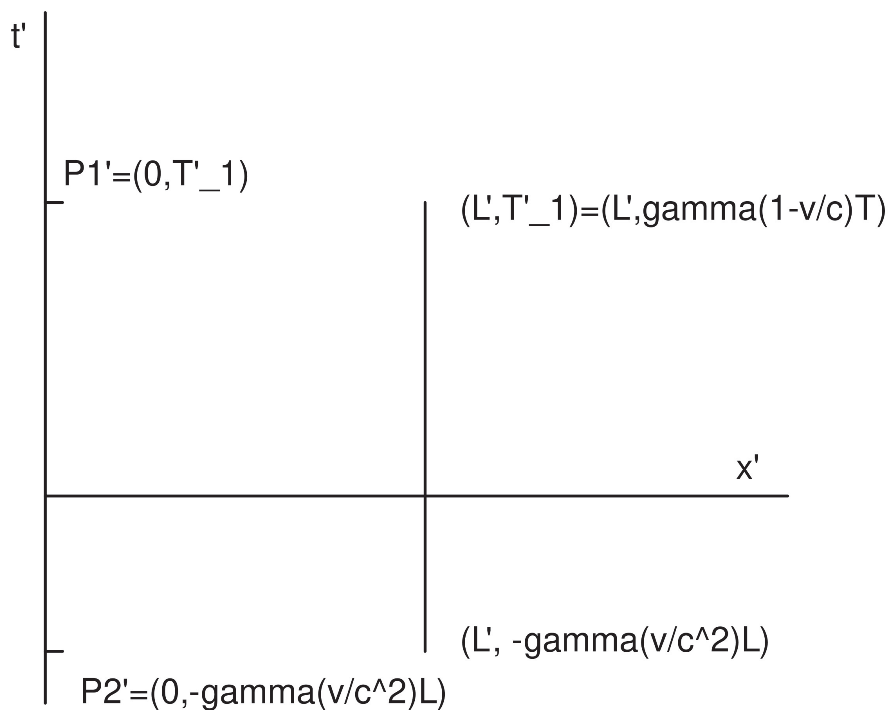

Figure 2. The points P1'and P2' shown in the (x',t')-plane. The time light

travels is the time from P2' to P1'.

In Figure 2 we have denoted $T_1' = \gamma(1 - \frac{v}{c})T = \sqrt{(c - v\frac{)}{(}c + v)}T$. This is the time we get from inserting the equation of light $t = \frac{x}{c}$ to the Lorentz Transform

$T_1' = t' = \gamma(t - (\frac{v}{c^2})x) = \gamma(T - (\frac{v}{c^2})\text{cT})$.

It is incorrect to think that this is the time light travels in the (x',t')-coordinates. Light starts in (x.t)-coordinates at the time $t = 0$. We must measure the time when light comes to the receiver in the (x',t')-coordinates by using a clock that is stationary at $L' = \gamma L$. This clock starts at the time $-\gamma(\frac{v}{c^2})L'$,m which is the time when light starts from the origin.

We notice from Figure 1 that the line from (0,0) to P1 has the same length and direction as the line from $(\gamma^2 L, (\gamma^2 - 1)\frac{L}{v})$ to $(L + \text{vT}, T)$. Both have the x=difference $\frac{\text{Tcv}}{(}c + v)$ and the t-difference $\frac{\text{Tc}}{(}c + v)$. Thus, the time value $T_1' = \gamma(1 - \frac{v}{c})T$ in the time coordinate of P1' is not the time light travels. It is only the time light travels from the image of $(\gamma^2 L, (\gamma^2 - 1)\frac{L}{v})$ to the image of $(L + \text{vT}, T)$. Light starts at the time 0 in the (x.t)-plane and the traveling time is from 0 to T along the line from $(L, 0)$ to $(L + \text{vT}, T)$. This is equal in length to the trip from P2 to P1 and the time T' that light travels in (x',t')-coordinates is the time difference between the points P2' and P1'. Thus,

$T' = \gamma(1 - \frac{v}{c})T + \gamma(\frac{v}{c^2})L = \gamma(1 - \frac{v}{c})T + \gamma(\frac{v}{c^2})(c - v)T = \gamma^{-1}T$.

Thus, the speed of light in R' to the positive x'-axis is

$c' = L\text{'}/T' = \frac{\gamma L}{(}\gamma^{-1}T) = \gamma^2\frac{L}{T} = \gamma^2(c - v)$.

The time $T'$is exactly the same as what we get by measuring the time in (x',t')-plane with a clock fixed at the origin of (x', t'), i.e., having the equation $x = \text{vt}$. Then

$T' = t' = \gamma(T - (\frac{v}{c^2})\text{vT}) = \gamma(1 - \frac{v^2}{c^2})T = \gamma^{-1}T$.

This is natural, we should be able to measure the speed of light by using one fixed clock and synchronizing the receiver's clock to the fixed clock.

The error Einstein makes is that he thinks that the projection on the t'-axis is $T' = t_2' - t_1'$. If this were the case, then we get

$c' = L\text{'}/T_1' = \frac{\gamma L}{(}\gamma(1 - \frac{v}{c})T) = (\frac{L}{T})(1 - \frac{v}{c})^{-1} = (c - v)(\frac{c}{(}c - v)) = c$.

However, this is wrong.

PART 3. THE FIELD EQUATION

Among other issues this part shows that the Einstein Equations, the field equation of GRT, does not allow any solutions that approximate Newtonian gravity in the simplest case of a point mass in empty space and have locally constant speed of light. This is fatal.

3.1 On the field equation in gravitation

Abstract:

The first section proves that Einstein's field equation in the General Relativity Theory is impossible because it does not give any spherically symmetric solutions in the situation of a point mass in an empty space. The second section explains why the field equation should not reduce to the Ricci scalar curvature as for a scalar field it leads to the the D'Alembert operator which has a wrong time dependency and an incorrect understanding of what the Laplace operator field equation really is.

1. The Einstein equations are incorrect

The Einstein equations are

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa_0 T_{ab} + \lambda g_{ab}. \tag{1}$$

Lemma 1. If a field gives the speed of light in vacuum as c at every point to every direction, then the field is a scalar field.

Proof Light travels along light-like world paths in the General Relativity Theory. They have $ds = 0$ in the space element

$$ds^2 = c^2 g_{00} dx_0^2 - g_{11} dx_1^2 - g_{22} dx_2^2 - g_{33} dx_3^2 \tag{2}$$

The speed of light to the direction of x_i is obtained by setting $dx_j = 0$, $j \neq i$, $j \in \{1, 2, 3\}$. Thus

$$ds^2 = 0 = c^2 g_{00} dx_0^2 - g_{ii} dx_i^2 \tag{3}$$

$$c^2 = \frac{g_{ii}}{g_{00}} \frac{dx_i^2}{dx_0^2} = \frac{g_{ii}}{g_{00}} \tag{3}$$

as the differentials dx_i are Euclidean. We get

$$g_{ii} = c^2 g_{00} \tag{4}$$

and we can define $\psi = c^{-1}\sqrt{g_{00}}$. This means that the field ψ is a scalar field. $\square$

Theorem 1. The Einstein equations do not have any scalar field solutions for a scalar field in the situation of a point mass in empty space.

Proof

The situation of a point mass in empty space is spherically symmetric. Therefore the scalar field ψ must be spherically symmetric. Let the field be $\psi(r, t)$

The nonzero elements of the metric tensor are $g_{00} = c^2\psi^2$, $g_{11} = -\psi^2$, $g_{22} = -r^2\psi^2$, $g_{33} = -r^2 \sin^2(\theta)\psi^2$.

The nonzero Ricci tensor entries for this scalar field in spherical coordinates (r, θ, ϕ) are:

$$R_{00} = \psi^{-1}\frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\psi^{-1}\frac{\partial\psi}{\partial r} + \psi^{-2}\left(\frac{\partial\psi}{\partial r}\right)^2 - \frac{3}{c^2}\psi^{-1}\frac{\partial^2\psi}{\partial t^2} + \frac{3}{c^2}\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2 \quad (5)$$

$$R_{11} = -\left(3\psi^{-1}\frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\psi^{-1}\frac{\partial\psi}{\partial r} - 3\psi^{-2}\left(\frac{\partial\psi}{\partial r}\right)^2 - \frac{1}{c^2}\psi^{-1}\frac{\partial^2\psi}{\partial t^2} - \frac{1}{c^2}\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2\right) \quad (6)$$

$$R_{22} = -r^2\left(\psi^{-1}\frac{\partial^2\psi}{\partial r} + \frac{4}{r}\psi^{-1}\frac{\partial\psi}{\partial r} + \psi^{-2}\left(\frac{\partial\psi}{\partial r}\right)^2 - \frac{1}{c^2}\psi^{-1}\frac{\partial^2\psi}{\partial t^2} - \frac{1}{c^2}\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2\right) \quad (7)$$

$$R_{33} = \sin^2 R_{22} \quad (8)$$

The Ricci scalar is

$$R = g^{ab}R_{ab} = \psi^2\left(6\psi^{-1}\frac{\partial^2\psi}{\partial r} + \frac{12}{r}\psi^{-1}\frac{\partial\psi}{\partial r} - 6\frac{1}{c^2}\psi^{-1}\frac{\partial^2\psi}{\partial t^2}\right)$$

and from (1)

$$-R = g^{ab}\left(R_{ab} - \frac{1}{2}Rg_{ab}\right) = g^{ab}\left(\kappa_0 T_{ab} + \lambda g_{ab}\right) \quad (9)$$

In the empty space outside the point mass $T_{ab} = 0$. The cosmological constant λ must be zero in the empty space outside the point mass because $\lambda g_{00} = \lambda\psi^2$ cannot match terms R_{00} and Rg_{00} that are of the form (5) and (9). Therefore the Einstein equations in this case require that R in (9) is zero and each R_{ii} for $i = 0, 1, 2, 3$ is zero. We set $c = 1$ for simplier notations.

First we eliminate the time derivatives by calculating the equation

$$R_{11} - \frac{R_{22}}{r^2} = 0 \quad (10)$$

It gives

$$\psi^{-1}\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\psi^{-1}\frac{\partial\psi}{\partial r} - 2\psi^{-2}\left(\frac{\partial\psi}{\partial r}\right) = 0 \quad (11)$$

Let

$$\psi' = \frac{\partial\psi}{\partial r} \quad (12)$$

$$y = \psi'\psi^{-2} \quad (13)$$

then

$$y' - \frac{1}{r}y = 0 \tag{14}$$

$$y = 2c_1 r = \psi'\psi^{-2} \tag{15}$$

where c_1 is a constant. Then

$$\psi = (c_1 r^2 + c_2) \tag{16}$$

Including the time depencence we can write

$$\psi = (c_1(t)r^2 + c_2(t)) \tag{17}$$

Next we calculate solve the time derivative from the equation

$$R_{00} + 3R_{11} = 0 \tag{18}$$

It yields

$$6\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2 = 8\psi^{-1}\frac{\partial^2\psi}{\partial r^2} + \frac{4}{r}\psi^{-1}\frac{\partial\psi}{\partial r} - 10\psi^{-2}\left(\frac{\partial\psi}{\partial r}\right)^2 \tag{19}$$

From (17) we get

$$\frac{\partial\psi}{\partial r} = -2c_1 r\psi^2 \tag{20}$$

$$\psi^{-1}\frac{\partial^2\psi}{\partial r^2} = -2c_1\psi + 8c_1^2 r^2\psi^2 \tag{21}$$

Inserting to (19) and simplifying gives

$$\left(\frac{\partial\psi}{\partial t}\right)^2 = -4c_1\psi^3 + 4c_1^2 r^2\psi^4 \tag{22}$$

Calculating

$$\frac{\partial\psi}{\partial t} = -(c_1' r^2 + c_2')\psi^2 \tag{23}$$

where $c_i' = dc_i/dt$, $i = 1, 2$. Then

$$(c_1' r^2 + c_2')^2 = -4c_1\psi^{-1} + 4c_1^2 r^2 = -4c_1(c_1 r^2 + c_2) + 4c_1^2 r^2 \tag{24}$$

Matching the coefficients of r^4, r^2 and r^0 in (24) gives the equations

$$(c')^2 = 0 \qquad 2c_1' c_2' = 0 \qquad (c_2')^2 = -4c_1 c_2 \tag{25}$$

Thus, $c_1(t) = c_1$ is a constant and

$$c_2' = \pm 2\sqrt{-c_1}\sqrt{c_2} \tag{26}$$

$$2\sqrt{c_2} = \pm 2\sqrt{-c_1}(t + c_3) \tag{27}$$

where c_3 is a constant. Then

$$c_2 = -c_1(t + c_3)^2 \tag{28}$$

We get as the only possible result from (10) and (18)

$$\psi = (c_1 r^2 - c_1(t + c_3)^2) \tag{29}$$

Lastly, we check if the solution satisfies $R_{00} = 0$. Inserting (20), (21) and

$$\frac{\partial \psi}{\partial t} = 2c(t + c_3)\psi^2 \tag{30}$$

$$\psi^{-1}\frac{\partial^2 \psi}{\partial t^2} = 2c_1 + 8c_1^2(t + c_3)^2\psi^2 \tag{31}$$

to (5) and simplifying gives

$$R_{00} = -12c_1\psi + 12c_1^2(r^2 - (t + c_3)^2)\psi^2 \neq 0 \tag{32}$$

Thus, the only solution from (10) and (18) does not satisfy Einstein's equations. $\square$

Another proof that the Einstein equations (1) fail in this situation is given in [2]. In the proof in 2] the scalar field ψ is allowed to depend on all variables (r, θ, ϕ, t). In [3] there is a proof that even if the field is not a scalar field but only spherically symmetric and does not depend on time, then also there are no solutions to the Einstein equations in the case of a point mass in empty space. In the proof in [3] the speed of light is allowed to be different in the direction of the gradient of the gravitational field but equal to c in vacuum in the directions orthogonal to the gradient of the gravitational field. That is certainly the minimum that can be demanded of a solution as we know that the speed of light is constant to all directions on the horizontal plane on the Earth. We conclude that the Einstein equations are incorrect. The derivation Einstein gave to (1) is not any valid derivation: he deduced it heuristically from quite questionable principles.

2. Where does the field equation come from?

We start from the conclusion that the field must be a scalar field so that the speed of light is constant. We can sum (1) setting $\lambda = 0$ as this term is of a wrong form. Then

$$g^{ab}R_{ab} - g^{ab}\frac{1}{2}Rg_{ab} = \kappa_0 g^{ab}T_{ab} \tag{33}$$

$$R - 2R = \kappa_0 T \tag{34}$$

$$R = -\kappa_0 T = -\frac{8\pi G}{c^4}T \tag{35}$$

where $T = g^{ab}T_{ab}$. For all scalar fields ψ holds

$$R = -6\psi^{-3}\Box\psi. \tag{36}$$

where the D'Alembert operator is

$$\Box\psi = \Delta\psi - c^{-2}\frac{\partial^2\psi}{\partial t^2} \tag{37}$$

Inserting (36) to (35) gives

$$\psi^{-1}\Box\psi = -4\pi G\frac{1}{3c^4}T\psi^2. \tag{38}$$

This equation also does not work. In the case of a point mass in empty space, $T = 0$, then we need a solution to

$$\Box\psi = 0 \tag{39}$$

There are some solutions, like

$$\psi(r,t) = -\frac{\rho_0}{r^2 - c^2t^2} \tag{40}$$

and

$$\psi(x,y,z,t) = \psi(x + y + z - \sqrt{3}ct) \tag{41}$$

but the solution should be close to the Newtonian potential.

The (correct) classical formula for a point mass M at the origin in empty space is

$$\Delta\psi = 0 \tag{42}$$

outside the origin. Δ is the Laplace operator.

Let us derive this formula. Let there be a mass M at the origin and a small test mass m_1 at the distance r from the origin in the Euclidean 3-space. The mass M creates a spherically symmetric time-independent gravitational field $\psi = \psi(r)$. The gradient points towards the origin. Writing the force $\bar{F} = F\bar{e}_r$, we have

$$F(r) = -m_1\frac{\partial\psi(r)}{\partial r} \tag{43}$$

where the negative sign is because $\bar{e}_r$ points away from the origin. The force spreads to all directions, but the force lines remain. Thus

$$\frac{(A(r+dr)F(r+dr) - A(r)F(r)}{dr} = 0 \tag{44}$$

where $A(r) = 4\pi r^2$ is the area of a 2-sphere of the radius r. Therefore we get

$$\frac{1}{A(r)}\frac{d}{dr}A(r)F(r) = 0 \tag{45}$$

which can be written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\left(-\frac{m_1\partial\psi}{\partial r}\right) = 0$$

thus

$$\Delta\psi(r) = 0 \tag{46}$$

outside the origin.

One often sees in the literature the formula:

$$\Delta\psi = -4\pi G\rho \tag{47}$$

where ρ is called mass density. This formula is correct, but it is a residue formula. It does not actually mean that ρ is mass density in the sense that some mass m is distributed with the density $\rho(r)$ to the area $r \in [0, R]$ and there are no point masses. Let us see what we get if $\rho(r)$ is radially symmetric mass density and there are no point masses creating singularities of the field ψ.

We place a small test mass m_1 to the point $z = h$ in the (x, y, z) coordinates. Let the total mass m is

$$m = \int_0^R 4\pi r^2 \rho(r) dr \tag{48}$$

for some R which we may at the end extend to infinity. The angle ϕ between the x and z axes ranges from $-\pi/2$ to $+\pi/2$. All points on a circle of radius r on the (x, y)-plane at the height $z = r \sin\phi$ have the same distance s to m_1. This distance s satisfies

$$s^2 = (r\cos\phi)^2 + (h - r\sin\phi)^2 \tag{49}$$

We take such circles as small masses of the size

$$dm(\phi) = 2\pi r \cos(\phi)\ r d\phi\ \rho(r) dr. \tag{50}$$

The gravitation force

$$dF(\phi) = Gm_1 \frac{2\pi r^2 \cos(\phi)}{s^2} d\phi\ \rho(r) dr \tag{51}$$

created by the mass $dm(\phi)$ is in the direction of $\bar{s}$ and from the test mass m_1 towards the small mass $dm(\phi)$. The force is not along the z-axis and we need to take a projection on the z-axis

$$dF_z(\phi) = \frac{h - r\sin(\phi)}{s} Gm_1 \frac{2\pi r^2 \cos(\phi)}{s^2} d\phi\ \rho(r) dr \tag{52}$$

This force we can integrate over the angle ϕ. The equation is

$$dF(h) = 2\pi G m_1 r^2 \rho(r) dr I \tag{52}$$

where

$$I = \int_{-\pi/2}^{\pi/2} \frac{(h - r\sin(\phi))\cos(\phi)}{s^3} d\phi \tag{53}$$

is easily integrated by the change of the variable to $x = \sin\phi$.

$$I = \int_{-1}^{1} a \frac{\frac{h}{r} - x}{(b-x)^{\frac{3}{2}}} dx \tag{54}$$

where

$$a = h^{-1}(2hr)^{-\frac{1}{2}} \tag{55}$$

$$b = \frac{1}{2}\frac{h}{r}\left(1 + \frac{r^2}{h^2}\right). \tag{56}$$

As

$$\int_{-1}^{1} \frac{a(\frac{h}{r} - x)dx}{(b-x)^{\frac{3}{2}}} = 2a(b-x)^{\frac{1}{2}}\left(\left(b - \frac{h}{r}\right)(b-x)^{-1} + 1\right) \tag{55}$$

we get

$$\begin{aligned} I = {} & 2a\left(\frac{h}{r} - b\right)\left((b-1)^{-\frac{1}{2}} - (b+1)^{-\frac{1}{2}}\right) \\ & -2a\left((b-1)^{\frac{1}{2}} - (b+1)^{\frac{1}{2}}\right). \end{aligned} \tag{56}$$

Inserting

$$\frac{h}{r} - b = \frac{h}{2r}\left(1 - \frac{r^2}{h^2}\right) \tag{57}$$

$$b \pm 1 = \frac{h}{2r}\left(1 \pm \frac{r}{h}\right)^2 \tag{57}$$

we get the final result

$$I = \frac{2}{h^2}. \tag{58}$$

Integrating the force dF_z over r gives the force $F(h)$ on the mass m_1

$$F(h) = \int dF_z = -4\pi G \frac{m_1}{h^2} \int_0^R \rho(r) r^2 dr. \tag{59}$$

Let $\rho(r)$ be a constant ρ. The force is Newtonian

$$F(h) = -4\pi G \frac{m_1}{h^2} \rho \frac{R^3}{3} = -G\frac{m_1 m}{h^2}. \tag{60}$$

The sign is negative because $\bar{F}(h) = F(h)\bar{e}_r$, which is also the reason why the force is the negative of the gradient of the field

$$F(h) = -\frac{\partial \psi(h)}{\partial h}. \tag{61}$$

The Laplace operator gives zero outside singularities

$$\Delta\psi(h) = \frac{1}{h^2}\frac{\partial}{\partial h} h^2 \frac{\partial}{\partial h}\psi(h)$$

$$= \frac{1}{h^2}\frac{\partial}{\partial h}h^2(-F(h)) = 0. \tag{62}$$

We see that outside point masses, which are singularities in Newton's gravitation theory, the Laplace operator vanishes. There is no requirement in the calculation above that R should be smaller than h or even finite. We can write

$$F(h) = \int dF_z = -4\pi G\frac{m_1}{h^2}\int_0^\infty \rho(r)r^2 dr. \tag{63}$$

Everything happens at one time t and no time derivatives are taken. We can simply add the time parameter and get

$$F(h,t) = \int dF_z = -4\pi G\frac{m_1}{h^2}\int_0^\infty \rho(r,t)r^2 dr. \tag{64}$$

The Laplace equation outside point masses is still

$$\Delta\psi(h,t) = 0. \tag{65}$$

Naturally, if we want to add a delay to the time, we can use a time difference equation inserting instead of $\rho(r,t)$ $\rho(r,t-s/c)$ with a suitable s, but differential-difference equations are difficult to work with.

We can express Δ in Cartesian coordinates

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x,y,z,t) = 0 \tag{66}$$

for a radially symmetric ψ. The form of the expression of (66) should logically be the same for any ψ. If so, then outside point masses we should get

$$\Delta\psi(\bar{h},t) = 0. \tag{67}$$

According to literature (67) is correct. We will prove it with point masses in (68)-(74). The solution of (67) is not radially symmetric and especially not $\psi \sim r^{-1}$ in the general case as Δ has other parts than the radial part.

The ρ in (47) is a convention that counts the residues of singularities in point masses in a volume bounded by a closed surface. The value of the residue is calculated as in the Gauss theorem for electromagnetism: a point mass at the origin in empty space gives the residue

$$\Delta\psi(r) = -4\pi GM\delta(\bar{r}) \tag{68}$$

where $\delta(\bar{r})$ is the Dirach delta. For any linear transform $\bar{r}' = \bar{r} - \bar{s}$ where $\bar{s}$ is some constant 3-vector holds

$$\Delta_{x,y,z} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} = \Delta_{x',y',z'} \tag{69}$$

Thus, changing from spherical coordinates to Cartesian, making the linear transform and chanring back we have

$$\Delta_{r,\theta,\phi} = \Delta_{x,y,z} = \Delta_{x',y',z'} \Delta_{r',\theta',\phi'} \tag{70}$$

For any constant vector $\bar{s}_i$ and new variable $\bar{r}' = \bar{r} - \bar{s}_i$ holds

$$\Delta_{r,\theta,\phi} \psi_i(\bar{r} - \bar{s}_i) = \Delta_{r',\theta',\phi'} \psi_i(\bar{r}'). \tag{71}$$

We define the potential as

$$\psi(\bar{r}) = \sum_i \psi_i(\bar{r} - \bar{s}_i). \tag{72}$$

Then

$$\Delta_{r,\theta,\phi} \psi_i(\bar{r}) = \sum_i \Delta_{r'_i,\theta'_i,\phi'_i} \psi_i(\bar{r}'_i) = -4\pi G \sum_i M_i \delta(\bar{r}'_i) \tag{73}$$

which is written as

$$\Delta\psi(\bar{}) = -4\pi G \rho(\bar{r}) \tag{74}$$

where

$$\rho(\bar{r}) = \sum_i M_i \delta(\bar{r} - \bar{s}_i). \tag{75}$$

Equation (74) is (47) and we see from (75) that ρ is not any continuous function.

Notice what this means to the equation (38) that we rewrite as

$$\Box\psi = -4\pi G \frac{1}{3c^4} T \psi^3. \tag{76}$$

In a time independent situation the equation is

$$\Delta\psi = -4\pi G \frac{1}{3c^4} T \psi^3. \tag{77}$$

If this equation holds and the force is r^{-2}, then there must be singularities. In the fifth chapter of Einstein's book [1] he presents Friedman's results where the mass density is assumed to be constant. The results given in [1] reveal that by constant density Einstein means that the values of $T_{ab}\psi^2$ are nonzero constants. Clearly Einstein and Friedman do not mean that the right side of (76) is nonzero because of singularities: the situation is as in the calculation (48)-(62) and Einstein should get zero as the Ricci scalar. Einstein gets something different meaning that his force is not r^{-2}. The tensor T_{ab} should describe the distribution and movement of matter, but if it is a continuous function, then it does not describe matter at all. It changes the curvature of the space from flat to non-flat. There seems to be some major understanding difference or misunderstanding in Einstein's theory of why the right side of (38) should have the matter term T.

Einstein's theory does create singularities, like a closed surface singularity in the Schwarzschild solution, but the residues in (47) and (75) are from point singularities. In [1] Einstein writes that he would like to remove singularities from his theory. If so, then (76) can give nonzero right side only if the force is not r^{-2}. Let us notice that if the force is not r^{-2}, then one should not use a spherical Laplace or D'Alembert operator. The coordinates should be for non-flat geometry, but equations (44)-(45) should still hold as force lines spread over the whole area of a 2-sphere. In suitable coordinates $\Delta\psi = 0$ should hold if there are no singularities. The matter term should come from residues of singularities. The singularites are sinks and sources of some flow. Einstein wanted to explain matter as surved space, but matter is not the gravitational field. Matter is singularities and something flows in and out.

The time dependency of $\Box$ in (76) is not motivated. It does not give solutions that are close to Newton's gravitation field. In (64) and (65) there is no time dependency, everything is calculated at the same time moment t and we can simply add the time variable to the density ρ that describes how mass bodies move. It is a question what the time dependency of $\Box$ is intended to model.

Finally let us notice that the Laplace operator assumes that unit 2-spheres have the area $4\pi r^2$. This is why there is division by r^2 and multiplication by r^2 in the radial part of Δ. This assumption means that the space geometry is flat. The geometry of the gravitational field is not flat in the Newtonian gravitation theory: close to a point charge the infinitesimal spheres of the gravitational geometry have the radius proportional to ψ, yet it is not expected that the force lines follow the gravitational geometry. The condition for the force lines in equation (44) is that the area of a sphere grows as r^2. Using $\Box$ means that one is assuming flat geometry and force lines spread in that geometry. For a scalar field the Ricci scalar reduces to $\Box$ by the equation (36). It seems that there is a flat space geometry in (1) in addition to the geometry of the gravitational field. We may ask if light follows the geodesics of the flat space geometry or geodesics of the gravitational field. Einstein claimed that it is the second alternative, but more likely it is the first alternative.

3. References

[1] A. Einstein, The Meaning of Relativity, Princeton University Press, last edition 1955.

[2] J. Jormakka, The Essential Questions in Relativity Theory, preprint, ResearchGate, 2023.

[3] J. Jormakka, Failure of the geometrization principle and some cosmological considerations, preprint, ResearchGate, 2023.

3.2 On the field equation in gravitation, part 2

1. The metric in can only be induced by a scalar field

The metric in the Relativity Theory is derived from Riemannian metric. Riemannian metric is induced by an inner product: the norm in local orthogonal coordinates X_i, $i = 1, \dots, n$ is defined as

$$R = \left(\sum_{i=1}^{n} x_i^2 \right)^{\frac{1}{2}} \tag{1}$$

thus the line element R satisfies

$$dR^2 = \sum_{i=1}^{n} dX_i^2 \tag{2}$$

where n is the dimension of the space. The local orthogonal coordinates can be defined by

$$dX_i = f_i(x)dx_i \tag{3}$$

where dx_i are the Cartesian coordinates of the Euclidean space R^n. The local coordinates can always be orthogonalized, therefore we can assume that the local coordinates are chosen as orthogonal Cartesian coordinates. When the local coordinates are orthogonal, there are no cross terms $dX_i dX_j$ in (2) because their inner product vanishes.

Let $n = 3$. We can change the Cartesian coordinates x_i, $i = 1, 2, 3$, into spherical coordinates

$$x = r\cos(\phi)\sin(\theta) \tag{4}$$

$$y = r\sin(\phi)\sin(\theta)$$

$$z = r\cos(\theta).$$

Then

$$r^2 = x^2 + y^2 + z^2 \tag{5}$$

$$dr = \frac{x}{r}dx + \frac{y}{r}dy + \frac{z}{r}dz \tag{6}$$

$$dr^2 = \frac{x^2}{r^2}dx^2 + \frac{y^2}{r^2}dy^2 + \frac{z^2}{r^2}dz^2 \tag{7}$$

$$+2\frac{xy}{r^2}dxdy + 2\frac{xz}{r^2}dxdz + 2\frac{yz}{r^2}dydz$$

$$\sin(\theta)d\theta = \frac{z}{r^2}dr - \frac{dz}{r} \tag{8}$$

$$\sin^2(\theta) = 1 - \frac{z^2}{r^2} = \frac{x^2 + y^2}{r^2} \tag{9}$$

$$r^2 d\theta^2 = \frac{z^2x^2}{r^2(x^2+y^2)}dx^2 + \frac{z^2y^2}{r^2(x^2+y^2)}dy^2 + \frac{x^2+y^2}{r^2}dz^2 \tag{10}$$

$$+2\frac{z^2xy}{r^2(x^2+y^2)}dxdy - 2\frac{z^2xz}{r^2}dxdz + 2\frac{z^2yz}{r^2}dydz$$

$$\frac{x}{y} = \cot(\phi) \tag{11}$$

$$\frac{1}{\sin^2(\phi)}d\phi = \frac{x}{y^2}dy - \frac{dx}{y} \tag{12}$$

$$\frac{1}{\sin^2(\phi)} = \frac{x^2+y^2}{y^2} \tag{13}$$

$$d\phi^2 = \frac{y^2}{(x^2+y^2)^2}dx^2 + \frac{x^2}{(x^2+y^2)^2}dy^2 - 2\frac{xy}{(x^2+y^2)^2}dxdy \tag{14}$$

$$r^2\sin^2(\theta)d\phi^2 = \frac{y^2}{x^2+y^2}dx^2 + \frac{x^2}{x^2+y^2}dy^2 - 2\frac{xy}{x^2+y^2}dxdy \tag{15}$$

Inserting (7), (10) and (15) we notice that

$$dr^2 + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2 = dx^2 + dy^2 + dz^2 \tag{16}$$

but if we weight the differentials at the left side of (16) with some functions, then the right side will not have the form as in (16). Especially, if we make the transfrom from spherical coordinates to Cartesian coordinates in the so-called Schwarzschild metric

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2d\theta^2 - r^2\sin^2(\theta)d\phi^2 \tag{17}$$

we get

$$ds^2 = A(r)dt^2 - (B(r)-1)dr^2 - dr^2 - r^2dtheta^2 - r^2\sin^2(\theta)d\phi^2 \tag{18}$$

$$ds^2 = A(r)dt^2 - (B(r)-1)dr^2 - dx^2 - dy^2 - dz^2 \tag{19}$$

$$= A(r)dt^2 - \left((B(r)-1)\frac{x^2}{r^2}+1\right)dx^2$$

$$-\left((B(r)-1)\frac{y^2}{r^2}+1\right)dy^2 - \left((B(r)-1)\frac{z^2}{r^2}+1\right)dz^2$$

$$-2(B(r)-1)\frac{xy}{r^2}dxdy - 2(B(r)-1)\frac{xz}{r^2}dxdz - 2(B(r)-1)\frac{yz}{r^2}dydz. \tag{20}$$

Equation (20) does not define a metric in local Cartesian coordinates. There cannot be cross terms $dxdy, dxdz, dydz$ because the Euclidean differentials dx, dy, dz are orthogonal and the inner product vanishes. The Schwarzschild metric (17) is not a valid metric in the Relativity Theory and it does not give the Minkowski metric as a limit when the local environment becomes infinitely small.

The Minkowski metric is a Riemannian pseudo-metric (because it is not positive definite) with the line element

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2. \tag{21}$$

This metric is flat and dt, dx, dy, dz are Euclidean differentials. We get the speed of light to the direction of the local coordinate X_i, $i = 1, 2, 3$, by setting $dX_j = 0$ for $j \neq i$ and setting $ds = 0$ because light travels on light-like world paths. Writing $dX_0 = dt$, we have

$$0 = ds^2 = c^2 dX_0^2 - dX_i^2 \tag{22}$$

$$\frac{dX_i^2}{dX_0^2} = c^2 \tag{23}$$

In the General Relativity Theory the local coordinates are expressed with Euclidean differentials:

$$dX_i = \sqrt{g_{ii}} dx_i \tag{24}$$

so that the metric in local Cartesian coordinates is in the form

$$ds^2 = g_{00} dx_0^2 - g_{11} dx_1^2 - g_{22} dx_2^2 - g_{33} dx_3^2 \tag{25}$$

The condition that the speed of light is c to the direction of the local coordinate X_i, $i = 1, 2, 3$, at the point x is

$$\frac{g_{ii}(x)}{g_{00}(x)} = c^2. \tag{26}$$

If the local coordinates X_i are orthogonal Cartesian coordinates, there are no cross terms $dxdy, dxdz, dydz$ in the expression (25). If there are such cross terms, the the local coordinates are not orthogonal Cartesian coordinates. We see that this is the case for the Schwarzschild metric.

The functions $g_{ii}(x)$ in (25) give the metric. The differentials $dx_i, i = 0, 1, 2, 3$ are Euclidean and $dx_i^2/dx_0^2 = 1$ for $i = 1, 2, 3$. In order to verify that the differentials are meant to be Euclidean, it is enough just to look at the Schwarzschild metric. There appears r and θ, which are Euclidean coordinates of the spherical coordinate system, and there appears dr, $d\theta$ and $d\phi$. They are Euclidean differentials of the spherical coordinate system. The metric is made by multiplying these Euclidean differentials by the functions $\sqrt{g_{ii}}$.

If g_{ii} are not constants, the geometry is curved, but locally a geometry is always flat. That is, the surface of the Earth is curved, but if we look at a disc with the radius of one kilometer, the surface appears quite flat. If we take an even smaller disc, the surface becomes still flatter. In a limit when the environment becomes infinitely small, the curvature of the space disappears: the tangent space is flat and in the Relativity Theory it must become a Minkowski space. The metric must give the following form line element at any chosen point $x = P$

$$ds^2 = c^2 A(P) dx_0^2 - A(P) dx_1^2 - A(P) dx_2^2 - A(P) dx_3^2 \tag{27}$$

where dx_0, dx_1, dx_2, dx_3 are Cartesian coordinates of the Euclidean space R^4. We can define $\psi(P)$ by the formula

$$\psi(P) = -\sqrt{A(P)} \tag{28}$$

at the point $x = P$. As P is any point, (28) defined $\psi(x)$ at any point. The metric is

$$ds^2 = c^2\psi(x)^2dx_0^2 - \psi(x)^2dx_1^2 - \psi(x)^2dx_2^2 - \psi(x)^2dx_3^2$$

$$ds^2 = c^2\psi(x,y,z,t)^2dt^2 - \psi(x,y,z,t)^2dx^2 - \psi(x,y,z,t)^2dy^2 - \psi(x,y,z,t)^2dz^2. \tag{29}$$

We can express the metric (29) in spherical coordinates as

$$ds^2 = c^2\psi(r,\theta,\psi,t)^2dt^2 - \psi(r,\theta,\psi,t)^2dr^2$$

$$-r^2\psi(r,\theta,\psi,t)^2dy^2 - r^2\sin^2(\theta)\psi(r,\theta,\psi,t)^2dz^2. \tag{30}$$

This is the most general form of a metric in the Relativity Theory.

We can especially look at a spherically symmetric situation. In a spherically symmetric situation, the solution (i.e., the metric) must be spherically symmetric. The most general spherically symmetric metric is:

$$ds^2 = c^2\psi(r,t)^2dt^2 - \psi(r,t)^2dx^2 - \psi(r,t)^2dy^2 - \psi(r,t)^2dz^2. \tag{31}$$

and the most general spherically symmetric metric in spherical coordinates is:

$$ds^2 = c^2\psi(r,t)^2dt^2 - \psi(r,t)^2dr^2 - r^2\psi(r,t)^2dy^2 - r^2\sin^2(\theta)\psi(r,t)^2dz^2. \tag{32}$$

In the situation of a point mass in empty space (or a spherical mass in empty space considered only in the space outside the mass) the Einstein equations

$$R_{ab} - \frac{1}{2}Rg_{ab} = \kappa_0 T_{ab} + \lambda g_{ab}. \tag{33}$$

simplify considerably: in the area where the space is empty holds $T_{ab} = 0$. We must set $\lambda = 0$ because the term λg_{ab} grows as $\psi(x)^2$ while R_{ab} grows as $\psi'\psi^{-1}$ and they cannot match. Multiplying (33) by g^{ab} and summing shows that $R = 0$.

$$g^{ab}R_{ab} - g^{ab}\frac{1}{2}Rg_{ab} = \kappa_0 g^{ab}T_{ab} \tag{34}$$

$$R - 2R = \kappa_0 T \tag{35}$$

$$R = -\kappa_0 T = -\frac{8\pi G}{c^4}T \tag{36}$$

where $T = g^{ab}T_{ab}$. We get three independent equations from (33): $R_{00} = 0$, $R_{11} = 0$, $R_{22} = 0$, but there is only one function $\psi(r,t)$ to solve in (32). It is shown in [4] that there are no solutions to the Einstein equations in this situation. It is also shown in [2] and [3] in different ways.

This means that there can be only one equation. Initially it might appear that the equation can be (36), but as $\psi(x)$ is a scalar function, we can simplify the equation with

$$R = -6\psi^{-3}\Box\psi. \tag{37}$$

where the D'Alembert operator is

$$\Box\psi = \Delta\psi - c^{-2}\frac{\partial^2\psi}{\partial t^2} \tag{38}$$

We get the equation

$$\psi^{-1}\Box\psi = -4\pi G\frac{1}{3c^4}T\psi^2. \tag{39}$$

which is similar to the field equation in Nordströmäs gravitation theory. In empty space (39) gives

$$\Box\psi = 0. \tag{41}$$

Equation (41) has the following radially symmetric time-dependent solution

$$\psi(r,t) = -\frac{\rho_0}{r^2 - c^2t^2} \tag{42}$$

Clearly, $\psi(r,t)$ in (42) is not close to the Newtonian gravitational field. Equation (41) and therefore (36) are wrong. Nothing can be saved fron Einstein's field equations (33). They are completely wrong.

2. Newton's gravitation potential is correct and the space is flat

Let us look at a situation when the space is empty outside the single mass body where the mass body is not a point mass. The mass body is a ball of constant mass density ρ centered at the origin and having the radius R and the mass m.

The situation is spherically symmetric, therefore the field must be radially symmetric. Measurements show that Newton's gravitational force is a good approximation. Let us assume that the force is well approximated in the range of r that interests us by the formula

$$F = G\frac{m_1 m}{r^\alpha} \tag{43}$$

where α is a constant and close to two. We are interested in proving that $\alpha = 2$.

We will assume that the geometry of the space is such that the length of a circle of radius r is well approximated in the range of r that interests us by the formula

$$L = \int_0^{2\pi} r^{\frac{\gamma}{2}}d\phi = 2\pi r^{\frac{\gamma}{2}} \tag{44}$$

where γ is a constant and close to two. Then the area of a sphere of radius r is well approximated by

$$A(r) = 4\pi r^{\gamma} \tag{45}$$

and the volume of a ball of radius r is well approximated by

$$V(r) = \frac{4\pi}{\gamma+1} r^{\gamma+1}. \tag{46}$$

We will assume that the space has Riemannian metric, so the square of the norm is

$$r^2 = x^2 + y^2 + z^2 \tag{47}$$

and only the area of an r-sphere grows faster (or slower) than in R^3. It is possible to construct such a space and we are interested in proving that $\gamma = 2$, the space is flat.

The mass m is distributed over the R-ball centered in the origin and having a constant density ρ, thus

$$m = \rho V(R). \tag{48}$$

The mass m_1 is a much smaller test mass that we will place to the z-coordinate to the place $(0, 0, h)$. The following spherical coordinates are a convenient choice for this calculation

$$x = r\cos(\phi)\cos(\beta) \qquad x = r\sin(\phi)\cos(\beta) \qquad z = r\sin(\beta) \tag{49}$$

The angle β is between the x-axis and z-axis. At a given value of β there is a circle with the radius $r\cos(\beta)$ with the length $2\pi r^{\frac{\gamma}{2}}$. The volume element having length $r^{\frac{\gamma}{2}} d\beta$ in the β-direction and dr in the radial direction is

$$dV(\beta) = 2\pi r^{\gamma} dr \cos(\beta) d\beta \tag{50}$$

and the mass is $dm(\beta) = \rho dV(\beta)$. The elements of this circular mass are at the distance s from m_1 where

$$s^2 = (r\cos(\beta)^2 + (h - r\sin(\beta))^2 = r^2 + h^2 - 2rh\sin(\beta) \tag{51}$$

This mass creates a gravitational force $dF(\beta, r)$ on m_1. The force from the elements of the mass are not in the direction of $-z$-axis, so we have to add a projection. It is the first multiplier at the right side in the formula below:

$$dF(\beta, r) = \frac{h - r\sin(\beta)}{s} G \frac{m_1 dm}{s^{\alpha}} \tag{52}$$

The force is towards the origin.

The integral over β is

$$dF(r) = 2\pi G m_1 \rho r^{\gamma} dr \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(h - r\sin(\beta))\cos(\beta)d\beta}{s^{\frac{\alpha+1}{2}}} \tag{53}$$

$$= 2\pi G m_1 \rho r^\gamma dr I$$

where $x = \sin(\beta)$ simplifies I to

$$I = \int_{-1}^{1} a \frac{(\frac{h}{r} - x)dx}{(b - x)^{\frac{\alpha+1}{2}}} \tag{54}$$

$$a = r(2hr)^{-\frac{\alpha+1}{2}} \tag{55}$$

$$b = \frac{r^2 + h^2}{2hr} \tag{56}$$

A calculation gives the following exact result

$$I = \frac{1}{\alpha - 1} \frac{1}{2r} h^{1-\alpha} I_2 \tag{57}$$

$$\begin{aligned} I_2 = &\left(1 + \frac{r}{h}\right)\left(1 - \frac{r}{h}\right)^{\alpha-2} - \left(1 - \frac{r}{h}\right)\left(1 + \frac{r}{h}\right)^{\alpha-2} \\ &- \left(1 - \frac{r}{h}\right)^{3-\alpha} + \left(1 + \frac{r}{h}\right)^{3-\alpha} \end{aligned} \tag{58}$$

From Taylor series we get the expression:

$$I = \frac{3 - \alpha}{\alpha - 1} 2h^{-\alpha} + p(\alpha) r^2 h^{-\alpha-2} + O(r^4 h^{-4}) \tag{59}$$

where

$$p(\alpha) = \frac{1}{2(\alpha - 1)} \left(20 - \frac{92}{3} + 15\alpha^2 - \frac{7}{3}\alpha^3\right) \tag{60}$$

If $\alpha = 2$, then $p(\alpha) = 0$ and the $O(r^4 h^{-4})$ term is also zero. Thus, $\alpha = 2$ gives an exact result as the first term of I.

We integrate $-dF(r)$ over r from zero to R and insert m from (48) and (46). The result is

$$F = G\frac{m_1 m}{h^\alpha} \frac{3 - \alpha}{\alpha - 1} + G\frac{m_1 m}{h^\alpha} \frac{\gamma + 1}{\gamma + 3} \frac{p(\alpha)}{2} \frac{R^2}{h^2} \left(1 + O(R^2 h^{-2})\right). \tag{61}$$

Let $h >> R$, then the second term in the right side becomes insignificant. The gravitational force created by a ball of radius R and mass m must approach the gravitational force created by a point mass with the size m. Thus

$$F = G\frac{m_1 m}{h^\alpha} \frac{3 - \alpha}{\alpha - 1} \to G\frac{m_1 m}{h^\alpha} \tag{62}$$

when h stays fixed and R approaches zero. This can only happen if

$$3 - \alpha = \alpha - 1 \tag{63}$$

that is, $\alpha = 2$. We have proven that the power α in Newton's formula must be two. It does not matter if we make α a function of h because we are not integrating over h in this calculation. Always (63) must hold in the limit, so $\alpha = 2$.

Let us remark that for $\alpha = 2$ the equation

$$F = G\frac{m_1 m}{h^2} \tag{64}$$

is exact. For any distance $h > R$ the mass of the shape of an R-ball with constant density always gives the same gravitational force as a point mass of the same size.

We did not get any result for the parameter γ from this calculation, but there is an easy way to conclude that in empty space $\gamma = 2$. Force lines from the gravitational force created by a point mass do not disappear in empty space. Thus

$$A(r+dr)F(r+dr) = A(r)F(r) \tag{65}$$

implying that $\gamma = \alpha = 2$, the space geometry is flat.

3. On the possibility of defining a local ether

Let us go to the classical field equation, derived in [4]

$$\Delta\psi(\bar{}) = -4\pi G\rho(\bar{r}) \tag{66}$$

where

$$\rho(\bar{r}) = \sum_i M_i \delta(\bar{r} - \bar{s}_i). \tag{67}$$

As explained in [4], ρ is not a continuous function. It is an approximation that describes a cloud of residues of point singularities coming from point masses M_i at locations $\bar{s}_i$. These point masses could be other mass bodies, but we will remain in the situation of a mass body at the origin being placed in what looks like empty space. But now we assume it is not empty space, it is a local ether containing extremely small point masses M_i.

A local ether can provide the local frame of reference where light has the local speedarguentm c at each point to each direction. A local ether that has some very small by mass can be kept around a mass body by the gravitational force, yet if the ether particles are very small, they do not need to rotate with a high speed in high altitude. There are several forces that can keep them above the ground. After all, our atmosphere is not blowing with high speed but seems to be staying quite well above the ground.

Local ether could possibly explain dark mass, assuming that there is needed such undetected mass as the arguments usually are based on the General Relativity Theory, which is false.

If there is such local ether with mass, it would mean that the gravitational force does not appear to follow the $\alpha = 2$ law exactly: ignoring the right side in (66) would lead to the conclusion that α is not precisely two. But it must be two.

The ether theory was discarded by Einsteinäs false claim that the Lorentz transfrom makes the speed of light constant c in every inertial frame. It does not. Einstein forgot to project (x', t') in the moving frame of the Lorentz transform to the t'-axis. The speed of light in the moving frame is not c, nor can it be made c by any linear transfrom. As Einstein's relativistic mass formula is also wrong and his proof of $E = mc^2$ is not a proof of anything, and his General Relativity Theory is totally wrong, it is time to look again at the ether theory, but the ether must be a local ether, a part of it moving with a mass body.

3. References

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3.3 Failure of the geometrization principle and some cosmological considerations

Abstract: Section 1 explains why the Schwarzschild solution is not a valid approximation for Newtonian gravitation. Section 2 shows that the Einstein equations do not have any solutions for the case of a point mass in empty space such that the speed of light in the direction orthogonal to the gradient of the gravitation field is constant. Section 3 explains why the geometrization principle as used by Einstein and Nordström is false: time cannot be treated similarly as space coordinates and especially the field equation cannot contain the Ricci scalar curvature. Section 3 briefly discusses how Newtonian gravitation potential could be modified and what are astronomical and cosmological implications if that is done.

1. Introduction

Special Relativity Theory [1] and General Relativity Theory [2] are refuted in my earlier papers [3]-[7]. The present paper is continuation to [7], as [7] and maybe the title of [5] may give a too positive impression of Nordström's theory. In the presented paper I explain why Nordström's and Einstein's way of geometrization of gravitation is wrong. This does not mean that all ideas in the geometrization principle are wrong, only that the field equation is wrong if it contains the Ricci scalar curvature. Time dependency must be made in another way, and Newtonian gravitation theory already has a valid way of time dependency if it is correctly understood.

Sections 1 explains why the Schwarzschils solution is not a valid approximation of Newtonian gravity and Section 2 shows that there is no way for that theory to approximate Newtonian gravitation even if we allow that in the radial direction the speed of light is different. The calculation in Section 2 is made in a similar way as in the Schwarzschild solution. Section 3 explains why the Ricci scalar curvature should not be included in a field equation for gravitation. The section also makes some consmological comments, like that the observed Hubble and CBR redshift can also be explained as a geometric redshift if the area of a r-sphere grows rather fast for in some cosmic level interval in an otherwise flat space.

2. GRT does not approximate Newton's gravitation theory

The Schwarzschild solution is an exact solution for Einstein's equations

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa_0 T_{ab} g_{ab} + \lambda g_{ab}. \tag{1}$$

in the situation of a single point mass in an empty space. The mass is placed to the origin of spherical coordinates. Outside the origin all entries T_{ab} of the energy-stress tensor are zero because these entries are derived from the matter distribution.

The Schwarzschild solution is constructed to approximate the Newtonian gravity in the same situation of a point mass in empty space. Newtonian gravitation potential created by a point mass M is time-independent and satisfies the equation

$$\Delta\psi(r) = \Delta\left(-GM\frac{1}{r}\right) = 4\pi GM\delta(r) \tag{2}$$

As the Newtonian gravitation field

$$\psi(r) = -GM/r$$

is time independent, we can write the field equation outside the origin as

$$\Box\psi(r) = 0 \tag{4}$$

The box $\Box$ is the D'Alembertian. In Cartesian coordinates and with the signs (+,-,-,-)

$$\Box = \partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2 = \partial_0^2 - \Delta. \tag{5}$$

The geometric form of (3) is

$$R = 0 \tag{6}$$

where R is the Ricci scalar curvature. For any scalar field ψ, and the Newtonian gravitation potential is a scalar, the Ricci scalar curvature is

$$R = -6\psi^{-3}\Box\psi. \tag{9}$$

The Schwarzshild solution approximates Newtonian gravity in the sense that $R = 0$. As the cosmological constant λ is intended to be zero in a flat space, and as a flat space in GRT means that $R = 0$, the Schwarzschild solution has $T_{ab} = 0$, $R = 0$ and $\lambda = 0$ and the Einstein equations reduce to

$$R_{ab} = 0. \tag{10}$$

It suffices to check that $R_{aa} = 0$, $a = 0, 1, 2, 3$, because in orthogonal coordinates, like the spherical coordinates, the off-diagonal entries R_{ab}, $a \neq b$, are zero.

The Schwarzschild solution approximates the Newtonian gravitation potential in several senses. The solution is time-independent and spherically symmetric, like the Newtonian potential. The difference between the potential field of the Schwarzschils solution in the radial direction and the (scaled) Newtonian potential decreases to zero when r grows to the infinity. This means that the gravitation force derived from the Schwarzschils solution approximates the Newtonian gravitation force when r grows to infinity.

In order to call a solution an approximation of something, it must converge to the approximated solution when something approaches infinity. This something in the case of a gravitation potential can only be that r approaches infinity. There is no sense to approximate Newtonian gravity in the case where r approaches zero as gravitation is the weakest of the four forces. The Schwarzschild solution does not approximate Newtonian gravitation force when r approaches zero: the solution has an event horizon. We see that the Schwarzschild solution can in a certain sense be called an approximation of Newtonian gravitation potential in the case of an empty space with a single point mass.

Yet, it is incorrect to consider the Schwarzschils solution as an approximation for the Newtonian gravitation in our solar system, and because of this problem it is more correct to say that no solution of the Einstein equation is an approximation Newtonian gravity in a sense applicable to the gravitation field of the Sun or the gravitation field of the Earth, both sufficiently well approximated by a single point mass in the empty space. This means that all experiments that are claimed to verify the General Relativity Theory are void, because the theory does not apply to the part of the universe where the experiments were made.

The problem is that the speed of light should be c at each point to each direction as it is in the (flat) Minkowski space metrics

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{11}$$

Light moves on light-like world paths and these paths are defined by the condition $ds = 0$. The speed of light to the direction e.g. of the y-axis is obtained by setting $dx = dz = 0$. Then

$$0 = c^2 dt^2 - dy^2 \qquad \text{implying} \qquad \mathrm{c}^2 = \frac{\mathrm{dy}^2}{\mathrm{dt}^2}. \tag{12}$$

This shows that the speed of light to the y-direction is $dy/dt = c$ as it should be. More generally, in a gravitation field, the metric is in spherical coordinates

$$ds^2 = g_{00}(ct)^2 - g_{11}dr^2 - g_{22}d\theta^2 - g_{33}d\phi^2 \tag{13}$$

where g_{ab} is the metric tensor in spherical coordinates ($g_{ab} = 0$ is $a \neq b$). In order for the speed of light to be c, we must have

$$1 = \frac{g_{11}}{g_{00}} \qquad r^2 = \frac{g_{22}}{g_{00}} \qquad r^2 \sin^2\phi = \frac{g_{33}}{g_{00}}. \tag{14}$$

Here $1 = g_{11}/g_{00}$ means that the speed of light is c because in the notation $x^0 = ct, x^2 = r, x^3 = \theta, x^4 = \phi$ used in the Schwarzschild solution, the time coordinate is $x^0 = ct$. In Cartesian coordinates the requirement is $g_{ii}/g_{00} = 1$ for $i = 1, 2, 3$.

The speed of light in the Schwarzschild solution is not constant to any direction r, θ or ϕ as is seen by the metric of the Schwarzschild solution

$$g_{00} = A(r) \qquad g_{11} - B(r) \qquad g_{22} = -r^2 \qquad g_{33} = -r^2 \sin^2\theta. \quad (15)$$

$$A(r) = \left(1 - \frac{r_s}{r}\right), B(r) = \left(1 - \frac{r_s}{r}\right)^{-1}$$

We may be willing to accept that the speed of light in the radial direction is not c, as it could be that the gravitation force changes the situation, but certainly in the direction orthogonal to the radial direction, the speed of light in vacuum should be c. In the orthogonal direction there is no gravitation force in the direction of the light beam. The speed of light in vacuum has been measured on the Earth in the horizontal direction with respect to the Earth in high precision

$$c = 299792458\frac{m}{s} \qquad (16)$$

Consider how it would fit our expectations adding to this measurement of c the following text:

This is the speed of light on the sea level in the particular location where the measurement was done. The speed of light in vacuum, even when measured in the direction perpendicular to the gravitation field gradient, is different in different gravitation field. It is different on a mountain top or if measured on the Moon. We just say it is a constant, but it is not constant.

Exactly what kind of a universal constant c would be in this case, and why would there be any sense in measuring it in such a precision. Clearly, the speed of light must be a constant in the direction orthogonal to the radial direction. The metric must be of the form

$$g_{00} = A(r) \qquad g_{11} = -B(r) \qquad g_{22} = -C(r)r^2 \qquad g_{33} = -C(r)r^2 \sin^2\theta$$

$$A(r) = \psi(r)^2 \qquad B(r) = \phi(r)^2 \qquad C(r) = \psi(r)^2. \qquad (17)$$

We will see in the next section that the Einstein equations do not have a solution that satisfies (17) and approximates the Newtonian gravitation potential in the same sense as the Schwarzschild solution.

2. GRT does not approximate Newton's gravitation theory in our solar system

Let us look for a time-independent and spherically symmetric solution that has $R = 0, \lambda = 0$ and is in the empty space with a point mass in the origin, i.e., $T_{ab} = 0$ outside the origin. We want a metric of the form (17). As g_{00}, g_{11}, g_{22} only depend on r and g_{33} depends on r and θ, the Ricci tensor entries are exceptionally short

$$R_{00} = \Gamma^1_{00}\left(-\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}\right) + \Gamma^1_{00,1} \tag{18}$$

$$\begin{aligned} R_{11} = \Gamma^1_{11}\left(\Gamma^0_{10} + \Gamma^2_{12} + \Gamma^3_{13}\right) - \left(\Gamma^1_{11}\right)^2 - \left(\Gamma^2_{12}\right)^2 - \left(\Gamma^3_{13}\right)^2 \\ -\Gamma^0_{10,1} - \Gamma^2_{12,1} - \Gamma^3_{13,1} \end{aligned}$$

$$R_{22} = \Gamma^1_{22}\left(\Gamma^0_{10} + \Gamma^1_{11} - \Gamma^2_{12} + \Gamma^3_{13}\right) + \Gamma^1_{22,1} - \left(\Gamma^3_{23}\right)^2 - \Gamma^3_{23,2}$$

$$R_{33} = \Gamma^1_{33}\left(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} - \Gamma^3_{13}\right) + \Gamma^1_{33,1} - \Gamma^3_{23}\Gamma^2_{33} + \Gamma^2_{33,2}$$

The Christoffel symbols needed above are

$$\begin{gathered} \Gamma^1_{00} = \phi^2\psi'\psi \qquad \Gamma^0_{10} = \psi'\psi^{-1} \qquad \Gamma^1_{11} = \phi'\phi^{-1} \\ \Gamma^3_{23} = \cot(\theta) \qquad \Gamma^2_{33} = -\sin(\theta)\cos(\theta) \end{gathered} \tag{19}$$

and if $C(r) = \psi(r)^2$

$$\begin{gathered} \Gamma^2_{12} = \frac{1}{r} + \psi'\psi^{-1} \qquad \Gamma^3_{13} = \frac{1}{r} + \psi'\psi^{-1} \\ \Gamma^1_{22} = -\phi^{-2}r(\psi^2 + r\psi'\psi) \qquad \Gamma^1_{33} = -\phi^{-2}r(\psi^2 + r\psi'\psi) \end{gathered} \tag{20}$$

while if $C(r) = 1$, we have the Schwarzschild solution where

$$\begin{gathered} \Gamma^2_{12} = \frac{1}{r} \qquad \Gamma^3_{13} = \frac{1}{r} \\ \Gamma^1_{22} = -\phi^{-2}r \qquad \Gamma^1_{33} = -\phi^{-2}r \end{gathered} \tag{21}$$

Let us first give the Schwarzschild solution. Let

$$y = \psi' \psi^{-1}. \tag{22}$$

The equations are

$$R_{00} = \phi^{-2}\psi'\psi \left(\phi'\phi^{-1} - y + \frac{2}{r} \right) + \frac{d}{dr}\left(\phi^{-2}\psi'\psi \right) = 0 \tag{23}$$

$$R_{11} = \phi'\phi^{-1} \left(y + \frac{2}{r} \right) - y^2 - \frac{2}{r^2} - \frac{d}{dr}\left(y + \frac{2}{r} \right) = 0$$

$$R_{22} = -\phi^{-2} r \left(y + \phi'\phi^{-1} \right) + \frac{d}{dr}(-\phi^{-2} r) + 1 = 0$$

The fourth equation $R_{33} = 0$ is $\sin^2(\theta)$ times $R_{22} = 0$ and can be ignored. The two first equations yield

$$\begin{aligned} y' &= \phi'\phi^{-1} y - y^2 - \frac{2}{r} y \\ y' &= \phi'\phi^{-1} \left(y + \frac{2}{r} \right) - y^2 \end{aligned} \tag{24}$$

which yields

$$\phi'\phi^{-1} = -y \tag{25}$$

Inserting this result to the last equation we get

$$\phi^2 = 2yr + 1 \tag{26}$$

Derivating

$$2\phi'\phi = 2y'r + 2y \tag{27}$$

and dividing by ϕ^2

$$\begin{aligned} -y = \phi'\phi^{-1} &= \frac{2y' + 2y}{2yr + 1} \\ y' &= -\frac{2}{r} y - 2y^2 \end{aligned} \tag{28}$$

which has the solution

$$\phi = \left(1 - \frac{r_c}{r}\right)^{-1/2} \tag{29}$$
$$\psi = \left(1 - \frac{r_c}{r}\right)^{1/2}$$
$$y = \frac{1}{2}\frac{r_c}{r^2}\left(1 - \frac{r_c}{r}\right)^{-1}$$

Let us try to solve the equations with $C(r) = \psi(r)^2$ in a very similar way. The equations are

$$R_{00} = \phi^{-2}\psi'\psi\left(\phi'\phi^{-1} - y + \frac{2}{r} + 2y\right) + \frac{d}{dr}\left(\phi^{-2}\psi'\psi\right) = 0 \tag{30}$$

$$R_{11} = \phi'\phi^{-1}\left(y + \frac{2}{r} + 2y\right) - y^2 - 2\left(\frac{1}{r} + y\right)^2 - \frac{d}{dr}\left(y + \frac{2}{r} + 2y\right) = 0$$

$$R_{22} = -\phi^{-2}r\left(\psi^2 + r\psi'\psi\right)\left(y + \phi'\phi^{-1}\right) + \frac{d}{dr}\left(-\phi^{-2}r\left(\psi^2 + r\psi'\psi\right)\right) + 1 = 0$$

The two first equations give

$$y' = \phi'\phi^{-1}y - 3y^2 - \frac{2}{r}y \tag{31}$$
$$y' = \phi'\phi^{-1}\left(y + \frac{2}{3r}\right) - y^2 - \frac{4}{3r}y \tag{32}$$

which yields

$$\phi'\phi^{-1} = -y - 3ry^2 \tag{33}$$

The third equation $R_{22} = 0$ gives

$$(1 + ry)\left(y - \phi'\phi^{-1} + \frac{1}{r}\right) - \frac{1}{r}\phi^2\psi^{-2}$$
$$+3y + ry' + 2ry^2 = 0 \tag{34}$$

Inserting (33) and simplifying gives

$$\phi^2 = \psi^2 \left(3r^2y^2 + 4ry + 1\right) \tag{35}$$

Derivating (35) and dividing by (35) gives $2\phi'\phi^{-1} = \phi^{-2}\frac{d}{dr}(\phi^2)$. Inserting (33) and simplfying gives the result

$$y' = -\frac{9r^3y^4 + 18r^2y^3 + 14ry^2 + 4y}{3r^2y + 2r} \tag{36}$$

$$y' = -3ry^3 - 4y^2 - \frac{6y^2}{3ry + 2} - \frac{8y}{3r^2y + 2r}$$

We are only interested in knowing if this solution approximates the Newtonian gravitation potential when r is large.

Let us try solving (36) with $y = \alpha r^{-2}$. The trial fails, we get

$$-2\alpha r^{-3} = -4\alpha r^{-3} + O(r^{-4}) \tag{37}$$

The result means that $y = \alpha r^{-2} + O(r^{-3})$ is not a solution of (36).

Let us try $y = \alpha r^{-1}$. Inserting this function (36) gives the equation

$$-\alpha r^{-2} = -\alpha r^{-2}(3\alpha + 4)\left(1 + \frac{2}{3\alpha + 2}\right) \tag{38}$$

There are solutions for three values of α, but not fof $\alpha = 1$ or $\alpha = -1/3$. Thus, $y = \alpha r^{-1} + O(r^{-2})$ is possible only for three values of α.

Solving the second order equation (33) for y yields two roots

$$y = -\frac{1}{3r} + O(\phi'\phi^{-1}) \tag{39}$$

$$y = -\phi'\phi^{-1} + O((\phi'\phi^{-1})^2) \tag{40}$$

The first root (39) is not possible because $\alpha = -1/3$ is not a solution of (36). Only (40) can be possible.

There are only two behaviors for ϕ for large r so that ϕ has the same r^{-1} behavior as the Newtonian gravitation potential when r grows large. Either $\phi = \beta r^{-1} + O(r^{-2})$ or $\phi = A + \beta r^{-1} + O(r^{-2})$. In the Schwarzschild solution it is the second alternative.

In the first alternative

$$
\begin{aligned}
\phi &= \beta r^{-1} + O(r^{-2}) \\
\phi' \phi^{-1} &= -r^{-1} + O(r^{-2}) \\
y &= r^{-1} + O(r^{-2}) \\
y = \alpha r^{-1} + O(r^{-2}) &\qquad \text{with} \qquad \alpha = 1
\end{aligned}
\tag{41}
$$

By (38) the value $\alpha = 1$ is not a solution for (36).

The second alternative gives

$$
\begin{aligned}
\phi &= A + \beta r^{-1} + O(r^{-2}) \\
\phi' \phi^{-1} &= -\beta A^{-1} r^{-2} + O(r^{-2}) \\
y &= -\beta A^{-1} r^{-2} + O(r^{-3}) \\
y = \alpha r^{-2} + O(r^{-3}) &\qquad \text{with} \qquad \alpha = \beta \mathrm{A}^{-1}
\end{aligned}
\tag{42}
$$

By (37) the function y is not a solution to (36).

The calculation shows that the Einstein equations do not have any solutions that approximate the Newtonian gravitation field in the most basic situation of a point mass in empty space and have constant speed of light in vacuum in the direction that is orthogonal to the gradient of the gravitation field. This failure means that GRT fails the experimental test.

A scalar field is the only possible mathematical form that gives a constant speed c to light in vacuum at every point and to every direction. GRT does not have any scalar field solutions that approximate Newtonian gravitation in the situation of a point mass in empty space, see [7] and [3]. We tried in this section to find a solution for GRT where the field is not a scalar field: the field we tried to find is different in the radial direction. But this effort also failed.

It is not possible to satisfy the four equations for $ab = aa$, $a = 0, 1, 2, 3$ in (1). The best that can be done is to sum these four equations with the weights g^{aa} and to obtain $R = g^{aa} R_{aa}$ and $T = g^{aa} T_{aa}$, the traces. When this is done, (1) gives the field equation in Nordström's gravitation theory:

$$
R = \alpha T \tag{43}
$$

where α is some constant.

Nordström's theory is a scalar field theory, the gravitation field is scalar and the speed of light is constant c to all directions at every point. In the next section we will investigate the idea of geometrization with Nordström's scalar gravitation theory and show where the geometrization ideas fail.

2. Time cannot be treated similarly to space coordinates in the field equation

Nordström's gravitation theory's [8] field equation is only a slight generalization to the classical result

$$\Box\phi = -4\pi G\rho(r) \tag{44}$$

It is a geometric theory and shows the geometrization ideas because $\Box$ is closely related to the Ricci scalar curvature by (9) for scalar fields. For time independent fields, Nordströ's field equation is exactly the Gauss-type equation in Newtonian gravitation theory. Also Nordström's equation of motion is also exactly the same as $F = ma$ if the field is time independent. Nordström's theory is not simply Newton's gravity: if gives correctly gravitational and acceleration time dilations.

But the field equation (44) seems incorrect if the field is time dependent. The D'Alembert operator has ∂_0^2, second time derivative. It is like in the wave equation and it could describe time dependency of a gravitational wave. One can derive the wave equation from the Maxwell equations. The Maxwell equations only model the field, not the charges. In a geometric gravitation theory the field is the space-time geometry. Is the mass in the gravitational field, i.e., in the space-time, or is it a separate entity in the matter part of the field equation? It seems that time dependency should also describe movement of mass etc. The wave equation does not mean that there is a four-dimensional space-time, but in a geometric theory of gravitation the field is space-time. The idea in the Ricci scalar curvature (related to $\Box$) is that the space has four dimensions. The Laplacian Δ for a function $\psi(r)$ shows how the operator is related to the geometry of the space:

$$\begin{aligned}\Delta\psi(r) &= \frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\psi(r) = \frac{1}{4\pi r^2}\frac{d}{dr}4\pi r^2\frac{d}{dr}\psi(r) \\ &= \frac{1}{A(r)}\frac{d}{dr}A(r)\frac{F(r)}{m} = 0\end{aligned} \tag{45}$$

where $F(r)$ is the gravitation force and $A(r)$ is the area of a sphere. This equation means that through any sphere of radius r goes the same amount of force lines coming from a point mass at the origin.

$$A(r)F(r) = \text{constant} \tag{46}$$

Thus, (45) does not come from any Euler-Lagrange equations (Einstein claimed to have derived the Einstein equations from Euler-Lagrange equations, but they do not come from the action S he used, see [7]). Equation (45) comes from the concept of force lines and the reason why Newtonian gravitational force depends on r as r^{-2} is that the area of a sphere in the Euclidean geometry is $4\pi r^2$. We can deduce that our world is 3-dimensional (not 4-dimensional) directly from the Newtonian gravitation force, or equivalently from the Newtonian gravitation potential. The D'Alembert operator, and the Ricci scalar, is for a 4-dimensional space. The operator $\Box$ fits naturally for a solution

$$\psi(r,t) = -\frac{GM}{\sqrt{x^2+y^2+z^2-(ct)^2}} \tag{47}$$

but this kind of a solution has no sense in our world. This is why time and space coordinates must not be treated in a similar way in the field equation of gravitation and the particular form of the geometrization idea that appears in the Einstein equations and Nordström's field equation (i.e., that there is R and $\Box$) is false. It is possible and sometimes useful to describe the gravitational field as a metric and to have this metric as time dependent, thus the theory has four coordinates, but curvature of the space should be 3-dimensional curvature.

It is easy to deduce what time dependency in a gravitational field should mean: masses move and cause a changing gravitational field. Masses can also change, like when a star is burning hydrogen to helium. The change of the gravitational field propagates with the speed of light. Nothing of this can be described with the time derivative in $\Box$. We can see that all of this is already modelled in the Newtonian field equation.

Let us consider a point mass M_i in the location $\bar{r}_i$ with respect to the origin. We observe the gravitation field at the location $\bar{r}$ with respect to the origin. The gravitation potential at $\bar{r}$ at the time t of the observer at $\bar{r}$ is

$$\psi(\bar{r},t) = -\frac{GM_i(t-c^{-1}||\bar{r}-\bar{r}_i||)}{||\bar{r}-\bar{r}_i||} \tag{48}$$

Let us define $(x_i,y_i,z_i) = \bar{r}_i$, $x' = x - x_i, y' = y - y_i, z' = z - z_i$, $r' = \sqrt{{x'}^2+{y'}^2+{z'}^2}$. By moving the origin to r_i, we have a spherically symmetric field

$$\psi(r',t) = -\frac{GM_i(t-c^{-1}r')}{r'} \tag{49}$$

This spherically symmetric field can be solved from

$$F(r',t) = G\frac{M_i(r',t-c^{-1}r')m}{r'^2} \tag{50}$$

$$\frac{d}{dr'}\psi(r',t) = \frac{F(r',t)}{m} = G\frac{M_i(r',t-c^{-1}r')}{r'^2}$$

We can describe all masses M_i by a density function

$$\rho(r',t) = \sum_i M_i(r', t - c^{-1}r') \tag{51}$$

This is possible: the right side is a function of t and r'. The density function is not necessarily what we think density function should be as it sums masses M_i at different times. But finding this density function simplifies the calculations.

$$\begin{aligned}
\frac{d}{dr'}\psi(r', t = G\frac{Vol * \rho(r',t)}{r'^2} &= G\frac{4\pi}{3}r'^3\frac{\rho(r',t)}{r'^2} \\
r'^2\frac{d}{dr'}\psi(r',t) &= G\frac{4\pi}{3}r'^3\rho(r\ddot{a},t) \\
\frac{d}{dr'}r'^2\frac{d}{dr'}\psi(r',t) &= G4\pi r'^2\rho(r',t) \\
\Delta'\psi(r',t) = \frac{1}{r'^2}\frac{d}{dr'}r'^2\frac{d}{dr'}\psi(r',t) &= G4\pi\rho(r',t)
\end{aligned} \tag{52}$$

Notice that in (r', θ', ϕ') coordinates $\psi(r',t)$ is spherically symmetric and

$$\Delta'\psi(r',t) = \frac{1}{r'^2}\frac{d}{dr'}r'^2\frac{d}{dr'}\psi(r',t) \tag{53}$$

while in (r, θ, ϕ) coordinates $\psi(r') = \psi(\bar{r} - \bar{r}_i, t)$ is not spherically symmetric

$$\Delta\psi(r,\theta,\phi,t) \neq \frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\psi(r,\theta,\phi,t) \tag{54}$$

but

$$\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2 = \partial_{x'}^2 + \partial_{y'}^2 + \partial_{z'}^2 = \Delta' \tag{55}$$

because $dx = dx', dy = dy', dz = dz'$. Therefore

$$\begin{aligned}
\Delta'\psi(r',t) &= \Delta'\psi(x',y',z',t) = G4\pi\rho(r',t) \\
&= \Delta\psi(\bar{r},t) = \Delta\psi(x,y,z,t) = G4\pi\rho(r',t)
\end{aligned} \tag{56}$$

We get the result

$$\Delta\psi(x,y,z,t) = 4\pi G\rho(x,y,z,t) \tag{57}$$

and notice that this result already includes the finite speed of light, movements of masses and the change of masses. We can solve $\phi(x,y,z,t)$ from this equation, but in order to relate this field to individual masses M_i, we have to use a formula that has time delays from the finite speed of light:

$$\psi(x,y,z,t) = \sum_i -GM_i(x,y,z,t-c^{-1}r)\frac{1}{r} \tag{58}$$

As a summary of this section, the geometrization principle as used by Einstein and Nordström is incorrect: time cannot be treated in the same way as space coordinates in the field equation and the field equation should not use the Ricci scalar. Curvature should be 3-dimensional curvature. Newtonian gravitational theory already has a way to treat time and this way includes the finite speed of light, if correctly used.

3. Curvature in 3-dimensions and some cosmological considerations

Let us again look at the equation (46)

$$A(r)F(r) = \text{constant}$$

We see that the fundamental property in the Newtonian field equation (57) is the force, not the potential. The potential is only more convenient as potentials are scalars and are additive while force is a vector and needs vector summing.

There are some reasons for investigating the possibility that the gravitational force from a point source is not growing as r^{-2} but has an additional term growing as r^{-4}. This case appeared in my calculations in [4] of the free fall of a test mass in a gravitational field of a larger mass. If Einstein's SRT were correct, a moving mass has a larger mass than a mass at rest. This increased mass leads to a different equation for the movement of the test mass compared to the case where the mass does not change because of velocity. Einstein's moving mass concept is derived from the Lorentz transform and that transform is incorrect, as I show in [3] and [4]. There is no theoretical justification for the moving mass concept because there is no reason why equations of motion should be Lorentz invariant (see [4]). Therefore, the moving mass concept should be dropped. However, there are experiments that claim to demonstrate the moving mass. In [4] it is shown that if the force is given an additional term growing as r^{-4}, then the equation for the moving test mass is exactly the same as if there is a moving mass. The calculation in [4] is done for gravitation potential. Experiments that claim to demonstrate moving mass are from particles in particle accelerators (the accelerator appears to need more energy than it should, and this is explained by energy needed for the moving mass). In particle accelerators, charged particles are accelerated by electric field. A static electric field has the same form as the gravitational field and if the force in one case needs an additional term, the same may be true in the other case. In [4] it is mentioned that we can either change the potential by adding a term r^{-3} or change the relation of force to potential.

Two other reasons for investigating the possibility of an additional term either in the gravitational force or in the potential are in [9] and [10]. In [9] it is shown that a small additional term breaks the double zero that forbids energy conserving elliptic orbits in a two-body problem in Newtonian physics. Such a term may also help to explain the precession of the perihelion of Mercury. Article [10] speculates that with an additional r^{-3} term a classical gravitational field can be connected with the two scalar Higgs fields.

In general, adding a term that grows as $r^{-\alpha}$ to the potential does not cause a major problem to the field equation (57). As

$$\Delta\left(-\frac{1}{r^\alpha}\right) = \alpha(\alpha-1)\frac{1}{r^{\alpha+2}} \tag{59}$$

we only need to add corresponding terms to the right side of (57)

$$\begin{aligned} \Delta\phi(x,y,z,t) &= 4\pi G\rho(x,y,z,t) \\ +GM\beta\alpha(\alpha-1)\sum_i &\frac{M_i(\bar{r}_i, t - c^{-1}||\bar{r}-\bar{r}_i||)}{||\bar{r}-\bar{r}_i||^{\alpha+2}} \end{aligned} \tag{60}$$

These terms do not nicely sum, so the vectors $\bar{r}$, $\bar{r}_i$ remain. The solution of (60) for an empty space with a point mass M at the origin is

$$\psi(r) = -\frac{GM}{r}\left(1 - \frac{\beta}{r^\alpha}\right) \tag{61}$$

However, (46) makes a compelling argument that $A(r)$ should be different from $4\pi r^2$ if there are additional terms in the force or potential. Let us focus only on a radially symmetric situation.

Let $\phi(r)$ be any field that we want to be the solution for the situation of a point mass M in empty space. Let us define

$$\mathcal{D} = \frac{1}{A(r)}\frac{d}{dr}A(r)\frac{d}{dr} \tag{62}$$

where

$$A(r) = \frac{4\pi GM}{\phi'(r)}. \tag{63}$$

Then

$$\mathcal{D}\phi(r) = 0 \tag{64}$$

and for any $\psi(r)$ holds

$$\mathcal{D}\psi(r) = \Delta\psi(r) - \frac{\Delta\phi(r)}{\Delta\phi(r)}\psi'(r) \tag{65}$$

$$\mathcal{D}\psi(r) = \Delta\psi(r) + \left(\frac{A(r)'}{A(r)} - \frac{2}{r}\right)\psi(r)'. \tag{66}$$

We can add a small r^{-3} term to the Newtonian potential. It does not much change the area of a sphere as is seen from (63). If we want that $A(r)$ grows clearly faster than $4\pi r^2$, then (63) shows how the field must grow. The right side of (61), though not a function of r, does approximate a function of r from a long enough distance if the mass distribution is sufficiently spherical.

Let us finish this article with three cosmological comments.

De Sitter's space is derived as a positive curvature solution to the Einstein equations. As the Einstein equations do not approximate Newtonian gravity in the basic scenario, these equations have no place as models in the universe. There is no sense in making cosmological deductions from de Sitter spaces, yet they are used as basic models of accelerating expansion of the universe.

Black holes are also derived from the Einstein equations and the same is true for them. Something similar to black holes also come in Newton's or Nordström's theories. If a small r^{-3} term is added to the potential, then gravitation becomes repulsive at a short distance from the mass center point. Such repulsive gravitation may explain e.g. supernova behavior: if enough matter is pushed by other matter into the area where gravitation is repulsive, then if in some way the outer layers of mass are decreased, they cannot keep the center mass from coming out and an explosion may follow.

One can explain the Hubble redshift and the redshift of cosmic background radiation without an expanding universe by inserting geometric redshift, i.e., if $A(r)$ increases faster than $4\pi r^2$ for some range of values $r_1 < r < r_2$, then between those values there is a redshift for emitted EM radiation. It is possible to insert a function that is very small if $r < r_1$ and will not be noticable close to our experiences, and gets also small when $r > r_2$ and will not cause any infinity problems, yet it gives a redshift without a Doppler effect, huge masses or acceleration.

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3.4 A New Look on Nordström's Gravitation Theory

Abstract: This article is not a historical look at a rejected theory. It aims to recover a working scalar theory of gravitation from Nordström's old theory because the General Relativity Theory has very serious flaws that cannot be fixed. Nordström's theory is not that bad and can work as a classical starting point for developing a quantum gravitation theory.

1. Introduction

Gunnar Nordström was the first to present a relativistic theory of gravitation [1] but unfortunately he decised to work on it with Einstein who was developing his own relativity theory [4][5]. It does seem that Einstein confused Nordström in many issue, and later he made false claims that Nordström's theory has serious flaws. Einstein was believed and work on that theory was stopped. These false claims are still circulated and believed, see [2][3]. I hope the present article shows that Nordström's theory is a quite good simple classical field theory for gravitation without any serious flaws. It can also be quantized quite easily and can work as a good starting point for quantum gravity. Instead, Einstein's both relativity theories have very serious flaws.

Nordström's gravitation theory is described in two published articles [1], see also [2] and [3]. Nordström's theory essentially contains only of two equations: the field equation and the dymanic equation.

2. The field equation in Nordström's gravitation theory

The field equation for classical Newtonian gravitation is usually derived by modifying the Gauss law in electro-magnetism

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \tag{1}$$

.

to match the Newtonian gravitation potential which so is similar to the Coulomb law. Calculus of residues gives the starting point

$$\nabla \cdot \frac{\bar{r}}{|r^3|} = 4\pi\delta(r) \tag{2}$$

Therefore

$$\nabla \cdot \left(GM\frac{\bar{e}_r}{r^2} \right) = 4\pi GM\delta(r) \tag{3}$$

The term $\bar{g} = GM\frac{\bar{e}_r}{r^2}$ is the acceleration in a gravitational field, thus

$$\bar{g} = \nabla\phi$$

Inserting this result and replacing $M\delta(r)$ by a continuous mass density $\rho(r)$ we get

$$\nabla \cdot \nabla\phi = \Delta\phi = 4\pi G\rho(r) \tag{4}$$

Nordström's theory's field equation is only a slight generalization to this classical result

$$\Box\phi = -4\pi G\rho(r) \tag{5}$$

The box $\Box$ is the D'Alembertian. In Cartesian coordinates and with the signs (+,-,-,-)

$$\Box = \partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2 = \partial_0^2 - \Delta. \tag{6}$$

In the second version of Nordström's theory, the field equation is written as

$$\phi^{-1}\Box\phi = -4\pi T_{matter} \tag{7}$$

but it is the same equation: $T_{matter} = G\rho\phi^{-1}$. We can write (5) in a geometric form by using the geometric concept of the Ricci scalar curvature R. It is useful to first scale the field so that is is a plain number, ϕ has the units m^2/s^2. The scaled field is

$$\psi = c^{-2}\phi. \tag{8}$$

For any scalar field ψ the Ricci scalar curvature is

$$R = -6\psi^{-3}\Box\psi. \tag{9}$$

Writing

$$T = \rho\psi^{-3} \tag{10}$$

$$\kappa = 12\pi Gc^{-2} \tag{11}$$

$$g = \psi^2 \tag{12}$$

the field equation (5) gets the form that intentionally mimics the field equation in the General Relativity Theory (GRT)

$$-\frac{1}{2}Rg = \kappa Tg \tag{13}$$

With the coordinates $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$ and signs $\eta_{00} = 1$, $\eta_{ii} = -1$, $i = 1,2,3$, $\eta_{ab} = 0$ if $a \neq b$ we get

$$g_{ab} = \eta_{ab} x^a x^b \tag{14}$$

and

$$g_{aa} = g \qquad \text{for} \quad \mathrm{a} =, 0, 1, 2, 3. \tag{15}$$

so, if we define the stress-energy tensor as $T_{bb} = T$, we can write (13) very much like the Eistein equations in GRT

$$-\frac{1}{2} R g_{ab} = \kappa T_{ab} g_{ab}. \tag{16}$$

What is missing in (16) from the field equation of GRT

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa_0 T_{ab} g_{ab} + \lambda g_{ab}. \tag{17}$$

in addition the λ-term (that Einstein initially did not want to the equation) is the Ricci tensor entry R_{ab}. Because of this entry, the Einstein equations (17) are six separate equations. When the field is a scalar field, only the diagonal entries are nonzero ($g_{ab} = 0$ it $a \neq b$ for a scalar field), but it is still four separate equations. This leads to a serious problem when trying to solve (17) for a scalar field. Let us explain the problem.

2. The General Relativity Theory really is wrong

For Cartesian coordinates the nonzero Ricci entries R_{aa} and the Ricci scalar $R = g^{aa} R_{aa}$ of a scalar field ψ are: (we write x_i instead of x^i so that indices are not confused with powers)

$$R_{00} = -\psi^{-1}\Box\psi + \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 + 3\psi^{-2}\left(\frac{\partial\psi}{\partial x_0}\right)^2 - 2\psi^{-1}\frac{\partial^2\psi}{\partial x_0^2}$$

$$R_{ii} = \psi^{-1}\Box\psi - \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 + \psi^{-2}\left(\frac{\partial\psi}{\partial x_0}\right)^2 - 2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2}$$

$$+4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \qquad \text{for} \qquad \mathrm{i} = 1,2,3$$

$$R = \psi^{-2}R_{00} - \psi^{-2}R_{11} - \psi^{-2}R_{22} - \psi^{-2}R_{33} = -6\psi^{-3}\Box\psi. \tag{18}$$

Let the space be empty with one point mass at the center. The Schwarzschild solution is an exact solution of (17) in this situation. In the derivation of the Schwarzschild solution the tensor $T_{ab} = 0$ is set to zero outside the origin and is zero in the calculation. Schwarzschild understood correctly, $T_{ab} = 0$ outside the origin because in GRT

$$T_{ab} = -2\frac{\delta L_{matter}}{\delta g^{ab}} + g_{ab}L_{matter} \tag{19}$$

If matter is all concentrated to the origin, then $T_{ab} = 0$ outside the origin and is zero in the calculation of the Einstein equations. Every $g_{aa} = g$ is equal. Thus,

$$R_{aa} = \frac{1}{2}Rg + \lambda g. \tag{20}$$

for every $a = 0, 1, 2, 3$. Let us take R_{ii} and R_{jj}, $i \neq j$, $i, j \in \{1, 2, 3\}$, cancel all common terms in $R_{ii} = R_{jj}$ coming from (20), and then we get an equation that cannot be satisfied.

First we subtract

$$R_{00} - R_{ii} = (\frac{1}{2}R + \lambda)(g - g) = 0 \tag{21}$$

and then we insert R_{00} and R_{ii} from (18) and move all terms that are common to every $i = 1, 2, 3$ to the left

$$\begin{aligned} R_{00} + \psi^{-1}\Box\psi - \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 - \psi^{-2}\left(\frac{\partial\psi}{\partial x_0}\right)^2 \\ = 2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2} - 4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \end{aligned} \tag{22}$$

The terms that are different for R_{ii} and R_{jj} give the equation

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial\psi}{\partial x_j}\psi^{-2}\right). \tag{23}$$

Equations (23) are solved by any function the form $\psi = \psi(\rho)$ where $\rho = \sum x_j$, but the solution we need is close to the radially symmetric Newtonian gravitation field in this special case of a single point mass in the origin, as that is the case in our solar system. See [6] for a proof that this is not possible. The problem can be explained easily. A solution for this situation must be close to the Newtonian gravitation potential, which is spherically symmetric. Thus, the solution is very closely spherically symmetric. Applying the left side of (23) to a function of

$r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ gives a function of the form $x_i^2 f(r) + h(r)$. The right side gives the function $x_j^2 f(r) + h(r)$ with the same f and h. We can see that they cannot cancel and no small addition to the Newtonian potential can give a large enough term that the equation (23) can hold. Instead, if we sum all three values of i together, as is done in calculation of R, the terms x_i^2 add to r^2 and R can be zero.

Let us locate the error in Einstein's derivation of the Einstein equations. He claimed to have obtained (17) from the Lagrangian

$$L = \frac{c^4}{16\pi G}\left(R - 2\lambda\right) - L_{matter}. \tag{24}$$

Our L_{matter} is zero outside the origin and we set $lambda = 0$. Then the Langangian is the Ricci scalar curvature R multiplied by a constant that we can forget. The Euler-Lagrange equations are

$$-\frac{\partial}{\partial_\mu}\left(\frac{\partial L}{\partial(\partial_\mu \psi)}\right) + \frac{\partial L}{\partial \psi} = 0 \tag{25}$$

We calculate these terms for the scalar field from (18).

$$\frac{\partial R_{ii}}{\partial(\partial_i \psi)} = -\psi^{-2} 2\partial_i \psi + 4\psi^{-2} 2\partial_i \psi = 6\psi^{-2}\partial_i \psi$$

$$\frac{\partial R_{ii}}{\partial(\partial_j \psi)} = -\psi^{-2} 2\partial_j \psi$$

$$\frac{\partial R_{ii}}{\partial(\partial_0 \psi)} = \psi^{-2} 2\partial_0 \psi$$

$$\frac{\partial R_{00}}{\partial(\partial_0 \psi)} = 6\psi^{-2}\partial_0 \psi$$

$$\frac{\partial R_{ii}}{\partial(\partial_0 \psi)} = 2\psi^{-2}\partial_0 \psi$$

$$\frac{\partial R}{\partial \psi)} = 18\psi^{-4}\Box\psi. \tag{26}$$

From these results we obtain for the Ricci scalar curvature R, which is our L in this case

$$R = \psi^{-2}(R_{00} - R_{11} - R_{22} - R_{33})$$

the result

$$\frac{\partial R}{\partial(\partial_\mu \psi)} = 0$$

for all $\mu = 0, 1, 2, 3$. Consequently the Euler-Lagrange equations reduce to

$$\frac{\partial R}{\partial \psi)} = 18\psi^{-4}\Box\psi = 0 \qquad (34)$$

i.e., to Nordström's field equation (5) in this empty space situation

$$\Box\psi = 0 \qquad (35).$$

Of course, we would get the same result by directly inserting R to (24), but the goal was to show that the Ricci entries in (17) really are not obtained from Euler-Lagrange equations.

The Lagrangian gives Nordström's field equation, not Einstein equations. Einstein must have solved the Euler-Lagrange equations incorrectly, though it is very difficult to imagine how he could have ever come to (17) from (24)-(25). As the error in GRT is now demonstrated by an example and also by Einstein's incomprehensible assumed derivation of (17) from (25) applied to (24), we can forget GRT. It is wrong. In [6] it is explained why the gravitational field must be a scalar field or the speed of light in vacuum is not c in every point to every direction. But we do not need this result in order to reject GRT. GRT is derived by incorrect calculation of variations: (25) does not give (17), and in our solar system the gravitational field is very close to the Newtonian potential. Such a field cannot be obtained as a solution of (17).

Also the Special Relativity Theory (SRT) is seriously wrong, see [6][7], Einstein did not project the time values to the time axis in the moving frame in the Lorentz transform as one must do. This error and the twin paradox in the Lorentz transform invalidate the whole SRT. Before going to Nordström's dynamical equation, let us look at time dilations.

3. Time dilations from Nordström's gravitation theory

Nordström's gravitation theory has the gravitational time dilation and the acceleration time dilation. The acceleration time dilation does not agree with Einstein's equivalence principle. Instead, it agrees with the observed time difference in GPS satellites.

The gravitational time dilation in Nordström's gravitation theory is simply that time time unit in a gravitational field ψ can be read from the metric ds^2

$$ds^2 = c^2\psi^2 dt^2 - \psi^2 dx^2 - \psi^2 dy^2 - \psi^2 + dz^2 \qquad (36)$$

The the gravitational field of a mass M makes the length of one second to the length

$$\psi^2 = \frac{GM}{c^2 r} = \frac{1}{c^2}\phi \tag{37}$$

in the distance of r from the mass center. This equation is scaled to give seconds.

The acceleration time dilation is derived as follows. Constant acceleration a leads to the distance $s = (1/2)at^2$ in time t. Work is force times length (or integral actually) $F = ma$, $W = Fs$, thus

$$\frac{W}{m} = as = \frac{1}{2}a^2t^2 = \frac{1}{2}v^2. \tag{38}$$

What time dilation this work density could cause? A gravitational field is also energy divided by mass and it produces the gravitational time dilation. A field

$$\phi = \frac{E_p}{m} \tag{39}$$

lenghtens one second to

$$\frac{1}{c^2}\phi. \tag{40}$$

The acceleration time dilation rule that gives the correct time for GPS satellite time difference is that the ratio work to mass

$$\frac{W}{m} = \frac{1}{2}v^2 \tag{41}$$

lenghtens one second to

$$\frac{v^2}{c^2} \tag{42}$$

This rule works not only in the GPS case but also explains the muon longer lifetime in the muon-laboratory experiment. The time dilation of SRT can be removed from the theory. The calculations for the GPS satellite clock delay are in [11].

4. The dynamic equation in Nordström's gravitation theory

The second equation that Nordström gave in his theory is a direct modification of

$$F = ma \qquad\qquad and \qquad\qquad F = \nabla\phi. \tag{43}$$

In GRT we write $\phi, b = \partial_b\phi$ where ∂_b is $\partial/\partial x_b$ (using a lower index for clarity). The notation $\dot{u_a}$ means a derivative with respect to the proper time

$$\dot{u_b} = \frac{d}{d\tau}u_b \tag{44}$$

where $\tau = c^2\psi dt$ is the proper time scaled by c^2, to get seconds remove the c^2, ψ is a plain number and dt is seconds. The dynamic equation is very much what is should be, a denotes acceleration and u is velocity

$$F = ma = -m\nabla\phi$$

The minus sign here is because the acceleration is towards decreasing r.

$$a = -\nabla\phi$$

$$\frac{d}{dt}u_b = -\phi_{,b}$$

$$\frac{c^2\psi}{c^2\psi}\frac{d}{dt}u_b = c^2\psi\frac{d}{d\tau}u_b = -\phi_{,b}$$

$$\psi\frac{d}{d\tau}u_b = -\psi_{,b}. \tag{45}$$

The only modification Nordström made is that if the field can change in time, there should be a term containing $\dot{\psi}$. He inserted it into the equation in a natural way

$$\frac{d}{d\tau}\psi u_b = -\psi_{,b}$$

$$\psi\dot{u_b} = -\psi_{,b} - \dot{\psi}u_b. \tag{46}$$

Equation (46) is the dynamical equation and it hardly could be anything else.

We come now to the first false claim: that the Nordström's theory predicts the precession of the perihelion of Mercury in the wrong direction. This hardly is possible. The gravitational field of the Sun is static, so $\dot{\psi} = 0$. The field equation (5) in Nordstrom's theory gives exactly the Newtonian gravitational potential and the dynamic equation is exactly the Newtonian dynamic equation.

How could Nordström's theory predict anything else than what the Newtonian theory says? Yet, I did find a strange calculation in [3] that claims it does. The problem of the precession of Mercury's perihelion is not solved by Nordström's two equations. I did write a text on this problem that may explain it a bit [8], no new theory is needed to explain the precession.

The second false claim against Nordström's gravitation theory is that light does not bend in his theory. Nordström did not consider light. He only gave the field equation and the dynamic equation for a test mass in a gravitational field. But Nordström naively believed that the Special Relativity Theory is correct (which it is not, see [6][7]) and in that theory light has (moving) mass and is attracted by gravitational fields. Thus, in Nordström's theory light would behave like a test mass. GRT has a different, and better, theory for lights movement: light moves along geodesics of the gravitational field. There is no problem in adding this postulate to Nordström's gravitational theory and then light moves in Nordström's theory just like in GRT. However, if we mean the light bending experiment by Eddington, then it is shown in [12] that light does not travel along geodesics of the gravitational field, see also [8] to be convinced that Einstein's geodesic Lagrangean is completely wrong: a freely falling mass in a spherical gravitational field does not accelerate, so should it be true that in Nordström's theory light does not bend, this would be a correct prediction by that theory. But Nordström did not consider this case at all. The bending of light in Eddington's experiment is caused by some medium around the Sun that bends light. Nordström's theory agrees with Eddington's experiment at least to the extent that it gives a gravitational field that is essentially the same as in Newton's theory while Einstein's GRT does not. Einstein's equations do not have any solution that can approximate the at least very closely Newtonian gravitation potential close to the Sun. Nordström's theory has exactly that potential.

5. A suggestion for further work

It is possible to generalize Nordström's field equation to give a different gravitational potential in the case of an empty space with a point mass in the origin. The following field equation may not be the most elegant, but it does the job.

$$\Box\phi - 6GM\beta\frac{1}{r^5} = -4\pi G\rho(r). \tag{47}$$

$$M = \int dV\rho$$

This field equation gives the solution

$$\phi = -\frac{GM}{r}\left(1 - \frac{\beta}{r^3}\right). \tag{48}$$

There is some reason for investigating a potential like in (48). Such a potential may explain how the moving mass of SRT can be removed from the theory, see [7], and it may help in explaining a strange issue in planet orbits related to the precession of Mercury's perihelion, see [8], but the real reason is that it may help to find a connection between the gravitational field and the Higgs field. I thouched this question in [9]. Reference [10] shows e.g. that Nordström's theory passes the Shapiro delay test while GRT fails it. In [11] there are calculations of the clock delation in GPS satellites and also a brief look at the muon-laboratory experiment.

Nordström's gravitation theory is as close to a valid geometrized gravitation theory as can be achieved. It is not correct. The field equation should have Δ, not $\Box$, but in a geometric theory Δ comes from R and R has $\Box$ as (9) shows. A fortcoming paper from the author will clarify this issue. The implications of this issue is that the geometrization idea fails.

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3.5 The error in the General Relativity Theory and its cosmological considerations

Abstract:

Einstein wrote a new chapter to the 1953 edition of his book *The Meaning of Relativity* on Friedman's results from the field equation in the General Relalitity claimed to have cosmological implications. The book, originally published in 1922, is based on Einstein's lectures of the Relativity Theory in Princeton. It is especially enlightening for getting an idea of how his views on the Relativity Theory had changed in his later life. They did not change much and the basic errors were never corrected by Einstein. This chapter to the 1953 edition demonstrates clearly that Einstein cheated intentionally, which gives this text special importance. The article explains in the first and second sections the model used by Friedman and what the basic error in the General Relativity Theory is. Section 2 gives the solutions of the Einstein equations in Friedman's case. Section three highlightens the unexplainable differences of the correctly calculated equations with the equations that Einstein gives in this chapter, indicating intentional fraud. The last section, section four, discusses the principles in the General Relativity Theory and finds them lacking.

1. The Friedman model for cosmological considerations

Chapter 5 in [1] states that Einstein presents Friedman's results, but the text is from Einstein's pen and the errors are on his responsibility. Friedman, and Einstein in his chapter, considers a scalalar field of the product type

$$\psi = A(r)G(t). \tag{1}$$

The Ricci tensor entries for this scalar field in spherical coordinates (r, θ, ϕ) are:

$$R_{00} = \frac{A''}{A} + \left(\frac{A'}{A}\right)^2 + \frac{2}{r}\frac{A'}{A} - \frac{3}{c^2}\frac{G''}{G} + \frac{3}{c^2}\left(\frac{G'}{G}\right)^2 \tag{2}$$

$$R_{11} = -\left(3\frac{A''}{A} - 3\left(\frac{A'}{A}\right)^2 + \frac{2}{r}\frac{A'}{A} - \frac{1}{c^2}\frac{G''}{G} - \frac{1}{c^2}\left(\frac{G'}{G}\right)^2\right) \tag{3}$$

$$R_{22} = -r^2\left(\frac{A''}{A} + \left(\frac{A'}{A}\right)^2 + \frac{4}{r}\frac{A'}{A} - \frac{1}{c^2}\frac{G''}{G} - \frac{1}{c^2}\left(\frac{G'}{G}\right)^2\right) \tag{4}$$

$$R_{33} = -r^2\sin^2(\theta)\left(\frac{A''}{A} + \left(\frac{A'}{A}\right)^2 + \frac{4}{r}\frac{A'}{A} - \frac{1}{c^2}\frac{G''}{G} - \frac{1}{c^2}\left(\frac{G'}{G}\right)^2\right) \tag{5}$$

The Ricci scalar is

$$R = g^{ab}R_{ab} = A^{-2}G^{-2}\left(6\frac{A''}{A} + \frac{12}{r}\frac{A'}{A} - 6\frac{1}{c^2}\frac{G''}{G}\right) \tag{6}$$

For easier verification of these formulas, the nonzero elements of the metric tensor are $g_{00} = c^2(AG)^2$, $g_{11} = -(AG)^2$, $g_{22} = -r^2(AG)^2$, $g_{33} = -r^2\sin^2(\theta)(AG)^2$, the sixteen nonzero Christoffel symbols are: $\Gamma^0_{00} = G'G^{-1}$, $\Gamma^0_{10} = A'A^{-1}$, $\Gamma^0_{11} = c^{-2}G'G^{-1}$, $\Gamma^0_{22} = c^{-2}r^2G'G^{-1}$, $\Gamma^0_{33} = c^{-2}r^2\sin^2(\theta)G'G^{-1}$, $\Gamma^1_{00} = c^{-2}A'A^{-1}$, $\Gamma^1_{10} = G'G^{-1}$, $\Gamma^1_{11} = A'A^{-1}$, $\Gamma^1_{22} = -(r + r^2A'A^{-1})$, $\Gamma^1_{33} = -sin^2(\theta)(r + r^2A'A^{-1})$, $\Gamma^2_{20} = G'G^{-1}$, $\Gamma^2_{21} = r^{-1} + A'A^{-1}$, $\Gamma^2_{33} = -\sin(\theta)\cos(\theta)$, $\Gamma^3_{30} = G'G^{-1}$, $\Gamma^3_{31} = r^{-1} + A'A^{-1}$, $\Gamma^3_{32} = \cot(\theta)$.

Let us define another metric tensor by giving the nonzero elements as $\gamma_{00} = c^2$, $\gamma_{11} = -1$, $\gamma_{22} = -r^2$, $\gamma_{33} = -r^2\sin^2(\theta)$. This is simply flat metric expressed in spherical coordinates. We notice that every R_{ii}, $i = 0, 1, 2, 3$, gives R_{ii}/γ_{ii} of the type

$$\frac{R_{ii}}{\gamma_{ii}} = a_1\frac{A''}{A} + a_2\left(\frac{A'}{A}\right)^2 + \frac{a_3}{r}\frac{A'}{A} + \frac{a_4}{c^2}\frac{G''}{G} + \frac{a_5}{c^2}\left(\frac{G'}{G}\right)^2 \tag{7}$$

where a_i, $i = 1,,,7$, are real numbers. What is important in (7) is that r and t separate

$$\frac{R_{ii}}{\gamma_{ii}} = f(r) + g(t). \tag{8}$$

From (6) we see that the term $Rg_{ii} = R\psi^2$ is also of the same type (7) and r and t separate.

Thus

$$R_{ab} - \frac{1}{2}Rg_{ab} = \gamma_{ab}\left(\frac{R_{ab}}{\gamma_{ab}} - \frac{1}{2}R\psi^2\right) \tag{8}$$

is of the type (7) and r and t separate.

We sum the Einstein equations

$$R_{ab} - \frac{1}{2}Rg_{ab} = \kappa_0T_{ab} + \lambda g_{ab} \tag{9}$$

setting $\lambda = 0$ as Einstein does in his chapter. The result is

$$g^{ab}R_{ab} - g^{ab}\frac{1}{2}Rg_{ab} = \kappa_0g^{ab}T_{ab} \tag{10}$$

$$R - 2R = \kappa_0T \tag{11}$$

$$R = -\kappa_0T = -\frac{8\pi G}{c^4}T \tag{12}$$

where $T = g^{ab}T_{ab}$. For all scalar fields ψ holds

$$R = -6\psi^{-3}\Box\psi. \tag{13}$$

Here $\psi = A(r)G(t)$. Inserting (13) to (12) gives

$$\psi^{-1}\Box\psi = -4\pi G\frac{1}{3c^4}T\psi^2. \tag{14}$$

The left side in (14) is of the type (7), therefore $T\psi^2$ must also be of the type (7). Einstein's chapter states that the density is constant in Friedman's model and the equations in the chapter show that T_{ab} only appears as fixed constants. Therefore the model must have set the tensor T_{ab} in the following way. The nonzero tensor values are

$$T_{00} = c^2 b_0 \qquad T_{11} = -b_1,$$

$$T_{22} = -r^2 b_2 \qquad T_{33} = -r^2\sin^2(\theta)b_3 \qquad \text{with} \qquad \mathrm{b}_3 = \mathrm{b}_2 \tag{15}$$

where b_i, $0, 1, 2$, are numbers. Notice that T_{33} has the same value $b_3 = b_2$ as T_{22}. It is because the equation

$$\frac{R_{ii}}{\gamma_{ii}} - \frac{1}{2}R\psi^2 = \frac{T_{ii}}{\gamma_{ii}} = b_i \tag{16}$$

is the same for $i = 2$ and $i = 3$.

In classical equations the mass density is ρ in the Poisson equation (i.e. Gauss equation)

$$\Delta\psi = -4\pi G\rho \tag{17}$$

but what is constant in Friedman's mode cannot be ρ. The density that can be constant is

$$\rho\psi^{-1} = \frac{1}{3c^4}T\psi^2 \tag{17}$$

because $T\psi^2$ gives the form (7).

We can now return to λ. Einstein implicitly sets $\lambda = 0$ in his chapter. That is the only choice. The term λg_{ii} is not of type (7). It is of type $F(r)H(t)$ because $g_{ii}/\gamma_{ii} = A(r)^2G(t)^2$. If λ is not zero, we get an equation of the type

$$f(r) + g(t) = F(r)G(t). \tag{18}$$

Equation (18) implies that either $F(r)$ and $f(r)$ are constants or $H(t)$ and $h(t)$ are constants. In order to see that, select two values t_1 and t_2, then

$$h(t_1) - h(t_2) = F(x)(H(t_1) - H(t_2)) \tag{19}$$

holds for any x. If $F(x)$ is not constant, $H(t)$ must be constant implying that $h(t)$ is constant, while if $F(x)$ is constant, then $f(x)$ must also be constant. The term λg_{ab} in the Einstein equations is an error. The term could only be $\lambda\gamma_{ab}$.

The Einstein equations in the situation of the chapter are

$$2\frac{A''}{A} + \frac{4}{r}\frac{A'}{A} - \left(\frac{A'}{A}\right)^2 = 3c^{-2}\left(\frac{G'}{G}\right)^2 - b_0 \tag{20}$$

$$\frac{4}{r}\frac{A'}{A}+3\left(\frac{A'}{A}\right)^2=2c^{-2}\frac{G''}{G}-c^{-2}\left(\frac{G'}{G}\right)^2-b_1 \tag{21}$$

$$2\frac{A''}{A}+\frac{2}{r}\frac{A'}{A}-\left(\frac{A'}{A}\right)^2=2c^{-2}\frac{G''}{G}-c^{-2}\left(\frac{G'}{G}\right)^2-b_2 \tag{22}$$

The last equation (22) from R_{33} is the same as (21) from R_{22}.

2. Solution of the Einstein equations

It will be shown in this section that equations (20)-(23) cannot be satisfied and that the behavior of all equations is exponential.

Nordström generalized the classical equation (17) into the form

$$\psi^{-1}\Box\psi=-4\pi T \tag{23}$$

and Einstein expanded this equation to several independent equations (in this case to three independent equations) of the type

$$R_{ii}-\frac{1}{2}g_{ii}=\kappa_0 T_{ii} \tag{24}$$

which can be summed into (23) as in (10), but this division into three separate equations does not work, shown clearly by the functions R_{ii} not being close to zero for $\psi=r^{-1}$ and because of this the method not working even for the Newtonian potential, see [2].

Especially the product form field $\psi=A(r)G(t)$ cannot give a solution to (20)-(22), but Friedman and Einstein thought that it could give some information of possible solutions. This is not the case. We can solve these equations and notice that their behavior is not at all similar to what the chapter claims it would be. We do not get the cosmological results that Einstein's chapter claims.

Let us start from the first equation (20). We solve separately the radial and the temporal parts as the equations separate.

$$2\frac{A''}{A}+\frac{4}{r}\frac{A'}{A}-\left(\frac{A'}{A}\right)^2=C \tag{25}$$

First we use the rule

$$\frac{d}{dr}\left(A'A^{\alpha}\right)=A''A^{\alpha}+\alpha\left(A'\right)^2A^{alpha-1} \tag{26}$$

to change (25) into

$$y'+\frac{2}{r}y=\frac{C}{4}f \tag{27}$$

where

$$y=A''A^{-\frac{1}{2}},\qquad f=\int ydr=2A^{\frac{1}{2}} \tag{28}$$

We have a second order differential equation

$$rf'' + 2f' - \frac{C}{4}rf \tag{29}$$

which is easily solved by the Laplace transform

$$-\frac{d}{ds}\left(s^2F - sf(0) - f'(0)\right) + 2(sF - f(0)) + \frac{C}{4}F' = 0 \tag{30}$$

$$F' = -\frac{f(0)}{s^2 - \frac{C}{4}} \tag{31}$$

$$F' = -\frac{f(0)}{\sqrt{C}}\left(\frac{1}{s - \sqrt{C}/2} - \frac{1}{s + \sqrt{C}/2}\right) \tag{32}$$

$$F = -\frac{f(0)}{\sqrt{C}}\left(\ln(s - \sqrt{C}/2) - \ln(s + \sqrt{C}/2)\right) \tag{32}$$

Using the rule: if $\mathcal{L}\{\mathrm{f(t)}\} = \mathrm{F(s)}$ then $\mathcal{L}\{\mathrm{tf(t)}\} = -\mathrm{F'(s)}$ we get

$$f(r) = \frac{f(0)}{\sqrt{C}}\frac{1}{r}\left(\exp\left(\frac{1}{2}\sqrt{C}r\right) - \exp\left(-\frac{1}{2}\sqrt{C}r\right)\right) \tag{33}$$

and $A = f^2/4$

$$A(r) = \frac{f(0)^2}{4C}\frac{1}{r^2}\left(\exp\left(\sqrt{C}r\right) - 2 + \exp\left(-\sqrt{C}r\right)\right). \tag{34}$$

Notice that if this function grows, then it grows exponentially.

The temporal equation in (20) is simply (C here is different than in (25))

$$\frac{G'}{G} = C. \tag{35}$$

The solution is $G(t) = C_1\exp(Ct)$. Again the growth is exponential.

The spatial part of (21) is a second order equation for $y = A'A^{-1}$

$$\frac{4}{r}\frac{A'}{A} + 3\left(\frac{A'}{A}\right)^2 = C \tag{35}$$

$$y^2 + \frac{4}{3r}y - \frac{C}{3} = 0 \tag{36}$$

$$y = -\frac{2}{3r} \pm \sqrt{\frac{4}{9r^2} + \frac{C}{3}} \tag{37}$$

$$\ln A = \int y dr \tag{38}$$

$$A = C_1 r^{-\frac{2}{3}}\exp\left(\pm\int\sqrt{\frac{4}{9r^2} + \frac{C}{3}}dr\right). \tag{39}$$

The solution is exponential.

The temporal part of (21) is (C is different than before)

$$2\frac{G''}{G} - \left(\frac{G'}{G}\right)^2 = C. \tag{40}$$

We use the method in(26)-(28)

$$y = G'G^{-\frac{1}{2}} \qquad f = \int ydt = 2G^{\frac{1}{2}} \tag{41}$$

$$f'' = \frac{C}{4}f \tag{42}$$

There is no need for a Laplace transform:

$$f = C_1 \exp\left(\frac{\sqrt{C}}{2}t\right) + C_2 \exp\left(-t\frac{\sqrt{C}}{2}t\right) \tag{43}$$

and $G(t) = f^2/4$. This is also exponential.

The spatial part of (22) is a bit more difficult to calculate. The equation is

$$2\frac{A''}{A} + \frac{2}{r}\frac{A'}{A} - \left(\frac{A'}{A}\right)^2 = C \tag{44}$$

It is almost (25), but when we follows the steps (26)-(31) we get

$$F'\left(-s^2 + \frac{C}{4}\right) - sF = 0 \tag{45}$$

Then

$$\frac{F'}{F} = -\frac{s}{s^2 + C/4} = -\frac{\sqrt{C}}{4}\left(\frac{1}{s - \sqrt{C}/4} + \frac{1}{s + \sqrt{C}/4}\right) \tag{46}$$

$$F = \left(s^2 - \frac{C}{4}\right)^{-\sqrt{C}/4}. \tag{47}$$

It can be difficult to make the inverse Laplace transform analytically (though it can be made numerically) for a freely chosen C, but for instance for $C = 16$ it can be made

$$F = \frac{1}{s^2 - C/4} = \frac{1}{\sqrt{C}}\left(\frac{1}{s - \sqrt{C}/2} - \frac{1}{s + \sqrt{C}/2}\right) \tag{48}$$

$$f(r) = \frac{1}{\sqrt{C}}\left(e^{t\sqrt{C}/2} - e^{-t\sqrt{C}/2}\right) \tag{48}$$

and $A(r) = f^2/4$. This is also exponential.

The temporal part of (22) is

$$2\frac{G''}{G} - \left(\frac{G'}{G}\right)^2 = C \tag{49}$$

We have already calculated this in (40)-(43).

We can finally look at (12), the equation of the Ricci scalar

$$R\psi^2 = -\kappa_0 T\psi^2 \tag{50}$$

$$6\frac{A''}{A} + \frac{12}{r}\frac{A'}{A} - 6\frac{G''}{G} = b_0 + b_1 + 2b_2 \tag{51}$$

$$\frac{A''}{A} + \frac{2}{r}\frac{A'}{A} = \frac{G''}{G} + \frac{b_0 + b_1 + 2b_2}{6} \tag{52}$$

The spatial part is

$$rA'' + 2A' - CrA = 0 \tag{53}$$

Laplace transforming we get as in (29)-(33)

$$A(r) = \frac{A(0)}{\sqrt{4C}}\frac{1}{r}\left(\exp(\frac{1}{2}\sqrt{4C}r - \exp(-\frac{1}{2}\sqrt{4C}r\right) \tag{54}$$

This solution is also exponential. The temporal part is

$$\frac{G''}{G} = C \tag{55}$$

and natually it is exponential.

As a conclusion, the equations (20)-(22) do not give the same answer confirming that the equation set (20)-(23) does not have solutions, but we also notice that the equations have a certain form and the solutions to them are exponential both in r and in t. Also the equation for the Ricci scalar gives exponential solutions.

3. Einstein's claims in his chapter

Einstein does not solve the equations. He claims that the equations are (equation (3) in chapter 5 in [1])

$$-\frac{1}{r}\left(\frac{A'}{A}\right)' + \left(\frac{A'}{Ar}\right)^2 = 0 \tag{56}$$

$$-\frac{2A'}{Ar} - \left(\frac{A'}{A}\right)^2 - BA^2 = 0 \tag{57}$$

and then he gets the solution (equations (3a) and (3b) in chapter 5 of [1])

$$A = \frac{c_1}{c_2 + c_3 r^2} \qquad B = 4\frac{c_2 c_3}{c_1^2} \tag{58}$$

The way he supposedly derived these formulas is given in a very hazy way, while the Einstein equations and the Ricci scalar equation can be quite easily solved by any technical student who did not sleep on lectures on the Laplace transform.

We notice that Einstein has moved in (56) the term r^{-1} to the second derivative of A

$$\frac{1}{r}\left(\frac{A'}{A}\right)' = \frac{1}{r}\frac{d}{dr}\left(\frac{A'}{A}\right)$$
$$= \frac{1}{r}\frac{A''}{A} - \frac{1}{r}\left(\frac{A'}{A}\right)^2 \tag{59}$$

In equations (20) and (22) there is the second derivative of A, but the term r^{-1} is only a coefficient in the term $A'A^{-1}$, see the form of the expression in equation (7). By doing these changes Einstein modified the equations so that he got the result that he wanted (58).

It is not possible that this is anything other than intentional fraud. In 1952 when this chapter was written Einstein had spent decades with the Einstein equations and must have known that they cannot be solved with $\psi = A(r)G(t)$ and that this ψ gives exponential time and space behavior. Growing exponential behavior is not possible because then the expansion speed of the universe very fast exceeds the speed of light in vacuum. The Einstein equations do not in any way support the concept of an expanding universe, though that is what Einstein implies in his chapter.

Einstein also must have known that the speed of light is constant c in vacuum only if ψ is a scalar field because the speed of light to direction x_i, $i = 1, 2, 3$ in Cartesian local coordinates can be read from the space element. Light travels along light like world paths in relativity theory, thus $ds = 0$ in

$$ds^2 = c^2 g_{00} dx_0^2 - g_{11} dx_1^2 - g_{22} dx_2^2 - g_{33} dx_3^2 \tag{60}$$

The speed of light to the direction of x_i is obtained by setting $dx_j = 0$, $j \neq i$, $j \in \{1, 2, 3\}$

$$ds^2 = 0 = c^2 g_{00} dx_0^2 - g_{ii} dx_i^2 \tag{69}$$

$$c^2 = \frac{g_{ii}}{g_{00}}\frac{dx_i^2}{dx_0^2} = \frac{g_{ii}}{g_{00}} \tag{61}$$

as the differentials dx_i are Euclidean. Thus

$$g_{ii} = c^2 g_{00} \tag{62}$$

and we can define $\psi = c^{-1}\sqrt{g_{00}}$ and the field ψ is a scalar field.

The Einstein equations do not have any scalar solutions that are close to the Newtonian gravitational field in the situation of a point mass in empty space even if we allow the field to depend on (r, θ, ϕ, t), see the proof in [2]. In this situation the tensor T_{ab} is zero outside the origin, and we must set $\lambda = 0$ also in this case. Then every R_{ii} must be zero, but it is not possible for any scalar field that approximates the Newtonian gravitation: the potential field r^{-1} gives zero Ricci scalar but the Ricci tensor entries R_{ii} are nowhere close to zero.

The Schwarzschild solution does not have a scalar field and the speed of light is not c at every point to every direction. As the constant speed of light was a central issue for Einstein, he must have known that the Einstein equations do not have any solutions where the speed of light is constant c in vacuum at each point to each direction and the solution approximates Newtonian gravitation. But Einstein pretended that this is the case.

I cannot avoid the conclusion that Einstein cheated on purpose.

4. Some general considerations

The field must be a scalar field so that the speed of light is constant. Einstein's equations do not work and must be discarded: equation (23) cannot be split into several equations by using Ricci tensor entries. But also (23), originally from Nordström seems wrong. In it the Laplace operator Δ in (17) has been replaced by the D'Alembert operator $\Box$. It gives an unfortunate time behavior for time dependent ψ. In engineering the D'Alembert operator typically appears in problems of waves and the seacher solution is usually of the type $\psi(s - bt)$ where b is a constant, often imaginary. Let us give some examples of this type of nonseparable solutions

$$\Box\psi = \Delta\psi - c^{-2}\frac{\partial^2\psi}{\partial t^2} \tag{63}$$

Then

$$\psi(r,t) = -\frac{\rho_0}{r^2 - c^2t^2} \tag{64}$$

and

$$\psi(x,y,z,t) = \psi(x + y + z - \sqrt{3}ct) \tag{65}$$

both satisfy

$$\Box\psi = 0 \tag{66}$$

This type of solutions seem to have no physical sense and cannot at least easily be matched with what the gravitational field should be. I do not say that the situation is necessarily hopeless, in electro-magnetism there is the Coulomb force that is formally similar to the Newtonian gravitational force. Possibly by making the gravitational field complex and identifying the field that we see as $\psi\psi^*$ one could find some usage for the D'Alembert operator. But I have started to think that the time simply is not similar to space coordinates and the whole geometrization principle is wrong. It is not at all clear if the metric of the space is metric of the gravitational field.

For instance, does light travel along geodesics of the gravitational field? Eddington's experiment does not prove it because light does bend in medium and there can well be some medium close to the Sun. Let us take an example. Put a rather long cylinder standing on a plane. Then smoothen it so that it can be seen as a smooth mountain. Take two points A and B on the opposite sides of the base circle of this cylinder. The shortest way from A to B is not over the mountain. It goes a half circle very close to the plane level. Let this cylinder be transparent and instead of filling the cylinder with some medium that can bend light, we let the geometry inside the cylinder be such that the lenght of a straight line from A to B through this cylinder is as long as the trip from A to B by climbing the mountain up and then climbing it down. Now we have space geometry corresponding to a gravitational field. The shortest path from A to B goes around the half circle and minimizing the straight path through the cylinder from A to B does give this half circle. Which way does light go? A normal observation is that it will not go around the half circle. It will go through the straight line even though it is the longer way. It only takes longer for light to travel the way. The half circle is a geodesic of the geometry. Light would follow geodesics of space geometry, so maybe space geometry is not gravitational field geometry.

Light bends when there is medium, because at many points light gets absorbed and re-emitted to all directions. The wave front bends if the speed of light is different to different directions. What is the case in vacuum?

There are very few results and principles in the relativity theory that are valid and have some sense. The Lorentz transform is wrong because one must take the projection of the time t' of the moving frame R' to the t'-axis and then the speed of light is not c in the moving frame. The "Lorentz"-transform that Einstein used in the Minkowski space and in making the Einstein equations Lorentz covariant is not the same transform, but is also does not make the speed of light constant in all moving frames. There is nothing to be gained by these coordinate transforms and the whole of the Special Relativity Theory is wrong. Einstein's relativistic mass and his $E = mc^2$ proof are wrong. The principle of light traveling on geodesics of the gravitational field is in question. All verifications of the relativity theory have other explanations and they do not verify the theory. The principle that the space is four dimensional is questionable. Probably the only result that still remains is the gravitational time delay, but undoubtedly it also has other explanations. The Relativity Theory is a very bad theory. What is interesting in chapter five of Einstein's book [1] is that in that chapter it is clear that Einstein intentionally cheated. It gives some perspective to the theory.

5. References

[1] A. Einstein, The Meaning of Relativity, Princeton University Press, last edition 1955.

[2] J. Jormakka, The Essential Questions in Relativity Theory, preprint, ResearchGate, 2023.

PART 4. THE GEODESIC LAGRANGEAN

Einstein had the idea that he could replace the Lagrangean if Newtonian mechanics, derived from'total energy, by a geodesic Lagrangean. Well, it does not work. A test mass falling freely in a gravitational field does not accelerate. That is fatal. Let us first look at the perihelion of Mercury problem, it nicely illustrates the problem with a geodesic Lagrangean. Then we will see some other strange results.

4.1 A better solution the the precession of Mercury's perihelion

Abstract: The article shows that the explanation that Einstein gave to the precession of the perihelion of Mercury is incorrect: the dynamic equations he used do not even accelerate a falling stone, they cannot be used as an improvement of Newtonian mechanics. Then the article derives a formula for the precession speed and shows why most of the precession of Mercury can be explained by gravitational forces from other planets. But these forces change in time, the last section calculates a long time average of the effect of Jupiter on Mercury's precession speed. This effect is about one hundred times smaller than the relatively short term effect that has been measured. This means that actually Mercury's long term precession is much smaller than it seems to us based on our relatively short time series when the precession has been measured. This long term precession effect is quite on the range of the unexplained small part of Mercury's precession and it might be a mechanism that has not been considered. The last section shows a serious error in the relativistic calculation of the precession speed of Mercury.

1. Introduction

The precession of Mercury's perihelion has been measured to 5600 arcseconds in a century. Of this figure known mechanisms can explain at most 5557 archseconds when the error bounds of the estimated precession for each mechanisms is taken to the maximum limit. Still 43 archseconds in a century remain unexplained and there must exist some unknown or overlooked mechanism or mechanisms. Einstein gave a formula derived from the General Relativity Theory. This formula gives exactly 43 archseconds, which is rather surprising as it means that all known mechanisms did reach the maximum error limits. A figure that is a bit higher than 43 archseconds in a century would be more believable. Einstein's formula also predicts very well the precession of the perihelion of Venus, but it is not equally accurate in the precession speed of the Earth. There is no known reason why the formula would be less accurate in some cases.

Einstein used in his calculation a dynamic equation derived from a geodesics of the Schwarzschild metric. The first section proves that this approach cannot be used to calculate corrections to Newtonian gravitational theory because the same method that Einstein used for Mercury gives a dynamic equation for a stone falling from the Pisa tower. A stone falling according to Einstein's dynamic equation does not accelerate at all. As the method fails to explain the old Pisa stone dropping experiment, which Newtonian gravity quite correctly explains for all practical purposes, it cannot be considered as valid method for calculating fine corrections to Mercury's orbit. Einstein's formula must be seen as heuristic: it gives good results in some cases (there are only few planets and moons), but lacks a sound theoretical basis.

The second section of the presented article derives an equation for the precession speed and shows with a simple model that the equation fits well to the gravitational effect of Jupiter in the rather short time period when the precession of Mercury has been measured. The whole precession cycle is over 23,000 years, therefore full precession cycles have never been measured scientifically.

The third section calculates a long term gravitational effect of a planet on the precession of Mercury. The result shows that Jupiter's long term effect on the precession speed of Mercury is about one hundred times smaller than Jupiter's effect on the relative short time period when Mercury's precession has been measured. The long term effect is about 54 archseconds in a century and such long term effects may explain the missing 43 archseconds in a century, a value that more likely is a bit bigger than 43.

The fourth section looks at the way Einstein's formula for precession is derived. The section shows that the Lagrangian is incorrectly calculated, $\mathcal{L}$ is not constant. This invalidates the calculation of precession speed. Then the section shows that the curve that Einstein's Lagrangian gives is not a rotating ellipse and that it gives an impossible relation for the impulse momentum. In short, the geodesic Lagrangean is completely wrong and useless.

2. The error in Einstein's calculation

In General Relativity dynamic equations of a test mass are Euler-Lagrange equations calculated from a geodesic Lagrangean

$$\mathcal{L} = \sqrt{g_{ab}\dot{x}^a\dot{x}^b} \qquad \text{where} \qquad \dot{x}^a = \frac{d}{d\tau}x^a \tag{1}$$

and τ is the proper time. The Lagrangian is chose to have the value $\mathcal{L} = 1$ as it simplifies calculating the Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial x^a} - \frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{x}^a} = 0. \tag{2}$$

In the calculation of the precession of the perihelion of Mercury Einstein derived the equation of motion from a geodesic in the Schwarzschild metric, probably because the gravitational field must approximate the Newtonian gravitational field around the Sun. The field that the Sun creates seems to be time-independent and spherically symmetric at least to some rather high degree of precision. The only time-independent and spherically symmetic solution to the Einstein equations that can be considered as approximating Newtonian gravity in some sense is the Schwarzschild metric.

The Schwarzschild metric is defined as

$$c^2d\tau^2 = A(r)dt^2 - B(r)dr^2 - r^2d\theta^2 - r^2\sin^2(\theta)d\phi^2 \tag{3}$$

where

$$A(r) = c^2\left(1 - \frac{r_s}{r}\right) \qquad B(r) = \left(1 - \frac{r_s}{r}\right)^{-1} \tag{4}$$

and r_s is a constant called Schwarzschild radius. This metric describes the gravitational field created by a mass at the origin. We will denote this mass by M.

Let us find the equation of motion for a test mass m falling straight to the mass center at the origin. This means that $\dot{\phi} = 0$ and $\dot{\theta} = 0$. The Lagrangean is

$$\mathcal{L} = \sqrt{A(r)\dot{t}^2 - B(r)\dot{r}^2}. \tag{5}$$

We get Euler-Lagrange equations only for t and for r. For t

$$\frac{\partial \mathcal{L}}{\partial t} = 0 \tag{6}$$

as the field is time-independent, while

$$\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{t}} = \frac{d}{d\tau}\left(2A(r)\dot{t}\right)(2\mathcal{L})^{-1} = 0. \tag{7}$$

Notice how nice it is that $\mathcal{L} = 1$, the division with the square root is division with one.

The equation (7) implies that $A(r)\dot{t} = C_1$, a constant. As $A(r) = B(r)^{-1}$ in (4)

$$B(r) = \frac{\dot{t}}{C_1}. \tag{8}$$

Taking the partial derivative with respect to r from (8) gives

$$\frac{\partial}{\partial r}B(r) = \frac{\partial}{\partial r}\frac{\dot{t}}{C_1} = 0. \tag{9}$$

but as $B(r)$ is only a function of r,

$$0 = \frac{\partial}{\partial r}B(r) = \frac{d}{dr}B(r) = B'(r). \tag{19}$$

Thus, $B(r) = C_2$, a constant. This observation does not agree with (4), but we pretend not to know what $B(r)$ is, let us continue. Then $A(r) = B(r)^{-1} = C_2^{-1}$ is also a constant and

$$\dot{t} = C_1 C_2^{-1} \tag{20}$$

shows that

$$t = C_1 C_2^{-1}\tau + C_3 \tag{21}$$

where C_3 is yet another constant. Calculating the Euler-Lagrange equation for r we get

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{\partial}{\partial r}(A(r)\dot{t}^2 - B(r)\dot{r}^2)(2\mathcal{L})^{-1} = \frac{\partial}{\partial r}(C_2^{-1}\dot{t}^2 - C_2\dot{r}^2)2^{-1} = 0 \tag{22}$$

and therefore

$$\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{t}} = \frac{d}{d\tau}(2B(r)\dot{r})(2\mathcal{L})^{-1} = C_2\frac{d}{d\tau}\dot{r} = 0 \tag{23}$$

where we used $\mathcal{L} = 1$. Thus

$$r = C_4\tau + C_5 \tag{24}$$

for some constants C_4 and C_5. Proper times cannot be directly observed, but we can observe

$$\frac{d}{dt}r = \frac{d\tau}{dt}\frac{dr}{d\tau} = C_1 C_2^{-1} c_4. \tag{25}$$

That is a linear equation, thus

$$\frac{d^2}{dt^2}r = 0. \tag{26}$$

The stone does not accelerate while freely falling in a gravitational field.

This is not the only problem in the Schwarzschild metric and General Relativity. The Schwarzschild metric is not a valid metric at all: writing it in local Cartesian coordinates there are cross terms $dx_i dx_j$, $i \neq j$. Such cross terms cannot appear in any Riemannian metric with orthogonal coordinates and Cartesian local coordinates are orthogonal. The Schwarzschild metric does not converge to a Minkowski metric when the local environment shrinks. This is fatal: when the local environment is made smaller, curvature of the space decreases. The tangent space is flat and it should be a Minkowski space. It is not for the Schwarzschild metric. This is the reason why the speed of light is not constant in the Schwarzschild metric. In the Schwarzschild metric the speed of light sent horizontally has a speed that depends on the altitude, it would be measurable. The Einstein equations do not allow any spherically symmetric solution that has locally constant speed of light in vacuum. For proofs of these statements see [1]-[5].

3. Deriving a formula for the precession speed

An ellipse is defined by

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \tag{27}$$

where a and b are semi-major and semi-minor axes, $a \geq b > 0$. The focus points are $(-c, 0)$ and $(c, 0)$, $c \geq 0$, and in this article the rotation center is at $(-c, 0)$. Eccentricity is defined as $e = c/a$. Notice that $b^2 = a^2 - c^2$. Coordinates (x, y) are centered at origin. Polar coordinates (r_1, ϕ) are centered at $(-c, 0)$, thus

$$r_1 = \sqrt{(x + c)^2 + y^2} = ex + a \tag{28}$$

$$r_1 \cos(\phi) = x + c \qquad\qquad r_1 \sin(\phi) = y. \tag{29}$$

Solving r_1 from (28) and (29)

$$r_1 = e(r_1 \cos(\phi) - c) + a \tag{39}$$

gives

$$r_1 = a\frac{1 - e^2}{1 - e\cos(\phi)}. \tag{40}$$

The orbital velocity for an orbit that is in the (r_1, ϕ) plane is

$$\dot{x}^2 + \dot{y}^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 \tag{41}$$

$$\dot{y} = -\frac{b^2}{a^2}\frac{x}{y}\dot{x} \qquad \text{if } y \geq 0 \tag{42}$$

$$r_1\dot{\phi} = \frac{b^2}{a}\frac{1}{y}\dot{x} \qquad \text{if } y \geq 0 \tag{43}$$

Kepler's law is that the the angular momentum

$$L = r_1^2\dot{\phi} \tag{44}$$

is constant. It does not follow from the equation of an ellpise. It follows from Euler-Lagrangian equations for a test mass m_1 circulating a spherically symmetric gravitational field created by a mass m_2 at $(-c, 0)$. The Lagrangean function for dynamic equations should normally be the sum of kinetic and potential energies

$$\mathcal{L} = E_k(t, q_i, \dot{q}_i) + E_p(t, q_i, \dot{q}_i) = E. \tag{45}$$

In order to find the dynamic equations, we minimize the action integral

$$S = \int_{t_1}^{t_2} \mathcal{L}dt = E(t_2 - t_1). \tag{45}$$

It is quite fine that the Lagrangean has a constant value like the total energy E, compare to Einstein's Lagrangean at (7). The Euler-Lagrange equations give the dynamic equations that keep the total energy at the constant value E. As an example, on the Earth surface the potential energy at the height s is $E_p = mgs$ and the kinetic energy is $E_k = (1/2)m\dot{s}^2$. We get the correct equation of motion from the Lagrangean

$$\mathcal{L} = E_k + E_p \tag{46}$$

$$\frac{\partial \mathcal{L}}{\partial s} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{s}} \tag{47}$$

$$mg = m\ddot{s}. \tag{48}$$

There is no sense in minimizing the integral over time of a function

$$\mathcal{L} = E_k(t, q_i, \dot{q}_i) - E_p(t, q_i, \dot{q}_i) = T - V \tag{48}$$

that does not have a lower bound. However, if we use radial coordinates, like (r_1, ϕ), then the acceleration is $-\ddot{r}_1$ because the r_1 vector points outside. Then we must write the Lagrangean as in (48), but it is only a question of the direction of r_1. Thus, in (r_1, ϕ) coordinates we write the Lagrangean as

$$\mathcal{L} = \frac{1}{2}m_1(\dot{r}_1^2 + r_1^2\dot{\phi}^2) - E_p(t, r_1, \phi). \tag{49}$$

Then Kepler's law

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m_1 \frac{d}{dt} r_1^2 \dot{\phi} = 0 \tag{50}$$

means that

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{\partial}{\partial \phi} E_p(t, r_1, \phi) = 0. \tag{51}$$

This is true only if m_1 is an insignificant test mass that does not disturb the field with its own field which is circulating on an elliptic orbit and for sure the position of m_1 depends on ϕ.

That is, Kepler's law is only approximatively true for planets orbiting the Sun. As Kepler's law is one of the postulates of Newtonian mechanics, it is difficult to understand why some people have thought that Newtonian mechanics should give an exact result for such a very small effect as the precession speed of Mercury and if it does not, then there would be needed a new theory like Einstein's geodesic Lagrangean.

Assuming that the potential energy is of the type

$$E_p = -GMm_1 \frac{1}{r_1} \tag{52}$$

Kepler's law holds, $r_1^2 \phi = L$ is constant and we can solve the Euler-Lagrange equation for r_1:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}_1} = m_1 \ddot{r}_1$$

$$\frac{\partial \mathcal{L}}{\partial r_1} = m_1 r_1 \dot{\phi}^2 - GMm_1 \frac{1}{r_1^2}$$

$$\ddot{r}_1 = r_1 \dot{\phi}^2 - GM \frac{1}{r_1^2} \tag{53}$$

By using Kepler's law

$$\frac{d^2}{d\phi^2}\frac{1}{r_1} = \frac{d}{d\phi}\left(-\frac{\dot{r}_1}{\dot{\phi}}\frac{1}{r_1^2}\right) = -\frac{dt}{d\phi}\frac{d}{dt}\frac{\dot{r}_1}{L}$$

$$= -\frac{1}{\dot{\phi}L}\ddot{r}_1 = -\frac{r_1^2}{L^2}\ddot{r}_1 \tag{54}$$

and inserting to (53) gives an equation that r_1 in (40) fulfills

$$\frac{d^2}{d\phi^2}\frac{1}{r_1} + \frac{1}{r_1} = GM\frac{1}{L^2}. \tag{55}$$

Thus, the solution is an ellipse (40) and the angular momentum L is constant. In a gravitational field created by a point mass M the value of L is

$$L = \sqrt{GM}\frac{b}{\sqrt{a}} \tag{55}$$

if we assume that M is at the focal point $(-c, 0)$.

The orbital period is calculated as

$$T = \int_0^T dt = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\dot{\phi}} d\phi = 2\int_{-a}^{a} \frac{1}{\dot{x}} dx = \frac{b^2}{aL} 2\int_{-a}^{a} \frac{r_1}{y} dx$$

$$= \frac{b^2}{aL} 2\frac{a}{b}\pi = 2\pi\sqrt{\frac{a^3}{Gm}} \tag{56}$$

for L as in (55).

If the mass M is not at the focal point, then the mass used in (55) is different and the orbital period (56) is also different. We could in principle find out where the Sun is related to the focal point by measuring the orbital period, but planets are so small compared to the Sun that this may be impossible in practice.

The exact position of the Sun is another issue that adds an error in the classical solution. We have placed the Sun at the focal point, but the Sun actually cannot be exactly at the focal point. We can see it by thinking of two equal masses m_1 and m_2 circulating each others. By symmetry, the focal point must be at the center of mass. If m_1 is insignificat test mass and m_2 practically infinite, then m_2 is at the focal point. Between these two extreme situations the placement of the focal point must move continuously depending on the ratio of the masses. As the ratio of the mass of a planet and the Sun is not zero, the Sun cannot be exactly at the focal point.

It is also impossible that the focal point of the Sun-planet system is at the center of mass. If this were the case, then considering the two-body system Sun-Jupiter the Sun would be circulating the focal point with the orbital period of Jupiter. This means that every other planet that circulates the Sun would also have to circulate the same focal point and it would have to have the orbital period of Jupiter. This is not the case, planets have quite different orbital periods. Therefore the Sun must be much close to the focal point than it is in the coordinates where the focal point is the center of mass. The Sun must be so close to the focal point that the planets can have different orbital periods, yet the Sun cannot be exactly at the focal point.

This means that the movements of the planets are not quite separated, there is some small influence through the movement of the Sun. The Sun is in an orbit with some acceleration and if we choose a coordinate system (r_1, ϕ) where the Sun is at the focal point, then the origin of the coordinate system (r_1, ϕ) is accelerating and there are additional forces affecting the Sun.

The Sun, like Jupiter and Saturn, is not a solid mass, it is a gas ball and it compresses if a force is applied. In an accelerating orbit, or a coordinate system where the Sun is fixed but the coordinate system's origin is in accelerated orbit, there are acceleration forces. They do not need to do any work if the mass body is solid, but a gas ball compresses and the force makes work against forces that try to keep the mass body as spherical. The gas ball acts as a spring that is compressed by a force, it stores energy and at some other point it releases this energy. Therefore the total energy is not as in (26). There is additionally compressed energy. A small planet is reasonably solid and will not compress. As it mimics the movement of the Sun around the focal point but does not store energy by compression, it will have a mismatch between potential and kinetic energy: the sum of these energies is not constant, This mismatch is solved by precession of the orbit of the planet. This effect is outside Newtonian mechanics and requires understanding of how the Sun compresses under an acceleration force.

We see many small effects that can cause that Newtonian mechanics cannot give a precise result for the precession speed of Mercury. We now proceed to derive an equation for the precession speed.

Let us assume that the coordinates (r_1, ϕ) rotate around the focal point with angular velocity ω:

$$\phi = \phi_1 + \omega t$$

and we assume that the orbit is sufficiently close to an ellipse in (r_1, ϕ_1)-coordinates, i.e.,

$$r_1 = a\frac{1-e^2}{1-e\cos(\phi_1)}.$$

We also assume the following conditions:

A1. All energy is in kinetic and potential energy, so no compression energy.

A2. The Sun is at the focal point of both planets we consider: Mercury and Jupiter.

A3. Kepler's law holds at the perihelion at $r_{1,min}$ and aphelion at $r_{1,max}$.

A4. The only effect causing precession of Mercury is that other planets change the gravitational force.

A3 means that the speed $v_{\phi_1,max}$ at the perihelion relates to the speed of $v_{\phi_1,min}$ at the aphelion as

$$v_{\phi_1,max} = \frac{r_{1,max}}{r_{1,min}} v_{\phi_1,min} \tag{57}$$

The speeds in the perihelion and aphelion in (r_1, ϕ) relate to the speeds in (r_1, ϕ_1) as

$$v_1 = v_{\phi_1,max} + r_{1,min}\omega \tag{58}$$

$$v_2 = v_{\phi_1,min} + r_{1,max}\omega$$

Using (57) we get

$$v_1^2 - v_2^2 = \frac{r_{1,max}^2 - r_{1,min}^2}{r_{1,min}^2} v_{\phi_1,min}^2 - (r_{1,max}^2 - r_{1,min}^2)\omega^2$$

$$= \frac{4ac}{(a-c)^2} v^2_{\phi_1,min} - 4ac\omega^2. \tag{59}$$

The assumption that in (r_1, ϕ_1) the orbit is an ellipse means that at the perihelion and aphelion where $y = 0$ we can calculate the centrifugal force as

$$\dot{x} = -\frac{a^2}{b^2} \frac{y}{x} \dot{y} \tag{58}$$

$$\ddot{x} = -\frac{a^2}{b^2} \frac{1}{x} \dot{y}^2 - \frac{a^2}{b^2} y \frac{d}{dt} \frac{\dot{y}}{x} \tag{59}$$

$$\ddot{x}|_{y=0} = \mp \frac{a^2}{b^2} \frac{1}{a} \dot{y}^2 \tag{60}$$

The absolute value of the centrifugal force at the perihelion is

$$F_{c,1} = m_1 \frac{a}{b^2} v_1^2 \tag{61}$$

and at the aphelion

$$F_{c,2} = m_1 \frac{a}{b^2} v_2^2$$

We assume that the gravitational force at the perihelion is

$$F_{g,1} = \alpha_1 G m_1 m_2 \frac{1}{r^2_{1,min}} \tag{62}$$

and at the aphelion

$$F_{g,2} = \alpha_2 G m_1 m_2 \frac{1}{r^2_{1,max}}$$

where α_1, α_2 describe the change of the gravitational force because of other planets. Thus,

$$v_1^2 - v_2^2 = G m_2 \frac{b^2}{a} \frac{\alpha_1}{(a-c)^2} - G m_2 \frac{b^2}{a} \frac{\alpha_2}{(a+c)^2} \tag{63}$$

$$= G m_2 \frac{1}{a} \frac{\alpha_1 (a+c)^2 - \alpha_2 (a-c)^2}{a^2 - c^2}. \tag{64}$$

From (61) and (62) comes

$$v_2 = \sqrt{G m_2} \sqrt{\frac{a-c}{a+c}} \sqrt{\frac{\alpha_2}{a}}. \tag{65}$$

Equations (59) and (64) give two expressions for the left side of the equations. Inserting (65) gives after some manipulation a second order equation for ω

$$\omega^2 - 2\omega \frac{b}{4ac} \sqrt{G m_2} \sqrt{\frac{\alpha_2}{a}} + \frac{b^2}{(4ac)^2} G m_2 \frac{\alpha_1 - \alpha_2}{a} \tag{66}$$

The solution is

$$\omega = \frac{b}{4ac}\sqrt{Gm_2}\sqrt{\frac{\alpha_2}{a}}\left(1-\sqrt{2-\frac{\alpha_1}{\alpha_2}}\right) \tag{67}$$

The values to be inserted to (67) are: the gravitation constant $G = 6.6743 * 10^{-11} m^3 kg^{-1} s^{-2}$, the mass of the Sun $m_2 = 1.9891 * 10^{30} kg$, for Mercury: semi-major axis $a = 5.7895 * 10^{10} m$, $e = 0.206$, $c = ea = 1.1926 * 10^{10} m$, semi-minor axis $b = \sqrt{a^2 - c^2} = 5.6653 * 10^{10} m$, $r_{a,max} = a + c$, $r_{a,min} = a - c$. The measured precession of 5600 archseconds in a century is $\omega = 8.6 * 10^{-12} s^{-1}$.

We can assume that α_1, α_2 are small and express them as $\alpha_i = 1 - \gamma_i$. In the first order

$$1-\sqrt{2-\frac{\alpha_1}{\alpha_2}} = \frac{1}{2}(\gamma_1 - \gamma_2) \tag{68}$$

and the first order approximation for ω is

$$\omega = \frac{b}{8ac}\sqrt{Gm_2}\sqrt{\frac{1}{a}}(\gamma_1 - \gamma_2).$$

Inserting numbers

$$\omega = 4.9 * 10^{-7}(\gamma_1 - \gamma_2)\ s^{-1}. \tag{69}$$

We notice that (70) is very much what we would expect: $\gamma_1 - \gamma_2$ should be $3.51 * 10^{-5}$ to give the measured value. Jupiter is about thousand times smaller than the Sun and its orbit is about ten times larger than that of Mercury, therefore the gravitational force from Jupiter to Mercury should be about 1/1000*100 of that of the Sun. This is just the 10^{-5} size. Let us make a very elementary estimation of the effect of Jupiter on Mercury's perihelion and aphelion. At the perihelion the gravitational field from Jupiter might be roughly

$$-Gm_J \frac{1}{a_J + r_{1,min}}$$

and at the aphelion roughly

$$-Gm_J \frac{1}{a_J - r_{1,max}}$$

where a_J is the semi-major axis of Jupiter, $a_J = 77.8473 * 10^{10} m$ and $m_J = 1.898 * 10^{27} kg$ is the mass of Jupiter. Then

$$\alpha_1(-Gm_2\frac{1}{r_{1,min}}) = -Gm_2\frac{1}{r_{1,min}} + Gm_J\frac{1}{a_J + r_{1,min}}$$

$$\alpha_1 = 1 - \frac{m_J}{m_2}\frac{r_{1,min}}{a_J + r_{1,min}} \tag{70}$$

and

$$\alpha_2 = 1 - \frac{m_J}{m_2}\frac{r_{1,max}}{a_J - r_{1,max}}$$

$$\gamma_1 - \gamma_2 = \frac{m_J}{m_2}\left(\frac{r_{1,min}}{a_J + r_{1,min}} - \frac{r_{1,max}}{a_J - r_{1,max}}.\right)$$

Notice that $\gamma_1 - \gamma_2 < 0$, so ω is negative, opposite to what we observe. We can ignore this issue because the example only demonstrates the strength of Jupiter's influence. We should put Jupiter and Mercury to different positions to get the direction of ω correct. Ignoring the sign, the strength is correct:

$$|\gamma_1 - \gamma_2| = \frac{m_J}{m_2}\frac{2a^2 - 2c^2 + 2ca_J}{(a_J + c)^2 - a^2} = 3.863 * 10^{-5}.$$

This gives the precession speed

$$\omega = 4.9 * 10^{-7}\frac{1}{2}3.863 * 10^{-5}\ s^{-1} = 9.46\ s^{-1}$$

which is not bad for such a simple approximation. Using better approximations 19th century astronomers managed to explain over 99% of the measured 5600 archseconds, mostly with the effect of the other planets.

Thus, tracking the positions of the other planets one can get quite good approximations for the measured precession speed of Mercury. The size of the measured ω is quite on the range of effects of planets, but here comes a caveat. The time series of Mercury's perihelions and aphelions is relatively short. Mercury is at the perihelion 415 times in a century and precise measurements have been made maybe for 500 years. There cannot be much more than some 2000 perihelion points in the record. Compare this to the presumed length of the precession cycle. With 5600 archseconds in a century one full cycle takes over 23,000 years. Nobody has ever measured a single full cycle. There is no good reason to assume that Mercury ever makes a full precession cycle. Instead, there is a reason to suspect that planet orbits only wobble and do not make full precession cycles: why else should the orbits of all planets be now pointing to roughly the same direction.

The question of what in reality is the precession speed of Mercury is not answered by experimental measurements. From measurements we only get the precession speed at this our time. In some thousand years the precession speed can be quite different. The planerary system has existed for billions of years. If there is a long term force, e.g., from Jupiter or other planets, that gives Mercury some small precession speed, then this speed continues in our times because of conservation of angular momentum, while we cannot see in our time any force that causes this precession. This may be the origin of the 43 archseconds.

Let us next calculate what is the long term effect of Jupiter on Mercury's precession. The result may be surprising: the long term effect is one hundred times smaller than the effect we see now. The forces that cause the effect now cancel each others when the observation time is very long, but all forces do not cancel, there remains long term effects. The long term effect of Jupiter on Mercury's perihelionic precession is quite in the range of this missing 43 archseconds. We get roughly 54 archseconds in a century and should remember that this 43 archseconds is the minimum unexplained precession component: the unexplained part can be longer because in order to get this 43 archseconds every explanation has been pushed to its limits, to explain as much as possible. It is not likely that every mechanism should explain up to its maximum limits.

3. Long term effect of a planet on the precession speed of Mercury

The average gravitational field at a place $(h, 0)$ caused by a mass body m moving on an elliptic orbit with constant angular momentum is

$$\phi_{ave}(x_1, x_2) = -Gm\frac{1}{W(x_1, x_2)}\int_{x_1}^{x_2}\frac{r_1}{y}\frac{1}{s}dx \tag{71}$$

where r_1 is in (28), y and x in (27)

$$s = \sqrt{(x-h)^2 + y^2} = r_1\sqrt{1 - x\frac{2(c+h)}{r_1^2} + \frac{h^2 - c^2}{r_1^2}} \tag{72}$$

and the weight W is from the orbital time formula

$$W(x_1, x_2) = \int_{x_1}^{x_2}\frac{r_1}{y}dx. \tag{73}$$

The integral (71) can be calculated to the desired precision from a series expansion of (72). Let the numbers in (71) correspond to the orbit of Jupiter.

The semi-major axis $a = 77.8473 * 10^{10}m$, semi-minor axis $b = 77.7549 * 10^{10}m$, $c = 3.79116 * 10^{10}m$, $e = 0.0487$. The values of h that are of interest to us are $h_1 = -(c + r_{1,min})$, $r_{1,min} = 4.5969 * 10^{10}m$ and $h_2 = r_{1,max} - c$, $h_2 = r_{1,max} = 6.982 * 10^{10}m$ are the perihelion and aphelion values for Mercury. We assume that the orbits of Mercury and Jupiter have the same focal point and the Sun is at this point. The parameter x in (72) ranges from $-a$ to a. We can insert the numbers to (72) and notice that the term under the square root is

$$\sqrt{1+z} = 1 + \frac{1}{2}z + \frac{3}{8}z^2 + O(z^3) \tag{74}$$

and that $|z|$ is smaller than 0.18. This means that in order to get two significant numbers (error in 10^{-3}) we need a second order approximation in (72). This precision is sufficient for us.

Integral (71) can be calculated with the transform

$$\int\frac{dx}{yr_1^k} = -\frac{a^k}{b^{2k}}\int\frac{(ez+1)^{k-1}dz}{\sqrt{1-z^2}} \tag{75}$$

where

$$z = -a\frac{x+c}{cx+a^2} \tag{76}$$

and either partially integrating or cancelling one term r_1 in the denominator.

In the second order approximation we need

$$\int \frac{dx}{y} = \frac{a}{b} \arcsin\left(\frac{x}{a}\right) \tag{77}$$

$$\int_{-a}^{a} \frac{dx}{y} = \frac{a}{b}\pi$$

$$\int \frac{dx}{yr_1} = \frac{a}{b^2} \arcsin\left(a\frac{x+c}{cx+a^2}\right) \tag{76}$$

$$\int_{-a}^{a} \frac{dx}{yr_1} = \frac{a}{b^2}\pi$$

$$\int \frac{dx}{yr_1^2} = \frac{a^2}{b^3}\left(\left(x+\frac{a^2}{c}\right)^{-1}\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{b}\arcsin\left(a\frac{x+c}{cx+a^2}\right)\right) \tag{77}$$

$$\int_{-a}^{a} \frac{dx}{yr_1^2} = \frac{a^2}{b^4}\pi$$

$$\int \frac{dx}{yr_1^3} = -\frac{a^3}{b^6}\left(-\left(1+\frac{1}{2}e^2\right)\arcsin(z) + \sqrt{1-z^2}\left(\frac{1}{2}e^2 z + 2e\right)\right) \tag{78}$$

$$\int_{-a}^{a} \frac{dx}{yr_1^3} = \frac{a^3}{b^6}\pi\left(1+\frac{1}{2}e^2\right)$$

$$\int \frac{dx}{yr_1^4} = \frac{a^4}{b^8}\left(-\left(1+3e^2\right)\arcsin(z) + \sqrt{1-z^2}\left(3e + \frac{3}{2}e^2 z + \frac{1}{3}e^3 z^2 + \frac{2}{3}e^3\right)\right) \tag{79}$$

$$\int_{-a}^{a} \frac{dx}{yr_1^4} = \frac{a^4}{b^8}\left(1+3e^2\right)\pi$$

$$\begin{aligned}\int \frac{Ax^2+Bx+C}{yr_1^2} &= A\frac{a^2}{c^2}\int\frac{dx}{y} + \left(B\frac{a}{c} - A\frac{2a^2}{c^2}\right)\int\frac{dx}{yr_1} \\ &+ \left(A\frac{a^4}{c^2} - B\frac{a^2}{c} + C\right)\int\frac{dx}{yr_1^2}\end{aligned} \tag{80}$$

$$\begin{aligned}\int_{-a}^{a} \frac{Ax^2+Bx+C}{yr_1^2} &= A\pi\frac{a}{e^2 b}\left(1 - 2\frac{a}{b} + \frac{a^3}{b^3}\right) \\ &+ \left(B\pi\frac{a}{c} - A\frac{2a^3}{c^2}\right) + C\pi\frac{a^2}{b^4}\end{aligned} \tag{81}$$

$$\int \frac{Ax^2+Bx+C}{yr_1^4} = A\frac{a^2}{c^2}\int \frac{dx}{yr_1^2} + \left(B\frac{a}{b} - A\frac{2a^2}{c^2}\right)\int \frac{dx}{yr_1^3}$$
$$+\left(A\frac{a^3}{c^2} - B\frac{a^2}{c} + C\right)\int \frac{dx}{yr_1^2} \tag{82}$$

$$\int_{-a}^{a} \frac{Ax^2+Bx+C}{yr_1^2} = A\pi\frac{a^2}{e^2b^4}\left(1 - 2\frac{a^2}{b^2}\left(1+\frac{1}{2}e^2\right) + \frac{a^4}{b^4}\left(1+\frac{3}{2}e^2\right)\right)$$
$$+B\pi\frac{a^3}{eb^6}\left(1+\frac{1}{2}e^2 - \frac{a^2}{b^2}\left(1+\frac{3}{2}e^2\right)\right)$$
$$+C\pi\frac{a^4}{b^8}\left(1+\frac{3}{2}e^2\right) \tag{83}$$

The second order approximation is

$$\frac{1}{s} = \frac{1}{r_1}\left(1 + (c+h)\frac{x}{r_1^2} + \frac{1}{2}(c^2-h^2)\frac{1}{r_1^2} + (c+h)^2\frac{x^2}{r_1^4}\right)$$
$$+\frac{1}{r_1}\left(-\frac{3}{4}(ch^2 - hc^2 + h^3 - c^3)\frac{x}{r_1^4} + \frac{3}{8}(c^2-h^2)^2\frac{1}{r_1^4}\right). \tag{84}$$

Integrating gives

$$\int_{-a}^{a} \frac{r_1 dx}{ys}$$
$$= \frac{a}{b}\pi - (c+h)\frac{a}{b^2}\pi\frac{e}{1-e^2} + \frac{1}{2}(c^2-h^2)\frac{a^2}{b^4}\pi$$
$$+\frac{3}{2}(c+h)^2\frac{a^2}{b^4}\pi\left(\frac{3}{2}\frac{a^4}{b^4} - \frac{a^2}{b^2} + \frac{e^2}{(1-e^2)^2}\right)$$
$$-\frac{3}{4}(ch^2 - hc^2 + h^3 - c^3)\frac{a^3}{b^6}\pi e\left(-\frac{1}{1-e^2} + \frac{1}{2} - \frac{3}{2}\frac{a^2}{b^2}\right)$$
$$+\frac{3}{8}(h^2-c^2)^2\frac{a^4}{b^8}\pi\left(1+\frac{3}{2}e^2\right) \tag{85}$$

Derivating the second order approximation of the field with respect to h gives an approximation of the force, but notice that we have not yet divided by W, so the result is not yet force. We wll drop terms with e^2 because the approximation has an error term of the size 10^{-3} and for Jupiter $e^2 = 2.3 * 10^{-3}$.

$$I = \frac{d}{dh}\int_{-a}^{a} \frac{r_1 dx}{ys}$$

$$
\begin{aligned}
&= -\frac{a}{b^2}\pi\frac{e}{1-e^2} - h\frac{a^2}{b^4}\pi \\
&+3(c+h)\frac{a^2}{b^4}\pi\left(\frac{3}{2}\frac{a^4}{b^4} - \frac{a^2}{b^2}\right) \\
&-\frac{3}{4}(2ch - c^2 + 3h^2)\frac{a^3}{b^6}\pi e\left(-\frac{1}{2} - \frac{3}{2}\frac{a^2}{b^2}\right) \\
&+\frac{3}{2}h(h^2 - c^2)\frac{a^4}{b^8}\pi
\end{aligned} \tag{86}
$$

Constant forces cancel when we calculate $v_1^2 - v_2^2$ in (63). Therefore we drop them:

$$
\begin{aligned}
I &= -h\frac{a^2}{b^4}\pi \\
&+3h\frac{a^2}{b^4}\pi\left(\frac{3}{2}\frac{a^4}{b^4} - \frac{a^2}{b^2}\right) \\
&-\frac{3}{4}(2ch + 3h^2)\frac{a^3}{b^6}\pi e\left(-\frac{1}{2} - \frac{3}{2}\frac{a^2}{b^2}\right) \\
&+\frac{3}{2}h(h^2 - c^2)\frac{a^4}{b^8}\pi
\end{aligned} \tag{87}
$$

We take the leading term of (87) as the other terms are clearly smaller:

$$
I = h\frac{a^2}{b^4}\pi\left(-1 + 3\frac{a^2}{b^2}\left(\frac{3}{2}\frac{a^2}{b^2} - 1\right)\right)
$$

$$
= h\frac{a^2}{b^4}\pi\left(-1 + 3\frac{a^2}{b^2}\left(\frac{1}{2}\frac{a^2}{b^2} - \frac{-e^2}{1-e^2}\right)\right)
$$

Dropping e^2 terms

$$
= h\frac{a^2}{b^4}\pi\left(-1 + \frac{3}{2}\frac{a^4}{b^4}\right)
$$

Dividing with W and obtaining the force

$$
W = \int_{-a}^{a}\frac{r_1 dx}{y} = \frac{a^2}{b}\pi
$$

$$
F = -Gm_1 m\frac{I}{W} = Gm1m2\frac{h}{b^3}\left(1 - \frac{3}{2}\frac{a^4}{b^4}\right)
$$

Since

$$
\frac{a^4}{b^4} = {\frac{1}{1-e^2}}^2 = 1 + 2e^2 + O(e^4)
$$

we simplify the force to

$$
F = -Gm1m\frac{h}{2b^3}. \tag{88}
$$

This force comes from potential of the type h^2, but that is because of the approximation that we used. The force has a fixed value at both values of h that we are interested in. We find a potential that is of the correct type

$$\psi = -Gm\frac{1}{r}$$

and gives the same force at h_1 and h_2. We now denote the values for Jupiter by an index. Thus, in (89) $b = b_J$, $m = m_J$, $c = c_J$ not to confuse them with the values for Mercury:

Thus, at $c_J + h = -r_{1,min}$

$$\psi_1 = -Gm\frac{-r_{1,min} - c_J}{2b^3}r_{1,min} \tag{89}$$

and at $r_{1,max} = c_J + h$

$$\psi_2 = -Gm\frac{r_{1,max} - c_J}{2b^3}r_{1,max} \tag{90}$$

Then we still have to get α_i as in (70) and γ_i.

$$\gamma_1 = \frac{m_J}{m_2}\frac{(-r_{1,min} - c_J)r_{1,min}^2}{2b_J^3}$$

$$\gamma_2 = \frac{m_J}{m_2}\frac{(r_{1,ax} - c_J)r_{1,min}^2}{2b_J^3}$$

Now we can estimate the size of the long term effect of Jupiter on the periheliotic precession of Mercury:

$$\gamma_1 - \gamma_2 = -\frac{m_J}{m_2}\frac{-r_{1,min}^3 - c_J r_{1,min}^2 - r_{1,max}^3 + c_J r_{1,max}^2}{2b_J^3}$$

$$= \frac{m_J}{m_2}\frac{a^3\left(1 + 3e^2 - 2e\frac{c_J}{a}\right)}{b_J^3}$$

Inserting numbers $\frac{m_J}{m_2} = 0.9542 * 10^{-3}$, $\frac{a^3}{b_J^3} = 4.127 * 10^{-4}$ and $1 + 3e^2 - 2e\frac{c_J}{a} = 0.8575$. The result is $\gamma_1 - \gamma_2 = 3.3625 * 10^{-7}$ and $\omega = 4.9 * 10^{-7} * 1.813 * 10^{-7}\ s^{-1} = 16.5 * 10^{-14}$ which is about 107 archseconds per century.

Jupiter's year is about 12 years, so the planet is at each place in its orbit every 12th year, but in the calculation we also assume that Mercury is at its perihelion and that this perihelion is in a particular place with Mercury's orbit pointing to the same direction as that of Jupiter. This assumption is not fully valid even today and when Mercury precesses more, this assumption cannot hold. Let us take half of 107 archseconds per century as a rough estimate to account for the angle between the semi-major axes of Mercury's orbit and Jupiter's orbit. Thus, the predicted precession is about 54 archseconds in a century.

This is my proposal for an unknown mechanism that can cause precession of Mercury's perihelion. There must exist some unknown or ignored mechanism that explains the 43 archseconds, and probably a bit more. Einstein's explanation cannot be correct. It is difficult to find some mechanisms that has not been considered, but there are very long time effects, all forces do not cancel even in a very long time. The solar system has had billions of years time and such long time effects have been compensated by precession because the energy budget must hold. If such a long term calculation shows that there is an energy inbalance, it must result to something that fixes it, like to very small precession. In a relatively short observation period, like some hundred years, we cannot see these long term mechanisms. The short term mechanisms are much stronger because forces do not cancel. Constant potential terms that come out of the integration in (85) do not mean constant potential in anything than in the average. At each time moment the potential that is shown as not dependent on h in (85) is a potential that has a clear gradient pointing to Jupiter. This is why there appears these hundred times larger forces than in the average.

4. A serious error in Einstein's formula for the precession

Einstein's calculation, or a form of it that seems to be used today for teaching students, can be found from Owen Biesel's paper [6]. The paper derives the Schwarzschild metric, but let as start from the point where the geodesic Lagrangean appears to the calculations

$$\mathcal{L} = -\left(1 - \frac{R_s}{r}\right)\dot{T}^2 + \left(1 - \frac{R_s}{r}\right)^{-1}\dot{r}^2 + r^2\dot{\phi}^2. \tag{91}$$

[6] says that $\mathcal{L} = -1$. If this is so, then he can use this Lagrangean instead of

$$\mathcal{L} = \sqrt{\left(1 - \frac{R_s}{r}\right)\dot{T}^2 - \left(1 - \frac{R_s}{r}\right)^{-1}\dot{r}^2 - r^2\dot{\phi}^2}. \tag{92}$$

as the square root term $(2\mathcal{L})^{-1}$ is 2^{-1}. Let us assume $\mathcal{L} = -1$ and calculate like [6]. [6] notices that

$$\frac{\partial \mathcal{L}}{\partial T} = \frac{\partial \mathcal{L}}{\partial \phi} = 0. \tag{93}$$

Therefore the Euler-Lagrange equations for T and ϕ give

$$\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{T}} = \frac{d}{d\tau}2\left(1 - \frac{R_s}{r}\right)\dot{T} = 0$$

$$\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{d}{d\tau}2r^2\phi = 0$$

and [6] gets Kepler's law and the energy conservation law:

$$L = r^2\dot{\phi} \tag{94}$$

$$E = \left(1 - \frac{R_s}{r}\right)\dot{T}$$

are constants. Then the paper again uses the assumption that $\mathcal{L} = -1$ inserting (94) to $\mathcal{L}$ and solving $\dot{r}^2$

$$\dot{r}^2 = E^2 - 1 + \frac{R_s}{r} - \frac{L^2}{r^2} + \frac{R_s L^2}{r^3} \tag{95}$$

Writing

$$\dot{r} = \frac{dr}{d\tau} = \frac{d\phi}{d\tau}\frac{dr}{d\phi} = \dot{\phi}r' = \frac{L}{r^2}r'$$

[6] gets (95) to the form

$$(r')^2 = \frac{E^2-1}{L^2}r^4 + \frac{R_s}{L^2}r^3 - r^2 + R_s r \tag{96}$$

There are four points when $r' = 0$, two of them being $R_+ = a+c$ and $R_- = a-c$, the aphelion and perihelion points of Mercury. Here $c = ea = 1.1926 * 10^{10}m$ and not the speed of light. One root is $r = 0$ and the fourth root [6] denotes by ϵ, but let us denote it by R_4 just to remind that it is meters. Thus

$$\frac{E^2-1}{L^2}r^4 + \frac{R_s}{L^2}r^3 - r^2 + R_s r = \frac{1-E^2}{L^2}r(R_+ - r)(r - R_-)(r - R_4). \tag{97}$$

[6] solves E^2 and L^2 using the two roots $R_+ = a+c$ and $R_- = a-c$. Then the paper calculates an integral

$$\int_{R_-}^{R_+} \frac{dr}{\sqrt{r(R_+ - r)(r - R_-)(r - R_4)}}$$
$$= \int_{R_-}^{R_+} \frac{dr}{r\sqrt{(R_+ - r)(r - R_-)(1 - R_4 r^{-1})}}.$$

A first order approximation is made

$$\frac{1}{\sqrt{1 - \frac{R_4}{r}}} = 1 + \frac{1}{2}\frac{R_4}{r} + \text{error term}.$$

Einstein's precession speed formula comes from the integral

$$I = \frac{R_4}{2}\int_{R_-}^{R_+} \frac{dr}{r^2\sqrt{(R_+ - r)(r - R_-)}} \tag{98}$$

The integral gives

$$I = \frac{1}{\sqrt{R_+ R_-}}\frac{\pi R_4}{4D}$$

where

$$D = \frac{R_+ R_-}{R_+ + R_-} = \frac{b^2}{2a}.$$

We used here $R_+ + R_- = (a+c) + (a-c) = 2a$ and $R_+ R_- = a^2 - c^2 = b^2$. Inserting D we get

$$I = \frac{1}{b} \frac{\pi R_4 2a}{4b^2} = \frac{R_4}{2} \frac{a}{b^3} \pi. \tag{99}$$

[6] notices that

$$\frac{L^2}{1-E^2} = \frac{R_+ R_{-1}}{1 - \frac{R_s}{D}}$$

and taking the constant from the polynomial (97) shows that

$$R_4 = \frac{R_s}{1 - \frac{R_s}{D}}.$$

The final result that gives the exact 43 missing arch seconds is

$$\phi_+ - \phi_- = \int_{R_-}^{R_+} \frac{dr}{r'} = \sqrt{\frac{L^2}{1-E^2}} \int_{R_-}^{R_+} \frac{dr}{\sqrt{r(R_+ - r)(r - R_-)(r - R_4)}}$$

Using the approximation the result is

$$\phi_+ - \phi_- = \frac{1}{\sqrt{1 - \frac{R_s}{D}}} \left(\pi + \frac{1}{\sqrt{R_+ R_-}} \frac{R_4}{2} \int_{R_-}^{R_+} \frac{dr}{r^2 \sqrt{(R_+ - r)(r - R_-)}} \right) \tag{100}$$

which gives Einstein's formula

$$\phi_+ - \phi_- = \frac{\pi}{\sqrt{1 - \frac{R_s}{D}}} \left(1 + 4 \frac{R_s}{1 - \frac{R_s}{D}} \right). \tag{101}$$

Thus, this formula does come from the Lagrangean, but it does not help. There is a serious error in the assumption that $\mathcal{L} = -1$. Let us assume it is so and calculate the Euler-Lagrange equation for r that [6] did not do. Thus,

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{1}{r^2 \left(1 - \frac{R_s}{r}\right)} \left(-R_s E^2 - R_s \dot{r}^2 + \frac{2L^2}{r} \left(1 - \frac{R_s}{r}\right)^2 \right) \tag{102}$$

and

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{r}} = \frac{1}{r^2 \left(1 - \frac{R_s}{r}\right)} \left(-2R_s \dot{r}^2 + 2\ddot{r} \left(1 - \frac{R_s}{r}\right) \right). \tag{103}$$

From (103) we get

$$\ddot{r} = \frac{R_s}{2r(r - R_s)} \left(-E^2 + \dot{r}^2 \right) - \frac{L^2}{r^3} + \frac{L^2 R_s}{r^4} \tag{104}$$

Inserting $\dot{r}^2$ from (95) to (104)

$$\ddot{r} = \frac{R_s}{2r(r-R_s)}\left(-1+\frac{1}{r}\left(R_s-\frac{2L^2}{R_s}+2L^2\right)+\frac{L^2}{r^2}\right) \tag{105}$$

We get another equation for $\ddot{r}$ by derivating (95) with respect to τ

$$2\dot{r}\ddot{r} = -\frac{R_s}{r^2}\dot{r}+2\frac{L^2}{r^3}\dot{r}-3\frac{R_sL^2}{r^4}\dot{r}$$

$$\ddot{r} = -\frac{R_s}{2r^2}+\frac{L^2}{r^3}-\frac{3}{2}\frac{R_sL^2}{r^4} \tag{106}$$

If $\mathcal{L}=-1$, then

$$-1+\frac{1}{r}\left(R_s-\frac{2L^2}{R_s}+2L^2\right)+\frac{L^2}{r^2}$$

equals

$$= \frac{2r(r-R_s)}{R_s}\left(-\frac{R_s}{2r^2}+\frac{L^2}{r^3}-\frac{3}{2}\frac{R_sL^2}{r^4}\right)$$

$$= -1+\frac{1}{r}\left(R_s+\frac{2L^2}{R_s}\right)-\frac{L^2}{r^2}+3\frac{R_sL^2}{r^3}$$

We see that they are not equal. The assumption $\mathcal{L}=-1$ is wrong. $\mathcal{L}$ is not constant and therefore the Euler-Lagrange equations for this geodesic Lagrangean are wrong. For a correct calculation of geodesics in the Schwarzschild metric,see [7]-[9]. The geodesic equations have long and difficult expressions.

Let us still investigate what is the curve that Einstein's geometric Lagrangean gives. It is not a rotating ellipse. A rotating ellipse has the formula

$$r = \frac{a(1-e^2)}{1-e\cos(\phi-\omega t)} \tag{107}$$

Assuming that ω is small, the orbital time when Mercury is circling the Sun is closely approximated by

$$T = 2\pi\sqrt{\frac{a^2}{GM}} \tag{108}$$

and L is closely approximated by

$$L = \sqrt{\frac{GM}{a}}b \tag{109}$$

Derivating r we get

$$r' = -\frac{e}{a(1-e^2)}\sin(\phi-\omega t)\left(1-\frac{\omega}{\dot{\phi}}\right)r^2.$$

There are two zeros in the range $0\le\phi\le\pi$. They are the zeros of $\sin(\phi-\omega t)$ and they are

$$\phi_- = 0 \qquad \phi_+ = \pi+\omega\frac{T}{2}.$$

The other zeros are not possible: $r = 0$ does not happen on the orbit of the ellipse and $1 - \omega(\dot{\phi})^{-1} = 0$ does not happen when ω is small.

Eliminating $\sin(\phi - \omega t)$ by using the following equation derived from (107)

$$\cos(\phi - \omega t) = \frac{1}{e}\left(1 - \frac{a(1-e^2)}{r}\right)$$

we get after some manipulation

$$r'^2 = \frac{r^2}{a^2(1-e^2)}(R_+ - r)(r - R_-)\left(1 - \frac{\omega}{\dot{\phi}}\right)^2$$

where $R_+ = a + c = a(1+e)$, $R_- = a - c = a(1-e)$. Thus

$$r' = \frac{1}{a\sqrt{1-e^2}} r\sqrt{(R_+ - r)(r - R_-)}\left(1 - \frac{\omega}{\dot{\phi}}\right)$$

Noticing that $a\sqrt{1-e^2} = b$

$$r' = \frac{1}{b} r\sqrt{(R_+ - r)(r - R_-)}\left(1 - \frac{\omega}{\dot{\phi}}\right) \tag{110}$$

This is an equation of a rotating ellipse.

Einstein has in (96)-(97)

$$r' = \sqrt{\frac{1-E^2}{L^2}} r\sqrt{(R_+ - r)(r - R_-)}\sqrt{1 - \frac{R_4}{r}}.$$

Noticing that

$$\sqrt{\frac{1-E^2}{L^2}} = \frac{\sqrt{1 - \frac{R_s}{D}}}{b} = \frac{1}{b}\sqrt{1 - \frac{2aR_s}{b^2}}$$

and that $R_4 = R_s/(1 - R_s/D)$ is very close to $R_s << r$ we can approximate

$$r'^2 = \frac{1}{b} r\sqrt{(R_+ - r)(r - R_-)}\sqrt{1 - \frac{2aR_s}{b^2}}\sqrt{1 - \frac{R_4}{r}}. \tag{111}$$

$$= \frac{1}{b} r\sqrt{(R_+ - r)(r - R_-)}\left(1 - R_s\left(\frac{a}{b^2} + \frac{1}{2r}\right)\right) + \text{error term} \tag{112}$$

Notice that (111) is not an equation of a rotating ellipse and that the approximation (112) comparing it to (110) gives a first order approximation

$$\frac{\omega}{\dot{\phi}} = R_s\left(\frac{a}{b^2} + \frac{1}{2r}\right) \tag{113}$$

which is totally impossible because though Kepler's law

$$L = r^2\dot{\phi} \tag{114}$$

need not hold exactly, it certainly is a very good approximation in the rotating coordinates (r, ϕ).

We calculate as in [6] eliminating $\dot{\phi}$ in (110) by (114)

$$\int_{\phi_-}^{\phi_+} d\phi = \int_{R_-}^{R_+} \frac{1}{\frac{dr}{d\phi}} dr = \int_{R_-}^{R_+} \frac{dr}{r'}$$

$$= b \int_{R_-}^{R_+} \frac{dr}{r\sqrt{(R_+ - r)(r - R_-)}\left(1 - \frac{\omega}{\dot{\phi}}\right)}$$

and notice that the insertation $x = (a/c)(r - a)$, $r_1 = r$, changes

$$\int_{R_-}^{R_+} \frac{dr}{r^\alpha \sqrt{(R_+ - r)(r - R_-)}} = \frac{a}{b} \int_{-a}^{a} \frac{dx}{y r_1^\alpha}$$

for any α. Taking a first order approximation

$$\left(1 - \frac{\omega r^2}{L}\right)^{-1} = 1 + \frac{\omega r^2}{L} + \text{error term}$$

we get a good approximation

$$\phi_+ - \phi_- = b \int_{-a}^{a} \frac{dx}{y r_1} + b\frac{\omega}{L} \int_{-a}^{a} \frac{r_1 dx}{y}.$$

We give some more formulas:

$$\int \frac{r_1 dx}{y} = -\frac{ca}{b}\sqrt{1 - \frac{x^2}{a^2}} + \frac{a^2}{b} \arcsin \frac{x}{a}$$

$$\int_{-a}^{a} \frac{r_1 dx}{y} = \frac{a^2}{b}\pi$$

$$\int \frac{r_1^2 dx}{y} = -\frac{c}{b}\left(2a^2 + \frac{1}{2}cx)\right)\sqrt{1 - \frac{x^2}{a^2}} + \frac{a^3}{b}\left(1 + \frac{1}{2}\frac{c^2}{a^2}\right) \arcsin \frac{x}{a}$$

$$\int_{-a}^{a} \frac{r_1^2 dx}{y} = \frac{a^3}{b}\pi\left(1 + \frac{1}{2}e^2\right)$$

The second formula we do not need here, but it is nice to know. We get

$$\phi_+ - \phi_- = b\left(\frac{\pi}{b} + \frac{\omega}{L}ab\pi\right)$$

and inserting L from (109)

$$\phi_+ - \phi_- = \pi + \pi\sqrt{\frac{a^3}{GM}}\omega.$$

The result is

$$\omega = \frac{\phi_+ - \phi_- - \pi}{\pi\sqrt{\frac{a^3}{GM}}} = \frac{\phi_+ - \phi_- - \pi}{\frac{1}{2}T}$$

as it should be, showing that the errors in the approximations cancel nicely.

In order to reject Einstein's formula, it is enough to compare (110) and (111). Whatever (111) is, it is not what it should be: a rotating ellipse. It gives an impossible result (113).

5. References

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4.2 A proof that light does not travel on the geodesics of the gravitationa field

Abstract:

Arthur Eddington's experiment in 1919 showed empirically that light bends close to the Sun. This experiment supposedly proved that Einstein was correct in the General Relativity Theory: light bends in a gravitational field. However, the General Relativity Theory claims that light travels on geodesics of the gravitational field. The article presents a simple mathematical proof that the geodesics of the gravitational field around the Sun, or around the Earth, are either on equipotential surfaces or they are radial. The proof calculates geodesics between two points by solving the Euler-Lagrange equations, i.e., by finding a minimal length path. Light does not follow geodesics of the gravitational field. This result is not surprising: already old-time sailors knew quite well that light does not follow equipotential surfaces of the Earth's gravitational field: light goes straight, the sea bends as the Earth is round, and because of this high masts disappear from sight, even from telescopic sight, when you go far enough to the sea.

1. Introduction

Arthur Eddington in 1919 announced having observed light beiding close to the Sun. Light wavefront does bend in an optically active medium and there are reasons to believe that around the Sun there is such a medium. This is the natural explanation for Eddington's findings. However, Eddinton's experiment has been taken as a verification that Einstein's idea in the General Relativity Theory is correct, that light bends in a gravitational field. In Einstein's General Relativity Theory light bends because it follows geodesics of the gravitational field. Thus, the theory claims more than that light only bends in a gravitational field. If light does not follow geodesics of the gravitational field, then the theory is wrong. The article shows that light does not follow geodesics of the gravitational field, the General Relativity Theory is wrong also in this claim.

In order to prove this we need a model for the gravitational field in the vicinity of the Sun. We cannot get such a model from the General Relativity Theory because of the following reason. It is proven in [3] and [4] that the gravitational field in the General Relativity Theory must be a scalar field so that the metric gives a Minkowski space to the tangent space at each point and the speed of light is constant at each point to each direction. This condition cannot be dropped because if c is not locally constant in a gravitational field, then the theory is in contradiction with its basic assumptions and also with measurements of the speed of light on the Earth. The Einstein equations do not have scalar field solutions that approximate Newtonian gravitation in the case of a point mass in empty space, see [3], [5] and [6]. Therefore the best and only gravitation field model that can be used for investigating if light follows geodesics of the gravitational field is to use the Newtonian gravitation potential field.

Einstein did use the Schwarzschild solution in several verifications of the General Relativity Theory, for instance, in his book [1] when discussing the precession of the perihelion of Mercury Einstein refers to the Schwarzschild solution. Unfortunately this solution cannot be used for any investigation of gravitation. It is shown in [10] that the Schwarzschild metric is not a valid metric and therefore the solution is not a valid solution to the Einstein equations. The Schwarzschild metric is not a metric, it is only a spherically symmetric differential form.

The article solves the Euler-Lagrange equations for a geodesic in the case of a point mass in empty space, a model that well fits to investigating gravitational light bending in the gravitational field of the Sun, and finds that a geodesic connecting two points that have the same distance r to the center of a point mass are segments of a circle with the radius r centered as the point mass. Needless to say, light does not make a half circle around a mass. It goes straight, as can be seen on the sea: the sea bends as the Earth is round, but light goes straight. This very old observation was probably the first indication that the Earth is round. The article also explains what are geodesics of the gravitational field of a point mass in the general case.

2. Calculation of geodesics

In a flat 2-dimensional space a geodesic between points A and B is solved by minimising

$$S = \int_A^B \sqrt{dx^2 + dy^2} = \int_a^b \sqrt{1 + y'^2} dx \tag{1}$$

where a and b are the x-coordinates of A and B. Minimizing S is done by solving the Euler-Lagrange equations. The Lagrangian is

$$L(x, y, y') = \sqrt{1 + y'^2} \tag{2}$$

and the Euler-Lagrange equations are

$$\frac{\partial L}{\partial y} - \frac{d}{dx}\frac{\partial L}{\partial y'} = 0. \tag{3}$$

In the General Relativity Theory, the length of a path from $A = (-s, 0, 0)$ to $B = (s, 0, 0)$ (in Cartesian coordinates) in a scalar gravitational field $\phi(x, y, z)$ centered at the origin and having the Newtonian form

$$\phi(r) = -\frac{GM}{r} \tag{4}$$

is

$$S = \int_A^B -\phi(x, y, 0)\sqrt{dx^2 + dy^2} = GM \int_{-s}^{s} \frac{\sqrt{1 + y'^2}}{\sqrt{x^2 + y^2}} dx. \tag{5}$$

Section 1 gives refereces that prove that the field must be scalar, that the General Relativity Theory does not give any solutions that can approximate the gravitational field around the Sun or the Earth, and that the Schwarzschild solution is wrong. We are left with the Newtonian gravitation potential (4) as the only reasonable model for a gravitational field around the Sun or the Earth.

But there is an even better reason why the gravitation field must be the Newtonian field. Section 2 of [4] proves that the Newtonian gravitation force and therefore Newtonian gravitation potential is the correct one. We repeat briefly the main argument of [4].

Let us take a radially symmetric gravitational field created by a mass m and we use a small test mass m_1 that is so small that the field can still be considered radial. We modify the Newtonian gravitation law to the form

$$F = G\frac{m_1 m}{h^{\alpha(h)}} \tag{5}$$

where h is the distance of m and m_1 and $\alpha(r)$ is an unknown function that that gives the exact gravitational force, not an approximation as Netwon's law may give. In a symmetric situation the force F must be radially symmetric, therefore such $\alpha(h)$ does exist.

In [4] it is proven that $\alpha(h) = 2$ exactly and everywhere. The area of a sphere $4\pi r^2$ is in [4] generalized to the form $4\pi r^{\gamma}$. [4] also proves in equation (65) that $\gamma = 2$, that is, empty space is flat.

The proof in [4] is given in the case where M is a 3-ball of constant density ρ centered at the origin. In equations (53)-(64) in [4] it is calculated that equation (5) gives the force that attracts the test mass m_1 as

$$F = \int_0^R 2\pi G\rho r^{\gamma} \int_{-\pi/2}^{\pi/2} \frac{h - r\sin(\alpha)}{s} \frac{\cos(\alpha)d\alpha}{s^{\alpha/2}} \tag{6}$$

where

$$s = \sqrt{r^2\cos^2(\alpha) + (h - r\sin(\alpha))^2} \tag{7}$$

is the distance of a point on the r-radius sphere around the origin. The number α in (6) is a constant that approximates the function $\alpha(s)$ in the 3-ball of radius R. Let R be very small. From (7) we see that α is very close to $\alpha(h)$. When R goes to zero, α in (6) is $\alpha(h)$ and as h is constant in the calculation in (53)-(61), we can set $\alpha = \alpha(h)$ in the 3-ball.

The result in (61)-(63) in [4] is that

$$F = G\frac{Mm}{h^{\alpha(h)}}\frac{3-\alpha(h)}{\alpha(h)-1} + O(R^2h^{-2}) \tag{8}$$

which converges to

$$F = G\frac{Mm}{h^{\alpha(h)}}\frac{3-\alpha(h)}{\alpha(h)-1} \tag{9}$$

when $R \to 0$. But when the 3-ball converges to a point mass, we must get (5). Therefore

$$\frac{3-\alpha(h)}{\alpha(h)-1} = 1 \tag{10}$$

for every value h. When $\alpha(h) = 2$, the $O(R^2 h^{-2})$ term in (8) is zero. This proves that the Newtonian gravitation law is exact for point masses, not an approximation. Thus, we have no choice but to use the Newtonian gravitation potential in a situation of a point mass in empty space.

We will first solve the length of horizontal paths, not geodesics. Let $y = a$, a constant, then $y' = 0$. We will write the integral with the parameter s instead of x and leave out $-GM$ as it does not matter here. This the integral is

$$S = \int_{-s}^{s} \frac{ds}{\sqrt{s^2 + a^2}} \tag{11}$$

The integral is easily calculated by a number of changes of the integration parameter. First $x = s/a$. The integral takes the form

$$S = \int_{-s/a}^{s/a} \frac{dx}{\sqrt{x^2+1}}. \tag{12}$$

We remove the bounds and take care of them later

$$I = \int \frac{dx}{\sqrt{x^2+1}}. \tag{13}$$

Change to $y = x^{-1}$

$$I = \int \frac{dy}{y\sqrt{1+y^2}}. \tag{14}$$

Change $\cot(\theta) = y$

$$I = \int \frac{d\theta}{\cos(\theta)}. \tag{15}$$

Change to $z = \sin(\theta)$

$$\begin{aligned} I = \int \frac{dz}{1-z^2} &= \frac{1}{2}\int \frac{dz}{1-z} + \frac{1}{2}\int \frac{dz}{1+z}. \\ &= -\frac{1}{2}\ln(1-z) + \frac{1}{2}\ln(1+z) \end{aligned} \tag{16}$$

Back to θ

$$I = -\frac{1}{2}\ln(1-\sin(\theta)) + \frac{1}{2}\ln(1+\sin(\theta)) \tag{17}$$

Back to y

$$I = -\frac{1}{2}\ln(1-(1+y^2)^{-\frac{1}{2}}) + \frac{1}{2}\ln(1+(1+y^2)^{-\frac{1}{2}}) \tag{18}$$

Back to x

$$I = -\frac{1}{2}\ln(1 - \frac{x}{\sqrt{x^2+1}}) + \frac{1}{2}\ln(1 + \frac{x}{\sqrt{x^2+1}}) \tag{19}$$

Back to s and inserting the bounds

$$S = \ln\left(\frac{\sqrt{s^2+a^2}+s}{\sqrt{s^2+a^2}-s}\right). \tag{20}$$

To get the length of the path we must multiply the result with GM. We notice that if $a = 0$, the length is infinite: the field has a pole at the origin.

Let us now calculate a geodesic. Let us generalize the Lagrangian by replacing the square root with power α for a reason that will be explained later. (It is not for the ease of calculation, we will just comment on something interesting.)

$$L(x, y, y') = \frac{(1+y'^2)^{\frac{1}{2}}}{(x^2+y^2)^{\alpha}} \tag{21}$$

The Euler-Lagrange equations are calculated as follows

$$\frac{\partial L}{\partial y} = -2\alpha y(x^2+y^2)^{-\alpha-1}(1+y'^2)^{\frac{1}{2}} \tag{22}$$

$$\frac{\partial L}{\partial y'} = (x^2+y^2)^{-\alpha}y'(1+y'^2)^{-\frac{1}{2}} \tag{23}$$

$$\begin{aligned}\frac{d}{dx}\frac{\partial L}{\partial y'} &= -2\alpha x(x^2+y^2)^{-\alpha-1}y'(1+y'^2)^{-\frac{1}{2}} \\ &\quad -2\alpha y'y(x^2+y^2)^{-\alpha-1}y'(1+y'^2)^{-\frac{1}{2}} \\ &\quad +(x^2+y^2)^{-\alpha}\frac{d}{dx}\left(y'(1+y'^2)^{-\frac{1}{2}}\right)\end{aligned} \tag{24}$$

Simplifying the Euler-Lagrange equations we get

$$\frac{d}{dx}\left(y'(1+y'^2)^{-\frac{1}{2}}\right) = 2\alpha(xy'-y)(x^2+y^2)^{-1}(1+y'^2)^{-\frac{1}{2}} \tag{25}$$

which still simplifies to

$$y'' = 2\alpha(xy'-y)(x^2+y^2)^{-1}(1+y'^2). \tag{26}$$

Despite the nasty looks, equation (26) is not difficult. We can find the solutions by simple reasoning. If $\alpha = 0$, then the equation reduces to

$$y'(1+y'^2)^{-\frac{1}{2}} = C \tag{27}$$

where C is a constant. Solving the simple second order equation of y' we get

$$y' = \pm\frac{C}{\sqrt{1-C^2}} \tag{28}$$

and we notice that there is no need for $\pm$ as C can be positive or negative. We get

$$y = \frac{C}{\sqrt{1-C^2}}x + B \tag{29}$$

where B is a constant. It is a straight line and the metric here is flat. This result encourages us to try $y = Cx$ and indeed, the Euler-Lagrange equations give zero for this y. It is a geodesic, i.e., locally it gives a minimum, but at the origin it gives infinity, so in a sense this geodesic gives the maximum if you continue it to the origin. It is the way mass bodies move in the field: to the center of the mass.

The real minimum for our situation is equally easy to find. Notice that with a potential $-GMr^{-1}$ the length of a circle with the center at the origin and the radius as r gives the same length

$$L = \pi GM \tag{30}$$

for a half a circle from $(-s, 0, 0)$ to $(s, 0, 0)$. This means that there must be a geodesic satisfying

$$r^2 = x^2 + y^2. \tag{31}$$

Thus

$$y' = -\frac{x}{y} \tag{32}$$

and indeed, from the equation

$$y'' = -\frac{1}{y} + \frac{xy'}{y^2} \tag{33}$$

$$2\alpha(xy' - y)(x^2 + y^2)^{-1}(1 + y'^2) = 2\alpha(xy' - y)r^{-2}\frac{r^2}{y^2} \tag{34}$$

$$= 2\alpha\left(-\frac{1}{y} + \frac{xy'}{y^2}\right). \tag{35}$$

The reason why I put α to the formula is seen in (35): only if $\alpha = \frac{1}{2}$ we get a solution. It is one more reason to think that the Newtonian gravitational potential has the correct power r^{-1}, as I have started to think that Newtonian gravitation theory is the more correct one. It only needs an interaction model, something like I made in [10].

This latter solution of the Euler-Lagrange equations is the minimum for our case. We can prove that it is the minimum and the only minimum. Let us take an arch between two angles ϕ_1, ϕ_2 in the radial coordinates (r, ϕ) of the (x, y)-plane. For two different values of r, the length of the arch is the same in this metric. We can make any small variation of the path by moving from r radially to some other r_1, then moving along an arch with the radius r_1 between some angles ϕ_1 and ϕ_2, and then moving radially back to r. The length of the arch is not changed. The radial parts add length and then the length grows. It grows whether r_1 is larger than r or smaller. This proves that a geodesic must always stay at one radius r. The geodesics between two points that are as far from the origin can only be arches from a circle that is centered at the origin in this metric.

Consider now if light travels along a geodesic in a metric induced by the Newtonian gravitation field. It does not. On the sea we see this very clearly: the sea bends because the Earth is a ball, but light goes straight. This is why we cannot see masts or towers from a long distance even though we use a telescope and visibility is good. Besides, how many times have you seen light make a circle or half a circle? The geodesic is half a circle.

2. A general solution of the geodesic of a point mass (or a spherical mass like the Sun)

Let us give a general solution to the geodesic problem in the case of a point mass. By the result in [4] section 2, a spherical mass gives the same force as a point mass of the same size. Therefore the discussion applies to the geodesics around the Sun.

People who have studied the General Relativity Theory (GRT) may think that one should use the geodesic equation. The geodesic equation is simply the Lagrange-Euler equations given with the Lagrangian on the variables $L(t, x_\mu(t), \dot{x}_\mu(t))$. This is unnecessary and complicates the analysis. Light always travels in GRT on light-like world paths. At each point the metric on the tangent space should be the metric of a Minkowski space, i.e., the speed of light should be locally constant c at each point to each direction. A scalar field, like the Newtonian gravitation potential, satisfies this condition. Therefore the speed of light is c at each point of the path and the time it takes for light to travel the path is obtained by dividing the length of the path by c. We only need to look for the shortest path, a 3-space geodesic. We do not need the time. By dispensing with time, we get only three variables: $x, y(x), y'(t)$. Keeping the time we have at least five variables $t, x(t), \dot{x}(t), y(t), \dot{y}(t)$. Euler-Langange equations are sufficiently difficult to solve already with three variables. There is no reason to have five variables.

Let us change the Euler-Lagrange equation (26) to polar coordinates $x = r\cos(\alpha)$, $y = r\sin(\alpha)$. Then

$$dx = dr\cos(\alpha) - r\sin(\alpha)d\alpha \tag{36}$$

gives

$$\frac{d\alpha}{dx} = \frac{1}{r}\cot(\alpha)\frac{dr}{dx} - \frac{1}{r\sin(\alpha)} \tag{37}$$

while

$$\frac{y}{x} = \tan(\alpha) \tag{38}$$

gives

$$y' = \tan(\alpha) + x\frac{1}{\cos^2(\alpha)}\frac{d\alpha}{dx} \tag{39}$$

$$y' = -\cot(\alpha) + \frac{1}{\sin(\alpha)}\frac{dr}{dx} \tag{40}$$

From these we calculate

$$\frac{(xy'-y)(1+y'^2)}{x^2+y^2} = -\frac{1}{r\sin^3(\alpha)} + \frac{3\cos(\alpha)}{r\sin^3(\alpha)}\frac{dr}{dx} \tag{41}$$
$$-\frac{1+2\cos^2(\alpha)}{r\sin^3(\alpha)}\left(\frac{dr}{dx}\right)^2 + \frac{\cos(\alpha)}{r\sin^3(\alpha)}\left(\frac{dr}{dx}\right)^3$$

Derivating y' we calculate y'' as

$$y'' = -\frac{1}{r\sin^3(\alpha)} + \frac{2\cos(\alpha)}{r\sin^3(\alpha)}\frac{dr}{dx} - \frac{\cos^2(\alpha)}{r\sin^3(\alpha)}\left(\frac{dr}{dx}\right)^2 + \frac{1}{\sin(\alpha)}\frac{d^2r}{dx^2} \tag{42}$$

Equation (26) takes the form

$$r\sin^2(\alpha)\frac{d^2r}{dx^2} = \cos(\alpha)\frac{dr}{dx} - (1-\cos^2(\alpha))\left(\frac{dr}{dx}\right)^2 + \cos(\alpha)\left(\frac{dr}{dx}\right)^3 \tag{43}$$

which in coordinates x, r is

$$\frac{d^2r}{dx^2} = \frac{x}{r^2-x^2}\frac{dr}{dx}\left(1+\left(\frac{dr}{dx}\right)^2 - \frac{r^2+x^2}{rx}\left(\frac{dr}{dx}\right)^3\right) \tag{44}$$

From (44) we can see that the Euler-Lagrange equations (26) have two easy solutions. If

$$1+\left(\frac{dr}{dx}\right)^2 - \frac{r^2+x^2}{rx}\left(\frac{dr}{dx}\right)^3 = 0 \tag{45}$$

then we can solve the roots of the second orded equation and get

$$\frac{dr}{dx} = \frac{r}{x} \qquad \text{or} \qquad \frac{\mathrm{dr}}{\mathrm{dx}} = \frac{\mathrm{x}}{\mathrm{r}} \tag{46}$$

In the second case

$$r^2 = x^2 + C \tag{47}$$

but this solution does not satisfy (44). In the first case where we get

$$r = Cx \tag{47}$$

which is the same as a solution

$$y = C_1 x \tag{48}$$

which we already found. This is the radial geodesic. The length of this geodesic from $r = r_2$ to $r = r_1$ is

$$L = GM\ln\left(\frac{r_2}{r_1}\right). \tag{49}$$

This is a geodesic, i.e., it is a minimum length curve in small variations, but if it reaches the origin, then the length is infinite.

The second type of solution to (44) is

$$\frac{dr}{dx} = 0 \tag{50}$$

This is the arch geodesic, which we also found earlier. The length of an arch of the angle β is

$$L = GM\beta. \tag{50}$$

Some trying is enough to show that is seems very difficult to find other solutions to (44). There is a reason for it. There is always a shortest path between two point A and B, but this path need not be differentiable at all points. Consider the arch and radial geodesics. Both are geodesics and they are in a right angle to each others, but as (44) is not a linear equation, we cannot superposition these solutions of (44) in order to create intermediate solutions to (44). We can prove that there are no smooth solutions to (44) and geodesics are composed of parts of arch and radial geodesics.

First we discuss how to remove the third space dimension. Consider two points A and B in the three dimensional space. The origin O is the third point. The point mass is located at the origin. These three points define a two-dimensional plane R^2. In order to get from A to B the geodesic cannot leave this plane because if it leaves the plane, it must return to the plane and the trip outside the plane is additional length to the path. We conclude that the geodesic is on the plane spanned by A, B and 0. In that plane the points A and B can be given polar coordinates $A = (r_1, \alpha_1)$, $B = (r_2, \alpha_2)$. We can choose the coordinates and naming of the points so that $r2 \geq r_1$ and $\alpha_2 \geq \alpha_1$.

In the plane any smooth path can be approximated by small parts that are either radial geodesics or arch geodesics between two points. We orient the path to be from A to B. The parts of the path that are on radial geodesics must sum to $r_2 - r_1$, else we cannot get from A to B. The parts that are radial geodesics must grow in r in order to lead towards r_2 because going forward and backward is unnessessary length. Therefore lengths of the parts of radial geodesics must sum to (49). The parts of the path that are on arch geodesics must sum to $\alpha_2 - \alpha_1$, else we cannot get from A to B. The parts of the path that are on arch geodesics must sum to $\alpha_2 - \alpha_1$, else we cannot get from A to B. The parts that are arch geodesics must grow in α in order to lead towards α_2 because going forward and backward is unnessessary length. Therefore he lengths of the parts of arch geodesics must sum to (50).

The length of the path does not change depending on how we select the small geodesic parts because we can without changing the length of the path reorder the parts so that the radial geodesic parts are first and then come the arch geodesic parts. The number of these geodesic parts also does not influence the length of the path. This constricted piece-wise smooth path has finitely many points where it is not differentiable, but in points when it is differentiable, it satisfies (44).

The smooth path that is approximated by this piece-wise smooth path does not satisfy (44), but it can be approximated to any desired precision by a piece-wise smooth path, we only need to add more pieces. Every approximation by piece-wise smooth path has the same length and these approximations converge to the smooth path. The length of the smooth path does not need to converge to the same number and usually does not. We can think about two examples: approximating the length of the hypotenuse in a straight angled triangle and approximating a smooth path in a metropol. In the first case, we can approximate the hypotenuse by small parts parallel to the sides of the triangle to any degree and the length these parts will always equal the sum of the sides, but the length of the hypotenuse is always shorter. In a metropol we can first use a map marking only the main streets and we can approximate a smooth path by going along these streets. The smooth path would require climbing over buildigs, it is always faster to go along the streets. Then we can take a better map that shows more streets and get a better approximation, and there is still a more detailed map showing more streets and giving an even better approximation. Always the smooth path requires climbing over buildings and gives a longer length. Thus, a piece-wise geodesic path can approximate the smooth path to any degree, but the length of the smooth path can be either shorter of longer than the lenght of the approximating piece-wise smooth path. Which case is it in (44)? It is the metropol map. The hypotenuse is not shorter than the sum of the sides because a line $y = cx + b$ is a solution to (26) only if $b = 0$. Equation (44) seems to define a smooth path that satisfies the Euler-Lagrange equations, but the path can be piece-wise smooth as points do not count in the length. There are no other smooth solutions to (44) than the radial and arch geodesics because if we take a proposed smooth geodesic and approximate it with the constructed piece-wise smooth path, the hypotenuse given by the proposed geodesic in every small triangle is longer than then sum of the sides in the constructed approximation meaning that the piece-wise smooth curve is shorter. We notice that any path that does not go backward and forward but from A always towards B gives the same length because it does not matter in what order the small geodesic parts are. A geodesic does not determine how a path goes, unless it a special case when either $r_1 = r_2$ or $\alpha_1 = \alpha_2$.

Let us now think about Eddington's experiment. Was the light beam coming from a distant star to the camera of Eddington in 1919 following a geodesic?

Firstly we notice that if light from a star bends and comes to Eddington's camera, then $r_1 \neq r_2$ and $\alpha_1 \neq \alpha_2$. Therefore a geodesic from the star to the camera can have taken any path. There is no way in verifying by the measurement if the observed path is a geodesic. If the observed path agrees with Einstein's prediction, it is because Einstein calculated the geodesic incorrectly from the Swhwarzschild solution.

Secondly, we have already noticed that if A and B have the same distance to the point mass, the the geodesic from A to B is an arch geodesic and light does not follow arch geodesics at least on the sea: sea bends while light goes straight. If light sent by a light-house would follow an arch geodesic, we would see the light-house signal much further. We do not. Light does not follow arch geodesics.

Thirdly, consider light from the distant star in Eddington's experiment. In order to get even close to the Sun, light from the star must travel very long very close to a geodesic of the radial type. If light travels along geodesics of the gravitational field, if would not get out of this geodesic as to get out of the minimum lenght path means taking a longer path. As a result the light beam from a distant star would hit the Sun and not bend. We could not see the star. Notice that we do not see stars behind the Moon because the Moon blocks the line of sight. The Moon also has a gravitational field, albeit a small one, but the strength of the field should not matter. It is a scaling issue.

Fourthly, light from the distant star started its trip several years before Eddington photographed the light and all this trip the direction of light was governed by some local reasons. With what mechanism this light would have minimized the length of the trip? Trip where? Light starting the trip did not have the goal of being photographed. Only after light arrives to B we may ask along what path this light came. Often light did not come through a straight path. It may have reflected from something, or light may have for instance bent close to the Sun because there is some optically active medium around the Sun. Light front does bend in optically active medium because there is different local speed of light in different places. In gravitational light bending the local speed of light should be constant. The bending would be caused by the hypothetical expansion of space geometry because of the gravitational field. Yet, light does not follow geodesics of the electro-magnetical fields. Why should if follow geodesics of a gravitational field? And it does not, the simple case of an arch geodesic from A to B shows it.

I have heard a claim that the geodesics aroud the Sun would be hyperbolic curves. This is not what I get. The different result that the Relativity Theory has on this issue may be due to the use of a solution that does not have constant speed of light at every point to every direction. The Einstein equations do not have any solutions that approximate Newtonian gravitation field in a situation of a point mass in empty space, a situation that is close enough to the gravitational field of the Sun. The solutions that there are, like Schwarzschild solution, have the local speed of light depending on the altitude, hardly an acceptable model. The Schwarzschild solution should not be used anywhere because the Schwarzschild metric is invalid as a pseudo-metric in the Relativity Theory, but Einstein used this solution in his verification of GRT. Inserting an invalid solution to the geodesic equation cannot be expected to give correct results. Therefore those results should be checked.

3. About the Relativity Theory

It was interesting for me to read from Einstein's book [1] that he knew that the geodesics are on equipotential surfaces. Thus, he must have known that the General Relativity Theory claim that light travels along geodesics of the gravitational field is wrong, because light certainly does not make a half circle around the Sun or the Earth. Yet, he allowed the claim to stay.

I made a similar observation in [2], a brief look at chapter 5 in Einstein's book [1] where Einstein presents Friedman's cosmological results. The results are wrong and Einstein must have known it, but he cheated on purpose. Just exactly how wrong GRT is see [3] and [4]: Einstein's field equations cannot be used as equations of a gravitational field, the Schwarzschild solution is wrong as it does not have a valid metric. That there are no solutions to Einstein equations in the case of a point mass in empty space is already proven in [5] and in a different way in [6], it is already mentioned in [7].

Shortly said, the General Relativity Theory (GRT) is wrong because the Einstein equations do not have any solutions that approximate Newtonian gravitation field in the situation of a point mass in empty space, which is a good approximation for the field of the Sun and the Earth. Because of this, none of the experiments that claim to have verified the General Relativity Theory can verify it: the theory does not give any gravitational field that applies to the situation in the experiment, so it cannot be verified by the experiment. There are always alternative explanations to results that are claimed to have verified GRT. Alternative explanations to some of these verifications of GRT are given in [8]-[10].

The Special Relativity Theory (SRT) is wrong because Einstein forgot to make a projection of (x', t') on the t'-axis in the Lorentz transform. The speed of light is not the same in all inertial frames of reference. When this is corrected, the whole SRT falls. I explain the problem e.g. in [12], [5] and [11].

Einstein's relativistic mass formula is wrong and his proof of $E = mc^2$ is not a proof of anything as is shown in [13]-[15].

For some time I thought that Nordström's gravitation theory might be workable (like in [8], [16], [17]), but it is also wrong: the time dependence of the D'Alembert operator is incorrect for gravitation, see [10].

Still a short time ago I thought that there are some correct claims in GRT, notably gravitational time dilation and bending of light in a gravitation field. In [11] I was even thinking that many principles of relativity can be kept. Now I do not think so any longer. Gravitational time dilation is measurable, but it is not time dilation, see [10] for an explanation to what really happens. The bending of light in a gravitational field is not true as I show in this article.

My conclusion now is that the Relativity Theory is false and no part of the theory is true as science. But it is a religion and religions seldom die out, no matter how many errors or absurdities you show in them.

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4.3 On light-like geodesics in the Schwarzschild metric

Abstract:

The article calculates light-like geodesics of the Schwarzschild metric. It is shown that the methods to calculate geodesics that are used in the Relativity Theory are incorrect.

1. Introduction

The so called Schwarzschild metric is defined as

$$c^2 d\tau^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2(\theta) d\phi^2 \tag{1}$$

where

$$A(r) = c^2 \left(1 - \frac{r_s}{r}\right) \qquad\qquad B(r) = \left(1 - \frac{r_s}{r}\right)^{-1} \tag{2}$$

and r_s is a constant, so called Schwarzschild radius. This differential form is not a valid metric. When we change it to Cartesian coordinates, it gives

$$c^2 d\tau^2 = A(r)dt^2 - (B(r)-1)dr^2 - \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2\right)$$

$$= A(r)dt^2 - (B(r)-1)dr^2 - \left(dx^2 + dy^2 + dz^2\right) \tag{3}$$

$$= A(r)dt^2 - ((B(r)-1)\frac{x^2}{r^2}+1)dx^2 - ((B(r)-1)\frac{y^2}{r^2}+1)dy^2 - ((B(r)-1)\frac{z^2}{r^2}+1)dz^2$$

$$-(B(r)-1)\frac{xy}{r^2}dxdy - (B(r)-1)\frac{xz}{r^2}dxdz - (B(r)-1)\frac{yz}{r^2}dydz.$$

Cartesian coordinates x, y, z are orthogonal and Riemannian metric is indused by an inner product. The inner product of orthogonal coordinates is zero. There must not be cross terms $dxdy, dxdz, dydz$ in the expression (3). The most general valid metric that is radially symmetric, does not depend on time, and has the speed of light constant at c is a scalar field

$$c^2 d\tau^2 = c^2 B(r)dt^2 - B(r)dx^2 - B(r)dy^2 + B(r)dz^2$$

$$= c^2 B(r)dt^2 - B(r)dr^2 - r^2 B(r)d\theta^2 - r^2 \sin^2(\theta) B(r) d\phi^2. \tag{4}$$

The failure of the Schawrzschild metric is local, in a small environment of a point. No global geometric or topological considerations matter: the metric (1) is not a valid metric. Notice also that light does not have the speed c to each direction at each point in the metric (1) and that the metric (1) does not give the Minkowski metric at the tangent space of a point as it should. The metric does not approximate Newtonian gravitation potential as the speed of light depends on the altitude in (1).

This article calculates an equation for light-like geodesics in the Schwarzschild metric and notices that the geodesics are not the same as in Newtonian gravitational potential. The article comments on some calculation methods for geodesics in the Schwarzschild metric presented in the Wikipedia page [18].

2. Calculation of the geodesics equation for the Schwarzschild metric

In a flat 2-dimensional space a geodesic between points A and B is solved by minimising

$$S = \int_A^B \sqrt{dx^2 + dy^2} = \int_a^b \sqrt{1 + y'^2} dx \tag{5}$$

where a and b are the x-coordinates of A and B. Minimizing S is done by solving the Euler-Lagrange equations. The Lagrangian is

$$L(x, y, y') = \sqrt{1 + y'^2} \tag{6}$$

and the Euler-Lagrange equations are

$$\frac{\partial L}{\partial y} - \frac{d}{dx} \frac{\partial L}{\partial y'} = 0. \tag{7}$$

This is essentially the same in any metric. In the General Relativity Theory (GRT) light is said to follow geodesics. We will focus on light. Light is said to travel always with the constant speed c. The Schwarzschild metric does not give light this constant speed, but we can take only the space part of the metric and require that light does have the speed c. Then minimizing the time on a path is the same as minimizing the length of the path in the space coordinates. The time is obtained by dividing the length by c. The function that we want to minimize is the length of the path in the space part of the metric (1), see equation (3)

$$S = \int_A^B \sqrt{(B(r) - 1)dr^2 + dx^2 + dy^2 + dz^2}. \tag{8}$$

We will require that in our geodesic $dz = 0$ and write (8) as in (5) with $r' = \frac{dr}{dx}$

$$S = \int_a^b \sqrt{(B(r) - 1){r'}^2 + 1 + {y'}^2} dx. \tag{8}$$

The Lagrangian is

$$L(x, r, r') = \sqrt{(B(r) - 1){r'}^2 + 1 + {y'}^2}. \tag{9}$$

We have y' in the equation, but y, y', y'' are all functions of x and r

$$y = \sqrt{r^2 - x^2} \tag{10}$$

$$y' = \frac{1}{r}(rr' - x) \tag{11}$$

$$y'' = -\frac{{y'}^2}{y} + \frac{1}{y}(r'^2 - 1) + \frac{r}{y} r'' \tag{12}$$

and we calculate $y = y(x, r)$. We write $B'(r) = \frac{dB(r)}{dx}$.

Then

$$\frac{\partial L}{\partial r} = \left(\frac{1}{2}B'(r)r'^2 - r\frac{y'^2}{y} + r'\frac{y'}{y}\right)L^{-1} \tag{13}$$

$$\frac{\partial L}{\partial r'} = \left((B(r)-1)r' + r\frac{y'}{y}\right)L^{-1} \tag{14}$$

$$\begin{aligned}\frac{d}{dx}\frac{\partial L}{\partial r'} = &\left(B'(r)r'^2 + (B(r)-1)r'' + r'\frac{y'}{y} + r\frac{y''}{y} - r\frac{y'^2}{y^2}\right)L^{-1}\\ &+\left((B(r)-1)r' + r\frac{y'}{y}\right)\frac{d}{dx}(L^{-1})\end{aligned} \tag{15}$$

where

$$\frac{d}{dx}(L^{-1}) = -\frac{1}{2}L^{-3}\left(B'(r)r'^3 + (B(r)-1)2r'r'' + 2yy''\right). \tag{16}$$

Inserting (12) and moving all terms containing r'' to the left side of the Euler-Lagrange equations (7) we get from a direct calculation (four terms cancel)

$$\begin{aligned}&r''\left((B(r)-1)(1-Mr') + \left(\frac{r}{y} - My'\right)\frac{r}{y}\right)\\ &= \left(\frac{r}{y} - My'\right)\left(\frac{y'^2}{y} - \frac{1}{y}(r'^2-1)\right) + \frac{1}{2}B'(r)r'^2(1+Mr')\end{aligned} \tag{17}$$

where

$$M = ((B(r)-1)r' + rr'y'y^{-1})L^{-2}. \tag{18}$$

.

Inserting M and simplifying

$$\begin{aligned}&r''\left((B(r)-1)\frac{1+y'^2-rr'y'y^{-1}}{(B(r)-1)r'(rr'-yy')+r}\frac{y^2}{r}+1\right)\\ &= \frac{1}{r}\left(y'^2 - r'^2 + 1\right)\\ &+\frac{1}{2}B'(r)r'^2\frac{2B(r)-1)r'^2+1+y'^2+rr'y'y^{-1}}{(B(r)-1)r'(rr'-yy')+r}\frac{y^2}{r}.\end{aligned} \tag{19}$$

The expression still simplifies with (10) and

$$\begin{aligned}rr' - yy' &= x\\ rr'y'y^{-1} &= \frac{1}{y^2}rr'(rr'-x)\end{aligned} \tag{20}$$

and we get the final equation

$$r''\left((B(r)-1)\frac{r^2-x^2-x(rr'-x)}{(B(r)-1)rr'x+r^2}+1\right) = \frac{1}{r}\left(\frac{(rr'-x)^2}{r^2-x^2} - r'^2 + 1\right) \tag{21}$$

$$+\frac{1}{2}B'(r)r'^2\frac{(2B(r)-1)r'^2+1)(r^2-x^2)+(2rr'-x)(rr'-x)}{(B(r)-1)rr'x+r^2}.$$

If $B(r)=1$, then (21) gives the straight line $y=Cx+D$ as a solution because always holds

$$r''=\frac{y}{r}y''+\frac{y'^2}{r}-\frac{1}{r}(r'^2-1) \tag{22}$$

while (21) for $B(r)=1$ gives

$$r''=\frac{y'^2}{r}-\frac{1}{r}(r'^2-1) \tag{23}$$

i.e., $y''=0$.

Newtonian gravitational potential field has two geodesics. One has $r'=0$, the geodesic is on an equipotential surface. The other one has $r=Cx$, it is radial. The one with $r'=0$ does not satisfy (21), thus the Schwarzschild solution does not resemble Newtonian gravitation potential also in the geodesics.

In General Relativity the Lagrangean for time-like world paths is different: there is the time part and then one can find a circular path for the Schwarzschild metric, but this method fails for a radial geodesic: the equation gives a nearly linear time dependence (i.e., no or too little acceleration) for a test mass falling to a mass center. Therefore this method is incorrect.

Instead, one can use (21) also for a mass point: it is the equation of the shortest path in the space coordinates of the gravitational geometry regardless of what the speed is. The Newtonian gravitation potential has the geodesic with $r'=0$, i.e., a circle around a mass point or a spherical mass. We do have geostationary satellites around the Earth, so an orbit that is very closely a circle around a spherical mass is certainly possible for a small test mass like a satellite. We conclude that according to (21) the Schwarzschild metric does not allow geostationary satellites and the Lagrangean that is used in the General Relativity Theory and which gives a circle geodesic for the Schwarzschild solution is wrong because it gives wrong results for a radial geodesic.

The reason why we get so messy geodesic equation as (21) is that the metric (1) is not a valid metric. The geodesic equation for the metric (4) of a scalar field

$$\psi(r)=\frac{1}{r} \tag{24}$$

is very simple to calculate. We can use variables x, y, y', thus:

$$L(x,y,y')=\sqrt{\psi(r)^2(1+y'^2)} \tag{25}$$

$$\frac{\partial L}{\partial y}=-y\psi^3(1+y'^2)^{\frac{1}{2}}$$

$$\frac{\partial L}{\partial y'}=y'\psi(1+y'^2)^{-\frac{1}{2}}$$

$$\frac{d}{dx}\frac{\partial L}{\partial y'} = -(xy' + yy'^2)\psi^3(1+y'^2)^{-\frac{1}{2}}$$
$$+y''\psi(1+y'^2)^{-\frac{1}{2}} - y''y'^2\psi(1+y'^2)^{-\frac{3}{2}}$$

The Euler-Lagrange equation is

$$y'' = \psi^2(1+y'^2)(xy' - y) \tag{26}$$

and it should be about as easy as this for a valid metric.

3. Serious errors in geodesics in GRT

One of the methods to calculate the geodesic in [18] is to use the Euler-Lagrange equations. It is made in the following way. They means here Einstein and other relativity people whose methods [18] describes.

They have a Lagrangian $T(\tau, x^\mu, \dot{x}^\mu)$. There is τ in the left side of (1), but τ in (1) is always zero for a light-like world path. For a light-like geodesic τ cannot be a proper time because according to the Relativity Time the proper time of a photon does not tick at all. Thus, τ can be considered as a variable used in the method without any deeper meaning.

There are four coordinates x^μ, so instead of finding the geodesic in the space coordinates, they try to find a geodesic with the Lagrangian

$$T = \sqrt{g_{mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}} \tag{27}$$

and the action to be minimized is

$$S = \int_a^b T d\tau. \tag{28}$$

Sometimes the Langangian in [18] lacks the square root and one does not know if they minimize the length or the square of the length. It is a different thing to minimize the norm or the square of the length. In (22) the Lagrangian minimizes the length, i.e., the norm of the length. Should we minimize the square of the length by using the Lagrangian

$$L(x, y, y') = \psi(r)^2(1+y'^2) \tag{29}$$

the Euler-Lagrange equation we get is

$$y'' = \psi^2(2xy' - y + yy'^2). \tag{30}$$

It is a different equation, it gives a different curve. I assume they minimize (28) as this is the usual way.

The serious error in [18] is that they assume that the Euler-Lagrange equations can be treated separately. They solve

$$\frac{\partial T}{\partial x^\mu} - \frac{d}{d\tau}\frac{\partial T}{\partial \dot{x}^\mu} = 0. \tag{31}$$

This is not possible for two reasons. One is that there is a constraint, the other one is that the equations do not separate even without a constraint.

First, the constraint issue. It is important with light-like world paths and less so (but does exist) with time-like world paths. In light-like world paths τ in (1) should always be exactly zero. If the equations separate and we solve separately $t(\tau)$, $r(\tau)$ and $\phi(\tau)$, then there is no reason to assume that the speed of light is c. We must impose a constraint on the Lagrangean that this is so: at every point of the path c-times the time differential must equal the vector sum of the space differentials. Constraints are imposed on the Lagrangean with a Lagrangeam multiplier:

$$L(\tau, x^\mu, \dot{x}^\mu, \lambda) = T(\tau, x^\mu, \dot{x}^\mu) - \lambda G(\tau, x^\mu, \dot{x}^\mu) \tag{31}$$

where G is the constraint and λ is an unsolved Lagrangian multiplier. With this constraint the equations do not separate and they have ten variables in each equation. With time-like world paths the speed must not exceed c, but usually speeds of mass bodies are so small that this constraint does not limit.

The other problem is that even without constraints, the equations do not separate. It can be shown in the simple three-dimensional example where $\psi = r^{-1}$, the Newtonian potential omitting $-MG$. Consider minimizing

$$S = \int_A^B \sqrt{\psi(r)^2 dx^2 + \psi(r)^2 dy^2 + \psi(r)^2 dz^2} \tag{32}$$

with the τ parameter. We look for a geodesic in the (x, y)-plane and set $dz = 0$. With τ the Lagrangean is

$$L(\tau, x, y, \dot{x}, \dot{y}) = (x^2 + y^2)^{-\frac{1}{2}} (\dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}. \tag{33}$$

A straightforward calculation gives the Euler-Lagrange equation

$$\frac{\partial L}{\partial x} - \frac{d}{d\tau}\frac{\partial L}{\partial \dot{x}} = 0. \tag{34}$$

gives the geodesic equation

$$\ddot{x}\dot{y} - \ddot{y}\dot{x} = r^{-2}(\dot{x}y - \dot{y}x)(\dot{x}^2 + \dot{y}^2). \tag{35}$$

The Euler-Lagrange equation

$$\frac{\partial L}{\partial y} - \frac{d}{d\tau}\frac{\partial L}{\partial y'} = 0. \tag{36}$$

gives the same geodesic equation (35). Thus, we can take the Euler-Lagrange equations separately but they do not separate the variables x and y. Equation (35) is harder to solve than (26).

Their method has the following problems. Firstly, what is to be minimized? For light τ in (1) is must be always zero (a light like world path), so how to minimize something that always must be zero? They are missing the speed of light constraint. The equations do not separate the variables.

The way to cope with these problems is first to restrict the geodesic to a plane. Then we must ignore the time dimension because for light the path that takes the shortest time is the shortest path in space coordinates, this way we handle to constraint of the speed of light being constant. This approach works also for time-like world paths. Finally when we now have only two space coordinates, we make one of them a function of the other and do not introduce any additional coordinates like τ. In this way the Euler-Lagrange equations can be written down and if the metric is something intelligible and not something like (1), then we can also solve the equations.

Another method in [18] uses the geodesic equation

$$\frac{d^2x^\mu}{dq^2} + \Gamma^{\lambda}_{\mu\ \nu}\frac{dx^\mu}{dq}\frac{dx^\nu}{dq}. \tag{31}$$

Notice that this is not the geodesic equation. In the equation there should be derivatives on the right side. However, [18] calculates a geodesic from (31). Naturally the result is wrong.

The geodisic equation has been derived in several ways, all wrong as the equation is wrong. Einstein derived if from from the Euler-Lagrange equations. We will see later that this derivation is incorrect, but let us first discuss the constraint issue.

Minimizing the action integral, the Wikipedia page [19] uses Hamiltonian principle to get separate equations. Inversely, if we cannot do this step, then the equations do not separate. There is no discussion of the constraint of the speed of light in [19]. Checking the referred Wikipedia page on Hamiltonian principle did not tell anything relevant in this issue. The principle discusses a case when there is one Euler-Lagrange equation. It is possible to solve a variational problem with an Euler-Lagrange equation, provided that the Lagrangian is correctly constructed, that we know and it is true.

Assuming that the Lagrangian lacks an essential constraint, then it will not solve the problem that one wants it to solve. I think here is such an issue. I think the equations for several coordinates do not separate because there is the speed of light constraint. This is because calculating a geodesic as I do is a correct way and it does not have any possibilities for missing a constraint, while the geodesics calculated in the ways in [18] look very different and they have a step that may be missing a constraint. Where exactly is it imposed to the Euler-Lagrange equations that light has the speed c?

A much worse problem is that the geodesic equation is wrong. The variables do not separate in (35), while the geodesic equation has only one second order time derivative. Let us see what we get from (35). It can be written in another form

$$\ddot{x}\dot{y} - \ddot{y}\dot{x} = r^{-2}(\dot{x}y - \dot{y}x)(\dot{x}^2 + \dot{y}^2)$$

$$\dot{y}(\ddot{x} + \psi^2 x(\dot{x}^2 + \dot{y}^2) = \dot{x}(\ddot{y} + \psi^2 y(\dot{x}^2 + \dot{y}^2) \tag{37}$$

where $\psi = r^{-1}$. Noticing that

$$\Gamma^a_{aa} = \frac{1}{2}g^{aa}g_{aa,a} = \eta^{aa}\eta_{aa}\psi^{-1}\partial_a\psi \tag{38}$$

$$\Gamma^a_{ab} = \frac{1}{2}g^{aa}g_{aa,b} = \eta^{aa}\eta_{aa}\psi^{-1}\partial_b\psi$$

$$\Gamma^a_{bb} = -\frac{1}{2}g^{aa}g_{bb,a} = -\eta^{aa}\eta_{bb}\psi^{-1}\partial_a\psi$$

we get

$$\Gamma^1_{11} = \psi^{-1}\partial_x\psi = -\frac{x}{r^2} = -\psi^2 x \tag{39}$$

$$\Gamma^1_{12} = \psi^{-1}\partial_y\psi = -\frac{x}{r^2} = -\psi^2 y$$

$$\Gamma^1_{22} = \psi^{-1}\partial_x\psi = \frac{x}{r^2} = \psi^2 x$$

With these we can write

$$\ddot{x} + \psi^2 x(\dot{x}^2 + \dot{y}^2) = \ddot{x} - \Gamma^1_{11}\dot{x}\dot{x} + \Gamma^1_{22}\dot{y}\dot{y} \tag{40}$$

but this is not the geodesic equation

$$\ddot{x}^c + \Gamma^c_{ab}\dot{x}^a\dot{x}^b \tag{41}$$

with $x^0 = t, x^1 = x, x^2 = y, x^3 = z$. We have set the differential of $dt = dz = 0$ and have only two variables $x^1 = x, x^2 = y$. We are missing Γ^1_{12} and Γ^1_{11} has the wrong sign. We get a similar equation for $\ddot{y}$. Though the expressions somewhat resemble (41), they are not geodesic equations and they are tied in (37). We cannot get a separate equation for $\ddot{x}$ and for $\ddot{y}$.

Einstein thought he had invented a trick to separate the second order differentials. He first wrote the Lagrangean as

$$L = \sqrt{g_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds}} \tag{42}$$

and then defined another variable τ satisfying

$$\frac{d\tau}{ds} = L \tag{43}$$

Let us use the notations

$$x^{c\prime} = \frac{dx^c}{ds} \tag{44}$$

$$\dot{x}^c = \frac{dx^c}{d\tau}$$

Then

$$x^{c\prime} = L\dot{x}^c. \tag{45}$$

We also assume that the coordinates are orthogonal, so $g_{ab} = 0$ if $a \neq b$ in this short refutation of the geodesic equation. Calculating the expressions for the Euler-Lagrange equations gives

$$\frac{\partial L}{\partial x^c} = \frac{(\partial_c g_{aa})x^{a\prime}x^{a\prime}}{2L}$$

$$\frac{\partial L}{\partial x^{c\prime}} = \frac{g_{cc}x^{a\prime}}{L} \tag{46}$$

and using (45)

$$\frac{\partial L}{\partial x^{c\prime}} = g_{cc}\dot{x}^a \tag{47}$$

$$\frac{\partial L}{\partial x^c} = L\frac{1}{2}\partial_c g_{aa}\dot{x}^a\dot{x}^a$$

The total derivative is now easy

$$\frac{d}{ds}\frac{\partial L}{\partial x^{c\prime}} = \frac{d\tau}{ds}\frac{d}{d\tau}g_{cc}\dot{x}^{a\prime} = L\dot{g}_{cc}\dot{x}^a + Lg_{cc}\ddot{x}^a. \tag{48}$$

The Euler-Lagrange equation for x^a is

$$\frac{1}{2}\partial_c g_{aa}\dot{x}^a\dot{x}^a - \dot{g}_{cc}\dot{x}^a - g_{cc}\ddot{x}^a = 0 \tag{49}$$

which gives

$$\ddot{x}^a - \frac{1}{2}g^{cc}\partial_c g_{aa}\dot{x}^a\dot{x}^a + g^{cc}g_{cc}\ddot{x}^a = 0 \tag{50}$$

Einstein claimed that $\dot{g}_{cc} = 0$, which is not true. He also claimed that the

$$\Gamma^c_{aa} = \frac{1}{2}g^{cc}\partial_c g_{aa} \tag{51}$$

which is also not true. Indeed

$$\Gamma^a_{bc} = \frac{1}{2}g^{ad}(g_{bd,a} + g_{dc,b} - g_{bc,a}) \tag{52}$$

and therefore

$$\Gamma^a_{aa} = \frac{1}{2}g^{aa}\partial_a g_{aa} \tag{53}$$

$$\Gamma^a_{bb} = -\frac{1}{2}g^{aa}\partial_a g_{bb}$$

and already for this reason the geodesic equation is false, but the worse thing is that $\dot{g}_{cc} \neq 0$. We can solve what g_{cc} is in a simple case of a scalar field in Cartesian coordinates: $g_{ii} = -\psi^2$, $i = 1, 2, 3$, $g_{00} = c^2\psi^2$. Then

$$L = \psi \sqrt{\sum_a (x^{a\prime})^2} \tag{54}$$

and

$$L = \psi L \sqrt{\sum_a (\dot{x}^a)^2} \tag{55}$$

Then

$$\frac{\partial L}{\partial x^{c\prime}} = \psi \dot{x}^c \tag{56}$$

but also

$$\frac{\partial L}{\partial x^{c\prime}} = \frac{\psi x^{c\prime}}{L} = \frac{\psi L \dot{x}^c}{\psi L \sqrt{\sum_a (\dot{x}^a)^2}}. \tag{57}$$

We get

$$\psi = \frac{1}{\sqrt{\sum_a (\dot{(}x^a)^2}} \tag{58}$$

and therefore

$$\dot{\psi} = -\sum_a \ddot{x}^2 \dot{x}^a \left(\sqrt{\sum_a (\dot{x}^a)^2} \right)^{-\frac{3}{2}}. \tag{59}$$

Clearly, $\dot{g}_{cc} = 2\psi\dot{\psi}$ for $i = 1, 2, 3$ does not vanish.

The second order differentials do not separate to different equations. There is no trick to do it. The best strategy is to reduce the variables to two and to make one dependent on the other.

Einstein took the concept of a geodesic from Riemannian manifolds. In a four-dimensional Riemannian manifold you would calculate the shortest path from the whole metric, but that metric is positive definite and it does not give cross terms for orthogonal coordinates like the Schwarzschild metric does. In a Riemanninan manifold the shortest path, a geodesic, is also a straight path in the sense that the Levi-Civita connection (the Christoffel symbols in the geodesic equation) give the same path. But in a pseudo-Riemannian manifold with a metric that is not positive definite the concept of a geodesic is hazy. It can be in a sense thought of as a straight path, but what is it minimizing? For light-like world paths the distance that is should be minimizing is always zero if the speed of light is always c.

There is no constraint between the coordinates in a geodesic of a four-dimensional Riemannian manifold, but in GRT there is a constraint, or should be, it is not imposed. The displacement in time should be c times the displacement in space. It is a constraint that should be imposed by a Lagrangian multiplier and it makes the equations nonseparable. Therefore the calculations of geodesics in the Schwarzschild metric in [18] are wrong. The correct way is as it is made here, minimizing only the three-dimensional space distance. Then the speed of light is by definition c as it is not at all in the calculations.

As for Einstein's trick by which he separated second derivatives to individual equations, that trick does not work. The geodesic equation is wrong as was shown in this section.

Eddington's measurements of light bending close to the Sun do not verify the calculations done for the Schwarzschild solution because the gravitational field close to the Sun is not the field from the Schwarzschild solution, or from any solution of Einstein's equations. We can verify in this article that the Schwarzschild solution does not approximate the Newtonian gravitation potential by noticing that (21) does not have a geodesic with $r' = 0$. Already the looks of (21) show that *God did not plan world like that.*

This error in the calculation of geodesics is only one of the long list of serious errors in the Relativity Theory, see [2]-[17].

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4.4 Error in Einstein's geodesic equation and the failure of his geometrization idea

Abstract:

The article shows serious errors in the derivation of the geodesic equation in General Relativity, finds two time-like geodesics for the Schwarzschild metric, one being rather acceptable and the other not at all, and argues that the geometrization of gravity is a wrong idea.

1. Einstein's geometrization idea and the geodesic equation

Newton's gravitation law can be derived as the Euler-Lagrange equation from mimimization of the total energy: using the total energy of a test mass m in a time-independent gravitational field ϕ divided by the test mass as the Lagrangean

$$E = E_p + E_k = m\phi + \frac{1}{2}mv^2 \tag{1}$$

$$L(t, x_i, \dot{x}_i) = \phi(x_1, x_2, x_3) + \sum_{i=1}^{3} \frac{1}{2}\dot{x}_i^2 \tag{2}$$

gives

$$\frac{\partial L}{\partial x_i} = \partial_i \phi \tag{3}$$

$$\frac{\partial L}{\partial \dot{x}_i} = \dot{x}_i \tag{4}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = \ddot{x}_i \tag{5}$$

The Euler-Lagrange equation

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = 0 \tag{6}$$

give

$$\partial_i \phi = \ddot{x}_i \tag{7}$$

which is Newton's $F = ma$

$$F = m\nabla\phi = m\ddot{x}_i = ma. \tag{8}$$

Einstein wanted to derive the gravitation law from minimizing the geodesic. He took the Langangean from the line element

$$ds^2 = c^2 g_{00} dt^2 - g_{11} dx^2 - g_{11} dy^2 - g_{33} dz^2 \tag{9}$$

where t, x, y, z are orthogonal local coordinates. Writing $c^2 d\tau = ds$, $(t, x, y, z) = (x^0, x^1, x^2, x^3)$, $dx^a - d\tau = \dot{x}^a$, setting $c = 1$, and allowing the coordinates to be nonorthogonal, we can write (8) as

$$d\tau = \sqrt{g_{ab} dx_a dx_b} \tag{10}$$

and dividing by $d\tau$ we get a Lagrangean that equals one

$$1 = L(\tau, x^a, \dot{x}^a) = \sqrt{g_{ab}\dot{x}^a\dot{x}^b}. \tag{11}$$

The situation that $L = 1$ simplifies the Euler-Lagrange equations

$$\frac{\partial L}{\partial x^c} = \partial_c g_{ab}\dot{x}^a\dot{x}^b\frac{1}{2L} = \frac{1}{2}\partial_c g_{ab}\dot{x}^a\dot{x}^b \tag{12}$$

$$\frac{\partial L}{\partial \dot{x}^c} = g_{cd}2\dot{x}^d\frac{1}{2L} = g_{cd}\dot{x}^d \tag{13}$$

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{x}^c} = g_{cd}\ddot{x}^d + \dot{g}_{cd}\dot{x}^d. \tag{14}$$

The Euler-Lagrange equations are

$$\frac{1}{2}\partial_c g_{ab}\dot{x}^a\dot{x}^b - g_{cd}\ddot{x}^d - \dot{g}_{cd}\dot{x}^d = 0. \tag{15}$$

Notice that the Lagrangean (11) is only for time-like world paths. For light-like world paths $ds = 0$ always, therefore $L = 0$ and this method cannot be used. For space-like world paths the sign must be changed as ds is negative.

Let us take orthogonal local coordinates, thus $g_{ab} = 0$ if $a \neq b$. Then (15) is

$$\frac{1}{2}\partial_c g_{aa}\dot{x}^a\dot{x}^a - g_{cc}\ddot{x}^c - \dot{g}_{cc}\dot{x}^c = 0. \tag{16}$$

Multiplying by $g^{cc} = g_{cc}^{-1}$ we get

$$\ddot{x}^c - \frac{1}{2}g^{cc}\partial_c g_{aa}\dot{x}^a\dot{x}^a + g^{cc}\dot{g}_{cc}\dot{x}^c = 0. \tag{17}$$

This is not the geodesic equation of the General Relativity Theory

$$\ddot{x}^c + \Gamma^c_{ab}\dot{x}^a\dot{x}^b = 0 \tag{18}$$

for two reasons.

The first reason is that there is a clear error in the derivation of the geodesic equation from the Euler-Lagrange equations because

$$\Gamma^c_{ab} \neq -\frac{1}{2}g^{cc}\partial_c g_{ab}. \tag{19}$$

The definition of the Christoffel symbol is

$$\Gamma^a_{bc} = \frac{1}{2}g^{ad}(g_{bd,a} + g_{dc,b} - g_{bc,a}). \tag{20}$$

For orthogonal coordinates $g_{ab} = 0$ if $a \neq b$ the equation is

$$\Gamma^a_{bc} = \frac{1}{2}g^{aa}(g_{ba,a} + g_{ac,b} - g_{bc,a}). \tag{21}$$

Setting $a = b = c$ we get

$$\begin{aligned}\Gamma^c_{cc} &= \frac{1}{2}g^{cc}(g_{cc,c} + g_{cc,c} - g_{cc,c}) \\ &= \frac{1}{2}g^{cc}g_{cc,c} = \frac{1}{2}g^{cc}\partial_c g_{cc}\end{aligned} \tag{22}$$

while if $b \neq c$, we first set $c = b$ in (21)

$$\Gamma^a_{bb} = \frac{1}{2}g^{aa}(g_{ba,a} + g_{ab,b} - g_{bb,a}). \tag{23}$$

and then change a to c

$$\Gamma^c_{bb} = \frac{1}{2}g^{cc}(g_{bc,c} + g_{cb,b} - g_{bb,c}). \tag{24}$$

and the terms g_{bc} and g_{cb} are zeros, thus

$$\Gamma^c_{bb} = -\frac{1}{2}g^{cc}g_{bb,c} = -\frac{1}{2}g^{cc}\partial_c g_{bb} \tag{25}$$

Using (22) and (24) we can write (17) as

$$\ddot{x}^c + \Gamma^c_{bb}\dot{x}^b\dot{x}^b - 2\Gamma^c_{cc}\dot{x}^c\dot{x}^c + g^{cc}\dot{g}_{cc}\dot{x}^c. \tag{26}$$

We see that the different signs in (22) and (24) already invalidate (18), but (18) also has the additional terms

$$\Gamma^c_{cb} = \frac{1}{2}g^{cc}\partial_b g_{cc} \tag{27}$$

which we can calculate from (21) by setting $a = c$ with $b \neq c$

$$\Gamma^c_{bc} = \frac{1}{2}g^{cc}(g_{bc,c} + g_{cc,b} - g_{bc,c}) = \frac{1}{2}g^{cc}\partial_b g_{cc}. \tag{28}$$

There are no such terms in (17). The confusion in the relativity theory is created by writing (10) with nonorthogonal coordinates and then identifying incorrectly the Christoffel symbols in the step from (17) to (18).

The second reason is that taking $\dot{g}_{cd} = 0$ selects a particular world path and restricts the equation to this particular world path. In the best case we find a solution and it satisties the imposed condition on the world path, then we limited the set of solutions but found a solution. In the worse case the calculation after requiring $\dot{g}_{cd} = 0$ in (17) does not give a solution that satisfies the imposed condition on the world path and imposing $\dot{g}_{cd} = 0$ in (17) is an error. We will see both situations with the Schwarzschild metric, for time-like world paths we have the first case, for light-like world paths we have the second case.

Notice especially that the condition $\dot{g}_{cd} = 0$ does not mean that the field is time independent. The field is time independent if $\partial_0 g_{cd} = 0$. If some g_{ab} or sum of several g_{ab} gives a function g that only depends explicitly on r, i.e., $g = g(r)$, and g is not constant, then the condition that every g_{ab} in (17) satisfies $\dot{g}_{ab} = 0$ implies that $\dot{r} = 0$. The world path with $\dot{r} = 0$ which is a circular orbit and imposing this condition in (17) means that we are only checking if there is a circular geodesic.

Limiting to $\dot{r} = 0$ is too restrictive, there are other interesting geodesics. If the field is time-independent, that is $\partial_0 g_{cd} = 0$, the Euler-Lagrange equation for the time parameter t is very easy. As $L = 1$ does not depend explicitly on t, the term $\partial L/\partial t = 0$ and there remains

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{t}} = 0. \tag{29}$$

Then

$$\frac{\partial L}{\partial \dot{t}} = g_{00} 2\dot{t}\frac{1}{2L} = g_{00}\dot{t} = C \tag{30}$$

where C is a constant. In this situation a radial geodesic gives a simple equation. We can find a radial geodesic to the direction of y by setting $r = y$, $x = z = 0$. The Lagrangian takes the form

$$L = \sqrt{g_{00}\dot{t}^2 - g_{22}\dot{y}^2}. \tag{31}$$

Then

$$\frac{\partial L}{\partial y} = \frac{1}{2L}\frac{dg_{00}(y)}{dy}\dot{t}^2 - \frac{1}{2L}\frac{dg_{22}(y)}{dy}\dot{y}^2. \tag{32}$$

Writing $g'_{ii} = dg_{ii}(y)/dy$ in the equation (32) gives

$$= \frac{C^2}{2}\frac{g'_{00}}{g_{00}^2} - \frac{1}{2}g'_{22}. \tag{33}$$

The term

$$\frac{\partial L}{\partial \dot{y}} = -\frac{1}{2L}g_{22}2\dot{y} = -g_{22}\dot{y} \tag{34}$$

and

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{y}} = -\dot{g}_{22}\dot{y} - g_{22}\ddot{y} = -\dot{y}g'_{22}\dot{y} - g_{22}\ddot{y}. \tag{35}$$

The Euler-Lagrange equation for y is

$$\ddot{y} + \frac{1}{2}\frac{g'_{22}}{g_{22}}\dot{y}^2 + \frac{C^2}{2}\frac{g'_{00}}{g_{00}^2 g_{22}} = 0. \tag{36}$$

We will see that this gives an interesting result for the Schwarzschild metric in the next section.

2. Circular and radial time-like geodesics of the Schwarzschild metric

The Schwarzschild metric is defined as

$$c^2 d\tau^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2(\theta) d\phi^2 \tag{37}$$

where

$$A(r) = c^2 \left(1 - \frac{r_s}{r}\right) \qquad\qquad B(r) = \left(1 - \frac{r_s}{r}\right)^{-1} \tag{38}$$

and r_s is a constant, so called Schwarzschild radius. In Cartesian coordinates (37) is

$$\begin{aligned} c^2 d\tau^2 &= A(r)dt^2 - (B(r)-1)dr^2 - \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2\right) \\ &= A(r)dt^2 - (B(r)-1)dr^2 - \left(dx^2 + dy^2 + dz^2\right) \end{aligned} \tag{39}$$

$$= A(r)dt^2 - ((B(r)-1)\frac{x^2}{r^2}+1)dx^2 - ((B(r)-1)\frac{y^2}{r^2}+1)dy^2 - ((B(r)-1)\frac{z^2}{r^2}+1)dz^2$$
$$-(B(r)-1)\frac{xy}{r^2}dxdy - (B(r)-1)\frac{xz}{r^2}dxdz - (B(r)-1)\frac{yz}{r^2}dydz.$$

The Lagrangian for (39) can be written as

$$L = \sqrt{A(r)\dot{t}^2 + (B(r)-1)\dot{r}^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2} = 1 \tag{40}$$

The Euler-Lagrange equation for the time in (30) gives $\dot{t} = CA(r)^{-1}$ for some constant C for all r.

We will first do the trick of simplifying (17) by demanding that $\dot{g}_{ab} = 0$, as this is what is done in order to get the geodetic equation. From (31) we see that the sum

$$g_{11} + g_{22} + g_{33} = B(r) + 2 \tag{41}$$

is a function of r. If $\dot{g}_{ii} = 0$ for $i = 1, 2, 3$, then $\dot{B}(r) = 0$, but as $B(r)$ is not a constant, this implies that $\dot{r} = 0$. Imposing $\dot{g}_{ab} = 0$ in (17) simplifies the Lagrangian to the form

$$L = \sqrt{A(r)\dot{t}^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2}. \tag{42}$$

Next let us calculate the Euler-Lagrange equation for y

$$\begin{aligned} \frac{\partial L}{\partial y} &= \partial_y A(r)\dot{t}^2 \frac{1}{2L} = \frac{y}{r} A'(r)\dot{t}^2 \frac{1}{2} \\ &= \frac{y}{2r} A'(r) \frac{C^2}{A(r)^2} \end{aligned} \tag{43}$$

$$\frac{\partial L}{\partial \dot{y}} = \frac{-2\dot{y}}{2L} = -\dot{y} \tag{44}$$

The Euler-Lagrange equation is

$$\frac{y}{2r} A'(r) \frac{C^2}{A(r)^2} + \ddot{y} = 0. \tag{45}$$

$$\ddot{y} = -\frac{y}{2r} A'(r) \frac{C^2}{A(r)^2}. \tag{46}$$

This equation has solutions as exponent functions, but they cannot give $r^2 = x^2 + y^2$. We must take a solution composed of sinuses and cosinuses. Because of the symmetry of a radially symmetric function, we can select x, y, z so that the solution is as nice as it can be. Thus, we select $z = 0$ and

$$y = r \sin(\sqrt{\frac{C^2 A'(r)}{2rA(r)^2}} \tag{47}$$

$$x = r \cos(\sqrt{\frac{C^2 A'(r)}{2rA(r)^2}}.$$

With sinusodial solutions, like (47), $\ddot{y}$ and y have opposite signs. The radius r is positive in (46). This means that $A'(r)$ must be positive in (46). We confirm from (38) that $A'(r)$ is positive.

From (47) follows that the radial velocity v is constant when r is constant

$$v^2 = \dot{x}^2 + \dot{y}^2 = r \frac{C^2 A'(r)}{2A(r)^2} \tag{48}$$

The centrifugal force is derived purely from geometry, so it has to be valid also in the relativity theory. For a cicular orbit $\dot{r} = \ddot{r} = 0$ and r^2 is constant

$$\frac{1}{2} \frac{d^2}{dt^2} r^2 = \dot{x}^2 + \dot{y}^2 + x\ddot{x} + y\ddot{y} = 0 \tag{49}$$

$$v^2 = \dot{x}^2 + \dot{y}^2 = -(x\ddot{x} + y\ddot{y}).$$

Inserting this to

$$\ddot{r} = \frac{\dot{x}^2 + \dot{y}^2}{r} + \frac{x\ddot{x} + y\ddot{y}}{r} - \frac{\dot{r}}{r^2}(x\dot{x} + y\dot{y}) = 0$$

$$\ddot{r} = \frac{v^2}{r} - \frac{v^2}{r} \tag{50}$$

we see that in order for the orbit to be a circle, there is needed acceleration $-v^2/r$, the second term to the right in (50), to compensate for the centrifugal acceleration, the first term to the right. This compensating acceleration comes from the gravitational force. The gravitational force F on a test mass m must therefore be

$$F = \frac{mv^2}{r} = \frac{C^2 A'(r)}{2A(r)^2} \tag{51}$$

Inserting $A(r)$ from (38) (where we set $c = 1$, as has been done in this calculation already earlier)

$$F = \frac{C^2}{2} \frac{A'(r)}{A(r^2)} = \frac{C^2}{2} \frac{r_s}{r^2} \frac{r^2}{(r - r_s)^2} = B \frac{1}{(r - r_s)^2} \tag{52}$$

which differs from Newton's gravitation force on higher powers of r. We see that the Schwarzschild metric does give a circular geodesic for time-like world paths and the gravitation force it predicts in (52) is not very different from the Newtonian gravitation force. That does not sound too bad, but let us look at a radial geodesic. First we calculate the Euler-Lagrange equation for y in the case when $\dot{r}$ is not zero.

From (40) we get

$$\frac{\partial L}{\partial y} = \frac{1}{2L} \left(\frac{y}{r} A'(r) \dot{t}^2 - \frac{y}{r} B'(r) \dot{r}^2 - (B(r) - 1) \frac{\partial \dot{r}^2}{\partial y} \right) \tag{53}$$

Inserting $A(r)\dot{t} = C$ from (30) and expanding

$$\frac{\partial L}{\partial y} = \frac{y}{2r} \frac{C^2 A'(r)}{A(r)^2} - \frac{y}{2r} B'(r) \dot{r}^2 + (B(r) - 1) \left(\frac{y}{r^2} \dot{r}^2 - \frac{1}{r} \dot{y} \dot{r} \right) \tag{53}$$

$$\frac{\partial L}{\partial \dot{y}} = -(B(r) - 1) \frac{y}{r} \dot{r} - \dot{y} \tag{54}$$

$$\begin{aligned} \frac{d}{d\tau} \frac{\partial L}{\partial \dot{y}} &= -\dot{r} B'(r) \frac{y}{r} \dot{r} + (B(r) - 1) \frac{\dot{r}}{r^2} y \dot{r} \\ & -(B(r) - 1) \frac{y}{r} \dot{y} \dot{r} - (B(r) - 1) \frac{y}{r} \ddot{r} - \ddot{y} \end{aligned} \tag{55}$$

The Euler-Lagrange equation for y is

$$\ddot{y} + y \left(\frac{C^2 A'(r)}{2rA(r)^2} + B'(r) \frac{\dot{r}^2}{2r} + (B(r) - 1) \frac{\ddot{r}}{r} \right) = 0 \tag{56}$$

The equation is the same for x and z, only replacing y with x or z, but we are now interested in a radial geodesic where $x = z = 0$ and $r = y$. The equation takes the form

$$B(y) \ddot{y} + B'(y) \frac{\dot{y}^2}{2} + \frac{C^2 A'(y)}{2A(y)^2} = 0 \tag{57}$$

Because $A(r) = B(r)^{-1}$, (57) simplifies further

$$\ddot{y} + \frac{1}{2}\frac{B'(y)}{B(y)}\dot{y}^2 + \frac{C^2}{2}\frac{A'(y)}{A(y)} = 0 \tag{58}$$

and still because $A(r) = B(r)^{-1}$

$$\ddot{y} - \frac{1}{2}\frac{A'(y)}{A(y)}\dot{y}^2 + \frac{C^2}{2}\frac{A'(y)}{A(y)} = 0 \tag{59}$$

It may be difficult to find all solutions to (59) for a general $A(y)$, but one solution is obvious:

$$y = C\tau + b \qquad\qquad r = y \tag{60}$$

solves (59) and therefore (56) for any $A(r)$, $B(r) = A(r)^{-1}$. But especially for $A(r)$ in the Schwarzschild solution we get

$$\frac{A'(y)}{A(y)} = r_s(y - r_s)^{-2} = r_s y_1^{-2} \tag{61}$$

$$\frac{2}{r_s} y_1^2 \ddot{y}_1 - \dot{y}_1^2 + C^2 = 0$$

which only has the solution $y_1 = C\tau + b_1$ and therefore $y = C\tau + b$. As $\dot{t} = CA(y)^{-1}$, we get

$$t = C\tau + r_s \ln\tau + t_0 \tag{62}$$

and the relation between t and y is nothing what you would expect. This radial geodesic does not seem as good as the circle geodesic we found before. In a gravitational field, a test mass should not move towards the mass center in (nearly) linear time! This geodesic is a valid geodesic and it shows that something is very wrong in the Schwarzschild metric or in the geometrization idea.

3. Concerning light-like geodesics

With light-like geodesics $d\tau$ in (9)

$$d\tau = \sqrt{g_{ab}dx_a dx_b}$$

is always zero and we cannot divide by $d\tau$. There are two possibilities. Either we separate the time term

$$c^2 g_{00} dt^2 = g_{ij} dx^i dx^j \tag{63}$$

where i and j range from 1 to 3, and form the Lagrangian as

$$L = \sqrt{\frac{g_{ij}}{c^2 g_{00}}\dot{x}^i \dot{x}^j} \tag{64}$$

In this case $L = 1$ and calculations are again simplified by this choice, but the problem with this Lagrangian is that if the speed of light is locally c everywhere, then

$$\frac{g_{ii}}{g_{00}} = c^2 \tag{65}$$

and the Lagrangian reduces to the Lagrangian of empty space: the geodesics are straight lines and light is not at all influenced by the gravitational field. This was certainly not Einstein's intention.

It is impossible to release the requirement that light has the constant speed c in vacuum at every point to every direction. If it does not, then the tangent space-time of the space-time is not a Minkowski space, but it has to be since the tangent space-time is flat. Also, c appears in many formulas in the relativity theory as a constant. If c is not constant, then it is a vector function. The formulas and calculations should treat c as a vector variable, but they do not.

We have to select the other choice which is to minimize the 3-dimensional path. Then the Lagrangian is

$$L = \sqrt{-g_{ij}\dot{x}^i\dot{x}^j} \tag{66}$$

and it is not one. We find the shortest space path and as light travels with the constant speed c, it also is the shortest time path. We will only focus on the two special geodesics, a circular and a radial.

In a circular geodesic for the Schwarzschild metric for light-like world paths $\dot{r} = 0$, thus instead of (40) we have from (66)

$$L = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}. \tag{67}$$

This is a Lagrangean for empty space. The Euler-Lagrange equations are $\ddot{x} = \ddot{y} = \ddot{z} = 0$ and the geodesics are straight lines. But we have imposed the condition that $\dot{r} = 0$ in order to get (67). Here we have the error mentioned in the first section: after the condition $\dot{g}_{ab} = 0$ is imposed, the calculations give a solution that does not allow $\dot{g}_{ab} = 0$. Therefore imposing $\dot{g}_{ab} = 0$ is an error. The Schwarzschild metric does not allow circular geodesics for light, which is correct as light does not follow circular paths around a mass point.

In a radial geodesic for the Schwarzschild metric for light-like world paths the Lagrangean (64) is

$$L = \sqrt{B(y)\dot{y}^2} \tag{68}$$

$$\frac{L}{\partial y} = B'(y)\dot{y}^2\frac{1}{2L} \tag{69}$$

$$\frac{L}{\partial \dot{y}} = B(y)\dot{y}\frac{1}{L} \tag{70}$$

$$\frac{d}{d\tau}\frac{L}{\partial \dot{y}} = B'(y)\dot{y}^2\frac{1}{L}$$

$$+B(y)\ddot{y}\frac{1}{L}-B(y)\dot{y}\left(\frac{1}{2}B'(y)\dot{y}^3+B(y)\ddot{y}\dot{y}\right)\frac{1}{L^3} \tag{71}$$

The Euler-Lagrange equation for y is satisfied for any $B(r)$:

$$\frac{1}{2}B'(y)\dot{y}^2L^2-B'(y)\dot{y}^2L^2-B(y)\ddot{y}L^2+\frac{1}{2}B(y)B'(y)\dot{y}^4+B(y)^2\ddot{y}\dot{y}^2=0$$

$$-\frac{1}{2}B(y)B'(y)\dot{y}^4-B(y)^2\ddot{y}\dot{y}^2+\frac{1}{2}B(y)B'(y)\dot{y}^4+B(y)^2\ddot{y}\dot{y}^2=0. \tag{72}$$

Thus, the Schwarzschild metric does allow radial geodesics for light-like world paths.

However, the Schwarzschild metric does not have constant speed of light at every point to every direction. Indeed, the speed of light depends on the altitude, this is in violation with experimental mesurements. The Schwarzschild metric does not converge to a flat Minkowski space when we approach a point, and it should: curvature vanishes when the environment becomes infinitely small. The metric (37) is not a valid metric because it has the cross terms $dxdy, dxdz, dydz$ in the expression (39) and Cartesian coordinates x, y, z are orthogonal and with orthogonal coordinates there cannot be cross terms, the inner product that induced the Riemanninan metric is zero. These are the reasons to reject the Schwarzschild metric.

Additionally one can mention the following: though the Schwarzschild metric has a circle geodesic that predicts a gravitational force that is not so much different from the Newtonian gravity law, there is a proof in [4] why Newton's gravitation force is the correct one. This proof has the following argument: there is a spherical mass M with constant mass density and a test mass m at a distance r from the center of the mass M. If the force F attracting the test mass is the same as when we let the radius of the spherical mass approach zero, then the force F must be ar^{-2} exactly for some a. In the Schwarzschild solution it may be questioned if the radius of the spherical mass can completely approach zero, but the proof applies to letting the radius vary in a large scale, if the force F does not depend on the radius of the spherical mass but only on its mass, then the force is r^{-2}.

4. The error in the Lagrangean (11)

If you wonder why a test mass freely falling towards the mass center on a radial geodesic does not seem to accelerate (or accelerates minimally) when solved from the Euler-Lagrange equations with the Lagrangian (11), then there is a simple reason. The Lagrangean (11) does not minimize time or length, it minimises the difference between t and y. Let us see this with a simple example, we take the Newtonian potential $\psi = r^{-1}$ with the line element

$$ds^2=c^2\psi^2dt^2-\psi^2dx^2-\psi^2dy^2-\psi^2dz^2. \tag{73}$$

The Lagrangian in the radial case when $x = z = 0$ expressed as derivatives of t instead of τ is

$$L = \psi\sqrt{c^2 - y'^2}. \tag{74}$$

The terms for the Euler-Lagrange equation for y are obtained as

$$\frac{\partial L}{\partial y} = \psi'(y)\sqrt{c^2 - y'^2} \tag{75}$$

$$\frac{\partial L}{\partial y'} = -\psi(y)y'\sqrt{c^2 - y'^2}^{-1}$$

$$\frac{d}{dt}\frac{\partial L}{\partial y'} = -\psi'(y)y'^2(c^2-y'^2)^{-1/2} - \psi(y)y''(c^2-y'^2)^{-1/2} - \psi(y)y''y'^2(c^2-y'^2)^{-3/2}$$

and after some simplifying we get the equation

$$y'' = -\frac{\psi'(y)}{\psi(y)}(c^2 - y'^2). \tag{76}$$

Inserting ψ gives

$$\frac{d}{dt}(yy') = yy'' + y'^2 = c^2 \tag{76}$$

Thus, $f' = y'y = c^2t + b$ and $2f = y^2$. We get $y^2 = c^2t^2 + 2bt + d$ and this shows that y that we get from a Lagrangian like (11) grows like t, not as $y = at^2$ as it should in accelerating motion.

The error is the Lagrangian in (11). Instead of this Lagrangian, we should use Lagrangian in (66). In [18] and [19] there are calculations of the geodesics of the Schwarzschild and Newtonian metrics. The Lagrangian (66) can very well be used for time-like world paths, it minimizes the space distance regardless of the speed. The Lagrangean (11) does not minimize anything relevant. Interestingly, with the correct Lagrangean (66) used in [18] we see that the Schwarzschild metric does not allow circular geodesics, thus the metric does not allow geostationary satellites. But we have them and they stay on the orbit. Even more interestingly, Einstein used the Schwarzschild solution in his calculations of the precession of the perihelion of Mercury. But clearly a metric that does not allow circular orbits has no place in the solar system.

4. Conclusions of Einstein's geometrization idea

The Cristoffel symbol terms in the geodesic equation do not equal the terms in the Euler-Lagrange equation derived from the Lagrangean (11). This means that the geodesic equation is not derived from minimization of the length of the space-time path as in the Lagrangian (11). Though it is possible to define a straight path to mean a solution of the geodesic equation, i.e., we can define the geodesic equation as a connection in the manifold, yet physically there should be some reason why the path is called the straight path. This reason in the relativity theory is that the geodesics are shortest paths in the sense of the

Lagrangean (11). They are not because the Euler-Lagrange equation does not give the geodesic equation. This part of the geometrization idea fails.

It is reasonable that the dynamic law, like Newton's $F = ma$, comes from some Euler-Lagrange equation, like $F = ma$ does come from minimization of energy. But it appears that the dynamic equation does not come from a geodesic in the metric of the gravitational field. First we should notice that the speed of light must be constant c in vacuum at every point and to every direction. This rule agrees with measurements of the speed of light and it is difficult to see how the relativity theory could accept that the speed of light is not locally constant. This requirement means that the gravitational field must be a scalar field and the most general spherically symmetric field has the line element

$$ds^2 = c^2\phi^2 dt^2 - \phi^2(dx^2 + dy^2 + dz^2). \tag{77}$$

It gives the Lagrangean of the type (11) as

$$L = \phi\sqrt{c^2\dot{t}^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2}. \tag{78}$$

There is no way to derive the Newtonian gravitation law $F = ma$, or something close to it, as a geodesic from this Lagrangean. This concludes that the geometrization idea does not work. Notice also that the Einstein equations do not have any solutions that approximate Newtonian gravity in the case of a point mass in empty space and that have locally constant speed of light at every point to every direction, i.e., that are scalar fields.

Einstein made an effort to realize the geometrization idea with tensor gravitational fields, notably with the Schwarzschild solution. We have in this article looked at some geodesics of this solution. Two cases need to be treated separately. Time-like geodesics can be treated with the Lagrangean (11) while light-like geodesics should be treated with the Lagrangean (64). Though we may find some exotic solution with the Lagrangean (62) because the Schwarzschild metric does not satisfy (63), the requirement that the speed of light must be locally constant cannot be easily released.

We found that for time-like world paths the Schwarzschild metric gives a rather reasonable circular geodesic, but it fails in the case of a radial geodesic: there is a radial geodesic, but a test mass does not accelerate when falling towards a point mass. With light-like geodesics the Schwarzschild metric gave no circular geodesics, which is good as light does not follow circular geodesics. Let us mention that the Newtonian potential, if treated as a geometry, does give a circular geodesic for light, which shows that the gemoetric method cannot be used for the Newtonian potential. The Schwarzschild solution did give a radial geodesic for light-like world paths. However, the speed of light is not constant on this path in the Schwarzschild metric.

As the geometrization idea seems incorrect, we need some better idea. My guess is that they key to this problem is the interaction of a field with a test mass: there is an exchange of messages. That is where we get a correct proof of

$E = mc^2$, gravitational time dilation, the Lorentz factor, mass growth formula and so on.

For finding a whole set of serious errors in the relativity theory, see my preprints [2]-[19], then, please, dump the theory. It is not only wrong, it is intentional cheating.

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