Erosion protection of Tokamak and magnetic force on single charge

Kuan Peng titang78@gmail.com
13 March 2024

Abstract: The walls of Tokamaks are heavily eroded by plasma which suggests that a mysterious force pushes the plasma to the wall. We have theoretically discovered this force and named it extra-force. This force appears only on single charges but not on current carrying wire, which is why the Lorentz force law does not contain it. This discovery not only brings new knowledge to electromagnetism, but also gives a solution against the erosion, which could improve the technology for controlled nuclear fusion. We have proposed an experiment to test this new force.

1. Introduction

A promising way to achieve controlled nuclear fusion is Tokamak which holds fusion fuel at around 100 million kelvins in a toroidal chamber. At this temperature the fusion fuel is in plasma state which is a mixture of positive ions and electrons. The plasma is confined with a powerful magnetic field which makes charged particles circling the confinement field lines. The coils of Tokamaks are so designed that the confinement field lines are parallel to the wall of the chamber and the Lorentz force is expected to retain the plasma not to hit the wall. However, the walls of running Tokamaks are heavily eroded by plasma as if charged particles do not completely obey the Lorentz force law. So, we suspect that on top of the Lorentz force a mysterious force is acted on the charged particles, which makes the particles escape the binding of the confinement field.

Previously, we have shown a flaw in the Lorentz force law, that it ignores the parallel-to-current force, see « From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current»[1]. This flaw suggests that the mysterious force could also be a force that the Lorentz force law ignores. In the following, we will derive the expression for the mysterious force by starting from the Coulomb's law that expresses the electrostatic force that a charge q_b exerts on another charge q_a:

$$\boldsymbol{F}_{ba} = \frac{q_b q_a}{4\pi\varepsilon_0} \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} \quad (1)$$

When q_a and q_b move, relativistic effects come into play and modify the Coulomb's force given in (1). There are two such effects : the dynamic effect and changing distance effect.

2. Relativistic effects

a. Changing distance effect

Changing distance effect means that when the distance between q_a and q_b changes, the force on q_a changes. Let $\boldsymbol{r}$ be the radial distance from q_b to q_a, $\boldsymbol{v}_a$ the velocity of q_a and $\boldsymbol{v}_b$ that of q_b, see Figure 1. The velocity of q_a relative to q_b is $\boldsymbol{v} = \boldsymbol{v}_a - \boldsymbol{v}_b$. Because the direction and magnitude of $\boldsymbol{r}$ vary with time, the force $\boldsymbol{F}_{ba}$ in (1) varies with time, see Figure 2.

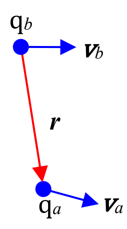


Figure 1

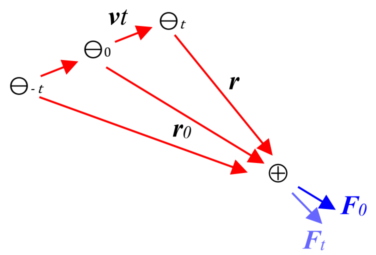


Figure 2

For computing the variation of $\boldsymbol{F}_{ba}$, we transform the factor $\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}$ in (1) with $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$, with t being the time and $\boldsymbol{r}_0$ the radial distance when $t = 0$:

$$\begin{aligned}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} &= \boldsymbol{r}|\boldsymbol{r_0}+\boldsymbol{v}t|^{-3} \\ &= \boldsymbol{r}((\boldsymbol{r_0}+\boldsymbol{v}t)^2)^{-\frac{3}{2}} \\ &= \boldsymbol{r}(\boldsymbol{r_0}^2+2\boldsymbol{r_0}\cdot\boldsymbol{v}t+(\boldsymbol{v}t)^2)^{-\frac{3}{2}} \\ &= \frac{\boldsymbol{r}}{|\boldsymbol{r_0}|^3}\left(1+\frac{2\boldsymbol{r_0}\cdot\boldsymbol{v}t+(\boldsymbol{v}t)^2}{\boldsymbol{r_0}^2}\right)^{-\frac{3}{2}}\end{aligned} \qquad (2)$$

For very small t, the magnitude of $\boldsymbol{v}t$ is very small before that of $\boldsymbol{r}_0$, then we do the linear expansion to (2) and introduce $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$, $\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}$ becomes:

$$\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} = \frac{\boldsymbol{r_0}+\boldsymbol{v}t}{|\boldsymbol{r_0}|^3}\left(1-3\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}t}{\boldsymbol{r_0}^2}\right) \qquad (3)$$

Introducing (3) into (1) we obtain the instantaneous force on q_a at time t :

$$\boldsymbol{F}_{ba} = \frac{q_b q_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left((\boldsymbol{r_0}+\boldsymbol{v}t)-3(\boldsymbol{r_0}+\boldsymbol{v}t)\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}t}{\boldsymbol{r_0}^2}\right) \qquad (4)$$

The charge q_a is moving, so we take the average force on q_a for the time interval $[-t_e, t_e]$ which equals the integral of (4) from $-t_e$ to t_e divided by $2t_e$:

$$\begin{aligned}\boldsymbol{F'}_{ba} &= \frac{1}{2t_e}\int_{-te}^{te}\boldsymbol{F}_{ba}dt \\ &= \frac{q_b q_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\frac{1}{2t_e}\int_{-te}^{te}\left(\boldsymbol{r_0}+\left(\boldsymbol{v}-3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}\right)t-3\boldsymbol{v}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}t^2\right)dt\right)\end{aligned} \qquad (5)$$

In the integral in (5), we have the three functions $1,\ t,\ t^2$ the integrals of which are :

$$\int_{-te}^{te} dt = 2t_e \qquad (6)$$

$$\int_{-te}^{te} tdt = t_e^2 - t_e^2 = 0 \qquad (7)$$

$$\int_{-te}^{te} t^2dt = \frac{1}{3}(t_e^3+t_e^3) = \frac{2}{3}t_e^3 \qquad (8)$$

We introduce (6), (7) and (8) into the integral in (5) and obtain :

$$\frac{1}{2t_e}\int_{-te}^{te}\left(\boldsymbol{r_0}+\left(\boldsymbol{v}-3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}\right)t-3\boldsymbol{v}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}t^2\right)dt = \boldsymbol{r_0} - t_e^2\boldsymbol{v}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2} \qquad (9)$$

The value of t_e is determined as the time that light takes to travel from q_b to q_a, which makes the changing distance effect a relativistic effect, see the equation (45) of «From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current»[1]. With c being the speed of light, t_e equals :

$$t_e = \frac{|\boldsymbol{r_0}|}{c} \qquad (10)$$

By combining (5), (9) and (10), we obtain the average force on q_a :

$$\boldsymbol{F}'_{ba} = \frac{q_b q_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} - \frac{|\boldsymbol{r_0}|^2}{c^2}\boldsymbol{v}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}\right) \quad (11)$$

b. Dynamic effect

Dynamic effect means that the intensity of the force is directly modified by the relative velocity. When the charge q_a moves relatively to q_b, the reference frame of q_a moves relatively to that of q_b and the time of q_a dilates by the factor $1/\sqrt{1-\frac{v^2}{c^2}}$ with respect to that of q_b. The force on q_a increases by the same factor due to dynamic effect, see the equation (8) in «From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current»[1]. Let ***F'*** be a static force and ***F"*** the force modified by the dynamic effect, the relativistic transformation for force is :

$$\boldsymbol{F}" = \frac{\boldsymbol{F}'}{\sqrt{1-\frac{\boldsymbol{v}^2}{c^2}}} \quad (12)$$

By applying (12) to (11), the modified force on q_a equals :

$$\boldsymbol{F}"_{ba} = \frac{q_b q_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} - \frac{|\boldsymbol{r_0}|^2}{c^2}\boldsymbol{v}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}\right)\left(1-\frac{\boldsymbol{v}^2}{c^2}\right)^{-\frac{1}{2}} \quad (13)$$

c. Force modified by the two effects

Because v^2/c^2 is very small before 1, we do the linear expansion to the term $\left(1-\frac{\boldsymbol{v}^2}{c^2}\right)^{-\frac{1}{2}}$:

$$\left(1-\frac{\boldsymbol{v}^2}{c^2}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{\boldsymbol{v}^2}{c^2} \quad (14)$$

We introduce (14) into (13) which then becomes :

$$\boldsymbol{F}"_{ba} = \frac{q_b q_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0}\left(1+\frac{1}{2}\frac{\boldsymbol{v}^2}{c^2}\right) - \frac{|\boldsymbol{r_0}|^2}{c^2}\boldsymbol{v}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}}{|\boldsymbol{r_0}|^2}\left(1+\frac{1}{2}\frac{\boldsymbol{v}^2}{c^2}\right)\right) \quad (15)$$

We neglect the terms of order $1/c^4$ and replace $\boldsymbol{v}$ with $\boldsymbol{v}_a - \boldsymbol{v}_b$, $\boldsymbol{F}"_{ba}$ is expressed as :

$$\boldsymbol{F}"_{ba} = \frac{q_b q_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} + \frac{\boldsymbol{r_0}}{2}\frac{(\boldsymbol{v}_a-\boldsymbol{v}_b)^2}{c^2} - \frac{|\boldsymbol{r_0}|^2}{c^2}(\boldsymbol{v}_a-\boldsymbol{v}_b)\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b)}{|\boldsymbol{r_0}|^2}\right) \quad (16)$$

$\boldsymbol{F}"_{ba}$ is the force that the charge q_b acts on q_a and modified by the relativistic dynamic effect and changing distance effect.

3. Couple of charges

a. Force a couple of charges acts on a single charge

The force $\boldsymbol{F}"_{ba}$ is a force that a single charge exerts on a single charge but not yet a magnetic force which is exerted by a neutral material carrying current. An elementary magnetic force is exerted by an elementary unit of current carrying wire which is a couple of charges formed by a fixed positive charge and a free electron. The couple of charges is neutral, the positive charge is denoted by $\oplus_b$, the electron by $\ominus_b$ and the couple of charges by $\oplus_b\ominus_b$. The force that $\oplus_b\ominus_b$ exerts on q_a is the elementary magnetic force exerted by a current carrying wire, see Figure 3.

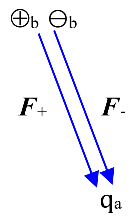


Figure 3

When the free electron is static the force that $\oplus_b\ominus_b$ exerts on q_a is zero because the electrostatic forces that $\oplus_b$ and $\ominus_b$ act on q_a cancel. When the free electron is moving the force from $\ominus_b$ is modified by the relativistic effects while that from $\oplus_b$ is not. In this case, the forces that $\oplus_b$ and $\ominus_b$ act on q_a cannot sum to zero, so $\oplus_b\ominus_b$ exerts a net force on q_a. This is how elementary magnetic force emerges from a neutral wire when the free electrons start to move.

Let us compute the elementary magnetic force by first computing the force $\oplus_b$ acts. The velocity of q_a is $\boldsymbol{v}_a$, the charge of $\oplus_b$ is q_b = e, it is static and $\boldsymbol{v}_b = 0$. We introduce these $\boldsymbol{v}_b$, q_b and $\boldsymbol{v}_a$ into (16) and get :

$$\boldsymbol{F}''_+ = \frac{eq_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} + \frac{\boldsymbol{r_0}}{2}\frac{\boldsymbol{v}_a{}^2}{c^2} - \frac{|\boldsymbol{r_0}|^2}{c^2}\boldsymbol{v}_a\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}_a}{|\boldsymbol{r_0}|^2}\right) \quad (17)$$

For the free electron $\ominus_b$, its velocity is $\boldsymbol{v}_b$ and its charge is q_b = -e. The velocity of q_a is still $\boldsymbol{v}_a$. We introduce these $\boldsymbol{v}_b$, q_b and $\boldsymbol{v}_a$ into (16) and obtain the force that $\ominus_b$ acts on q_a :

$$\boldsymbol{F}''_- = -\frac{eq_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} + \frac{\boldsymbol{r_0}}{2}\frac{(\boldsymbol{v}_a-\boldsymbol{v}_b)^2}{c^2} - \frac{|\boldsymbol{r_0}|^2}{c^2}(\boldsymbol{v}_a-\boldsymbol{v}_b)\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b)}{|\boldsymbol{r_0}|^2}\right) \quad (18)$$

The sum of (17) and (18) equals :

$$\begin{aligned}\boldsymbol{F}_1 &= \boldsymbol{F}''_+ + \boldsymbol{F}''_- \\ &= \frac{eq_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} + \frac{\boldsymbol{r_0}}{2}\frac{\boldsymbol{v}_a{}^2}{c^2} - \frac{|\boldsymbol{r_0}|^2}{c^2}\boldsymbol{v}_a\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}_a}{|\boldsymbol{r_0}|^2}\right) \\ &\quad - \frac{eq_a}{4\pi\varepsilon_0|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0} + \frac{\boldsymbol{r_0}}{2}\frac{(\boldsymbol{v}_a-\boldsymbol{v}_b)^2}{c^2} - \frac{|\boldsymbol{r_0}|^2}{c^2}(\boldsymbol{v}_a-\boldsymbol{v}_b)\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b)}{|\boldsymbol{r_0}|^2}\right) \\ &= \frac{eq_a}{4\pi\varepsilon_0c^2|\boldsymbol{r_0}|^3}\left(\frac{\boldsymbol{r_0}}{2}\left(\boldsymbol{v}_a{}^2 - (\boldsymbol{v}_a-\boldsymbol{v}_b)^2\right) - \left(\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a-\boldsymbol{v}_b)\left(\boldsymbol{r_0}\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b)\right)\right)\right)\end{aligned} \quad (19)$$

We develop the terms $\boldsymbol{v}_a{}^2 - (\boldsymbol{v}_a-\boldsymbol{v}_b)^2$ and $\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a-\boldsymbol{v}_b)\left(\boldsymbol{r_0}\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b)\right)$ in (20) and (21) :

$$\boldsymbol{v}_a{}^2 - (\boldsymbol{v}_a-\boldsymbol{v}_b)^2 = 2\boldsymbol{v}_a\cdot\boldsymbol{v}_b - \boldsymbol{v}_b{}^2 \quad (20)$$

$$\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a-\boldsymbol{v}_b)\left(\boldsymbol{r_0}\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b)\right) = \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) + (\boldsymbol{v}_a-\boldsymbol{v}_b)(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) \quad (21)$$

By introducing (20) and (21) into (19), we get :

$$\boldsymbol{F}_1 = \frac{eq_a}{4\pi\varepsilon_0c^2|\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) - \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a-\boldsymbol{v}_b)(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) - \frac{\boldsymbol{r_0}\boldsymbol{v}_b{}^2}{2}\right) \quad (22)$$

$\boldsymbol{F}_1$ is the elementary magnetic force that the couple of charges $\oplus_b\ominus_b$ acts on the single charge q_a.

b. Force a couple of charges acts on another couple of charges

Suppose that the object at the point *a* is a couple of charges, $\oplus_a\ominus_a$. The force that $\oplus_b\ominus_b$ acts on $\oplus_a\ominus_a$ equals the force that $\oplus_b\ominus_b$ acts on $\oplus_a$ plus that on $\ominus_a$. Let us first compute the force on $\oplus_a$ which is a fixed positive charge, so q_a = e and $\boldsymbol{v}_a$ = 0. We introduce these q_a and $\boldsymbol{v}_a$ into (22) and get :

$$\boldsymbol{F}_{1+} = \frac{e^2}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) - \frac{\boldsymbol{r_0}\boldsymbol{v}_b{}^2}{2}\right) \quad (23)$$

⊖ₐ is a moving electron, its velocity is $\boldsymbol{v}_a$ and its charge is q_a = - e. We introduce these q_a and $\boldsymbol{v}_a$ into (22) and get :

$$\boldsymbol{F}_{1-} = -\frac{e^2}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) - \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a - \boldsymbol{v}_b)(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) - \frac{\boldsymbol{r_0}\boldsymbol{v}_b{}^2}{2}\right) \quad (24)$$

The sum of these two forces equals :

$$\begin{aligned}\boldsymbol{F}_2 &= \boldsymbol{F}_{1+} + \boldsymbol{F}_{1-}\\ &= \frac{e^2}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) - \frac{\boldsymbol{r_0}\boldsymbol{v}_b{}^2}{2}\right)\\ &\quad - \frac{e^2}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) - \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a - \boldsymbol{v}_b)(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) - \frac{\boldsymbol{r_0}\boldsymbol{v}_b{}^2}{2}\right)\end{aligned} \quad (25)$$

$$\boldsymbol{F}_2 = \frac{e^2}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(-\boldsymbol{r_0}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) + \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) + \boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_b)\right) \quad (26)$$

$\boldsymbol{F}_2$ is the elementary magnetic force that a couple of charges in a current exerts on that of another current. Notice that $\boldsymbol{F}_2$ is the magnetic force on a couple of charges while $\boldsymbol{F}_1$ in (22) is that on a single charge. For the ions in a Tokamak we must use $\boldsymbol{F}_1$, not $\boldsymbol{F}_2$.

In fact, we have already derived $\boldsymbol{F}_2$ in « From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current»[1], see the equation (54) in this article. But, in the demonstration I have separately computed the dynamic effect for ⊕ₐ⊖ₐ and ⊕ᵦ⊖ᵦ and the changing distance effect for ⊕ₐ⊖ₐ and ⊕ᵦ⊖ᵦ, then, I have bluntly added the two effects together, which is not rigorous. Here, the derivation of $\boldsymbol{F}_2$ is mathematically rigorous.

4. Force from current-element

a. b is a current element

In a Tokamak the magnetic force on an ion is acted by a coil, so we will compute the force that a current element $d\boldsymbol{I}_b$ acts on an ion q_a. Suppose $d\boldsymbol{I}_b$ contains n free electrons which move at the mean velocity $\boldsymbol{v}_b$. Then, $d\boldsymbol{I}_b$ equals :

$$d\boldsymbol{I}_b = I_b d\boldsymbol{l} = -n\cdot e\boldsymbol{v}_b \quad (27)$$

We express $\boldsymbol{v}_b$ with $d\boldsymbol{I}_b$:

$$\boldsymbol{v}_b = -\frac{d\boldsymbol{I}_b}{n\cdot e} \quad (28)$$

The number of couple of charges equals that of free electrons, so we introduce (28) into (22) and get :

$$\boldsymbol{F}_1 = \frac{eq_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{r_0}\left(\boldsymbol{v}_a\cdot\left(-\frac{d\boldsymbol{I}_b}{n\cdot e}\right)\right) - \left(-\frac{d\boldsymbol{I}_b}{n\cdot e}\right)(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a - \boldsymbol{v}_b)\left(\boldsymbol{r_0}\cdot\left(-\frac{d\boldsymbol{I}_b}{n\cdot e}\right)\right) - \frac{\boldsymbol{r_0}}{2}\boldsymbol{v}_b\left(-\frac{d\boldsymbol{I}_b}{n\cdot e}\right)\right) \quad (29)$$

We multiply both sides of (29) with n and cancel the e's, we obtain :

$$n\boldsymbol{F}_1 = \frac{q_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(-\boldsymbol{r_0}(\boldsymbol{v}_a\cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) + (\boldsymbol{v}_a - \boldsymbol{v}_b)(\boldsymbol{r_0}\cdot d\boldsymbol{I}_b) + \frac{\boldsymbol{r_0}}{2}(\boldsymbol{v}_b\cdot d\boldsymbol{I}_b)\right) \quad (30)$$

b. Expression with double vector product

Using the double vector product identity, $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = -\boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B}) + \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C})$, we transform the term $-\boldsymbol{r_0}(\boldsymbol{v}_a \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r_0} \cdot \boldsymbol{v}_a)$ in (30) into :

$$-\boldsymbol{r_0}(\boldsymbol{v}_a \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r_0} \cdot \boldsymbol{v}_a) = \boldsymbol{v}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r_0}) \quad (31)$$

We denote n·$\boldsymbol{F}_1$ with $\boldsymbol{F}_c$, introduce (31) into (30) and obtain :

$$\boldsymbol{F}_c = n\boldsymbol{F}_1 = \frac{q_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{v}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r_0}) + (\boldsymbol{v}_a - \boldsymbol{v}_b)(\boldsymbol{r_0} \cdot d\boldsymbol{I}_b) + \frac{\boldsymbol{r_0}(\boldsymbol{v}_b \cdot d\boldsymbol{I}_b)}{2}\right) \quad (32)$$

$\boldsymbol{F}_c$ is the magnetic force that the current element $d\boldsymbol{I}_b$ acts on the single charge q_a.

c. Lorentz force

Today, magnetic force is computed with the Lorentz force law. Let q_a be a single charge moving at the velocity $\boldsymbol{v}_a$ in the magnetic field $\boldsymbol{B}$. According to the Biot–Savart law the magnetic field that $d\boldsymbol{I}_b$ creates on the charge q_a equals :

$$d\boldsymbol{B} = \frac{d\boldsymbol{I}_b \times \boldsymbol{r_0}}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3} \quad (33)$$

Then the Lorentz force on q_a equals :

$$\begin{aligned}\boldsymbol{F}_{\text{Lorentz}} &= q_a \boldsymbol{v}_a \times d\boldsymbol{B} \\ &= \frac{q_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3} \boldsymbol{v}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r_0})\end{aligned} \quad (34)$$

d. Difference between Fc and the Lorentz force

Let us subtract (32) with (34), the difference between F_c and the Lorentz force equals :

$$\begin{aligned}\boldsymbol{F}_c - \boldsymbol{F}_{\text{Lorentz}} = &\frac{q_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(\boldsymbol{v}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r_0}) + (\boldsymbol{v}_a - \boldsymbol{v}_b)(\boldsymbol{r_0} \cdot d\boldsymbol{I}_b) + \frac{\boldsymbol{r_0}(\boldsymbol{v}_b \cdot d\boldsymbol{I}_b)}{2}\right) \\ &- \frac{q_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3} \boldsymbol{v}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r_0})\end{aligned} \quad (35)$$

This force is the part of magnetic force that the Lorentz force law ignores. We denote this force as $\boldsymbol{F}_e$ and call it "extra-force". We rearrange (35) and obtain :

$$\boldsymbol{F}_e = \boldsymbol{F}_c - \boldsymbol{F}_{\text{Lorentz}} = \frac{q_a}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left((\boldsymbol{v}_a - \boldsymbol{v}_b)(\boldsymbol{r_0} \cdot d\boldsymbol{I}_b) + \frac{\boldsymbol{r_0}(\boldsymbol{v}_b \cdot d\boldsymbol{I}_b)}{2}\right) \quad (36)$$

Because $\boldsymbol{F}_e$ is missing in the Lorentz force law, it could be the mysterious force that pushes the plasma to the wall.

e. Value of the extra-force

The value of the extra-force is evaluated with respect to the Lorentz force. Let us take a current element $d\boldsymbol{I}_b$ and a free electron in another current element $d\boldsymbol{I}_a$. Suppose that $d\boldsymbol{I}_a = d\boldsymbol{I}_b$ and the velocities of free electrons are equal, $\boldsymbol{v}_a = \boldsymbol{v}_b$. The radial distance $\boldsymbol{r}_0$ is perpendicular to $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$. The reference frame is defined by the unit vectors $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$, then we have :

$$\begin{aligned}\boldsymbol{r_0} &= r_0 \boldsymbol{e}_x \\ d\boldsymbol{I}_b &= dI_b \boldsymbol{e}_y \\ \boldsymbol{v}_a &= -v_a \boldsymbol{e}_y \\ \boldsymbol{v}_b &= -v_b \boldsymbol{e}_y\end{aligned} \quad (37)$$

The Lorentz force that $d\boldsymbol{I}_b$ acts on one electron of $d\boldsymbol{I}_a$ is computed with (34) and (37) for $q_a = -e$:

$$\begin{aligned}\boldsymbol{F}_{\text{Lorentz}} &= \frac{-e(-v_a dI_b r_0)}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\boldsymbol{e}_y \times (\boldsymbol{e}_y \times \boldsymbol{e}_x) \\ &= -\frac{e v_a dI_b r_0}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\boldsymbol{e}_x\end{aligned} \quad (38)$$

The extra-force that $d\boldsymbol{I}_b$ exerts on the same electron is computed with (36) and (37):

$$\begin{aligned}\boldsymbol{F}_e &= \frac{-e}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\left(0 * (\boldsymbol{r_0} \cdot d\boldsymbol{I}_b) - \frac{v_b dI_b}{2} r_0 \boldsymbol{e}_x\right) \\ &= \frac{1}{2}\frac{e v_a dI_b r_0}{4\pi\varepsilon_0 c^2 |\boldsymbol{r_0}|^3}\boldsymbol{e}_x \\ &= -\frac{\boldsymbol{F}_{\text{Lorentz}}}{2}\end{aligned} \quad (39)$$

So, in this case the extra-force equals minus half the Lorentz force.

f. Force that a current element exerts on another current element

Let us compute the force that a current element $d\boldsymbol{I}_b$ exerts on another current element $d\boldsymbol{I}_a$. Suppose that $d\boldsymbol{I}_a$ contains m free electrons and $d\boldsymbol{I}_b$ contains n, then $d\boldsymbol{I}_a$ contains m $\oplus_a\ominus_a$ and $d\boldsymbol{I}_b$ contains n $\oplus_b\ominus_b$. Because $\boldsymbol{F}_2$ is the force that one $\oplus_b\ominus_b$ acts on one $\oplus_a\ominus_a$, the force that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$ equals m·n times $\boldsymbol{F}_2$. So, we multiply both sides of (26) with m·n and obtain :

$$\boldsymbol{F}_{Iba} = mn\boldsymbol{F}_2 = \frac{1}{4\pi\varepsilon_0 c^2 |\boldsymbol{r}|^3}\big(-\boldsymbol{r}(d\boldsymbol{I}_a \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r} \cdot d\boldsymbol{I}_a) + d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)\big) \quad (40)$$

$$\boldsymbol{F}_{Iba} = \frac{1}{4\pi\varepsilon_0 c^2 |\boldsymbol{r}|^3}\big(d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r}) + d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)\big) \quad (41)$$

$\boldsymbol{F}_{Iba}$ is the magnetic force that the current element $d\boldsymbol{I}_b$ acts on the current element $d\boldsymbol{I}_a$.

In « From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current»[1], the Lorentz force law and the Biot–Savart law have been correctly derived from (41). So, equation (41) is correct. Since (41) is derived from equation (22), equation (22) must be correct too and $\boldsymbol{F}_I$ can be safely used for the ions in a Tokamak.

5. Force in Tokamak

a. Extra-force in a Tokamak

If the extra-force is the mysterious force, it must point to the wall. The total extra-force on an ion is computed by integrating the extra-force over the entire confinement coil. Let us take a current element $d\boldsymbol{I}_b$, the mean velocity of the free electrons in $d\boldsymbol{I}_b$ is $\boldsymbol{v}_b$, the velocity of the ion q_a is $\boldsymbol{v}_a$ and the radial distance from $d\boldsymbol{I}_b$ to q_a is $\boldsymbol{r} = \boldsymbol{r}_0$. For computing the extra-force on q_a we introduce these $\boldsymbol{v}_b$, q_b, $\boldsymbol{v}_a$ and $\boldsymbol{r}$ into (36) and get :

$$\begin{aligned}\boldsymbol{F}_E &= \oint \boldsymbol{F}_e \\ &= \frac{q_a}{4\pi\varepsilon_0 c^2}\left(\oint \frac{\boldsymbol{v}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} - \oint \frac{\boldsymbol{v}_b(\boldsymbol{r} \cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} + \oint \frac{\boldsymbol{r}(\boldsymbol{v}_b \cdot d\boldsymbol{I}_b)}{2|\boldsymbol{r}|^3}\right)\end{aligned} \quad (42)$$

This integral is for a given time, so $\boldsymbol{v}_a$ the velocity of the charge q_a is constant over the integration and is extracted from the first integral :

$$\oint \frac{\boldsymbol{v}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} = \boldsymbol{v}_a \oint \frac{\boldsymbol{r} \cdot d\boldsymbol{I}_b}{|\boldsymbol{r}|^3} \quad (43)$$

The integral in (43) $\oint \frac{\boldsymbol{r} \cdot d\boldsymbol{I}_b}{|\boldsymbol{r}|^3} = 0$ because the confinement coil is a closed loop. Then, equation (42) becomes :

$$\boldsymbol{F}_E = \frac{q_a}{4\pi\varepsilon_0 c^2}\left(-\oint \frac{\boldsymbol{v}_b(\boldsymbol{r}\cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} + \oint \frac{\boldsymbol{r}(\boldsymbol{v}_b\cdot d\boldsymbol{I}_b)}{2|\boldsymbol{r}|^3}\right) \quad (44)$$

This equation shows that the direction of $\boldsymbol{F}_E$ is determined by the sign of the charge q_a. However, $\boldsymbol{F}_E$ cannot easily be computed for the entire coil. So, we will compute the extra-force that a cross section of the toroidal coil exerts on one ion. Near the wall, the extra-force from the other part of the toroidal coil becomes negligible before that of the taken cross section and the direction of the total extra-forces is the same as that from the taken cross section.

Suppose that the taken cross section is a circular coil which we plot in Figure 4. The reference frame is determined by the unit vectors $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$, the local unit vectors at the point of $d\boldsymbol{I}_b$ are $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$. The ion has the charge q_a and is at the position $X_0\boldsymbol{e}_x$. $d\boldsymbol{I}_b$ is at the position $R\boldsymbol{e}_r$. The local unit vectors $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ equal :

$$\boldsymbol{e}_r = \cos\theta\,\boldsymbol{e}_x + \sin\theta\,\boldsymbol{e}_y \quad (45)$$
$$\boldsymbol{e}_\theta = -\sin\theta\,\boldsymbol{e}_x + \cos\theta\,\boldsymbol{e}_y \quad (46)$$

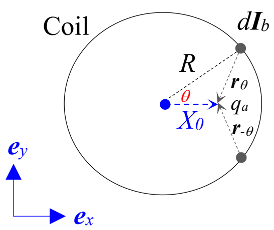


Figure 4

The radial distance from $d\boldsymbol{I}_b$ to q_a equals the position of q_a minus the position of $d\boldsymbol{I}_b$:

$$\boldsymbol{r} = X_0\boldsymbol{e}_x - R\boldsymbol{e}_r \quad (47)$$

$d\boldsymbol{I}_b$ is expressed as :

$$d\boldsymbol{I}_b = I_b dl\boldsymbol{e}_\theta \quad (48)$$

The mean velocity of the free electrons $\boldsymbol{v}_b$ is opposite to $d\boldsymbol{I}_b$ and is expressed as :

$$\boldsymbol{v}_b = -v_b\boldsymbol{e}_\theta \quad (49)$$

For the integral $\oint \frac{\boldsymbol{v}_b(\boldsymbol{r}\cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3}$ in (44) the term $\boldsymbol{r}\cdot d\boldsymbol{I}_b$ is computed with (47) and (48) :

$$\begin{aligned}\boldsymbol{r}\cdot d\boldsymbol{I}_b &= (X_0\boldsymbol{e}_x - R\boldsymbol{e}_r)\cdot(I_b dl\boldsymbol{e}_\theta)\\ &= X_0 I_b dl\boldsymbol{e}_x\cdot\boldsymbol{e}_\theta - RI_b dl\boldsymbol{e}_r\cdot\boldsymbol{e}_\theta \qquad (50)\\ &= X_0 I_b dl\boldsymbol{e}_x\cdot\boldsymbol{e}_\theta\end{aligned}$$

The dot product $\boldsymbol{e}_x\cdot\boldsymbol{e}_\theta$ is computed with (46) :

$$\boldsymbol{e}_x\cdot\boldsymbol{e}_\theta = -\sin\theta \quad (51)$$

By combining (49), (50) and (51) we get :

$$\begin{aligned}\boldsymbol{v}_b(\boldsymbol{r}\cdot d\boldsymbol{I}_b) &= (-v_b\boldsymbol{e}_\theta)X_0 I_b dl(-\sin\theta)\\ &= X_0 v_b I_b \sin\theta\, dl\boldsymbol{e}_\theta\end{aligned} \quad (52)$$

For the integral $\oint \frac{\boldsymbol{r}(\boldsymbol{v}_b\cdot d\boldsymbol{I}_b)}{2|\boldsymbol{r}|^3}$ in (44), we compute the dot product $\boldsymbol{v}_b\cdot d\boldsymbol{I}_b$ in (53) with (48) and (49). Then $\boldsymbol{r}(\boldsymbol{v}_b\cdot d\boldsymbol{I}_b)$ is expressed in (54) :

$$\boldsymbol{v}_b \cdot d\boldsymbol{l}_b = -v_b I_b dl \quad (53)$$
$$\boldsymbol{r}(\boldsymbol{v}_b \cdot d\boldsymbol{l}_b) = -v_b I_b\, \boldsymbol{r} dl \quad (54)$$

Introducing (52) and (54) into (44) we get :

$$\begin{aligned} \boldsymbol{F}_E &= \frac{q_a}{4\pi\varepsilon_0 c^2}\left(-\oint \frac{X_0 v_b I_b \sin\theta\, dl \boldsymbol{e}_\theta}{|\boldsymbol{r}|^3} - \oint v_b I_b \frac{\boldsymbol{r} dl}{2|\boldsymbol{r}|^3}\right) \\ &= -\frac{q_a v_b I_b}{4\pi\varepsilon_0 c^2}\oint \left(X_0 \sin\theta\, \boldsymbol{e}_\theta + \frac{\boldsymbol{r}}{2}\right)\frac{dl}{|\boldsymbol{r}|^3} \end{aligned} \quad (55)$$

We take advantage of the symmetry of the circle and add the value of $\left(X_0 \sin\theta\, \boldsymbol{e}_\theta + \frac{\boldsymbol{r}}{2}\right)$ for the angle θ with that for the symmetrical angle -θ :

$$\begin{aligned} &\left(X_0 \sin\theta\, \boldsymbol{e}_\theta + \frac{\boldsymbol{r}}{2}\right)_\theta + \left(X_0 \sin\theta\, \boldsymbol{e}_\theta + \frac{\boldsymbol{r}}{2}\right)_{-\theta} \\ &= (X_0 \sin\theta\, \boldsymbol{e}_\theta + X_0 \sin(-\theta)\, \boldsymbol{e}_{-\theta}) + \frac{1}{2}(X_0\boldsymbol{e}_x - R(\boldsymbol{e}_r)_\theta + X_0\boldsymbol{e}_x - R(\boldsymbol{e}_r)_{-\theta}) \\ &= X_0 \sin\theta\,(\boldsymbol{e}_\theta - \boldsymbol{e}_{-\theta}) + \frac{1}{2}\big(2X_0\boldsymbol{e}_x - R((\boldsymbol{e}_r)_\theta + (\boldsymbol{e}_r)_{-\theta})\big) \end{aligned} \quad (56)$$

With (45) and (46) we get the symmetrical unit vectors $(\boldsymbol{e}_r)_{-\theta}$ and $\boldsymbol{e}_{-\theta}$ and the values of $(\boldsymbol{e}_r)_\theta + (\boldsymbol{e}_r)_{-\theta}$ and $\boldsymbol{e}_\theta - \boldsymbol{e}_{-\theta}$:

$$(\boldsymbol{e}_r)_{-\theta} = \cos\theta\, \boldsymbol{e}_x - \sin\theta\, \boldsymbol{e}_y \quad (57)$$
$$\boldsymbol{e}_{-\theta} = \sin\theta\, \boldsymbol{e}_x + \cos\theta\, \boldsymbol{e}_y \quad (58)$$
$$\begin{aligned} (\boldsymbol{e}_r)_\theta + (\boldsymbol{e}_r)_{-\theta} &= \cos\theta\, \boldsymbol{e}_x + \sin\theta\, \boldsymbol{e}_y + \cos\theta\, \boldsymbol{e}_x - \sin\theta\, \boldsymbol{e}_y \\ &= 2\cos\theta\, \boldsymbol{e}_x \end{aligned} \quad (59)$$
$$\boldsymbol{e}_\theta - \boldsymbol{e}_{-\theta} = -2\sin\theta\, \boldsymbol{e}_x \quad (60)$$

We introduce (59) and (60) into (56) and get :

$$\begin{aligned} &\left(X_0 \sin\theta\, \boldsymbol{e}_\theta + \frac{\boldsymbol{r}}{2}\right)_\theta + \left(X_0 \sin\theta\, \boldsymbol{e}_\theta + \frac{\boldsymbol{r}}{2}\right)_{-\theta} \\ &= -2X_0 \sin\theta \sin\theta\, \boldsymbol{e}_x + \frac{1}{2}(2X_0\boldsymbol{e}_x - 2R\cos\theta\, \boldsymbol{e}_x) \\ &= -2X_0 \sin^2\theta\, \boldsymbol{e}_x + (X_0 - R\cos\theta)\boldsymbol{e}_x \end{aligned} \quad (61)$$

The integral in (55) is over a circle. Because of the symmetry we integrate over half a circle, that is, from θ = 0 to θ = π. The elementary length of a circle is $dl = Rd\theta$. We introduce (61) and $dl = Rd\theta$ into (55) and reset the boundary of the integral, we get :

$$\begin{aligned} \boldsymbol{F}_E &= -\frac{q_a v_b I_b}{4\pi\varepsilon_0 c^2}\int_0^\pi \left(\frac{-2X_0 \sin^2\theta + (X_0 - R\cos\theta)}{|\boldsymbol{r}|^3}\right) R d\theta \boldsymbol{e}_x \\ &= \frac{q_a v_b I_b}{4\pi\varepsilon_0 c^2 R}\left(2\frac{X_0}{R}\int_0^\pi \frac{\sin^2\theta}{|\boldsymbol{r}/R|^3} d\theta - \int_0^\pi \frac{X_0/R - \cos\theta}{|\boldsymbol{r}/R|^3} d\theta\right)\boldsymbol{e}_x \end{aligned} \quad (62)$$

Let us simplify the form of $\boldsymbol{F}_E$ with the function K in (63), then $\boldsymbol{F}_E$ is written in (64) :

$$K = 2\frac{X_0}{R}\int_0^\pi \frac{\sin^2\theta}{|\boldsymbol{r}/R|^3} d\theta - \int_0^\pi \frac{X_0/R - \cos\theta}{|\boldsymbol{r}/R|^3} d\theta \quad (63)$$
$$\boldsymbol{F}_E = \frac{q_a v_b I_b}{4\pi\varepsilon_0 c^2 R} K \boldsymbol{e}_x \quad (64)$$

For determining K we have numerically computed its second part function $f(X_0)$ which cannot be determined analytically :

$$f(X_0) = -\int_0^{\pi} \frac{X_0/R - \cos\theta}{|\boldsymbol{r}/R|^3} d\theta \quad (65)$$

And plotted its values in Figure 5 :

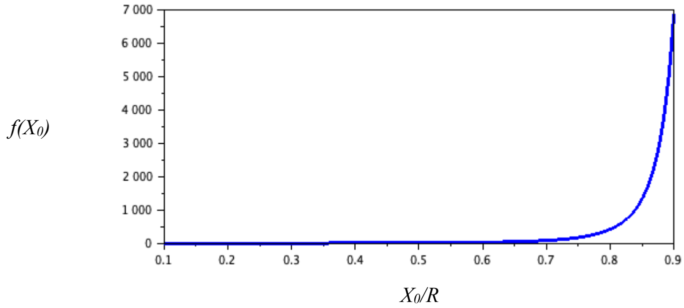


Figure 5

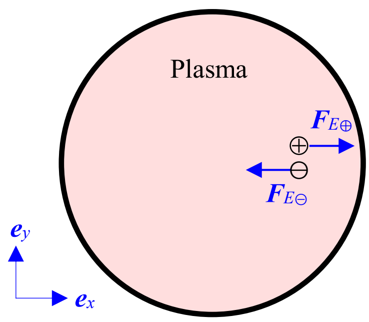
Figure 5 shows that the function $f(X_0)$ is positive for $0 < X_0 < R$. On the other hand, the value of $\int_0^{\pi} \frac{\sin^2\theta}{|\boldsymbol{r}/R|^3} d\theta$ is positive. So, the function K is positive, see (63). For positive ions, $q_a > 0$, then the extra-force $\boldsymbol{F}_{E\oplus}$ is in the same direction as the unit vector $\boldsymbol{e}_x$, see (64) :

$$\boldsymbol{F}_{E\oplus} = \left|\frac{ev_b I_b}{4\pi\varepsilon_0 c^2 R} K\right| \boldsymbol{e}_x \quad (66)$$

For electrons, $q_a < 0$ and the extra-force $\boldsymbol{F}_{E\ominus}$ is in the opposite direction as $\boldsymbol{e}_x$:

$$\boldsymbol{F}_{E\ominus} = -\left|\frac{ev_b I_b}{4\pi\varepsilon_0 c^2 R} K\right| \boldsymbol{e}_x \quad (67)$$

As $\boldsymbol{e}_x$ is directed to the nearest current element on the wall, the positive ions are attracted by the wall while the electrons are pushed away from the wall, see Figure 6. In consequence, the wall is eroded by positive ions, not by electrons.



Tokamak chamber section

Figure 6

b. Keeping the plasma away from the wall

We could protect the wall if we could repel the positive ions. But, if an electromagnetic force can repel positive ions, the same force will forcefully attract electrons and the electrons would erode the wall instead of positive ions. So, it seems impossible to repel the plasma from the wall.

But I have figured out a way to do so. Let us imagine a cloud of positive ions, in the middle of which is a negative charge. Because the negative charge attracts the positive ions, the cloud would not expand. Can we apply this principle to limit the expansion of the plasma in a Tokamak?

A cloud of plasma would expand under its proper pression. But if it contains an excess of electrons it becomes negatively charged and would interact with the extra-force like a negative cloud. More precisely, the electrons in excess would drift outward due to the electrostatic force among them. At equilibrium, the electrons in excess would concentrate on the surface of the cloud and rearrange within the plasma such that the electric potential is constant. In a Tokamak's chamber, the surface of the cloud is a thin layer near the wall which we call the skin of the plasma.

How would the skin interact with the extra-force? Let us take a bunch of particles from the skin which contains m positive ions and n electrons. The extra-force on one positive ion is given by (66) and that on m positive ions equals :

$$m\boldsymbol{F}_{E\oplus} = m\left|\frac{ev_b I_b}{4\pi\varepsilon_0 c^2 R}K\right|\boldsymbol{e}_x \quad (68)$$

The extra-force on one electron is given by (67) and that on n electrons equals :

$$n\boldsymbol{F}_{E\ominus} = -n\left|\frac{ev_b I_b}{4\pi\varepsilon_0 c^2 R}K\right|\boldsymbol{e}_x \quad (69)$$

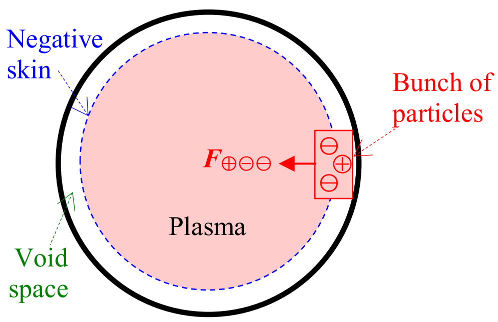
Then, the extra-force on the bunch of particles equals :

$$\begin{aligned}\boldsymbol{F}_{\oplus\ominus\ominus} &= m\boldsymbol{F}_{E\oplus} + n\boldsymbol{F}_{E\ominus} \\ &= (m-n)\left|\frac{ev_b I_b}{4\pi\varepsilon_0 c^2 R}K\right|\boldsymbol{e}_x\end{aligned} \quad (70)$$

Because the skin is richer in electron, n is bigger than m, so m - n < 0 and :

$$\boldsymbol{F}_{\oplus\ominus\ominus} = -\left|(m-n)\frac{ev_b I_b}{4\pi\varepsilon_0 c^2 R}K\right|\boldsymbol{e}_x \quad (71)$$

$\boldsymbol{F}_{\oplus\ominus\ominus}$ is the extra-force on the bunch and is opposite to $\boldsymbol{e}_x$, which means that the bunch would be repelled by the wall, creating a void space between the wall and the skin, see Figure 7.



Tokamak chamber section

Figure 7

However, there can be many objections to this point of view. For example, why don't the positive ions move to the wall as the extra-force pushes them in this direction? In fact, the positive ions and the electrons of the bunch being very close and the electrons more numerous, the electrostatic force of the electrons is much stronger than the extra-force and the positive ions would stay near the electrons in spite of the extra-force.

Figure 7 shows a bunch in the skin that contains one positive ion and two electrons. Because $\boldsymbol{F}_{\oplus\ominus\ominus}$, the net extra-force on the bunch, points away from the wall and the positive ions is locked with the two electrons, the positive ions will follow the two electrons and move away from the wall. In consequence, the three particles will move as a whole under the force $\boldsymbol{F}_{\oplus\ominus\ominus}$. As all bunches of the skin would move as a whole, the skin itself would move away as a whole leaving a void space near the wall.

Another objection would be : is the extra-force strong enough to hold the skin away? I think that, because the plasma is already strongly confined by the confinement field, the charged particles that could escape the confinement field plus the extra-force would be rare. Only experiment could determine how much particles could make to the wall. We reasonably expect that the negatively charged skin would give a good protection to the wall.

Moreover, we have a way to still improve the protection by negatively charging the wall itself. This way, on top of the extra-force, the wall would act a negative electrostatic force on the skin which would be pushed further away making the void space larger. This idea shows that, in the case where you do not agree with the extra-force, you can still create a void space near the wall by negatively charging the wall.

For fine-tuning the protection, we now have two adjustable parameters: the quantity of electrons in excess in the plasma and the quantity of negative charge in the wall. The real key of this technique of protection is adding electrons to the plasma.

6. How to test the extra-force

Because the extra-force is not predicted by the Maxwell's theory and has eluded experimentation for more than one and half century, the physical community would not accept this new force except it is shown in experiment. In order to test the action of the extra-force I have designed an experiment which will show the influence of the extra-force on the electron beam of a cathode ray tube.

The cathode ray tube is placed outside and in the middle of a very long solenoid. According to the Maxwell's theory, the magnetic fields inside and outside a long solenoid are parallel to the solenoid. Moreover, outside and in the middle region the magnetic field is zero. Let $\boldsymbol{B}$ be the magnetic field and $\boldsymbol{v}_a$ the velocity of the electrons of the beam. The cathode ray tube is parallel to the solenoid so that $\boldsymbol{v}_a$ is parallel to the solenoid. Because $\boldsymbol{B}$ is also parallel to the solenoid, $\boldsymbol{B}$ and $\boldsymbol{v}_a$ are parallel vectors and the Lorentz force on the electrons which is $q_a\boldsymbol{v}_a \times \boldsymbol{B}$ equals zero. In consequence, the electron beam should not be deflected and the spot on the screen of the cathode ray tube made by the electron beam should stay still in the center, see Figure 8.

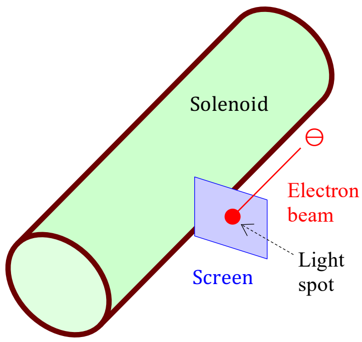


Figure 8

The total extra-force that the solenoid exerts on one electron is computed with (44). The charge of an electron is $q_a = -e$, a current element of the solenoid is $d\boldsymbol{I}_b$. We introduce these $\boldsymbol{v}_a$, q_a and $d\boldsymbol{I}_b$ into (44) and get :

$$\boldsymbol{F}_E = -\frac{e}{4\pi\varepsilon_0 c^2}\left(-\oint \frac{\boldsymbol{v}_b(\boldsymbol{r}\cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} + \oint \frac{\boldsymbol{r}(\boldsymbol{v}_b\cdot d\boldsymbol{I}_b)}{2|\boldsymbol{r}|^3}\right) \quad (72)$$

As $\boldsymbol{F}_E$ is not zero, the electron beam should be deflected and the spot on the screen should move accordingly. For testing the action of the extra-force, we will vary the current in the solenoid to make the extra-force vary with the current. If the extra-force is real, the light spot would move in synchroneity with the current, see Figure 8. So, the result of this experiment would be :

- If the spot moves on the screen, the extra-force exists and the Maxwell's theory is not fully correct.
- If the spot does not move, the extra-force does not exist.

The interior of a solenoid is similar to the interior of the toroidal chamber of a Tokamak. So, it is interesting to test the extra-force in the interior of a large solenoid. The result of this experiment would be the same as that in the exterior.

7. Discussion

In this article we have derived four forces :

1. The force that one single charge exerts on another single charge, see (16). We call it "charge-to-charge force".
2. The force that one current element exerts on a single charge, see (32). We call it "current-to-charge force".
3. The difference force between the "current-to-charge force " and the Lorentz force on a single charge, see (36). We call this force "extra-force".
4. The force that one current element exerts on another current element, see (41). Because this force is different from the Lorentz force we call it "current-to-current force".

The charge-to-charge force is derived from the Coulomb's law and thus, respects the Newton's third law. The current-to-charge force and current-to-current force are both sums of charge-to-charge forces and thus, respect the Newton's third law too. In the contrary, the Lorentz force law does not respect the Newton's third law and is flawed.

Tokamaks are designed with the most accurate electromagnetic theory today : the Maxwell's theory. In spite of the highest care taken in the design, Tokamaks still suffer from erosion. But we are unable to find a solution with the Maxwell's theory. So, the Maxwell's theory is not sufficiently correct for current to plasma interaction.

We have derived the new extra-force in (36) which, according to our computation, pushes the plasma to the wall. So, we have understood how erosion occurs, which enables us to propose a technical solution to prevent erosion. This solution consists of adding more electrons into the plasma which would become negatively charged and be repelled by the extra-force. So, we have not only improved the understanding of electromagnetism, but also brought real technical progress.

For showing the existence of the extra-force we have proposed an experiment using solenoid and cathode ray tube. Previously, I have done a similar experiment which is shown in this video https://www.youtube.com/watch?v=DXu88IreBj4. The explanation of this experiment is in «Aharonov–Bohm effect in CRT experiment » [2]. But in this experiment the electron beam is perpendicular to the solenoid rather than parallel.

The experiments with solenoid are important because they would validate the extra-force which is a magnetic force in a region where the Maxwell's theory predicts no magnetic force. The reason that the Lorentz force law does not contain the extra-force is that this law was established with the data of former experiments which were all done with current carrying wires. A conductor wire is formed by free electrons and positive ion. Although the extra-forces on the positive and negative charges exist within the wire, they are opposite and cancel out. This is why the extra-force does not appear on a neutral wire. In consequence, the Lorentz force law cannot contain the extra-force because the latter has never been reported in any experimental data and was known to no one.

Because of its founding experiments, the subject of the Lorentz force law is current carrying wires in closed loop. Single charge has never been tested for Lorentz force and thus is not the subject of the Lorentz force law. A physical law is valid only for its subject but not for things that are not its subject. So, the Lorentz force law is valid only for current carrying wires in closed loop, but not for single charges.

A more radical way to make swift progress is to directly add electrons in a functioning Tokamak and see if the plasma separates from the wall in someplace. Because there are many working Tokamaks all over the world, many teams can do this experiment with the existing equipment without having to create new one and then, these experiments can be done fast. The validation of this concept would accelerate the achievement of controlled nuclear fusion. So, I highly recommend interested teams to perform the experiments. The first successful team would get the most credit for this important step to the future.

References

1. Kuan Peng, 2023, « From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current»
   Researchgate.net: From Coulomb's force to magnetic force and experiments that show magnetic force parallel to current
   https://pengkuanem.blogspot.com/2023/09/from-coulombs-force-to-magnetic-force.html
   https://www.academia.edu/106863205/From_Coulombs_force_to_magnetic_force_and_experiments_that_show_magnetic_force_parallel_to_current
2. Kuan Peng, 2015, « Aharonov–Bohm effect in CRT experiment PDF, Word with video »
   https://www.academia.edu/12052541/Aharonov_Bohm_effect_in_CRT_experiment_Video_included_ or
   https://pengkuanem.blogspot.com/2015/04/aharonovbohm-effect-in-crt-experiment.html

Letter to readers

Dear readers,

I have derived the extra-force because I thought that the magnetic force on a single charge could be different from the Lorentz force. Why did I have that intuition? We know that currents are composed of positive charges and moving electrons. So, the Lorentz force is surely the difference between the force on the positive charges and that on the moving electrons. Because the moving electrons have a velocity while the positive charges do not, it is surely the velocity that modifies the electrostatic force. The only way physical quantities could be modified in moving frame is through relativity. So, I have tried to modify electrostatic force with relativity and fortunately, was successful in deriving the 4 magnetic forces.

I have proposed to inject additional electrons into functioning Tokamak to protect its wall. This technique would not only make Tokamak work longer, but also allow higher temperature of the plasma because the latter would no longer touch the wall. This technique would increase the density of the plasma because the extra-force is a compressing force so that more plasma can be loaded. In consequence, the confinement time, temperature and density would all be improved at once, making the Lawson criterion and controlled nuclear fusion closer to reach.

The Maxwell's theory is very successful for explaining the majority of electromagnetic phenomena, which makes us believe that it can correctly predict all electromagnetic phenomena. However, there are things it does not predict but really exist, for example, the extra-force. When these things happen, for example, the erosion of the wall of Tokamaks, we do not understand why they occur, let alone to fix their consequence if disastrous. So, it is important to correct the Maxwell's theory when a flaw is found.

The missing of the extra-force is a flaw in the Maxwell's theory and its discovery a breakthrough for electromagnetism. So, this article topples the Maxwell's theory again and will be no doubt rejected by all respectable journals. In consequence, I will not submit it for publication but only post online. I'm sure that you, my online readers, would have the wise to recognize its value and would promote it either by sharing it with your colleagues or by performing the proposed experiments. If you succeed in doing the experiments, you will get honorable credit for your contribution.

Kuan Peng, 13 March 2024